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Understanding of Meyer and Stephens's operator *o* as a multi-operational one

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***Abstract*—In this paper, the idea of an extended operator *o* introduced in the literature on modelling weakly nonlinear circuits by Meyer and Stephens is revisited. The mathematically precise definitions of this operator for the successive Volterra series terms are given. Furthermore, the exhaustive formal and illustrative descriptions of these definitions are also presented. Finally, the possibility of a reverse formulation for the convolution operations occurring in descriptions of weakly nonlinear circuits is reported.**

***Keywords*—Operators and operations, operator *o*, Volterra series, weakly nonlinear circuits, modelling harmonic and intermodulation distortion.**

I. Introduction

NARAYANAN introduced in [1] an operator which he called *o* operator. He used it in description of a linear part of a mildly nonlinear bipolar transistor circuit. That is this operator was applied by him to describe a circuit part whose evaluation fully relies upon using only the first term of a Volterra series [2]. The strictly nonlinear part of his circuit was modelled directly by the second, third, and higher order Volterra series terms. Altogether, this mixed description was used by Narayanan in his method of harmonic and intermodulation distortion analysis presented in [1].

Narayanan's operator *o* was defined for transferring time signals from one set to another. That is it was defined in the time domain.

Referring to as the description presented in [1], Meyer and Stephens extended the linear operator *o* to such a mapping which enables an input-output description of a weakly nonlinear circuit in a mixed way [3]. In other words, they used their extended operator *o* in [3] to connect the input voltages or currents of a given circuit taken in the time domain with the functions describing this circuit in the multi-dimensional frequency domains. Thereby, they believed to find a new form of notation for the Volterra series which uses their operator *o*.

As shown in [4] and [5], the above approach regarding extension of the Narayanan's operator *o* is faulty. The objective of this paper is to find means for improvement of the concept of this operator. Furthermore, we refine here the Meyer and Stephens's definition of the operator *o* presented in [3] to make it mathematically precise. Apart from this, we present also the detailed descriptions of the refined Meyer and Stephens's definitions of the operator *o* for the successive terms in the Volterra series, leading in fact to a family of operators. These descriptions explain formally how to understand the defining expressions derived for these operators. It is shown that they must be understood and interpreted in terms of the compositions of operations. That is we can say that the operators considered are the multi-operational ones.

For better understanding of the operators discussed, their illustrative descriptions are also given in this paper. It is shown that they can be considered in terms of a development of an imprecise idea of the operator *o* presented in [3].

Finally, we show that it is possible to formulate a reverse notation for the convolution operations which are used in description of weakly nonlinear circuits. Note that this notation can be useful in some applications.

The remainder of the paper is organized as follows. In the next section, we present derivations leading to the mathematically precise definitions of the operator *o* that can be used to write down the successive terms of the Volterra series. For final defining expressions, the exhaustive formal as well as illustrative descriptions are given. In section III, we present another means of writing down the convolution operations occurring in descriptions of mildly nonlinear circuits. We call here this means of notation a reverse one. The paper ends with a conclusion.

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II. Definition of Extended Operator *O* by Meyer and Stephens Revisited

Meyer and Stephens claimed in [3] that a weakly nonlinear circuit can be described by a Volterra in the following form

$$
\begin{aligned} y(t) = {} & A_1(f)\,o\big(x(t)\big) + A_2(f_1,f_2)\,o\big(x(t)\big)^2 + \\ & + A_3(f_1,f_2,f_3)\,o\big(x(t)\big)^3 + \ldots \end{aligned} \tag{1}
$$

with the use of their extended operator *o*. In (1), the functions $y(t)$ and $x(t)$ of time variable t stand for the circuit output and input signals, respectively. Moreover, the functions $A_1(f)$, $A_2(f_1,f_2)$, and $A_3(f_1,f_2,f_3)$ of one frequency variable f or of two f_1, f_2 or of three f_1, f_2, f_3, respectively, represent the corresponding nonlinear transfer functions of the first, second, and third order of the circuit considered. They are the one-, two-, and three-dimensional Fourier transforms of the corresponding nonlinear circuit impulse responses of the first-, second-, and third-order [6]. The operator *o* occurring in (1) was imprecisely defined by Meyer and Stephens in the following way: "the operator sign indicates that the magnitude and phase of each term in x^n is to be changed by the magnitude and phase of $A_n(f_1, f_2, ..., f_n)$".

Observe that the representation given by (1) can be viewed as a mixed time-frequency version of the standard Volterra series which is formulated as [2], [6]

$$\begin{aligned} y(t) &= \int_{-\infty}^{\infty} a_1(\tau)x(t-\tau)d\tau + \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_2(\tau_1,\tau_2)x(t-\tau_1)x(t-\tau_2)d\tau_1 d\tau_2 + \\ &+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 + \dots , \end{aligned} \tag{2}$$

where the functions $a_1(\tau)$, $a_2(\tau_1,\tau_2)$, and $a_3(\tau_1,\tau_2,\tau_3)$ of the corresponding auxiliary time variables are the nonlinear circuit impulse responses of the first-, second-, and third-order [2], [6], respectively. Further, note from (2) that the Volterra series is simply a sum of the multidimensional convolutions.

So, comparison of (1) with (2) shows that the following relations

$$A_1(f)o(x(t)) = \int_{-\infty}^{\infty} a_1(\tau)x(t-\tau)d\tau \quad , \tag{3a}$$

$$\begin{aligned} A_2(f_1,f_2)o(x(t))^2 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_2(\tau_1,\tau_2)x(t-\tau_1)\cdot \\ &\cdot x(t-\tau_2)d\tau_1 d\tau_2 \quad , \end{aligned} \tag{3b}$$

$$\begin{aligned} A_3(f_1,f_2,f_3)o(x(t))^3 &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_3(\tau_1,\tau_2,\tau_3)\cdot \\ &\cdot x(t-\tau_1)x(t-\tau_2)x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 \quad , \end{aligned} \tag{3c}$$

and so on, should hold.

Looking now at the expressions in (3a), (3b), and (3c), we see that the operator *o* occurring therein is not the same. Obviously, it indicates operation of carrying out a convolution. But, this convolution is one-dimensional in the first case, two-dimensional in the second case, and three-dimensional in the third one. Therefore, these cases will be treated separately in what follows, retaining however the original notation *o*, as assumed in [3]. The dimensionality of the convolution operator *o* will follow from the context.

To achieve the homogeneity regarding the variables on both sides of (3a), (3b), and (3c), that is to have both the time and frequency variables on the right-hand sides of these relations (as is the case on their left-hand sides), we introduce therein, instead of the nonlinear impulse responses $a_1(\tau)$, $a_2(\tau_1,\tau_2)$, and $a_3(\tau_1,\tau_2,\tau_3)$, their inverse multidimensional Fourier transforms [2], [6] given by

$$a_1(\tau) = \int_{-\infty}^{\infty} A_1(f)\exp(j2\pi\tau t)df \quad , \tag{4a}$$

$$\begin{aligned} a_2(\tau_1,\tau_2) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_2(f_1,f_2)\exp(j2\pi f_1\tau_1)\cdot \\ &\cdot\exp(j2\pi f_2\tau_2)df_1 df_2 \quad , \end{aligned} \tag{4b}$$

$$\begin{aligned} a_3(\tau_1,\tau_2,\tau_3) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_3(f_1,f_2,f_3)\exp(j2\pi f_1\tau_1)\cdot \\ &\cdot\exp(j2\pi f_2\tau_2)\exp(j2\pi f_3\tau_3)df_1 df_2 df_3 \quad . \end{aligned} \tag{4c}$$

Let us now consider in more detail the above substitution in the case of (3a). So, substituting (4a) into (3a) gives

$$\begin{aligned} A_1(f)o(x(t)) = (a_1 ox)(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_1(f)\cdot \\ &\cdot df\exp(j2\pi f\tau)x(t-\tau)d\tau \quad . \end{aligned} \tag{5}$$

Note the use of an alternative notation $(a_1 ox)(t)$ for $A_1(f)o(x(t))$ in (5). Such notation manner is oft used to emphasize the fact, that, regarding our case, the result of operation indicated by the operator *o* is a function of a time variable *t*.

Observe now that rearranging the terms in (5) and introducing a new variable $\tau' = t - \tau$, we get

$$\begin{aligned} A_1(f)o(x(t)) &= -\int_{-\infty}^{\infty} A_1(f)\exp(j2\pi ft)\cdot \\ &\cdot\int_{\infty}^{-\infty} x(\tau')\exp(-j2\pi f\tau')d\tau' df \quad . \end{aligned} \tag{6}$$

In the next step, recognize a Fourier transform of $x(t)$ in (6) and denote it by $X(f)$. Using this, we can rewrite (6) in the following form

$$A_1(f)o(x(t)) = \int_{-\infty}^{\infty} A_1(f)X(f)\exp(j2\pi ft)df \quad . \tag{7}$$

By substituting (4b) into (3b), rearranging then the terms, and introducing new variables $\tau_1' = t - \tau_1$ and $\tau_2' = t - \tau_2$, we obtain

$$\begin{aligned} A_2(f_1,f_2)o(x(t))^2 &= (a_2 ox^2)(t) = \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_2(f_1,f_2)\exp(j2\pi f_1 t)\exp(j2\pi f_2 t)\cdot \\ &\cdot\left(\int_{\infty}^{-\infty} x(\tau_1')\exp(-j2\pi f_1\tau_1')d\tau_1'\right) \\ &\cdot\left(\int_{\infty}^{-\infty} x(\tau_2')\exp(-j2\pi f_2\tau_2')d\tau_2'\right)df_1 df_2 \quad . \end{aligned} \tag{8}$$

Note now that recognizing the Fourier transforms of the signal $x(t)$ in (8) allows to rewrite the latter as

$$A_2(f_1,f_2)o(x(t))^2 = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_2(f_1,f_2)X(f_1)\cdot \cdot X(f_2)\exp(j2\pi f_1 t)\exp(j2\pi f_2 t)df_1 df_2 \quad . \tag{9}$$

Finally, substituting (4c) into (3c), rearranging afterwards the terms, and introducing new variables $\tau_1' = t-\tau_1$ and $\tau_2' = t-\tau_2$, $\tau_3' = t-\tau_3$, we arrive at

$$\begin{aligned} A_3(f_1,f_2,f_3)o(x(t))^2 &= (a_3 o x^3)(t) = \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_3(f_1,f_2,f_3)\exp(j2\pi f_1 t)\cdot \\ &\cdot\exp(j2\pi f_2 t)\exp(j2\pi f_3 t)\cdot \\ &\cdot\left(\int_{\infty}^{-\infty} x(\tau_1')\exp(-j2\pi f_1\tau_1')d\tau_1'\right)\cdot \\ &\cdot\left(\int_{\infty}^{-\infty} x(\tau_2')\exp(-j2\pi f_2\tau_2')d\tau_2'\right)\cdot \\ &\cdot\left(\int_{\infty}^{-\infty} x(\tau_3')\exp(-j2\pi f_3\tau_3')d\tau_3'\right)df_1 df_2 df_3 \quad . \end{aligned} \tag{10}$$

And similarly as before, recognizing the Fourier transforms of the signal $x(t)$ in (10), we get

$$\begin{aligned} &A_3(f_1,f_2,f_3)o(x(t))^3 = \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} A_3(f_1,f_2,f_3)X(f_1)\cdot \\ &\cdot X(f_2)X(f_3)\exp(j2\pi f_1 t)\exp(j2\pi f_2 t)\cdot \\ &\cdot\exp(j2\pi f_3 t)df_1 df_2 df_3 \end{aligned} \tag{11}$$

respectively.

Looking now at the right-hand sides of (7), (9), and (11), we see that they are nothing else than the inverse multidimensional Fourier transforms (for their definitions see, for example, [6]) of the following functions: $A_1(f)X(f)$ of one frequency variable, $A_2(f_1,f_2)X(f_1)X(f_2)$ of two frequency variables, and $A_3(f_1,f_2,f_3)X(f_1)X(f_2)X(f_3)$ of three frequency variables. Using this allows us to rewrite (7), (9), and (11) in a more compact form as

$$A_1(f)o(x(t)) = F^{-1}\{A_1(f)X(f)\}, \tag{12}$$

$$A_2(f_1,f_2)o(x(t))^2 = F_2^{-1}\{A_2(f_1,f_2)X(f_1)X(f_2)\}, \tag{13}$$

$$A_3(f_1,f_2,f_3)o(x(t))^3 = F_3^{-1}\{A_3(f_1,f_2,f_3)X(f_1)\cdot \cdot X(f_2)X(f_3)\}, \tag{14}$$

where $F_i^{-1}\{\cdot\}$, $i=1,2,3,\ldots$, means the inverse i-dimensional Fourier transform, with a shorter notation $F^{-1}\{\cdot\}=F_1^{-1}\{\cdot\}$ applied in (12) for the standard one-dimensional transform.

Finally, to achieve the homogeneity of the variables on both sides of (12), (13) and (14), which was postulated above, we introduce the Fourier transform of the circuit input signal $x(t)$, that is $X(f)=F\{x(t)\}$, in the aforementioned relations. This leads to

$$A_1(f)o(x(t)) = F^{-1}\{A_1(f)\cdot F\{x(t)\}\}, \tag{15}$$

$$A_2(f_1,f_2)o(x(t))^2 = F_2^{-1}\{A_2(f_1,f_2)\cdot \cdot F_{f_1}\{x(t)\}\cdot F_{f_2}\{x(t)\}\}, \tag{16}$$

$$A_3(f_1,f_2,f_3)o(x(t))^3 = F_3^{-1}\{A_3(f_1,f_2,f_3)\cdot \cdot F_{f_1}\{x(t)\}\cdot F_{f_2}\{x(t)\}\cdot F_{f_3}\{x(t)\}\}, \tag{17}$$

where $F_{1f_k}\{\cdot\}$ stands for the one-dimensional Fourier transform, in which the frequency variable is denoted as f_k, $k=1,2,3,\ldots$. A shorter notation $F_{1f}\{\cdot\}=F$ is used in (15).

Taking into account these final relations given by (15), (16), and (17) in (1), we can rewrite (1) in the following form

$$\begin{aligned} y(t) &= F_1^{-1}\{A_1(f)F_{1f}(x(t))\} + \\ &+ F_2^{-1}\{A_2(f_1,f_2)F_{1f_1}(x(t))F_{1f_2}(x(t))\} + \\ &+ F_3^{-1}\{A_3(f_1,f_2,f_3)F_{1f_1}(x(t))F_{1f_2}(x(t))\cdot \\ &\cdot F_{1f_3}(x(t))\} + \ldots \quad . \end{aligned} \tag{18}$$

Note further that the relations (15), (16), and (17) allow us to define precisely what the operator *o* does mean in the Volterra series (1) written in the notational convention assumed in [3]. In words, we can express this as follows.

a) **Description of the operator *o* in the first component of (1).** Relation (15) constitutes a definition of this operator. It says that we take first the standard Fourier transform of the circuit input signal and multiply it by its first order nonlinear transfer function (linear one). Then, we calculate the standard inverse Fourier transform of the result of this multiplication.

b) **Description of the operator o in the second component of (1).** Relation (16) constitutes a definition of this operator. It says that we take first two Fourier transforms of the circuit input signal calculated in point a) above and treat their arguments, being the frequency variables, as two independent ones. Then, we multiply these transforms getting a function of two variables. Afterwards, we multiply the product obtained by the circuit second order nonlinear transfer function. And finally, we calculate the inverse two-dimensional Fourier transform of the result of the above multiplication.
c) **Description of the operator o in the third component of (1).** Relation (17) constitutes a definition of this operator. It says that we take first three Fourier transforms of the circuit input signal calculated at point a) above and treat their arguments, being the frequency variables, as three independent ones. Then, we multiply them getting a function of three variables. Afterwards, we multiply the product obtained by the circuit third order nonlinear transfer function. Finally, we calculate the inverse three-dimensional Fourier transform of the result of the above multiplication.

Let us now make two remarks regarding the above descriptions. First, as already stated before, the o operators from points a), b), and c) do not mean the same. Second, they do not mean performing only one operation. They constitute compositions of more than one operations. That is they are multi-operational operators. For example, the operator o in case a) means a successive performing of three operations (calculation of the standard Fourier transform, multiplication, and calculation of the standard inverse Fourier transform).

Apart from the formal descriptions in a), b), and c), we can also describe more illustratively the operations performed by the o operators from the above points. That is in the following way.

1) **Illustrative description of the operator o in the first component of (1).** We calculate the Fourier transform of the circuit input signal to have a distribution of its magnitudes and phases in the frequency domain. Then, these magnitudes are modified through multiplication by the magnitudes of the circuit linear transfer function. But the signal transform phases are modified by adding to the phases of the circuit linear transfer function. And both the modifications are carried out in the whole range of frequencies. At the end, this "image" of the first component of the circuit output signal in the frequency domain is transformed into the time domain.
2) **Illustrative description of the operator o in the second component of (1).** We take two Fourier transforms of the circuit input signal calculated in point 1) above and treat their arguments, being the frequency variables, as two independent ones. Then, we form all the possible pairs of values for these frequency variables (their Cartesian products). Next, we multiply the circuit input signal Fourier transform calculated at the first frequency of the aforementioned pair by the circuit input signal Fourier transform evaluated at its second frequency. Obviously, we do this for all the possible frequency pairs (Cartesian products). The resulting function can be interpreted as an "image" in the frequency domain of all the possible interactions (products) of the circuit input signal on its quadratic nonlinearity. In other words, this complex-valued function have sense of a distribution of the shares of the frequency products in a two-dimensional frequency domain. Its magnitudes are modified through multiplication by the magnitudes of the circuit nonlinear transfer function of the second order, but the phases of this function are modified by adding to the phases of the circuit nonlinear transfer function mentioned, in the whole range of values of the product frequencies. The resulting function of the two frequency variables can be viewed as an "image" of the second component of the circuit output signal in the frequency-product domain. At the end, this "image" of the second component of the circuit output signal is transformed into the time domain.
3) **Illustrative description of the operator o in the third component of (1).** In this case, we take three Fourier transforms of the circuit input signal calculated at point 1) above and treat their arguments, being the frequency variables, as three independent ones. Then, we form all the possible triads of values for these frequency variables (their Cartesian products). Next, we multiply the circuit input signal Fourier transform calculated at the first frequency of the aforementioned triad by the circuit input signal Fourier transform evaluated at its second frequency, and also by the circuit input signal Fourier transform evaluated at its third frequency. Obviously, we do this for all the possible frequency triads (Cartesian products). Further, similarly as before, the resulting function can be interpreted as an "image" in the frequency domain of all the possible interactions (products) of the third degree (order) of the circuit input signal on its cubic nonlinearity. In other words, this complex-valued function have sense of a distribution of the shares of the frequency products of the third order in a three-dimensional frequency domain. Its magnitudes are modified through multiplication by the magnitudes of the circuit nonlinear transfer function of the third order, but the phases of this function are modified by adding to the phases of the circuit nonlinear transfer function mentioned, in the whole range of values of the product frequencies. The resulting function of three frequency variables can be viewed as an "image" of the third component of the circuit output signal in the frequency-product domain considered. At the end, this "image" of the third component of the circuit output

signal is transformed into the time domain.

Finally in this section, we remark that the illustrative descriptions presented above in points 1), 2), and 3) can be viewed as a development of the imprecise idea that "the operator sign indicates that the magnitude and phase of each term in x^n is to be changed by the magnitude and phase of $A_n(f_1, f_2, ..., f_n)$" from [3].

III. Reverse Notation for Convolution Operations in Description Of Weakly Nonlinear Circuits

Consider now another, equivalent means of writing down the convolution operations, we call here a reverse notation for these operations. To this end, let begin with the linear convolution as written on the right-hand side of (3a). Note that introducing a new variable $\tau' = t - \tau$ and after some manipulations, we get

$$\begin{aligned}(a_1 \otimes x)(t) &= \int_{-\infty}^{\infty} a_1(\tau)x(t-\tau)d\tau = \\ &= -\int_{\infty}^{-\infty} a_1(t-\tau')x(\tau')d\tau' = \\ &= \int_{-\infty}^{\infty} x(\tau')a_1(t-\tau')d\tau' = \\ &= \int_{-\infty}^{\infty} x(\tau)a_1(t-\tau)d\tau = (x \otimes a_1)(t)\end{aligned} \tag{19}$$

for the right-hand side of (3a). In (19), the symbol $\otimes$ is used to denote a linear convolution (one-dimensional convolution integral).

The result in (19) shows that the linear convolution $(a_1 \otimes x)(t)$ can be also written using the reverse notation. That is as $(x \otimes a_1)(t)$ (with the reverse order of occurrence of the functions $a_1(t)$ and $x(t)$).

Applying similar kind of variable substitutions and manipulations, as those used in derivation of (19), to the two-dimensional convolution on the right-hand side of (3b) and to the three-dimensional convolution on the right-hand side of (3c), we arrive at

$$\begin{aligned}(a_2 \otimes_{(2)} x)(t) &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_2(\tau_1,\tau_2)x(t-\tau_1)\cdot \\ &\cdot x(t-\tau_2)d\tau_1 d\tau_2 = ((a_2 \otimes x)\otimes)(t) = \\ &= (x \otimes (x \otimes a_2))(t)\end{aligned} \tag{20}$$

and

$$\begin{aligned}&(a_3 \otimes_{(3)} x)(t) = \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} a_3(\tau_1,\tau_2,\tau_3)x(t-\tau_1)x(t-\tau_2)\cdot \\ &\cdot x(t-\tau_3)d\tau_1 d\tau_2 d\tau_3 = (((a_3 \otimes x)\otimes x)\otimes)(t) = \\ &= (x \otimes (x \otimes (x \otimes a_3)))(t)\ ,\end{aligned} \tag{21}$$

respectively. In (20) and (21), the symbols $\otimes_{(2)}$ and $\otimes_{(3)}$ stand for the two-dimensional and the three-dimensional convolution integral, accordingly. Moreover, the lower index 2 by a_2 or 3 by a_3 means that this is a function of two or three time variables, respectively.

The results $(a_2 \otimes_{(2)} x)(t) = ((a_2 \otimes x)\otimes)(t)$ in (20) and $(a_3 \otimes_{(3)} x)(t) = (((a_3 \otimes x)\otimes x)\otimes)(t)$ in (21) show that the multi-dimensional convolutions can be viewed as the compositions of the one-dimensional ones. Furthermore, the relations $(a_2 \otimes_{(2)} x)(t) = (x \otimes (x \otimes a_2))(t)$ in (20) and $(a_3 \otimes_{(3)} x)(t) = (x \otimes (x \otimes (x \otimes a_3)))(t)$ in (21) show how the reverse notation should be understood in the case of the multi-dimensional convolutions. We see that then, apart from the change of the order of the functions involved, we realize it by a composition of the one-dimensional convolutions.

IV. Conclusion

We hope that many discussions and derivations presented in this paper, which regard the extended operator *o* introduced in [3], will make this concept more transparent and the mathematics related with it precise.

Note also that this paper being the third one in a series completes the results of the two previous ones [5] and [7].

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