Development and Research of Concentrator-Sonotrode with Increased Radiating Surface

Sergei S. KHMELEV, Vladimir N. KHMELEV, Roman N. GOLYKH, Andrey V. SHALUNOV

Biysk Technological Institute (branch) of the AltSTU
659305, Biysk, Russia; e-mail: ssh@bti.secna.ru

This paper presents the results of studies on functional possibilities of the optimization of geometric sizes and the design development of specialized resonance concentrating link (concentrator-sonotrode) with enlarged radiating surface. Developed theoretical model allows to determine the value of longitudinal and transverse sizes of each part of concentrating link providing the achievement of required features of the ultrasonic vibrating systems (gain factor of the unit and its resonance frequency). To verify the efficiency of designed model, the geometric sizes of resonance concentrating link were determined using the finite-element complex, which showed that the disagreement did not exceed 10%. The efficiency of proposed model at the determining of size and resonance characteristics of concentrating link was proved by the experiments.

Theoretical and experimental studies helped to optimize the size of concentrating link while the vibrating system developed on its base enabled the enlargement of radiating surface without decreasing the radiation intensity for the realization of technologies of cavitation treatment of liquid media.

Keywords: ultrasound, ultrasonic vibrating system, resonance concentrating link, concentrator, gain factor.

1. Introduction

The ultrasonic technological equipment applied nowadays is based on general principles of design. The equipment has similar structure. Any ultrasonic technological apparatus consists of the electronic generator and the ultrasonic vibrating system. Vibrating system is a device which provides the transformation of energy of electrical oscillations coming from the generator into elastic mechanical vibrations, their amplification and introduction into processed media. The concentrators that are velocity transformers realize the amplification of the amplitude of mechanical vibrations to the value suitable for different technological processes.

The concentrators of ultrasonic vibrations are rods of variable cross-section connected to the transducer by input part of larger cross-section. The increase of vibration amplitude generated by the piezoelectric transducer due to the application of the concentrators helps to accomplish cavitation processing of liquid media, ultrasonic welding, dimensional processing of brittle materials, and other ultrasonic technological processes.

2. Problem statement

Operating principle of the concentrator is based on the increase of the amplitude of vibrational displacement of the rod due to reduction of its cross-section according to the law of conservation of movement. Many studies (MERKULOV, 1957; 1959) demonstrated that the most perspective concentrators among applied ones are stepwise concentrators with radial transition zone. Figure 1 shows the scheme of the stepwise-radial concentrator.
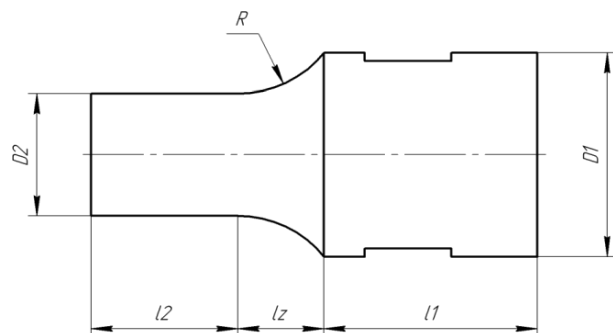

Fig. 1. General view of the stepwise-radial concentrator.

The calculations of the lengths l_1, l_2, l_z may be realized on the base of well-known calculation procedures described in detail in (MERKULOV, 1957; 1959).

The concentrators used in modern ultrasonic apparatuses have high gain factor and strength characteristics matching the piezoelectric transducer with liquid media. However, the application of the concentrators with high gain factor (about 10) is not always possible in practice, as the reduction of the diameter of output zone decreases strength characteristics of transition zone and decreases area of radiating surface. The attempts to enlarge input cross-section of the concentrator do not solve the problem, because it requires increasing the size of the piezoelectric transducers, that is not always possible due to the limits of the diameters of produced piezoelectric elements.

At the same time, one solution of considered problem is known; it is based on the use of concentrating link connecting classic concentrator with additional mass resonating on its own frequency.

For the first time such construction was proposed in (PESHKOVSKY, PESHKOVSKY, 2008; PESHKOVSKY *et. al.*, 2007) and it was a base for the design of the ultrasonic vibrating systems used in a number of high-production chemical cavitation reactors. The application of similar construction allows to obtain required gain factor at equal input and output diameters of the link.

Figure 2 shows the main points of the link construction. The zone with the diameter D_1 is input and it is chosen equal to the output diameter of the piezoelectric transducer. At practical application the linear sizes l_1, l_2, l_3 are chosen according to the sizes of the stepwise-radial concentrator. The diameter D_1 is intermediate and it determines necessary gain factor. The size l_4 usually corresponds to the size l_2 and the influence of the angle α on the parameters of amplification is not considered. The size l_5 is chosen from the condition, that $l_4 + l_5 = \lambda/2$, where λ is a wavelength in the material of concentrating link. The choice of the output diameter D_3 usually corresponds to the diameter D_1. Thus, its value should be determined by the requirements of the technological processes and it requires further studying.
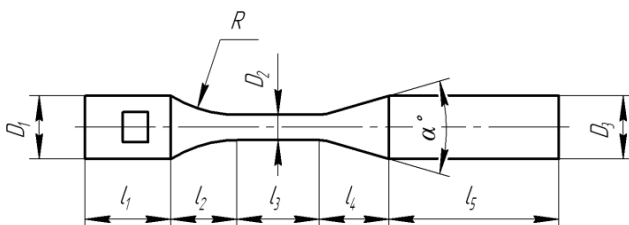

Fig. 2. Structural diagram of concentrating resonating link.

A longitudinal ultrasonic wave passes from input end of the link to cross-section diameter D_2 (located on border of zones l_2 and l_3). Amplitude of vibrating displacement is increased at zones l_1 and l_2, because cross-section is decreased (according to conservation law of momentum). The bigger the difference of diameters D_1 and D_2 of cross-sections link is, the more displacement amplitude increases.

The displacement amplitude is not changed at passes longitudinal wave from zones l_2 and l_3 to output end. But the area of radiating surface increases.

A reactive component of mechanical impedance of link is seeking to zero because at selected lengths l_3, l_4, l_5 a primary wave from low cross-section of concentrator added up in phase with a wave from output end of link.

A mechanical stress of zone l_3 is caused by equal displacement amplitudes of low cross-section of concentrator and output end of link.

Unfortunately, the concentrating link uniting the classic concentrator with the resonating mass does not have wide practical application. It is caused by the absence of theoretical and experimental investigations aimed at working-out of procedural recommendations on the optimization of the construction to achieve maximum gain factor in different conditions, at different diameters and sizes of radiating surface.

Thus, the presence of a large number of transition and longitudinal zones of various diameters in concentrating resonating link requires detailed studies of the construction for revealing of optimum relation of zone lengths and diameters at given ratios of the areas of input and output ends of the concentrator.

This paper is devoted to the studies of concentrating resonating link.

3. Theoretical calculations

To make theoretical model, which is able to provide theoretical calculations of concentrating resonating link, we proceed from the assumptions, that:

a) in the rod plane wave is propagated, i.e. stress and velocity of particles along the area of optional cross-section are constant;

b) there is no transverse contraction;

c) vibrations of the construction are harmonic.

In this case, the distribution of amplitudes of vibrational displacement of the zones with small diameters of considered link depending on longitudinal coordinate can be described by the following equation of longitudinal vibrations of the thin rod (1):

$$S(x)\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial}{\partial x}\left(S(x)\frac{\partial u}{\partial x}\right), \qquad (1)$$

where $S(x)$ is the cross-section area of concentrating resonating link in longitudinal coordinate x; u is the value of vibrational displacement of the link in the point with longitudinal coordinate x; c is the propagation velocity of longitudinal vibrations in the material.

As described resonance link is a linear system, it is true statement that all zones of the link vibrate with the same frequency ω:

$$-\omega^2 S(x)U = c^2 \frac{\partial}{\partial x}\left(S(x)\frac{\partial U}{\partial x}\right), \qquad (2)$$

where U is the amplitude of vibrational displacement.

For further consideration, the link is divided into several zones of the following types:

1. The zone of the uniform cross-section

$$S(x) = \text{const.}$$

The value of vibrational displacements can be presented as follows:

$$U(x) = A\cos(kx) + B\sin(kx),$$

where k is the wave number of concentrating link.

2. The zone with the linear profile. The area of cross-section is represented by the following dependence on the coordinate:

$$S(x) = \left(\sqrt{S_0} + \frac{\sqrt{S_1} - \sqrt{S_0}}{L}x\right)^2, \qquad (3)$$

where S_0 is the area of the input end, S_1 is the area of the output end, L is the length of the zone. The value of vibrational displacements is:

$$U(x) = AG_{1,S_0,S_1,k,L}(x) + BG_{2,S_0,S_1,k,L}(x). \quad (4)$$

3. The zone of the link with the radial profile. The area of the cross-section is presented by the following dependence on the coordinate:

$$S(x) = S_1\left(\frac{d_1 + R - \sqrt{R^2 - (L-x)^2}}{d_1}\right)^2, \quad (5)$$

where S_1 is the area of the output end, d_1 is the diameter of the output end, L is the length of the zone, R is the radius of curvature of the longitudinal profile.

This zone is needed to increase the amplitude of the vibrations. Radial profile reduces the value of mechanical stress.

The value of the vibrational displacements is represented in the following way:

$$U(x) = AE_{1,S_0,S_1,k,L}(x) + BE_{2,S_0,S_1,k,L}(x). \qquad (6)$$

Functions G and E are defined from the solution of the Eq. (2).

Using stated representations, the system of equations from the following system of boundary conditions of the equality of displacements and stresses at the borders of the zones (uniform cross-section – radial – uniform cross-section – with linear profile – radial) is formed:

$$A_0 = U_0, \qquad (7)$$

$$A_0\cos(kL_1) + B_0\sin(kL_1) = A_1E_{1,S_0,S_1,k,L}(0) + B_1E_{2,S_0,S_1,k,L}(0), \qquad (8)$$

$$-kA_0\sin(kL_1) + kB_0\cos(kL_1) = A_1E'_{1,S_0,S_1,k,L}(0) + B_1E'_{2,S_0,S_1,k,L}(0), \qquad (9)$$

$$A_1E_{1,S_0,S_1,k,L}(L_2) + B_1E_{2,S_0,S_1,k,L}(L_2) = A_2, \quad (10)$$

$$A_1E'_{1,S_0,S_1,k,L}(L_2) + B_1E'_{2,S_0,S_1,k,L}(L_2) = kB_2, \quad (11)$$

$$-kA_2\sin(kL_3) + kB_2\cos(kL_3) = A_3G'_{1,S_0,S_1,k,L}(0) + B_3G'_{1,S_0,S_1,k,L}(0), \qquad (12)$$

$$A_2\cos(kL_3) + B_2\sin(kL_3) = A_3G_{1,S_0,S_1,k,L}(0) + B_3G_{1,S_0,S_1,k,L}(0), \qquad (13)$$

$$A_3G_{1,S_0,S_1,k,L}(L_4) + B_3G_{1,S_0,S_1,k,L}(L_4) = A_4, \quad (14)$$

$$A_3G'_{1,S_0,S_1,k,L}(L_4) + B_3G'_{1,S_0,S_1,k,L}(L_5) = kB_4, \quad (15)$$

$$A_4G'_{1,S_0,S_1,k,L}(L_5) + B_4G'_{1,S_0,S_1,k,L}(L_5) = 0. \qquad (16)$$

The solution of obtained system of Eqs. (7)–(16) by Gauss method relative to constant coefficients $A_1, \ldots, A_4, B_1, \ldots, B_4$ allows determining the distribution of vibration amplitudes along the concentrating resonance link.

The obtained amplitude distribution of longitudinal vibrational displacements along the acoustic axis of the concentrating link is shown in Fig. 3 at different lengths of the zone l_4 and the diameters of the input and output cross-sections (10, 15, 27 mm).
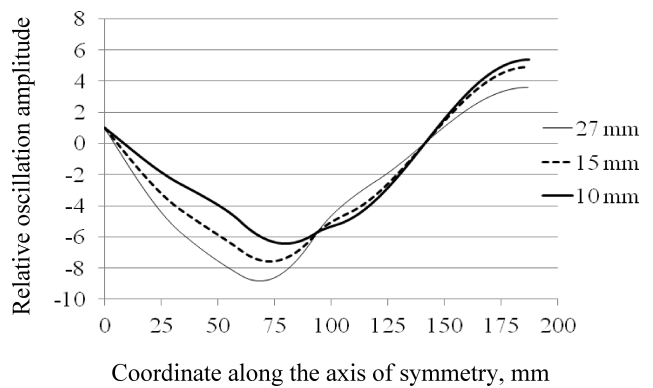

Fig. 3. Distribution of the amplitudes of longitudinal vibrations of concentrating link at different lengths of the zone l_4.

Obtained distributions of the vibration amplitudes along the concentrating resonance link allow to conclude that the choice of length of the zone l_4 at

constant total length of amplifying link can provide 1.5 times larger gain factor without changes of the diameters D_2 and D_3.

This is because added mass (zone l_4) will be decreased at decreasing of length l_4.

Figure 4 shows the gain factor dependence on the ratio of the areas of input (S_{in}) and output (S_{out}) stepwise-radial concentrator cross-sections obtained by using developed model.
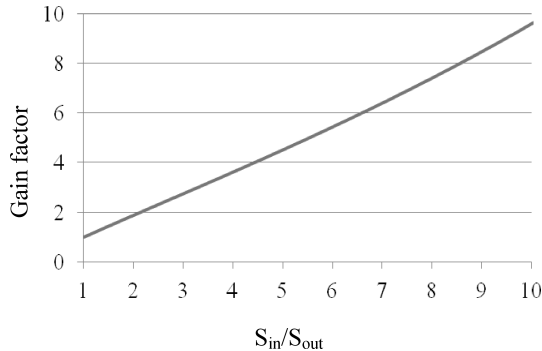

Fig. 4. Gain factor dependence on the ratio of the areas of input (S_{in}) and output (S_{out}) stepwise-radial concentrator cross-sections.

When determining relations between the areas of input and output cross-sections of the concentrating link, which equal to the relation of the areas of the zones of maximum and minimum cross-section (l_5 and l_3) of developed amplifying link, it is possible to achieve the increased gain factor (in this case equals to 5.7) by the optimization of the length of the zone l_4.

However, due to 6.25 times increase of the area of the output end in considered link it is possible to achieve more than 6 times rising of entered acoustic energy, that proves high efficiency of above mentioned concentrating link. Figure 5 and Fig. 6 show the dependences of gain factor on the diameter and length of the zone l_4.
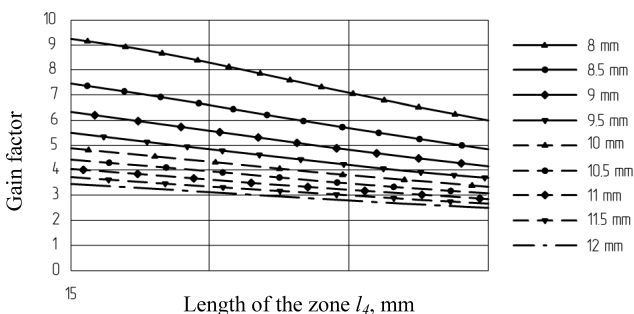

Fig. 5. Dependence of the gain factor on the length of the zone l_4 at different diameters D_2.

Presented dependences allow to reach the conclusion that the diameter D_2 of the zone l_3 the most essentially influences the gain factor.

However, the attempt to reduce the diameter from 10 to 8 mm aimed at increasing in gain factor in two
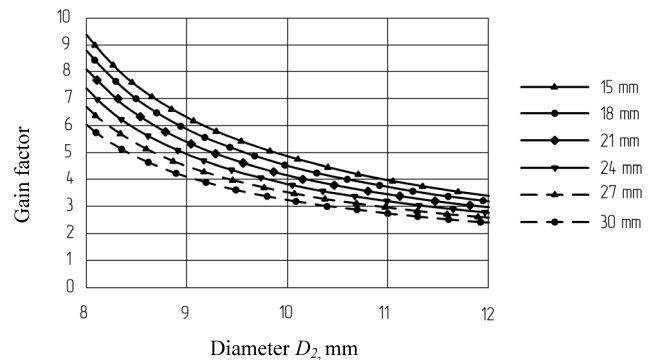

Fig. 6. The dependence of gain factor on the diameter of the zone l_3 at different diameters of the zone l_4.

times is inadmissible as it leads to the growth of mechanical stress, which can cause the breakage of the link. That is why optimum diameter providing allowable mechanical stress is about 10 mm. At the same time, the analysis of the model demonstrates the possibility of raising gain factor in 6...7 times by the reduction of the length of the zone l_4 (the increase of the angle α, see Fig. 2).

Thus, developed model allows perfecting the construction of the concentrating unit by the determining of optimum design factors.

4. The modelling of concentrating link by finite element method

Because of consistent carrying out of all stages of the calculations of proposed model, the main design sizes of the concentrating link are determined. To verify adequacy of proposed model the comparison with the calculations obtained by direct numerical method of finite elements was made (KHMELEV *et. al.*, 2003).

For this purpose, the 3D solid model of concentrating link was developed by means of the system of automated solid design, further exported to CAE system.

As initial sizes of the 3D model were as follows: input diameter $D_1 = 25$ mm (it equals to the output diameter of the piezoelectric transducer further used for experimental studies), $D_2 = 10$ mm, the output diameter $D_3 = 25$ mm. The zones l_1, l_2, l_3 are calculated by the design procedure given in (MERKULOV, 1957; 1959). At this $l_1 + l_2 + l_3 = \lambda/2$, the zone l_4 is chosen equal to the zone l_2. The zone l_5 is calculated on condition that $l_4 + l_5 = \lambda/2$. As a result, the total length of the construction is resonant and equals to $2\lambda/2$. The resonant frequency used in the calculation is equal to 30,000 Hz.

For CAE modelling the modal type of analysis was chosen, the geometric model was decomposed down into 65000 elements and the tetrahedral type of finite element set. The model material was chosen to meet the following specifications: density is 4500 g/m^3,

Young's modulus is $1.12 \cdot 10^{11}$ Pa, Poisson ratio is 0.429, what corresponds to the properties of titanium alloy Ti-5Al.

After that, the concentrating links with the step of 0.5 mm around these basic sizes were modelled. Obtained results are shown in Fig. 7. Figure 7a shows the dependence of gain factor on the diameter D_2. Figure 7b shows the dependence of gain factor on the length of the zone l_4.
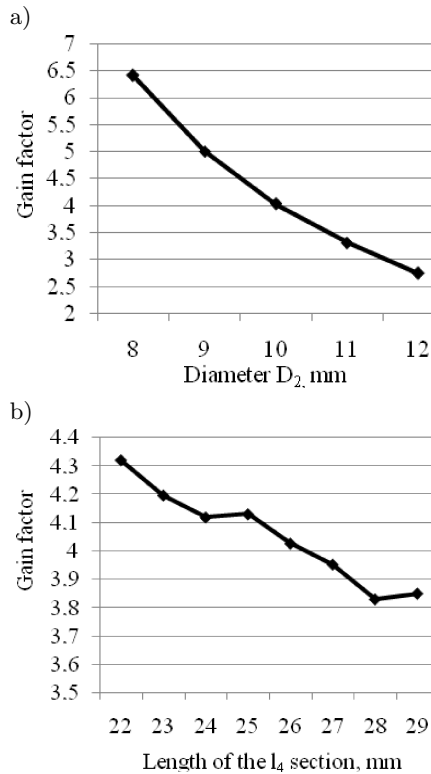
a)

b)

Fig. 7. Results of modelling by finite element method.

From the dependence shown in Fig. 7a it can be assumed that if the diameter D_2 is 8 mm, maximum gain factor can be achieved at constant diameter of the output part. However, as it was mentioned above, such diameter of middle part of the link and its further reduction leads to the increase of mechanical stress in this zone, and finally causes breakage. That is why optimum diameter satisfying the conditions of the achievement of maximum gain factor at mechanical stresses, which do not exceed threshold value for the material of the link, equals 10 mm.

The dependence of the gain level (Fig. 7a) is a smooth and monotonic function. This is because primary wave (radiated from the input end) has almost constant phase difference with the wave reflected from the transition zone and the output end.

From the dependence shown in Fig. 7b it is evident that at fixed size of the zone D_2 the highest gain factor is achieved at minimum length of the zone l_4 of the concentrating link. The conditions of passing of longi-

tudinal waves along the link at changing of length l_4 change because the phase difference between the primary and the reflected waves also changes. Thus, the dependence of the gain level from the length of the zone l_4 has local extremes.

While modelling, the length of the concentrating link was determined for the purpose of assignment of exact value of proper resonance frequency of the construction. For instance, resonance frequency of the construction is 29,900 Hz, which differs from the estimated value at the 3D modelling stage in 100 Hz.

Thus, at the stage of modelling optimal lengths of all zones of the concentrating link at set resonance frequency were determined and chosen that allowed to produce the concentrating link for ultrasonic technological apparatuses.

As a result of the calculations, it is ascertained that the difference between the results of modelling by finite element method and by proposed model is not more than 10%. The presence of such error is caused by the fact that proposed model does not take into account transverse radial vibrations of the concentrating link, as longitudinal vibrations are the most interesting for the realization of ultrasonic technologies. Nevertheless, this model is satisfactory for the calculations of main geometric and resonance parameters of the concentrating link.

5. Experimental studies

The experimental studies on determining of performance specifications of developed link were devoted to the measurements of proper resonance frequency of the construction and gain factor of the ultrasonic vibrating system, amplitude of mechanical vibrations at the output end of the link and measurements of electroacoustic efficiency of the ultrasonic apparatus.

Figure 8 shows the appearance of the concentrating link and the ultrasonic device "Alyona" (KHMELEV *et. al.*, 2007) using the vibrating system with such link.

Fig. 8. Appearance of developed concentrating link and ultrasonic apparatus.

As a source of ultrasonic vibrations, the piezoelectric transducer consisting of 2 piezoceramic rings of standard size $24 \times 12.5 \times 6.35$ mm is used. It has

frequency-lowering radiating cover plate made of aluminum alloy V95 (U.S. designation – A97075) and frequency-lowering reflecting cover plate made of steel 45 (U.S. designation – M1044). The connection of the transducer with the concentrating link is carried out by pin joint.

The measurements of resonance frequency of the vibrating system and gain factor were carried out by the piezoelectric probe (KHMELEV *et. al.*, 2014) with dry point contact at the power supply of the ultrasonic vibrating system from the low-voltage generator.

The sensitivity of this piezoelectric probe is 0.86 V/μm (KHMELEV *et. al.*, 2014).

The resonance frequency was defined by maximum value of vibration amplitude; the gain factor was defined as a ratio of maximum vibration amplitude on radiating end of the ultrasonic vibrating system to vibration amplitude on back reflecting cover-plate of the piezoelectric transducer. For the measurements results refer to Table 1. Relative tolerance of results does not exceed 5%. Thus, the measurements results show that measured value of the gain factor of the concentrating link is 4.8, which in its turn varies in no more than 5% from the value of the gain factor obtained by theoretical calculations. The discrepancy of the definition of the resonance frequency is 0.6%.

Amplitude of mechanical vibrations at the end of the concentrating link was measured by stroboscope method (LEONOV *et. al.*, 2007). For this purpose, we used the stroboscope for measuring the amplitude of mechanical vibrations and microscope. Measurements of vibration amplitude during the operation of the apparatus showed that the amplitude of mechanical vibrations at the output end of the concentrating link was 40–45 μm. The obtained data also proved the values of gain factor of the ultrasonic vibrating system with the concentrating link, at that the obtained value of amplitude is sufficient for the realization of many ultrasonic technologies.

As the presence of diametrical vibrations of the concentrating unit was not considered, there is a special interest to study the distribution of vibrations of the cylindrical zone l_5. Figure 9 shows the distribution of vibration amplitude obtained by described above method.
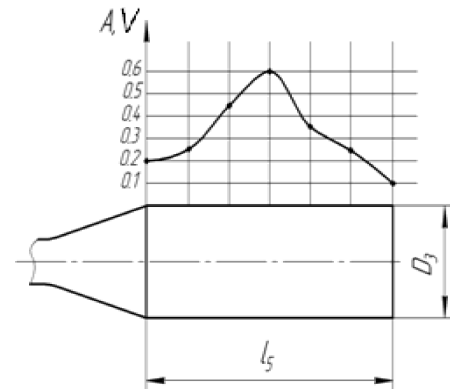

Fig. 9. Distribution of vibration amplitude along the cylindrical zone l_5.

As it is evident from the graph, the concentrating link has high level of transverse vibrations (maximum value of the amplitude does not exceed 5 μm at the operation of the apparatus), that can cause additional energy output from the side surface. This part can be easily defined by the experiments; we partly immerse the cylindrical zone into water.

Total (at overall immersion of the cylindrical zone) efficiency of the ultrasonic device was determined by calorimetric method. This method is one of the most widely used and suitable for the determination of available acoustic power of the ultrasonic devices intended for the operation in liquid and liquid-dispersed media (KHMELEV *et al.*, 2007). The procedure of measurements is based on the practical realization of the calorimetric method standardized by the IEC. Measurement of electroacoustic power was carried out by the quality analyzer of electric power. Two types of liquids – tap water and engine oil (SAE10-W40) – were measured. The number of sets of measurements for each type of liquid was 10, than the values were averaged.

For the measurements results refer to Table 2. Relative tolerance of the results does not exceed 15%.

Table 1. The measurement results obtained by the piezoelectric probe.

Link	Amplitude at the end [V]	Amplitude at the back reflecting cover-plate reflector [V]	Resonance frequency [kHz]	Gain factor
Piezoelectric transducer	2.4	1.6	29.71	1.5
+ concentrating link	**5.8**	**0.8**	**29.76**	**7.25**

Table 2. Results concerning measurement of the unit efficiency, as obtained by the calorimetric method.

Liquid description	Unit input power [W]	Acoustic power [W]	Efficiency [%]
Water	390	244	61.5
Engine oil	410	235	57.3

From obtained results it is evident that acoustic power radiated by the apparatus at the use of the piezoelectric transducer with the diameter of 25 mm with considered concentrating link exceeds radiated power (approximately 4 times) at the application of vibrating systems with the traditional concentrator. It proves the fact that studied type of the concentrating link allows bringing out more energy to processed medium. The value of the efficiency of the ultrasonic device with the concentrating link is rather high and it corresponds to the values of the efficiency of the ultrasonic apparatuses with traditional concentrators and mushroom-shaped working tools.

6. Summary and conclusion

The performed work resulted in studying a new type of the concentrating resonance link for the ultrasonic vibrating systems. As a result of theoretical calculations main dependences of the lengths of the zones composing the concentrating link were revealed, under given initial conditions we determined the conditions providing maximum gain factor of the link. On the base of the results of the theoretical investigations, improved concentrating resonance link for the ultrasonic vibrating system was developed, and new technological apparatus for such system was produced. The results of experimental studies of developed ultrasonic technological device helped to verify the possibility of generation ultrasonic vibrations with specified amplitude (up to 45 μm) and provide 4 times increase of acoustic energy entered to processed media (when the efficiency was more than 60%).

Acknowledgment

References

1. KHMELEV V.N., LEBEDEV A.N., TSYGANOK S.N. (2003), *Computer-Aided Design of Ultrasonic Oscillating Systems* [in Russian], Tula State University Journal: Selecta by the Members of the Second International Electronic Scientific and Technical Conference, pp. 14–18, Tula.

2. KHMELEV V.N., LEONOV G.V., BARSUKOV R.V., TSYGANOK S.N., SHALUNOV A.V. (2007), *Ultrasonic Multi- and Single-Purpose Stimulators of Processes in Industry, Agriculture and Housekeeping* [in Russian], Altai State Technical University Publisher, Biysk.

3. KHMELEV V.N., TSYGANOK S.N., LEVIN S.V., DEMYANENKO M.V., SHAKURA V.A. (2014), *Development and Investigating of Piezo-Transducer for Control of Amplitude of Radiating Surfaces Ultrasonic Oscillations* [in Russian], Polzunovsky vestnik, **2**, 88–91.

4. LEONOV G.V., KHMELEV V.N., SAVIN I.I., ABRAMENKO D.S. (2007), *Vibration Amplitude Measuring Method*, Russian Federation Patent #2292530, ipc g01h 9/00.; the applicant and patent holder is I.I. Polzunov Altai State Technical University., No. 2005111032/28, application of 14.04.05; published on 27.01.07.

5. MERKULOV L.G. (1957), *Theory of Compound Concentrators* [in Russian], Acoustic Journal, **3**, 3.

6. MERKULOV L.G., KHARITONOV A.V. (1959), *Theory and Design of Compound Concentrators* [in Russian], Acoustic Journal, **5**, 2.

7. PESHKOVSKIY S., FRIEDMAN M., HAWKINS W. (2007), *Ultrasonic Rod Waveguide-Radiator*, United States Patent 7156 201, January 2, 2007.

8. PESHKOVSKY S., PESHKOVSKY A. (2008), *High Capacity Ultrasonic Reactor System*, International Application no.: PCT/US2008/068697, International Filing Date: 30.06.2008.