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Active Noise Control with a Single Nonlinear Control Filter for a Vibrating Plate with Multiple Actuators

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Vibrating plates can be used in Active Noise Control (ANC) applications as active barriers or as secondary sources replacing classical loudspeakers. The system with vibrating plates, especially when nonlinear MFC actuators are used, is nonlinear. The nonlinearity in the system reduces performance of classical feedforward ANC with linear control filters systems, because they cannot cope with harmonics generated by the nonlinearity. The performance of the ANC system can be improved by using nonlinear control filters, such as Artificial Neural Networks or Volterra filters.

However, when multiple actuators are mounted on a single plate, which is a common practice to provide effective control of more vibration modes, each actuator should be driven by a dedicated nonlinear control filter. This significantly increases computational complexity of the control algorithm, because adaptation of nonlinear control filters is much more computationally demanding than adaptation of linear FIR filters.

This paper presents an ANC system with multiple actuators, which are driven with a single nonlinear filter. To avoid destructive interference of vibrations generated by different actuators the control signal is filtered by appropriate separate linear filters. The control system is experimentally verified and obtained results are reported.

Keywords: active noise-vibration control, active structural acoustic control, adaptive control, nonlinear-control.

1. Introduction

Vibrating plates are potentially very useful for Active Noise Control applications in industrial environments. The plates can be used as secondary sound sources, as replacement for classical loudspeakers, but also can be used as active barriers (Fahy, Gardonio, 2007; Hansen, Snyder, 1997; Rdzanek, Zawieska, 2003), where usually single or double plates (Pietrzko, 2009) are placed between the noise source and the area where the noise sound pressure level (SPL) should be reduced. Rectangular plates are frequently used (Zawieska, Rdzanek, 2007; Gorski, Kozupa, 2012), but also other plate shapes, including circular plates (Zawieska *et al.*, 2007; Rdzanek *et al.*, 2011; Leniowska, 2011) and triangular plates (Barański, Szela, 2008), are useful for some applications and are investigated in the literature. Also more complex structures like L-jointed plates, T-shaped plates (Keira *et al.*, 2005) or four connected plates (Liu *et al.*, 2010), are of scientific interest.

Plates are more resistant to harsh environment conditions than classical loudspeakers. However, plates are more difficult to control. The common problems include irregular multimodal response, the need to use multiple actuators on a single plate to effectively excite multiple plate modes, and a nonlinear response. The nonlinear response is caused by vibrations of not ideally clamped plate (El Kadri *et al.*, 1999; Saha *et al.*, 2005), but also might be caused by frequently used d33 effect of MFC patches if they are employed as actuators (Stuebner *et al.*, 2009; Mazur, Pawełczyk, 2011a). In this paper, the latter problem is mitigated by using EX1 electrodynamic actuators. However, the former problem still needs to be approached, because it may significantly degrade performance of a linear feedforward ANC system. The degradation is especially visible for simple deterministic signals, when active control is expected to be very successful. For active control of such plants a feedback system could be used, which has inherited capability to compensate to some extend for plant nonlinearity. However, if a reference signal is available in advance, a generally better performance can be obtained with a feedforward system, although it should have a nonlinear architecture. The nonlinear feedforward active noise control is more computationally demanding, than a linear control. Application of multiple actuators for a single plate additionally increases the number of required operations, and a dedicated control filter is used for each actuator.

This paper presents a solution with multiple actuators, which are driven by a single nonlinear ANC filter. The control signal being the output of such filter is then appropriately filtered by separate linear filters used to drive subsequent actuators. These filters are tuned to avoid destructive interference of vibrations, which might occur if control of the actuators were not coordinated.

2. Nonlinear feedforward control

There are many possible approaches to nonlinear feedforward control. For a simple and well modeled nonlinearity it can be possible to filter the control signal by an inverse model. This approach is commonly used for semi-active vibration control using MR dampers. However, for many plants it is very hard to find an appropriate inverse model for a required frequency band if the plant itself is nonminimum phase and with a delay. These are common problems for ANC applications. A popular alternative approach is to use black-box input-output models obtained with Artificial Neural Networks (Hansen, Snyder, 1997). Another approach, which can be employed for Active Noise/Vibration Control problems is to use nonlinear filters, linear with respect to parameters. A large number of algorithms fall into this category, including Volterra FXLMS (Tan, Jiang, 2001), FSLMS (Das, Panda, 2004; George, Panda, 2012) and Generalized FLANN (George, Panda, 2013).

For easier implementation, nonlinear filters, linear with respect to parameters, can be expressed as a sum of Hammerstein models with arbitrary nonlinear functions F_k (Mazur, Pawełczyk, 2011a; 2013):

$$u_c(i+1) = \sum_{k=1}^{K} W_{c,k}(z^{-1}) \, F_k\left(x(i), x(i-1), \ldots, x(i-(N-1))\right), \quad (1)$$

where $u_c(i+1)$ is the value of the c-th control signal at the $i+1$ sample, $x(i)$ is the reference signal, $W_{c,k}(z^{-1})$ is the linear finite response filter for the c-th control signal and the k-th nonlinear function F_k, z^{-1} is the one-sample delay operator. The number of nonlinear functions is equal to K and the number of secondary paths is equal to C.

Figure 1 shows the block diagram of a multichannel control system with such nonlinear control filter. The bank of F_k nonlinear functions converts the reference signal, $x(i)$, into a vector $\mathbf{x}(i) = [x_1(i), x_2(i), x_3(i)]^{\mathrm{T}}$. This vector is next filtered by a bank of linear $W_{c,k}(z^{-1})$ FIR adaptive filters. These filters can be grouped into a matrix of FIR filters $\mathbf{W}(z^{-1})$:

$$\mathbf{W}(z^{-1}) = \begin{bmatrix} W_{1,1}(z^{-1}) & W_{1,2}(z^{-1}) & \cdots & W_{1,K}(z^{-1}) \\ W_{2,1}(z^{-1}) & W_{2,2}(z^{-1}) & \cdots & W_{2,K}(z^{-1}) \\ \vdots & \vdots & \ddots & \vdots \\ W_{C,1}(z^{-1}) & W_{C,2}(z^{-1}) & \cdots & W_{C,K}(z^{-1}) \end{bmatrix}. \quad (2)$$

Outputs of $W_{c,k}(z^{-1})$ adaptive control filters form a vector of control signals $\mathbf{u}(i) = [u_1(i), u_2(i), \ldots, u_C(i)]^{\mathrm{T}}$. The signals are then are used to drive a vector of secondary paths $\mathbf{S} = [S_1, S_2, \ldots, S_C]^{\mathrm{T}}$. The adaptation algorithm uses the vector of reference signals $\mathbf{x}(i)$ and the scalar error signal $e(i)$ to adapt weights of $\mathbf{W}(z^{-1})$ control filters, as described in the following Section. The P stands for the primary path in the ANC system.

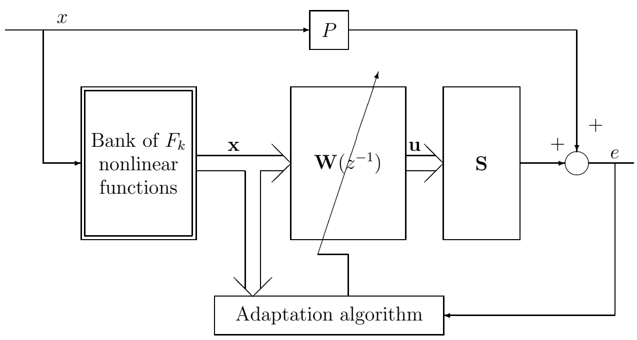

Fig. 1. Multichannel ANC system with Hammerstein nonlinear control filters.

3. Adaptation of control filter parameters

When the filters are linear with respect to parameters the classical adaptive control adaptation algorithms, like LMS, Affine Projection or RLS, with appropriate modifications to improve convergence properties can be employed. Otherwise, more complex algorithms such as genetic algorithms (Górski, Morzyński, 2013) or memetic algorithms should be employed.

The Normalized Leaky LMS algorithm takes the form (Elliott, 2001):

$$\mathbf{w}_{c,k}(i+1) = \alpha \mathbf{w}_{c,k}(i) - \mu \frac{\mathbf{x}^*_{c,k}(i)}{\sum_{c=1}^{C}\sum_{k=1}^{K} {\mathbf{x}^*_{c,k}}^{\mathrm{T}}(i)\mathbf{x}^*_{c,k}(i) + \zeta} e^*_c(i), \quad (3)$$

where $0 \ll \alpha < 1$ is the leakage coefficient, μ is the convergence coefficient, and ζ is a parameter protecting against division by zero in case of lack of excitation. In (3) $\mathbf{x}^*_{c,k}(i) = [x^*_{c,k}(i), x^*_{c,k}(i-1), \ldots, x^*_{c,k}(i-(M-1))]^{\mathrm{T}}$ is a vector of regressors of the reference signal for the LMS algorithm, M is the order of an FIR model of the secondary path. For notation simplicity it is assumed that orders of all secondary paths are the same. The symbol $e^*_c(i)$ stands for the error signal for the c-th secondary path. The error signal, $e(i)$, is to be minimized in the square sense by the LMS algorithm. Some additional modifications to the LMS algorithm, like variable step size, can also be used to enhance convergence properties (Bismor, 2012).

In contrary to electrical noise cancellation or speech enhancement (see, e.g. (Latos, Pawełczyk, 2010)) for active noise/vibration control applications the filter outputs drive the secondary path (acousto-electric or vibro-acousto-electric), the algorithm must be modified to guarantee convergence (Pawełczyk, 2008). The most popular modification is filtration of the reference signal by a model of the secondary path, what results in obtaining the well-known Filtered-x LMS algorithm (Elliott, 2001; Kuo, Morgan, 1996). Figure 2 shows a block diagram of the multichannel FXLMS algorithm, when used for adaptation of nonlinear filters as in (1). The ANC error signal is used as the error signal for the LMS algorithm $e^*_c(i) = e(i)$.

Because each reference signal must be filtered by models of corresponding secondary paths, the Filtered-x structure involves a number of numerical operations for such application, where multiple reference signals are generated from a single reference with a bank of F_k filters. The Filtered-error LMS (FELMS) algorithm is more appropriate in that case (Fig. 3). In the FELMS

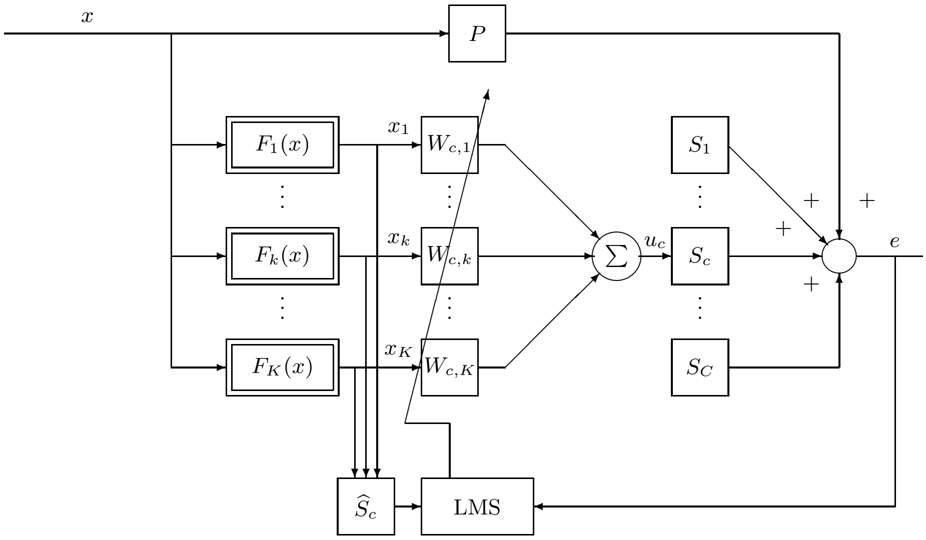

Fig. 2. An excerpt of the ANC system with Hammerstein nonlinear control filters and multichannel FXLMS algorithm for the c-th control channel.

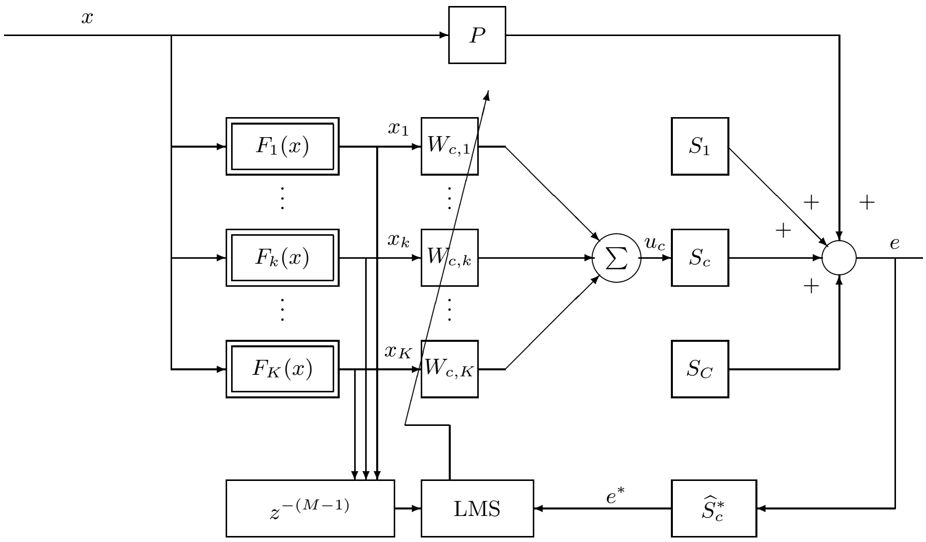

Fig. 3. An excerpt of the ANC system with Hammerstein nonlinear control filters using FELMS algorithm for the c-th control channel.

algorithm the multiple reference signals are simply delayed $x^*_{c,k}(i) = x_k(i-(M-1))$, but the error signal is obtained as:

$$e^*_c(i) = \widehat{\mathbf{s}}_c(i)^{\mathrm{T}}\mathbf{e}(i), \tag{4}$$

where

$$\widehat{\mathbf{s}}_c(i) = [\widehat{s}_{c,M-1}(i), \widehat{s}_{c,M-2}(i), \ldots, \widehat{s}_{c,0}(i)]$$

is a time-reversed model of the c-th secondary path,

$$\mathbf{e}(i) = [e(i), e(i-1), \ldots, e(i-(M-1))]^{\mathrm{T}}$$

is a vector of regressors of the error signal.

4. Control of multiple actuators with a single nonlinear filter

The Filtered-error structure reduces the number of numerical operations needed for filtering every reference signal by a secondary path model. However, when multiple actuators are used for the same plate, multiple nonlinear control filters are involved. This significantly increases computational cost. Such problem can be reduced by using a single nonlinear control filter. However, the same output cannot be used to drive all actuators because of potential destructive interference of vibrations generated by different actuators. The control signals for different actuators should be properly filtered for a positive interference to occur.

Figure 4 shows the resulting structure. The output of nonlinear ANC control filter is filtered by the E_c filter dedicated for each actuator. Dependent of E_c filter choice, the filtration of the error signal by $\widehat{H}^*$ filter might be required. However, in the proposed system such filter is avoided, because the E_c filters are tuned to linearize the total secondary path phase response.

Because the secondary path models may change in time, for instance they strongly depend on plate temperature (Mazur, Pawełczyk, 2011b), E_c filters are made adaptive. Figure 5 shows the control system used for adaptation of E filter weights (Mazur, Pawełczyk, 2013). This system compares the obtained actual response to the response of desired secondary path model H, and updates the weights appropriately. When the desired path have a linear phase response, filtration by $\widehat{H}^*$ is not needed and delaying the reference signals by D steps, where D is the delay of the desired path H, is sufficient for convergence.

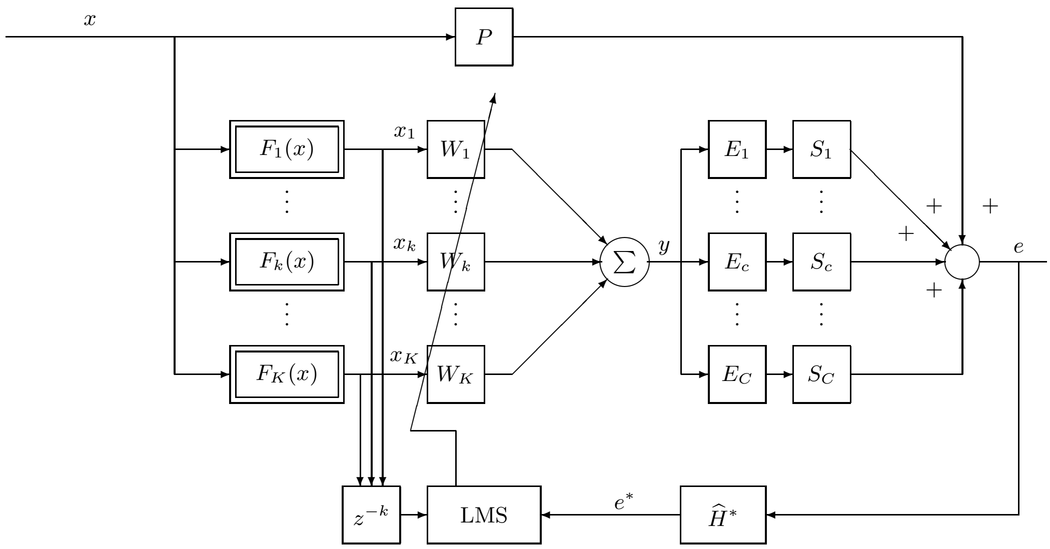

Fig. 4. ANC system with Hammerstein nonlinear control filters using single nonlinear control filter.

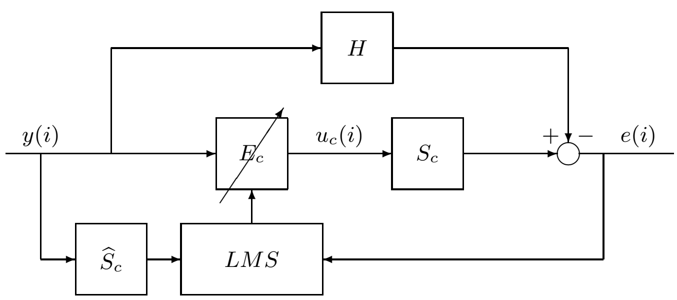

Fig. 5. Single-channel adaptive sound radiation control system.

Table 1 shows the asymptotic computational complexity of basic steps for the proposed algorithm and for the control system with separate per-actuator nonlinear filters, with FXLMS and FELMS adaptation algorithm. The order notion, O, is used. In case of per-actuator nonlinear filters with FXLMS adaptation algorithm each step have asymptotic computational complexity proportional to the number of channels multiplied by the number of nonlinear functions and the order of FIR filters. The FELMS adaptation algorithm reduces the computational complexity of the filtration by the secondary path model. In the proposed algorithm there is only one nonlinear filter, and the computational complexity does not depend on the number of actuators. The computational complexity of added additional steps does not depend on the number of nonlinear functions, and the overall asymptotic computational complexity is reduced from $O(KCA)$ to $O(KA + CA)$, where $A = \max(N, M, N_E, N_K)$.

Table 1. Asymptotic computational complexity of presented algorithms.

	NFXLMS	NFELMS	proposed algorithm
Nonlinear filter			
Output calculation	$O(KCN)$	$O(KCN)$	$O(KN)$
Reference/error filtration	$O(KCM)$	$O(CM)$	$O(M)$
Filter adaptation	$O(KCN)$	$O(KCN)$	$O(KN)$
Linear per-actuator filters			
Output calculation	–	–	$O(CN_E)$
Reference/error filtration	–	–	$O(CM_E)$
Filter adaptation	–	–	$O(CN_E)$

5. Experimental results

The control system has been applied to reduce noise transmitted through a fully-clamped aluminum plate of dimensions 400 mm × 500 mm × 1 mm from a small enclosure to a laboratory room (Fig. 6). The noise was generated by a loudspeaker located in the enclosure, and the goal of the control system was to reduce sound pressure level at specified area around an error microphone in the laboratory room.

Three NXT EX-1 actuators (of 5 W power) were mounted on the plate (see Fig. 6). Positions of the actuators were chosen to maximize the minimal eigenvalue of the controllability Gramian matrix for first 25 plate modes (see Fig. 7) (Wrona, Pawełczyk, 2013).

Both the nonlinear ANC filter and per-actuator linear control filters were implemented on a single microprocessor system (Fig. 8). However, for larger systems, where many vibrating plates are used, those separate

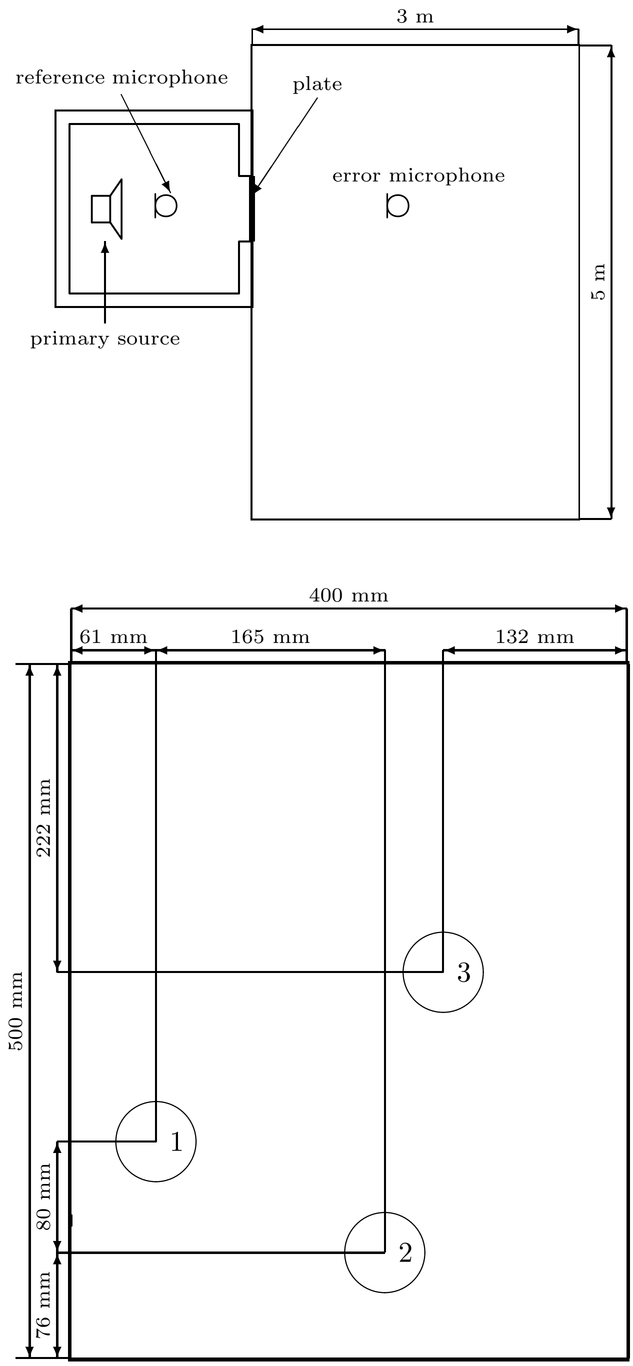

Fig. 6. Laboratory setup (top) and EX-1 actuators installed on the plate (bottom).

functions can be implemented on separate microprocessor systems and the $y(i)$ control signal(s) is then passed between those systems.

The error and reference microphones were connected to 16-bit ADCs with synchronous sampling by using microphone amplifiers and 8th order Butterworth low-pass anti-aliasing filters with 600 Hz cut-off frequency. The sampling frequency was set to 2 kHz (0.5 ms sampling period). Four 16-bit ZOH DACs with synchronous sampling were used to control loudspeaker in the enclosure and three EX-1 actuators. As reconstruction filters, 8th order Butterworth low-pass filters with 600 Hz cut-off frequency were used. The DACs and ADCs sampling processes were not synchronous – the DAC outputs were updated just after ADC conversion, after approximately 1.44 μs.

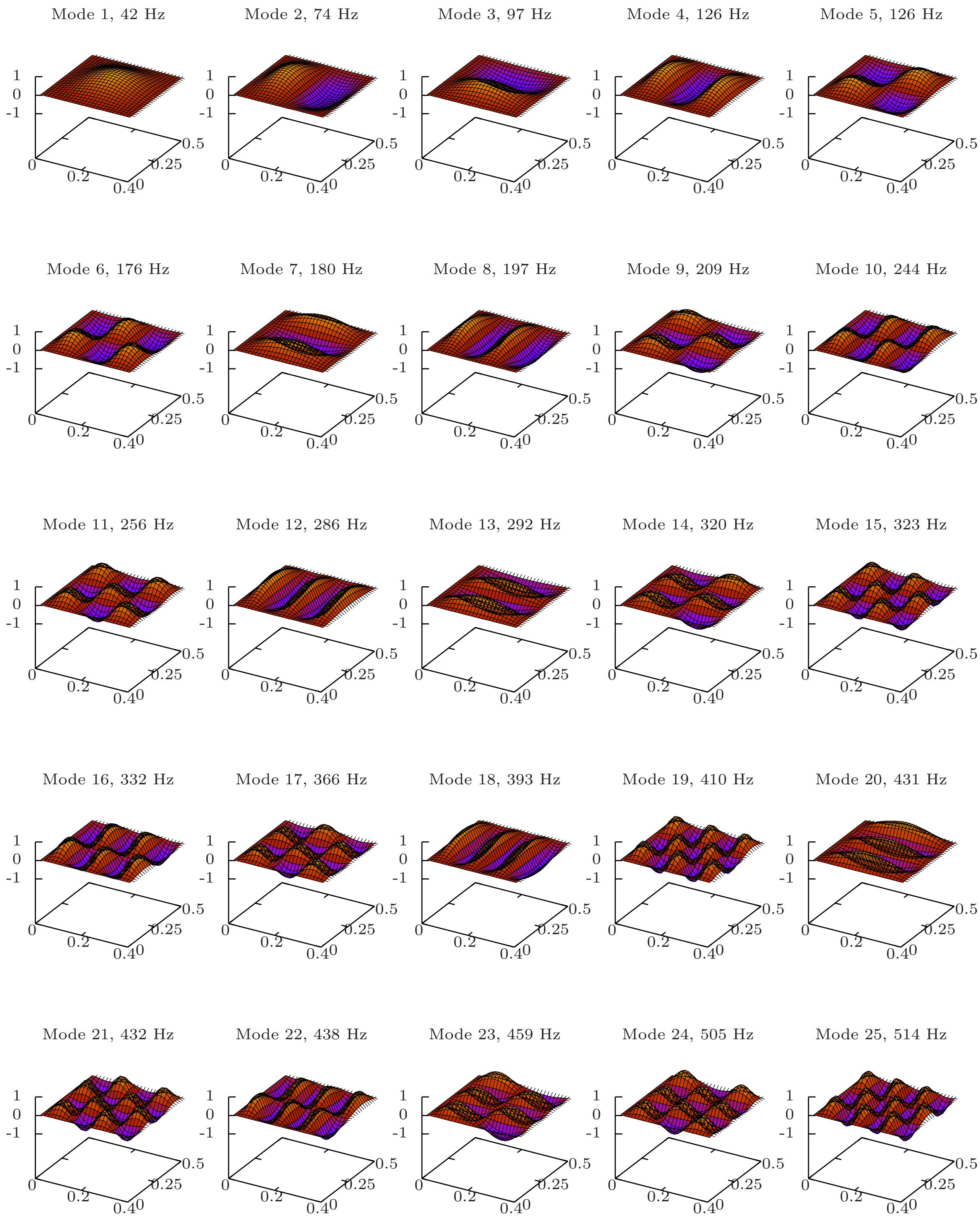

Fig. 7. First 25 mode shapes for used 0.5 m × 0.4 m plate. The vertical axis shows the vibration amplitude normalized to $[-1, 1]$ range.

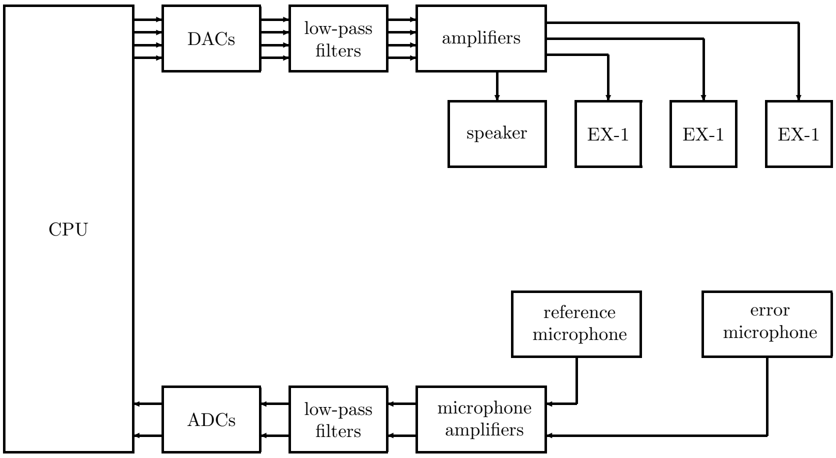

Fig. 8. Implementation of the ANC system.

The system presented in Fig. 4 was used as the nonlinear control system. The desired secondary path response H was equal to z^{-16}. The order of the E linear filters was set to $N_E = 256$. The order of control filter was set to $N = 256$. The parameters of NLMS adaptation algorithm were set to $\alpha = 1$, $\mu = 0.05$, $\zeta = 10^{-12}$. The first 5 functions of trigonometric expansion used in the FSLMS algorithm were used as the F_k functions:

$$\begin{aligned} F_1(x) &= x, & F_2(x) &= \sin(\pi x), \\ F_3(x) &= \cos(\pi x), & F_4(x) &= \sin(2\pi x), \\ F_5(x) &= \cos(2\pi x). \end{aligned} \tag{5}$$

Figure 9 shows the Power Spectral Density (PSD) of the error microphone signal for classical linear feedforward FXLMS ANC system, for different levels of 155 Hz tonal noise. For comparison, PSD of the noise

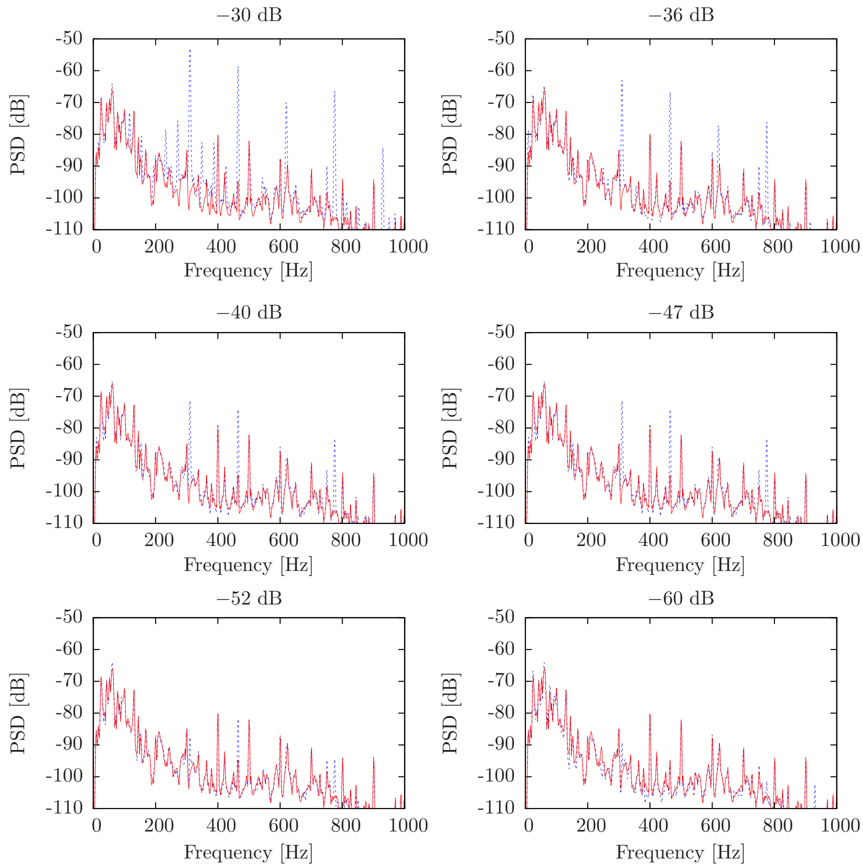

Fig. 9. PSD of error microphone signal for FXLMS ANC system for different levels of 155 Hz tonal noise and noise floor level.

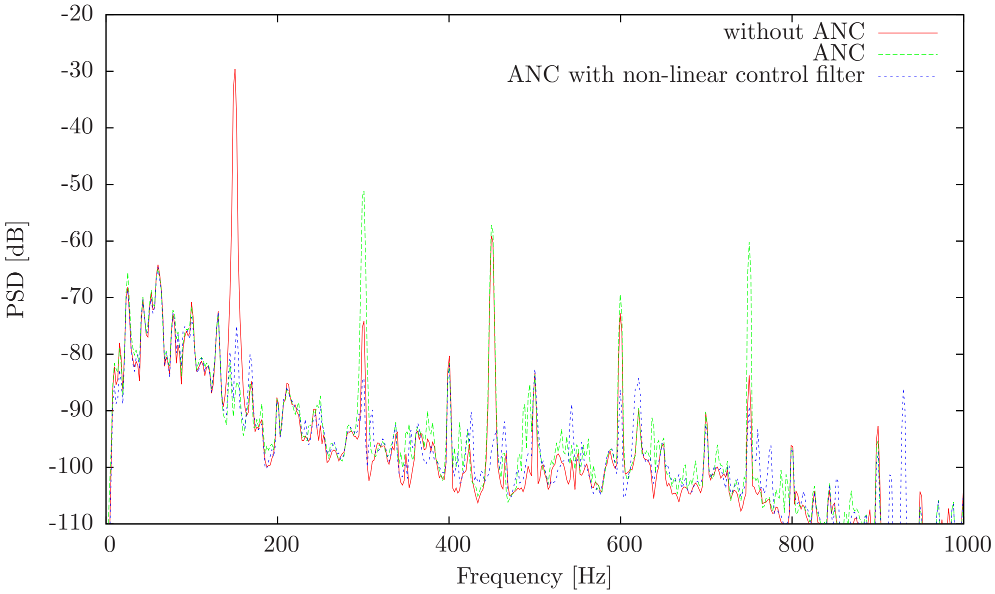

Fig. 10. PSD of error microphone signal for different control strategies for 155 Hz tonal noise.

floor signal (without the primary noise and with disabled ANC) is included in each plot. The PSD of the primary noise (without ANC but with the noise floor) is not plotted for clarity of the figures, but above each plot the power of 155 Hz tone without ANC is presented.

For small noise powers the nonlinear artifacts are not visible or they have powers comparable to the noise floor power. For higher powers, −30 dB and −36 dB, the harmonics are clearly visible and they limit the noise reduction level.

The performance of the proposed nonlinear control system for the 155 Hz tone is shown is Fig. 10. The FXLMS ANC achieves 18.9 dB reduction of the SPL. However, because the human hearing system is more sensitive to higher frequencies, the harmonics caused by the nonlinearity are even heard as louder. After A-weighting the SPL reduction of the FXLMS system is only 11.0 dB. The nonlinear feedforward control system provides 28.3 dB reduction of the SPL and even 30.6 dB when A-weighted.

6. Conclusions

By using linear filters dedicated for each actuator bonded to a single plate, it is possible to use only one nonlinear control filter to efficiently drive all the actuators. The nonlinear filters are very computationally demanding, and reduction of their number saves computational load, which can be spent for implementing more complex filters.

The per-actuator linear control filters were used also to linearize phase response of the secondary paths. This allowed for application of the FELMS, simplified even to the Delayed LMS algorithm for adaptation of nonlinear control filter weights. This provides additional reduction of numerical operations. Efficiency of the proposed approach is particularly evident, when considering A-weighted noise reduction results.

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