www.czasopisma.pan.pl PAN www.journals.pan.pl
POLSKA AKADEMIA NAUK

jet INTL JOURNAL OF ELECTRONICS AND TELECOMMUNICATIONS, 2016, VOL. 62, NO. 3, PP. 267-272
Manuscript received July 5, 2016; revised August, 2016. DOI: 10.1515/eletel-2016-0036

On Modelling AM/AM and AM/PM Conversions via Volterra Series

Andrzej Borys and Wiesław Sieńko

***Abstract*—In this paper, we present the expressions, not published up to now, that describe the AM/AM and AM/PM conversions of communication power amplifiers (PAs) via the Volterra series based nonlinear transfer functions. Furthermore, we present a necessary and sufficient condition of occurrence of the nonzero values of AM/PM conversion in PAs. Moreover, it has been shown that Saleh's approach and related ones, which foresee nonzero level of AM/PM conversion, are not models without memory. It has been also shown that using a polynomial description of a PA does not lead to a nonzero AM/PM conversion. Moreover, a necessary condition of occurrence of an AM/AM conversion in this kind of modelling is existence of at least one nonzero polynomial coefficient associated with its odd terms of degree greater than one.**

***Keywords*—AM/AM and AM/PM conversions, nonlinear distortion, power amplifiers, Volterra series.**

I. Introduction

POWER amplifiers (PAs) installed on satellites are main source of nonlinear distortion generated in satellite communication links [1]. Because of the obvious need for minimization of satellite power consumption, these amplifiers are forced to work at or near saturation regions of their characteristics. This causes, however, a significant increase of nonlinear distortion compared to the cases in which PAs would work exclusively in their linear ranges of operation. Therefore, the influence of these distortions on an appropriate behaviour of a satellite link must be alleviated. There are different techniques for doing this like pre- or post-distortion and equalization, to mention the two most popular. In all of them, accurate and workable models of PAs are necessary for their successful implementation. A good survey of these models is presented in [2]. In the literature, it is assumed that they can be divided into two separate groups (families): the one incorporating memory effects and the second not doing this. A notable representative of the first group is a model using the description via Volterra series [3]. The second family models the PA nonlinear distortion by giving the levels of amplitude-to-amplitude (AM/AM) and amplitude-to-phase (AM/PM) conversions occurring in it. Here, a notable representative is a model developed by Saleh [4].

In this paper, we refer to the above groups and show that the above taxonomy must be revised. Further objectives of this work are the following:

Andrzej Borys and Wiesław Sieńko are with the Department of Marine Telecommunications, Faculty of Electrical Engineering, Gdynia Maritime University, Gdynia, Poland; emails: {a.borys, w.sienko}@we.am.gdynia.pl.

1. derivation of expressions for evaluation of AM/AM and AM/PM conversions with the use of Volterra series based nonlinear transfer functions [3] of PAs,
2. showing that a necessary condition for occurrence of the AM/PM conversion in a PA is the existence of a memory in its nonlinear characteristic,
3. the Saleh's model and related ones, which foresee nonzero level of AM/PM conversion, are in fact models that incorporate memory effects (they are not models without memory as is thought).

II. Derivation of Expressions for AM/AM and AM/PM Conversions via Volterra Series

Let us begin with definitions of the AM/AM and AM/PM conversions. And to this end, assume that a modulated bandpass signal of the form

$$\begin{aligned} x_B(t) &= \mathrm{Re}\left\{x_S(t)\exp\left(j\left(\omega_c t+\alpha_B(t)\right)\right)\right\} = \\ &= x_S(t)\,\mathrm{Re}\left\{\exp\left(j\left(\omega_c t+\alpha_B(t)\right)\right)\right\} = \\ &= x_S(t)\frac{\exp\left(j\left(\omega_c t+\alpha_B(t)\right)\right)+\exp\left(-j\left(\omega_c t+\alpha_B(t)\right)\right)}{2} \end{aligned} \tag{1}$$

is applied to a PA.

In (1), f_c means frequency of a carrier, $\omega_c = 2\pi f_c$, t a time variable, and $j=\sqrt{-1}$. Further, the symbol $\mathrm{Re}\{\cdot\}$ in (1) denotes the operation of taking the real value of a complex number. Moreover, the bandpass signal $x_B(t)$ (subscript B stands for bandpass) contains a slowly varying real-valued baseband signal $x_S(t)$ (subscript S here stands for slowly varying) that modulates the carrier amplitude. The carrier phase changes with time according to a function $\alpha_B(t)$. This function, similarly as $x_S(t)$, represents also a slowly varying baseband signal.

In what follows, $(\cdot)^*$ will be used for denoting the complex conjugate value of a given complex number.

Under assumptions underlying the PA response description in terms of the so-called AM/AM and AM/PM characteristics (conversions), for more details see, for example, [5], we can express the PA output signal as

$$\begin{aligned} y_B(t) &= Y_B(t)\,\mathrm{Re}\left\{\exp\left(j\left(\omega_c t+\beta_B(t)\right)\right)\right\} = \\ &= A\left(x_S(t)\right)\mathrm{Re}\left\{\exp\left(j\left(\omega_c t+\alpha_B(t)+\Phi\left(x_S(t)\right)\right)\right)\right\}\ . \end{aligned} \tag{2}$$

In (2), $Y_B(t)=A(x_S(t))$ and $\beta_B(t)=\alpha_B(t)+\Phi(x_S(t))$ are the carrier amplitude and its phase, respectively, at the amplifier output. It is assumed that the function $A(x_S(t))$ is not a linear function of $x_S(t)$. That is $A(x_S(t))\neq a\cdot x_S(t)$, where a stands for a real-valued constant. Moreover, it is assumed that an additional phase component $\Phi(x_S(t))$ in (2) is nonzero and depends upon the slowly varying baseband signal $x_S(t)$. This means that we have to do here with a kind of amplitude modulation expressed by the nonlinear characteristic $A(x_S(t))$ and with a phase modulation expressed by the function $\Phi(x_S(t))$, both caused by the signal $x_S(t)$. Clearly, because of this, we refer to $A(x_S(t))$ as the AM/AM characteristic and to $\Phi(x_S(t))$ as the AM/PM conversion.

In the well-known Saleh's model [4], the above characteristics are approximated in the following way:

$$A(x_S(t))=\frac{a_1x_S(t)}{1+a_2(x_S(t))^2},\quad \Phi(x_S(t))=\frac{b_1(x_S(t))^2}{1+b_2(x_S(t))^2},\tag{3}$$

where the coefficients a_1 and a_2 as well as b_1 and b_2 assume real values and need adjustment to the measured data for a given amplifier.

Apart of the above convention of using AM/AM and AM/PM characteristics in modelling PAs for RF applications, conventional input-output descriptions of nonlinear systems that use polynomials, Volterra series, Hammerstein or Wiener models, or combined ones are also exploited [2]. The polynomial descriptions are used for describing systems without memory (memoryless), opposite to the remaining ones, which are applied to systems with memory. With regard to this fact, note that the AM/AM and AM/PM means of description is attributed to the former ones (memoryless) [2].

In what follows, we will take a closer look at the above widespread opinion. To this end, we will try to find the form of functions $A(x_S(t))$ and $\Phi(x_S(t))$ with the use of Volterra series based nonlinear transfer functions [3] describing a PA. Advantage of this approach lies in its generality. Namely, any Volterra series reduces to an ordinary polynomial, when its kernels (nonlinear impulse responses) become the multidimensional Dirac impulses [6]. Then, we get an usual (memoryless) polynomial description. We will exploit this fact later in our discussions.

Thus, let a PA be described by the Volterra series as

$$\begin{aligned}y(t)&=h^{(0)}+\int_{-\infty}^{\infty}h^{(1)}(\tau)x(t-\tau)d\tau+\\&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h^{(2)}(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1d\tau_2+\\&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h^{(3)}(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)\cdot\\&\cdot x(t-\tau_3)d\tau_1d\tau_2d\tau_3+\ldots\quad.\end{aligned}\tag{4}$$

In (4), the input and output signals of PA are denoted by $y(t)$ and $x(t)$, respectively. Further, $h^{(0)}$ is a constant (dc component), but $h^{(1)}(\tau)$, $h^{(2)}(\tau_1,\tau_2)$, $h^{(3)}(\tau_1,\tau_2,\tau_3)$, and so on, are, respectively, the first order (linear), second order, third order, and so on, nonlinear impulse responses (Volterra kernels) of PA analyzed.

At this point note also that if the kernels $h^{(1)}(\tau)=h_1\cdot\delta(\tau)$, $h^{(2)}(\tau_1,\tau_2)=h_2\cdot\delta(\tau_1,\tau_2)$, $h^{(3)}(\tau_1,\tau_2,\tau_3)=h_3\cdot\delta(\tau_1,\tau_2,\tau_3)$, and so on, become the multidimensional Dirac impulses in (4), then, according to [6], this relation reduces to an usual polynomial

$$y(t)=h_0+h_1\cdot x(t)+h_2\cdot(x(t))^2+h_3\cdot(x(t))^3+\ldots,\tag{5}$$

where h_0, h_1, h_2, h_3, and son on, mean some constants being real numbers.

Substituting $x(t)=x_B(t)$ given by (1) into (4) leads to

$$\begin{aligned}&y(t)=h^{(0)}+\\&+\int_{-\infty}^{\infty}h^{(1)}(\tau)\mathrm{Re}\left\{x_S(t-\tau)\exp\left(j\left(\omega_c(t-\tau)+\alpha_B(t-\tau)\right)\right)\right\}d\tau+\\&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h^{(2)}(\tau_1,\tau_2)\cdot\mathrm{Re}\left\{x_S(t-\tau_1)\exp\left(j\left(\omega_c(t-\tau_1)+\alpha_B(t-\tau_1)\right)\right)\right\}\cdot\\&\cdot\mathrm{Re}\left\{x_S(t-\tau_2)\exp\left(j\left(\omega_c(t-\tau_2)+\alpha_B(t-\tau_2)\right)\right)\right\}d\tau_1d\tau_2+\\&+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}h^{(3)}(\tau_1,\tau_2,\tau_3)\mathrm{Re}\left\{x_S(t-\tau_1)\cdot\right.\\&\left.\cdot\exp\left(j\left(\omega_c(t-\tau_1)+\alpha_B(t-\tau_1)\right)\right)\right\}\cdot\\&\cdot\mathrm{Re}\left\{x_S(t-\tau_2)\exp\left(j\left(\omega_c(t-\tau_2)+\alpha_B(t-\tau_2)\right)\right)\right\}\cdot\\&\cdot\mathrm{Re}\left\{x_S(t-\tau_3)\exp\left(j\left(\omega_c(t-\tau_3)+\alpha_B(t-\tau_3)\right)\right)\right\}\cdot\\&\cdot d\tau_1d\tau_2d\tau_3+\ldots\quad.\end{aligned}\tag{6}$$

Observe now that the following inequalities: $B_{B1}<<f_c$ and $B_{B2}<<f_c$ hold, where B_{B1} and B_{B2} stand for bandwidths of the baseband signals $x_S(t)$ and $\alpha_B(t)$, respectively. Moreover, see that we can choose the range of variables τ_i, $i=1,2,3,....$, in (6) much less than $1/B_{B1}$ and $1/B_{B2}$, that is $\tau_i<<1/B_{B1}$ and $\tau_i<<1/B_{B2}$, $i=1,2,..$, because the input-output characterization of any PA is very close to a polynomial description. But, as we already learned from the explanation regarding (5), in such a case, the nonlinear impulse responses in (6) are approximately the Dirac impulses. Taking into account the above two facts in (6), we can rewrite it as

$$\begin{aligned}y(t)&\cong h^{(0)}+\sum_{n=1}^{N}\underbrace{\int_{-\infty}^{\infty}..\int_{-\infty}^{\infty}}_{n\text{ times}}h^{(n)}(\tau_1,..,\tau_n)\cdot\\&\cdot\prod_{i=1}^{n}\mathrm{Re}\left\{x_S(t)\exp(j\alpha_Bt)\cdot\exp\left(j\omega_c(t-\tau_i)\right)\right\}d\tau_i\quad,\end{aligned}\tag{7}$$

where we have also restricted ourselves to retaining only the first N+1 components in the Volterra series description of a PA.

Note now that using (1) we can rewrite the products occurring in the iterated integrals of (7) in the following way

$$\begin{aligned}&\prod_{i=1}^{n}\mathrm{Re}\left\{x_S(t)\exp\left(j\alpha_B(t)\right)\cdot\exp\left(j\omega_c\left(t-\tau_i\right)\right)\right\}=\\&=\left(\frac{x_S(t)}{2}\right)^n\prod_{i=1}^{n}\left[\exp\left(j\left(\omega_c\left(t-\tau_i\right)+\alpha_B(t)\right)\right)+\right.\\&\left.+\exp\left(-j\left(\omega_c\left(t-\tau_i\right)+\alpha_B(t)\right)\right)\right]=\left(\frac{x_S(t)}{2}\right)^n\cdot\\&\cdot\sum_{m=0}^{C(n,m)}\exp\left(j\left(\left(n-2m\right)\left(\omega_c t+\alpha_B(t)\right)+\omega_c\sum_{C(n,m)}\tau_i\right)\right)\end{aligned}, \quad (8)$$

where the symbol $C(n,m)$ means combination of all the n non-conjugated terms in the second row of (8) with the n-m conjugated ones occurring in the third row of (8), of which products give the same resulting (product) frequency. Moreover, $\sum_{m=0}^{C(n,m)}(\cdot)$ denotes the operation of summation over all the distinct product terms in such a way that a given product is taken $C(n,m)=n!/\left(m!(n-m)!\right)$ times. Further, the symbol $\sum_{C(n,m)}\tau_i$ in (11) stands for a sum of the auxiliary time variables τ_i related with one of the distinct product frequencies mentioned above. Note that there are in each case $C(n,m)=n!/\left(m!(n-m)!\right)$ such combinations.

By substituting (8) into (7), we arrive at

$$\begin{aligned}&y(t)\cong h^{(0)}+\sum_{n=1}^{N}\left(\frac{x_S(t)}{2}\right)^n\underbrace{\int_{-\infty}^{\infty}..\int_{-\infty}^{\infty}}_{n\text{ times}}h^{(n)}(\tau_1,..,\tau_n)\cdot\\&\cdot\sum_{m=0}^{C(n,m)}\exp\left(j\left(\left(n-2m\right)\left(\omega_c t+\alpha_B(t)\right)+\omega_c\sum_{C(n,m)}\tau_i\right)\right)\prod_{i=1}^{n}d\tau_i\ .\end{aligned}\quad (9)$$

In what follows, we include a passband filter (with the center frequency f_c) at the PA output as a part of its model. By virtue of this filter, all the products in (9) related with the frequencies different from $\pm f_c$ will be filtered out. This means that we must substitute $n-2m=\pm1$ in (9) and omit all the other components. (Obviously, the relation $n-2m=\pm1$ will be fulfilled only for odd values of n.) Doing so, we get

$$\begin{aligned}&y_B(t)=\sum_{n=1,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n\underbrace{\int_{-\infty}^{\infty}...\int_{-\infty}^{\infty}}_{n\text{ times}}h^{(n)}(\tau_1,..,\tau_n)\cdot\\&\cdot\left[C\left(n,(n-1)/2\right)\cdot\exp\left(j\left(\left(\omega_c t+\alpha_B(t)\right)+\omega_c\sum_{C(n,(n-1)/2)}\tau_i\right)\right)+\right.\\&\left.+C\left(n,(n+1)/2\right)\cdot\exp\left(-j\left(\left(\omega_c t+\alpha_B(t)\right)+\omega_c\sum_{C(n,(n+1)/2)}\tau_i\right)\right)\right]\cdot\\&\cdot\prod_{i=1}^{n}d\tau_i\ ,\end{aligned}\quad (10)$$

where we denoted now by $y_B(t)$ the PA output signal after passing through the bandpass output filter.

The Volterra series based nonlinear transfer functions of a nonlinear system are defined as the multidimensional Fourier transforms of its corresponding nonlinear responses [3]. That is as

$$\begin{aligned}&H^{(n)}(f_1,..,f_n)=\underbrace{\int_{-\infty}^{\infty}..\int_{-\infty}^{\infty}}_{n\text{ times}}h^{(n)}(\tau_1,..,\tau_n)\cdot\\&\cdot\exp\left(-j2\pi f_1\tau_1\right)\cdot\cdot\exp\left(-j2\pi f_n\tau_n\right)d\tau_1\cdot\cdot d\tau_n\end{aligned}.\quad (11)$$

Taking into account (11) in (10), we obtain

$$\begin{aligned}&y_B(t)=\sum_{n=1,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n\cdot\left[C\left(n,(n-1)/2\right)\cdot\right.\\&\cdot H^{(n)}\left(\chi_n^+(\pm f_c)\right)\cdot\exp\left(j\left(\omega_c t+\alpha_B(t)\right)\right)+\\&+C\left(n,(n+1)/2\right)\cdot H^{(n)}\left(\chi_n^-(\pm f_c)\right)\cdot\\&\left.\cdot\exp\left(-j\left(\omega_c t+\alpha_B(t)\right)\right)\right]\ ,\end{aligned}\quad (12)$$

where $\chi_n^+(\pm f_c)$ and $\chi_n^-(\pm f_c)$ denote such the frequency sets $\{f_1,..,f_n\}$ whose elements $f_i,\ i=1,2,...,n,$ can assume only the values $+f_c$ or $-f_c$, and whose sums give the value $+f_c$ or $-f_c$, respectively.

To proceed further, observe that the coefficients $C\left(n,(n-1)/2\right)$ and $C\left(n,(n+1)/2\right)$ occurring in (12) assume the same values. Moreover, observe that the following

$$\begin{aligned}&H^{(n)}\left(\chi_n^-(\pm f_c)\right)\cdot\exp\left(-j\left(\omega_c t+\alpha_B(t)\right)\right)=\\&=\left(H^{(n)}\left(\chi_n^+(\pm f_c)\right)\cdot\exp\left(j\left(\omega_c t+\alpha_B(t)\right)\right)\right)^*\end{aligned}\quad (13)$$

holds.

Therefore, we can write

$$G(x_S(t),H^{(n)},f_c,n,N)=\sum_{n=1,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n C(n,(n-1)/2)\cdot$$
$$\cdot H^{(n)}\left(\chi_n^+(\pm f_c)\right)=\left[\sum_{n=1,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n C(n,(n+1)/2)\cdot\right.\tag{14}$$
$$\left.\cdot H^{(n)}\left(\chi_n^-(\pm f_c)\right)\right]^* = G^*(x_S(t),H^{(n)},f_c,n,N)\ .$$

A new complex-valued function $G(x_S(t),H^{(n)},f_c,n,N)$ has been defined in (14). Let us denote the magnitude and phase of this function as $\left|G\left(x_S(t),H^{(n)},f_c,n,N\right)\right|$ and $\varphi_G\left(x_S(t),H^{(n)},f_c,n,N\right)$, respectively. With the use of this notation, we can rewrite (12) in the following way:

$$y_B(t)=2\cdot\left|G(x_S(t),H^{(n)},f_c,n,N)\right|\cdot$$
$$\cdot\cos\left(\omega_c t+\alpha_B(t)+\varphi_G\left(x_S(t),H^{(n)},f_c,n,N\right)\right)\ .\tag{15}$$

Comparison of (2) with (15) shows that their form is the same. So, this validates the means of modelling of AM/AM and AM/PM conversions formulated in (2). Moreover, it follows from this comparison that the expressions describing the AM/AM and AM/PM characteristics, that is $A\left(x_S(t)\right)$ and $\Phi\left(x_S(t)\right)$, are given by

$$A\left(x_S(t)\right)=2\cdot\left|G(x_S(t),H^{(n)},f_c,n,N)\right|\tag{16a}$$

and

$$\Phi\left(x_S(t)\right)=\varphi_G\left(x_S(t),H^{(n)},f_c,n,N\right),\tag{16b}$$

respectively.

III. Discussion

We begin this section with formulation of the following theorem.

Theorem 1. The necessary and sufficient condition for occurrence of the nonzero values of AM/PM conversion in a PA is the existence of a memory in its nonlinear characteristic.

Proof: We construct the proof of this theorem by showing that the opposite would lead to a contradiction. To this end, consider the function $G(x_S(t),H^{(n)},f_c,n,N)$ given by (14). Next, assume that all the arguments $H^{(n)}$, $n=2,3,...,$ in this function are some real numbers. That is $H^{(1)}=h_1$, $H^{(3)}=h_3$, and so on. Note that this, according to the discussion underlying (5), means a description of a PA by a memoryless polynomial. With such values of $H^{(n)}$, $G(\cdot)$ is a real-valued function. That is its phase $\varphi_G(\cdot)$ is identically equal to zero. So, in other words, AM/PM conversion cannot be nonzero in this case.

When a PA is modelled by a Volterra series (being a description with memory [3]), the complex-valued arguments $H^{(n)}$, $n=2,3,...,$ in (14) make $G(x_S(t),H^{(n)},f_c,n,N)$ a complex-valued function having nonzero phase $\varphi_G(\cdot)$. So, the existence of a memory in characterization of a PA is also a sufficient condition. And this ends the proof.

Theorem 2. The Saleh's model and related ones, which foresee nonzero level of AM/PM conversion, are characterizations incorporating memory effects. That is they are not models without memory.

Proof: Proof of this theorem follows directly from the relations (3), (16), and Theorem 1.

Note that in view of Theorem 2 the taxonomy of PAs presented in [2] must be revised.

Theorem 3. Modelling of transferring characteristics of a PA by a purely polynomial model does not lead to a nonzero AM/PM conversion. In this case, the necessary condition of occurrence of an AM/AM conversion is the existence of at least one nonzero nonlinear transfer function $H^{(n)}=h_n$, $n=3,5,...,$ of a PA.

Proof: Note that proof of the first part of this theorem follows directly from Theorem 1. To show the validity of its second part, let us rewrite (14) in the following form

$$G(x_S(t),H^{(n)},f_c,n,N)=h_1\cdot\frac{x_S(t)}{2}$$
$$+\sum_{n=3,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n C(n,(n-1)/2)\cdot h_n\tag{17}$$

for the "without memory" case considered in this theorem.

Substituting (17) into (16a) gives

$$A\left(x_S(t)\right)=h_1\cdot x_S(t)+$$
$$+2\cdot\sum_{n=3,\text{ odd}}^{N}\left(\frac{x_S(t)}{2}\right)^n C(n,(n-1)/2)\cdot h_n\ .\tag{18}$$

Clearly, $h_1\cdot x_S(t)$ in (18) exhibits the linear part of a PA characteristic. But, the nonlinear part of it, responsible for the AM/AM conversion, is represented by the expression in the second row in (18). This expression has a nonzero value if at least one of the coefficients h_n is nonzero. And this ends the proof.

IV. Novelty of Results Presented and Comparison with Hitherto Approaches

In this section, we discuss importance of the results achieved and compare them with the related ones, hitherto published in the literature. Regarding these issues here, we will be more precise than in Introduction because we know now all the details of the derivations presented in the previous two sections.

First, our approach of applying the Volterra series to analyze distortion-oriented and measurement-based models of AM/AM and AM/PM conversions in PAs, like the Saleh's model, is used in this paper for the first time in the literature. Obviously, there are publications, as for example [7-9], in which the Volterra series is used to model an entire bandpass RF (radio frequency) communication system containing PAs.

Surprisingly, however, in the case of PAs being system's internal elements, another kind of modeling is applied in the articles mentioned above. That is not a natural approach, which involves application of the Volterra series for description of a mildly nonlinear circuit, in this case, a PA. But, note that the descriptions of devices and systems by the Volterra series are very popular among people designing RF amplifiers. For example, see [10].

In [7-9], the so-called quadrature model of a nonlinear device exhibiting the AM/AM and AM/PM conversions (in short, the quadrature model) for modelling PAs has been used. It consists of three functional blocks: two of them are described by memoryless nonlinearities and the third one is a 90 degree shifter. The memoryless nonlinear blocks are connected with each other in parallel, except that one of them is preceded by the block of a 90 degree shifter. The first branch of the connection represents an in-phase component, but the second one is a quadrature component of the model considered. The memoryless nonlinearities mentioned above are real-valued and can be expanded in power series (with real-valued coefficients). For more details regarding this model, see, for example, [7].

Note that the quadrature model of a PA described briefly above is not a physically-oriented one. That is it does not follow from any physical relations which describe the PA. It is an a priori model of which parameters are adjusted to the measured data. So, it can be classified as a behavioural or a black box model of the PA. However, the well-known disadvantage of such models is that they do not relate their parameters with any physical quantities describing the devices modelled.

Opposite to this, the Volterra series model of a mildly nonlinear device relates its nonlinear transfer functions (or equivalently its nonlinear impulse responses) with the physical quantities associated with this device, like resistances, capacitances, inductances etc. And it is clear that this property is very valuable for the PA designer because it enables analytical analysis of the dependence of the AM/AM and AM/PM characteristics of PA upon the aforementioned physical quantities.

At this point, we remark also that using the mathematical tools and approach, based on the functional analysis, which was exploited by Sandberg in [11], it is possible to prove that the quadrature model of PA is a model containing memory. This was done by one of the authors of this paper in [12].

Second, in all of the derivations presented in this paper, we do not refer to as the notion of an equivalent low-pass circuit (system) of a bandpass one. So, in this respect, this is a quite different approach from that which was used in [7]. Only common thing for both of them is the usage of the Volterra series.

By the way, note at this point that a method using the notion of an equivalent low-pass circuit of a bandpass circuit and the Volterra series to model a weakly nonlinear satellite communication system have been presented previously by the authors of this paper in [13]. Furthermore, note that the method used in [13] differs also from that exploited in [7] because it does not use the quadrature model for PA. In it, PA is described by the Volterra series.

Third, using the Volterra series for description of the entire mildly nonlinear communication system as well as for the PA alone makes the approach presented in this paper more transparent and consistent than all the other ones [7-9].

Fourth, the approach of this paper that resigns from the use of an equivalent low-pass circuit of a bandpass one is clear and transparent. So, thereby, it is more suitable for teaching than those proposed in [7-9].

Fifth, to our best knowledge, the formulae (16a) and (16b) expressing the AM/AM and AM/PM characteristics of a PA through its Volterra series based nonlinear transfer functions have been derived for the first time in this paper (and also in a conference paper [13] by applying a little bit different method).

Sixth, other related results achieved in this paper, which are summarized in an a concise form through three short theorems given in section III, are new, too. Their novelty lies in the fact that they give rise to revision of the taxonomy and views presented in [2] and other papers regarding the Saleh's model [4]. Simply, the latter model cannot be classified as a model without memory.

V. Final Remark

The measurements carried out on PAs show that the AM/PM conversion occurring in them cannot be neglected. In this paper, we have shown how this conversion is related with the Volterra series based nonlinear transfer functions of a PA calculated at the carrier frequency.

Admittedly, the Volterra series based models of PAs are not available in the literature. On one hand, they can be easily obtained from the existing ones, published up to now in the literature like the quadrature model [7] and its predecessors [14], [15], Saleh's model [4], and the others mentioned in [2]. They will be however, achieved on this way, only models with the nonlinear transfer functions being fixed complex numbers (adjusted to the measured data).

But, on the other hand, for getting the nonlinear transfer functions in analytical forms as functions of physical quantities, new investigations will be needed. Obviously, they will be connected with the physical modelling of PAs.

References

[1] G. Maral and M. Bousquet, *Satellite Communications Systems: Systems, Techniques and Technology*. Chichester, United Kingdom: John Wiley & Sons, 2009.

[2] J. Joung, C. K. Ho, K. Adachi, and S. A. Sun, "Survey on power-amplifier-centric techniques for spectrum- and energy-efficient wireless communications," *IEEE Communications Surveys & Tutorials*, vol. 17 pp. 315-333, 2015.

[3] M. Schetzen, *The Volterra and Wiener Theories of Nonlinear Systems*, New York: John Wiley & Sons, 1980.

[4] A. A. M. Saleh, "Frequency-independent and frequency-dependent nonlinear models of TWT amplifiers," *IEEE Trans. on Communications*, vol. 29, pp. 1715-1720, 1981.

[5] M. Jeruchim, P. Balaban, and K. Sam Shanmugan, *Simulation of Communication Systems: Modeling, Methodology, and Techniques*. New York: Kluwer, 2002.

[6] A. Borys, "Consideration of Volterra series with excitation and/or impulse responses in form of Dirac impulses," *IEEE Trans. on Circuits and Systems – II: Express Briefs*, vol. 57, pp. 466-470, 2010.

[7] S. Benedetto, E. Biglieri, and R. Daffara, "Modeling and performance evaluation of nonlinear satellite links - a Volterra series approach," *IEEE Trans. on Aerospace and Electronic Systems*, vol. 15, pp. 494-507, 1979.

[8] S. Benedetto and E. Biglieri, "Nonlinear equalization of digital satellite channels," *IEEE J. Sel. Areas Commun.*, vol. 1, pp. 57–62, 1983.

[9] G. Colavolpe and A. Piemontese, “Novel SISO Detection Algorithms for Nonlinear Satellite Channels,” *IEEE Wireless Communications Letters*, vol. 1, pp. 22-25, 2012.
[10] S. Ganesan, E. Sánchez-Sinencio, and J. Silva-Martinez, “A highly linear low-noise amplifier,” *IEEE Transactions on Microwave Theory and Techniques*, vol. 54, pp. 4079-4085, 2006.
[11] I. W. Sandberg, “Bounded inputs and the representation of linear system maps,” *Circuits, Systems, and Signal Processing*, vol. 24, pp. 103-115, 2005.
[12] A. Borys, “Saleh’s model of AM/AM and AM/PM conversions is not a model without memory effect,” submitted for publication.
[13] Borys and W. Sieńko, “On nonlinear distortions in satellite communication links and their equalization,” in *Proceedings of the 1st International Conference on Innovative Research and Maritime Applications of Space Technology IRMAST*, Gdańsk, 2015, pp. 161-166.
[14] A. Kaye, D. George, and M. Eric, “Analysis and compensation of bandpass nonlinearities for communications,” *IEEE Trans. on Commun. Technol.*, vol. 20, pp. 365-372, 1972.
[15] G. Heiter, “Characterization of nonlinearities in microwave devices and systems,” IEEE Trans. Microwave Theory Tech., vol. 21, pp. 797-805, 1973.