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Application of Interval Arithmetic to Production Planning in a Foundry

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Abstract

A novel approach for treating the uncertainty about the real levels of finished products during production planning and scheduling process is presented in the paper. Interval arithmetic is used to describe uncertainty concerning the production that was planned to cover potential defective products, but meets customer's quality requirement and can be delivered as fully valuable products. Interval lot sizing and scheduling model to solve this problem is proposed, then a dedicated version of genetic algorithm that is able to deal with interval arithmetic is used to solve the test problems taken from a real-world example described in the literature. The achieved results are compared with a standard approach in which no uncertainty about real production of valuable castings is considered. It has been shown that interval arithmetic can be a valuable method for modeling uncertainty, and proposed approach can provide more accurate information to the planners allowing them to take more tailored decisions.

Keywords: Application of IT to the foundry industry, Production planning, Scheduling, Uncertainty, Interval arithmetic

1. Introduction

At the present times, manufacturers are struggling with increasingly demanding customers. Orders are accepted in small batches, lead times are short, and there is a risk of defective production for newly designed products, there is a risk of defective production (need of remanufacturing or the necessity of overproduction due to the likelihood of faulty products). In addition, customers can change the parameters of orders, or delivery conditions or and requirements for further processing. Because of this these factors, uncertainty becomes an integral part of the decision-making process at all levels of management activities, including production planning and scheduling.

A lot of different models have been defined in the literature to support production planning and scheduling in various industries. An extensive review of such models for foundry industry has been provided by the authors in [9]. However, it should be underlined

that the problem of addressing the uncertainty in those models still remains virtually unsolved. Recently the authors have proposed an approach in which fuzzy theory was used to describe uncertainty regarding the demand for castings [7]. However, in that case, the final results had to be defuzzified before they were presented to a planner. This study is focused on the uncertainty concerning defective products, i.e. those that do not meet quality requirements of a customer. Interval arithmetic is applied to describe the uncertain number of finished products that can be delivered to the end customer as valuable ones. Moreover, the results will be kept in the form of intervals and they will be presented to the planner in this form as well. Thanks to this the planner will obtain more complete information about possible number of finished products and the possible costs of production.

The paper is organized as follows: Section 2 presents the concept of interval arithmetic and its application to production planning and scheduling described in the literature. In section 3 a lot-sizing and scheduling model presented by the authors in [4] is

adapted to the requirements of interval arithmetic and a discussion about its advantages and limitations is provided. Section 4 presents the results and the interpretation of computational experiments involving the use of a specialized version of a genetic algorithm that was adjusted to solve interval optimization problem. Finally, the conclusions and directions for future research in this area are shown in Section 5.

2. Interval arithmetic and its application to production planning

Interval arithmetic was proposed by Moore [7] in 1966, initially to treat rounding errors of computer programs. However, it is worth to notice, that the first steps in this field have been made ten years earlier by Warmus, Polish mathematician and computer scientist. Since then it has been applied to many different fields, like construction engineering, chemical engineering, robotics, global optimization and pattern recognition.

Real interval A is defined as [4][5]:

$$a = [\underline{a}, \overline{a}]$$

where $\underline{a}$ is the lower bound of the interval a, $\overline{a}$ is its upper bound and a is the set $\{x \in \Re : \underline{a} \leq x \leq \overline{a}\}$.

Basic arithmetical operators are defined as follows:

$$a + b = [\underline{a} + \underline{b}, \overline{a} + \overline{b}]$$

$$a - b = [\underline{a} - \overline{b}, \overline{a} - \underline{b}]$$

$$a \cdot b = [\min(\underline{a} \cdot \underline{b}, \underline{a} \cdot \overline{b}, \overline{a} \cdot \underline{b}, \overline{a} \cdot \overline{b}), \max(\underline{a} \cdot \underline{b}, \underline{a} \cdot \overline{b}, \overline{a} \cdot \underline{b}, \overline{a} \cdot \overline{b})]$$

$$a / b = [\min(\underline{a} / \underline{b}, \underline{a} / \overline{b}, \overline{a} / \underline{b}, \overline{a} / \overline{b}), \max(\underline{a} / \underline{b}, \underline{a} / \overline{b}, \overline{a} / \underline{b}, \overline{a} / \overline{b})]$$

If two intervals need to be ranked, which is the case e.g. in solving optimization problems, it can be done using a formula:

$$a \leq b \Leftrightarrow \underline{a} \leq \underline{b} \wedge \overline{a} \leq \overline{b}$$

However, when two intervals overlap it is more practical to use a formula based on the middles of intervals:

$$a \leq b \Leftrightarrow m(a) \leq m(b)$$

where middle point $m(a)$ can be calculated as:

$$m(a) = \underline{a} + (\overline{a} - \underline{a}) / 2$$

After the review conducted by the authors it one has to admit that interval arithmetic has not been widely used for solving problems of production planning and scheduling, especially when compared e.g. with the fuzzy number theory. However, in last few years few papers on job scheduling problem with interval time parameters have been published. Lei [5] introduced an interval job shop scheduling problem in which uncertain processing times were described using intervals. He proposed a genetic algorithm to solve the problem. Lei and Guo [6] dealt with a dual-resource constrained (DRC) job shop problem with interval processing time and heterogeneous resources. They proposed an efficient variable neighborhood search algorithm to solve this problem. Han et al. [3] proposed an evolutionary algorithm with a new crossover to solve multi-objective blocking lot-streaming flow shop scheduling with interval processing times. Finally, Pereira [8] described a single machine scheduling with interval processing times and total weighted completion time objective. In order to solve the problem, he used a branch and bound method.

The authors could not found any paper regarding the lot sizing and scheduling problem, so the study presented in this paper may be regarded as a pioneering one in this field.

3. Interval lot-sizing and scheduling model with variable finished products

In [2] the authors described many different models developed to solve production and scheduling problem in foundries. Some researchers focused on the problem of production sequencing and for its definition they usually provided some variants of the classic job shop scheduling problem, but most recently the research is mainly focused on the furnace utilization, as it is usually a bottleneck in a production process. A variant of multi-item lot sizing and scheduling problem is usually used to solve such problems. In [5] the authors presented and discussed a variant provided by Araujo et al. for an automated foundry in Brazil [6].

In this study, we will extend that model to precisely describe the number of defective products. In the classic, crisp models it is assumed that the production of defective products is added to the production that fully meets the quality requirements of the customers. To ensure that the demand for a given customer will be satisfied the production is planned with some surplus that may vary from 3-5% for the castings for which the technology is well-known, up to 20% for the castings with a complex production technology or newly introduced products. In result at the end of the production period we can have some extra valuable products that can be supplied to the customer, and we do not have to plan their production in the subsequent periods. That is why the concept of a variable number of finished products is introduced in this point and intervals are used to describe its value.

The number of finished product that may be supplied to the customer is expressed as:

$[\underline{x}_{it}, \overline{x}_{it}]$ - number of valuable items i produced in sub-period,

where $\underline{x}_{it}$ is a minimal value (when all the extra castings are faulty) and $\overline{x}_{it}$ is a maximal value (when there are no faulty products at all).

The mixed integer programming model for the decision variables defined above looks as follows:

Minimize

$$\sum_{i=1}^{I} \sum_{t=1}^{T} (h_{it}^{-} \cdot [\underline{I}_{it}^{-}, \overline{I}_{it}^{-}] + h_{it}^{+} \cdot [\underline{I}_{it}^{+}, \overline{I}_{it}^{+}]) + \sum_{k=1}^{K} \sum_{n=1}^{N} (st_k \cdot z_n^k) \quad (1)$$

subject to

$$[\underline{I}_{i,t+1}^{+}, \overline{I}_{i,t+1}^{+}] - [\underline{I}_{i,t-1}^{-}, \overline{I}_{i,t-1}^{-}] + \sum_{n=1}^{N} \sum_{k=1}^{K} [\underline{x}_{it}, \overline{x}_{it}] \cdot a_i^k$$

$$- [\underline{I}_{i,t+1}^{+}, \overline{I}_{i,t+1}^{+}] + [\underline{I}_{i,t-1}^{-}, \overline{I}_{i,t-1}^{-}] \geq d_{it}, \qquad i = 1, ..., I, t = 1, ..., T \quad (2)$$

$$\sum_{i=1}^{I} w_i \cdot \overline{x}_{in} \cdot a_i^k + st_k \cdot z_n^k \leq C y_n^k, \quad k = 1, ..., K, n = 1, ..., N \quad (3)$$

$$z_n^k \geq y_n^k - y_{n-1}^k, \quad k = 1, ..., K, n = 1, ..., N \quad (4)$$

$$\sum_{k=1}^{K} y_n^k = 1, \quad n = 1, ..., N \quad (5)$$

$$\underline{I}_{it}^{-}, \bar{I}_{it}^{-}, \underline{I}_{it}^{+}, \bar{I}_{it}^{+}, \underline{x}_{it}, \bar{x}_{it} \geq 0, \quad i = 1, ..., I, t = 1, ..., T \qquad (6)$$

$$\underline{I}_{it}^{-}, \bar{I}_{it}^{-}, \underline{I}_{it}^{+}, \bar{I}_{it}^{+}, \underline{x}_{it}, \bar{x}_{it} \in \mathfrak{I}, \quad i = 1, ..., I, t = 1, ..., T \qquad (7)$$

$$\underline{I}_{i0}^{-}, \bar{I}_{i0}^{-}, \underline{I}_{i0}^{+}, \bar{I}_{i0}^{+} = 0, \; i = 1, ..., I \qquad (8)$$

where:

T – number of days in planning horizon,

N – number of subperiods (loads of the furnace),

d_{it} - demand for item i in day t,

w_i - weight of item I,

$a_i^k = 1$, if item i is produced from alloy k, otherwise 0,

st_k - setup cost for alloy k,

C - loading capacity of the furnace,

h_{it}^{-}, h_{it}^{+} - penalty for delaying (–) and storing (+) production of item i in day t,

$[\underline{I}_{it}^{-}, \bar{I}_{it}^{-}]$ - interval (uncertain) number of items i delayed at the end of day t,

$[\underline{I}_{it}^{+}, \bar{I}_{it}^{+}]$ - interval (uncertain) number of items i stored at the end of day t,

$z_n^k = 1$, if there is a setup (resulting from a change) of alloy k in sub-period n, otherwise 0,

$y_n^k = 1$, if alloy k is produced in n in sub-period, otherwise 0.

The goal (1) is now defined as the sum of the interval costs of delayed production, interval storage costs of finished goods and the setup cost, if the alloy is changed during furnace load.

Equation (2) compares valuable finished production in a given period, delayed production and production finished to early, both expressed in the form of intervals with the demand for casting i in day t. Constraint (3) ensures that maximum number of produced castings will not exceed furnace capacity - this time only upper bounds of intervals are taken into consideration, as alloy has to be provided equally for valuable and defective products. Similarly, to the standard model, constraint (4) sets variable z_n^k to 1, if there is a change in alloys in the subsequent subperiods, while constraint (5) ensures that only one alloy is produced in each subperiod. Finally, constrains (6)-(8) ensures that the lower and upper bounds in intervals describing production are nonnegative integer numbers, and the stored and delayed production levels are 0 at the beginning of the production planning process.

After optimization of the presented model the planner obtains the lower and the upper approximation of amount of valuable finished products along with the lower and the upper approximation of the cost function (1).

4. Computational experiments with genetic algorithm

In order to show a practical application of the proposed approach we will use the same test problem, as suggested by by Araujo et al. [2][1]. The number of items to be planned is I=50 that are made out of K=10 different alloys. Planning horizon is T=5 days with N=10 subperiods. Demand d_{it} has been generated in a random way from the range [10,60], weight of castings w_i from the range [1,30] and the setup for alloy st_k from the range of [5,10]. The only change introduced to the test data is the percentage value of

defective products and it was generated for each order from the range of [1%-20%].

We used a genetic algorithm similar to the one described in [9], however two changes have to be made. First of all, the representation of solutions has to comply with interval values of production. An exemplary representation for 5 subperiods is shown in Fig. 1. The first three rows represent the interval for valuable items produced in subsequent subperiods, the next three rows represent the numbers identifying different orders, and the last row repents the number identifying the alloy type.

i	1	2	3	4	5
$x1_i$	[9,12]	[90,97]	[16,20]	[20,25]	[32,36]
$x2_i$	[50, 55]	[14,18]	[66,69]	[28,30]	[64,70]
$x3_i$	[31,33]	[35,42]	[61,68]	[81,95]	[15,17]
$o1_i$	3	8	5	6	1
$o2_i$	4	6	3	8	2
$o3_i$	2	9	2	10	4
a_i	1	2	3	2	1

Fig. 1. Solution representation in GA adjusted to interval approach

The same genetic operators as in [10] were used, i.e. one-point crossover and three types of the mutation operator: first mutation that alters the number of produced items (x vectors), second mutation that can alter the orders (o vectors) and third mutation altering the alloy type (vector a). However, additional constraint had to be introduced to the first mutation – if lower bound for interval became higher than its upper bound, the bounds were exchanged to get a proper interval.

The genetic algorithm with a population of 50 solutions was run for 50,000 generations, and it took ca. 180 seconds to get a final result (machine with Intel i7-2630QM processor). Since GA is a heuristic that every time can deliver slightly different results, we repeated the computations 20 times. Next, we compared the results for the interval approach with the standard approach in which both variables and parameters were represented as crisp numbers, and the production represents the number of valuable products (nett production) excluding any extra product that was manufactured to cover the expected number of defective products and treating them as defective ones in advance. The results for both the standard and the interval approach are gathered in Table 1.

Table 1.
Results achieved by GA for production planning with the interval and the standard approach.

Approach		Overall production volume	Cost function
Interval	Best	[8,265; 8,686]	[45,487; 48,695]
	Average	[7,743; 8,183]	[52,333; 56,128]
Standard	Best	7,897	59,121
	Average	7,672	63,453

The results achieved in the experiments confirmed our expectations. As some part of the customers' demand can be satisfied by extra products that were planned for production to cover potentially defective items, both overall valuable production volume and the production cost function are better in the interval approach when compared to the standard approach. The overall valuable production is higher by 4.5-9.9% for the best result and by 0.9-6.6% on average. The cost function occurred to be better by 21.4-29.9% for the best result and 13.1-21.2% for average result.

5. Conclusions

In this paper an innovative approach to production planning and scheduling in a foundry has been presented. Interval arithmetic was applied in order to describe uncertainty regarding the real number of valuable products that met customers' quality requirements. Thanks to this approach the planner receives more complete information about the potential number of finished products manufactured in a given planning horizon, as well as the more accurate estimation of the potential costs.

Certainly, the uncertainty in the proposed model can be reduced with the progress of the planning process. At the end of each subperiod the planners will know the exact number of products that can be qualified as good and delivered to the end customer. In result, at the end of the planning horizon the planner will have crisp numbers instead of intervals. But by analyzing the production values stored in the form of intervals it acquires additional information that can be used for more precise decision making concerning e.g. necessary inventory levels for raw materials or cores that have to be prepared earlier.

In further research, we will extend the tests to larger instances of the problem with even higher value of uncertainty about the potential defective production. Other uncertainties may be introduced to the model including the load of the furnace, machine breakdowns or a human factor. Also, the genetic algorithm should be improved to give more stable results, i.e. to reduce the distance between average and the best results. Nevertheless, is has been shown, with compliance to the latest trends in the world literature dedicated to production planning and scheduling, that interval arithmetic can be a valuable method for modeling uncertainty that the planners have to deal with in everyday decision making process.

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