www.czasopisma.pan.pl PAN www.journals.pan.pl

ARCHIVES OF ACOUSTICS
Vol. **43**, No. 1, pp. 21–29 (2018)

DOI: 10.24425/118077

Nonlinear Excitation of the Non-Wave Perturbations by the Magnetoacoustic Waves in the Non-Isentropic Plasma

Anna PERELOMOVA

Faculty of Applied Physics and Mathematics
Gdansk University of Technology
Narutowicza 11/12, 80-233 Gdansk, Poland; e-mail: anpe@mif.pg.gda.pl

(*received July 3, 2017; accepted November 6, 2017*)

Nonlinear excitation of slow modes by the planar magnetosonic perturbations in a plasma is discussed. Plasma is an open system due to radiation and external heating. This may stipulate enhancement of wave perturbations and hence the acoustical activity of plasma. Plasma is assumed to be a homogeneous ideal gas with infinite electrical conductivity. The straight magnetic field is orthogonal to the velocity of fluid's elements. Nonlinear excitation of the non-wave modes (that is, the Alfvén and the entropy modes) by periodic and aperiodic planar magnetoacoustic perturbations, is discussed. The sawtooth wave and the small-magnitude harmonic wave are considered as examples of periodic in time perturbations. The conclusions concern acoustically active and thermally unstable flows as well.

Keywords: magnetoacoustic waves; dynamics of plasma; nonlinear acoustics.

1. Introduction

The recent interest in studies of magnetohydrodynamic (MHD) perturbations in the solar atmosphere is concentrated on their diagnostic applications and role in heating and acceleration of atmospheric plasma (Liu, Ofman, 2014). Plasma in general is an open system with inflow of external energy and radiative losses. There are also mechanical reasons for dumping due to friction and compressibility of a gas, that is, due to the shear and bulk viscosity. Modelling of MHD perturbations and attendant phenomena gives hope of getting information about parameters of the solar plasma which can not be measured directly.

For a long time, theoretical studies have been focused on the linear dynamics of wave MHD perturbations (De Moortel, Hood, 2004). That is reasoned by the low Mach numbers of the MHD perturbations. The well-known conclusion is that weakly nonlinear phenomena in flows accumulate with time (Rudenko, Soluyan, 1977). This may lead not only to crucially new features of wave propagation (including formation of shocks in weakly dispersive flows) but to establishment of new equilibrium thermodynamic parameters of the background and to the bulk flows in a plasma. The understanding of that has given rise to reinforced efforts in studying of nonlinear dynamics of MHD perturbations (Nakariakov *et al.*, 2000; Kelly, Nakariakov, 2004; Ruderman, 2013). In this way, the nonlinear distortions of wave MHD perturbations in the course of their propagation are theoretically predicted in many important particular cases.

The most intriguing factor which impacts on nonlinear dynamics, is the thermal and acoustic (isentropic) instabilities of a plasma. These kinds of instability are observed in many applications concerning the solar chromosphere, interstellar gases and planetary nebulae including interstellar clouds and solar prominence formation. They are also important in fluid flows in tokamaks. Instability is conditioned by the external heating of a plasma (Field, 1965; Nakariakov *et al.*, 2000). Magnetoacoustic perturbations in acoustically active plasma enhance in the course of propagation, if attenuation is weak. Relaxation of thermodynamic processes often leads to the similar properties of sound in fluid flows with inflow of energy. The examples are gases with excited vibrational degrees of molecules and chemically reacting gases (Osipov, Uvarov, 1992; Molevich, 2001). The heating-cooling function which disturbs adiabaticity of a flow, includes various terms responsible for external heating and cooling due to radiation and other reasons. The nonlinear dynamics of magnetoacoustic waves in acoustically active and dissipative plasma has been studied analytically by Chin *et al.* (2010). They have concluded about possibility of self-organisation of wave MHD disturbances. The magnitude of magnetoacoustic shock autowaves is completely prescribed by the thermodynamic properties of equilibrium plasma. It is independent of the initial or boundary conditions at a transducer (Kelly, Nakariakov, 2004).

The nonlinear effects of MHD waves, that is, excitation of non-wave modes in their field, seems to be still unexplored domain, though the nonlinear interaction of different branches of magnetohydrodynamic waves has been considered by numerous authors (Petviashvili, Pokhotelov, 1992; Sagdeev, Galeev, 1969; Shukla, Stenflo, 1999). This study discovers some peculiarities of nonlinear interaction of the magnetoacoustic perturbations with the non-wave modes. We consider magnetoacoustic heating in a plasma due to the heating-cooling function in the general form. Acoustic heating is actually an enhancement of the entropy mode which is excited by sound in a nonlinear flow with attenuation (Rudenko, Soluyan, 1977; Hamilton, Morfey, 1998). It is a slow non-wave fluid motion, hence, entropy perturbations do not follow sound and form in fact a new quasi-stationary equilibrium state of a plasma. They are of major importance in the long-scale predictions and may be an indicator of the energy which associates with fast MHD perturbations. While quick perturbations carry energy which propagates with the wave speed, the entropy mode develops slowly. The entropy perturbations may be the only source of information about the heating-cooling function and about thermodynamic processes which occur in a plasma, as well as about magnetoacoustic perturbations which excite them. One may expect that the magnetoacoustic heating occurs unusually in the acoustically active plasma. That happens to all acoustically active media (Osipov, Uvarov, 1992; Molevich, 2001). We do not consider mechanical and shear viscosity of a plasma, its thermal conductivity, and its finite electrical conductivity. These effects contribute to attenuation and dispersion in a fluid flow and are well studied. Instantaneous magnetoacoustic streaming and heating due to these mechanisms have been studied by the author (Perelomova, 2016a; 2016b). In this study, we derive the instantaneous dynamic equations responsible for generation of the entropy and the Alfvén modes by planar magnetoacoustic perturbations and discuss them in the context of periodic excitation at a transducer.

2. Wave and non-wave modes in a planar flow

The system of MHD equations consists of the conservation equations in the differential form. They are: continuity equation, momentum equation, electrodynamic equations and energy balance equation:

$$\begin{aligned}
&\frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0,\\
&\rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t}=-\operatorname{grad}p+\mu_0\mathbf{H}\times\operatorname{rot}\mathbf{H},\\
&\frac{\partial\mathbf{H}}{\partial t}=\operatorname{rot}(\mathbf{v}\times\mathbf{H}),\\
&\operatorname{div}\mathbf{H}=0,\\
&\frac{\mathrm{d}p}{\mathrm{d}t}-\gamma\frac{p}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}t}=(\gamma-1)L(p,\rho),
\end{aligned}\tag{1}$$

where $\mathbf{v}$, p, ρ, $\mathbf{H}$, are the plasma's velocity, pressure, density, the magnetic field strength, respectively, and μ_0 is the magnetic permeability of free space ($\frac{\mathrm{d}}{\mathrm{d}t}$ denotes the substantial derivative). The system relates to an ideal gas with the caloric equation of state

$$e=\frac{p}{(\gamma-1)\rho},$$

where e is the internal energy of a gas, and γ is the adiabatic constant. $L(p,\rho)$ is the heating-cooling function which may disturb adiabaticity of fast perturbations in a plasma. In general, it includes incoming and outcoming parts. Following Nakariakov *et al.* (2000), it is assumed to be a function of pressure and density.

The magnetic field $\mathbf{H}=H_z(x,t)\mathbf{k}$ is orthogonal to the velocity of gas particles $\mathbf{v}=v_x(x,t)\mathbf{i}$ which is directed along axis x. The leading-order linear equations follow from Eqs. (1):

$$\begin{aligned}
&\frac{\partial\rho'}{\partial t}+\rho_0\frac{\partial v}{\partial x}=-\rho'\frac{\partial v}{\partial x}-v\frac{\partial\rho'}{\partial x},\\
&\frac{\partial v}{\partial t}+\frac{1}{\rho_0}\frac{\partial p'}{\partial x}+\frac{1}{\rho_0}\frac{\partial h'}{\partial x}=-v\frac{\partial v}{\partial x}+\frac{\rho'}{\rho_0^2}\frac{\partial p'}{\partial x}+\frac{\rho'}{\rho_0^2}\frac{\partial h'}{\partial x},\\
&\frac{\partial p'}{\partial t}+c^2\rho_0\frac{\partial v}{\partial x}-(\gamma-1)(L_p p'+L_\rho\rho')\\
&\qquad=-v\frac{\partial p'}{\partial x}-\gamma p'\frac{\partial v}{\partial x}\\
&\qquad\quad+(\gamma-1)(0.5L_{pp}p'^2+0.5L_{\rho\rho}\rho'^2+L_{p\rho}p'\rho'),\\
&\frac{\partial h'}{\partial t}+2h_0\frac{\partial v}{\partial x}=-v\frac{\partial h'}{\partial x}-2h'\frac{\partial v}{\partial x},
\end{aligned}\tag{2}$$

where h denotes the magnetic pressure,

$$h=\mu_0H^2/2,$$

and

$$L_p=\frac{\partial L}{\partial p},\qquad L_\rho=\frac{\partial L}{\partial\rho},\qquad L_{pp}=\frac{\partial^2L}{\partial p^2},$$
$$L_{\rho\rho}=\frac{\partial^2L}{\partial\rho^2},\qquad L_{p\rho}=\frac{\partial^2L}{\partial p\partial\rho}$$

are evaluated at equilibrium state (p_0, ρ_0). Hence, we consider them as some constants. All variables represent a sum of unperturbed quantity, marked by subscript 0 ($v_0=0$) and primed disturbance.

The dispersion relations determine all possible kinds of a fluid's motion. They follow from the linearised Eqs. (1). All perturbations are represented by a sum of planar waves proportional to $\exp(i\omega(k)t - ikx)$, where k designates the wave number of any individual planar wave, and ω is its frequency:

$$f'(x,t) = \int\limits_{-\infty}^{\infty} \widetilde{f}(k)\exp(i\omega(k)t - ikx)\,\mathrm{d}k,$$

($\widetilde{f}(k)\exp(i\omega(k)t) = \widetilde{f}(k,t)$ denotes the Fourier transform of $f'(x,t)$, so as $\widetilde{f}(k,t) = \frac{1}{2\pi}\int\limits_{-\infty}^{\infty} f(x,t)e^{ikx}\,\mathrm{d}x$). The dispersion relations in a planar flow of a magnetic fluid reflect the solvability of the linearised Eqs. (1). They are

$$\omega_{1,2} = \pm c_m k - ic_m B, \qquad \omega_3 = 0, \qquad \omega_4 = i(2c_m B - (\gamma - 1)L_p), \tag{3}$$

where

$$c_m = \sqrt{c^2 + c_A^2}, \qquad c = \sqrt{\frac{\gamma p}{\rho}}, \qquad c_A = \sqrt{\frac{2h}{\rho}}$$

designate the magnetosonic speed, the sound speed in an unmagnetised gas, and the Alfvén speed, all evaluated at the equilibrium state (p_0, ρ_0). The first two roots ω_1, ω_2 determine the magnetosonic waves of different direction of propagation, that is, fast MHD waves. The parameter

$$B = \frac{(\gamma - 1)}{2c_m^3}\left(c^2 L_p + L_\rho\right)$$

is responsible for attenuation or enhancement of MHD wave, if it differs from zero. It originates from the deviation of quick perturbations from isentropic. We suppose that attenuation or amplification of magnetoacoustic wave is small during its period,

$$|B| \ll \omega/c_m$$

and arrive at the conclusion that a gas is acoustically active under the condition

$$c^2 L_p + L_\rho > 0, \tag{4}$$

which has been discovered in early studies of non-isentropic flows (Field, 1965). In this case, fast MHD perturbations in the linear flow enhance in the course of propagation. The third root ω_3 relates to the magnetic Alfvén mode, and the last one, ω_4, corresponds to the entropy mode. This last mode exists in all fluid flows, not specifically magnetic. The dispersion relations in Eqs. (3) are evaluated with accuracy up to terms proportional to the first powers of L_p,L_ρ. The second derivatives of L do not contribute to the dispersion relations in view of the fact that they stand by quadratic nonlinear terms.

The total perturbation is represented by a sum of specific disturbances which in fact are eigenvectors of the correspondent matrix operator:

$$\begin{pmatrix}\rho'\\ v\\ p'\\ h'\end{pmatrix} = \begin{pmatrix}\sum\limits_{i=1}^{4}\rho_i\\ \sum\limits_{i=1}^{4}v_i\\ \sum\limits_{i=1}^{4}p_i\\ \sum\limits_{i=1}^{4}h_i\end{pmatrix},$$

$$\begin{pmatrix}\rho_1\\ v_1\\ p_1\\ h_1\end{pmatrix} = \begin{pmatrix}1\\ \dfrac{c_m}{\rho_0} - \dfrac{c_m B}{\rho_0}\displaystyle\int \mathrm{d}x\\ c^2 - 2c_m^2 B\displaystyle\int \mathrm{d}x\\ c_m^2 - c^2\end{pmatrix}\rho_1,$$

$$\begin{pmatrix}\rho_2\\ v_2\\ p_2\\ h_2\end{pmatrix} = \begin{pmatrix}1\\ -\dfrac{c_m}{\rho_0} - \dfrac{c_m B}{\rho_0}\displaystyle\int \mathrm{d}x\\ c^2 + 2c_m^2 B\displaystyle\int \mathrm{d}x\\ c_m^2 - c^2\end{pmatrix}\rho_2, \tag{5}$$

$$\begin{pmatrix}\rho_3\\ v_3\\ p_3\\ h_3\end{pmatrix} = \begin{pmatrix}1\\ 0\\ -\dfrac{L_\rho}{L_p}\\ \dfrac{L_\rho}{L_p}\end{pmatrix}\rho_3,$$

$$\begin{pmatrix}\rho_4\\ v_4\\ p_4\\ h_4\end{pmatrix} = \begin{pmatrix}1\\ \left(\dfrac{2c_m B}{\rho_0} - \dfrac{(\gamma-1)L_p}{\rho_0}\right)\displaystyle\int \mathrm{d}x\\ c^2 - c_m^2\\ c_m^2 - c^2\end{pmatrix}\rho_4.$$

Index i denotes the ordering number of a specific mode. The rows which distinguish excess densities corresponding to third and fourth roots,

$$P_3\begin{pmatrix}\rho'\\ v\\ p'\\ h'\end{pmatrix} = \rho_3, \qquad P_4\begin{pmatrix}\rho'\\ v\\ p'\\ h'\end{pmatrix} = \rho_4,$$

take the forms:

$$P_3 = \begin{pmatrix} 1 + \dfrac{L_\rho}{(c_m^2 - c^2)L_p} \\ 0 \\ 0 \\ -\dfrac{1}{c_m^2 - c^2} - \dfrac{L_\rho}{(c_m^2 - c^2)^2 L_p} \end{pmatrix}^{\mathrm{T}},$$

$$P_4 = \begin{pmatrix} -\dfrac{L_\rho}{(c_m^2 - c^2)L_p} \\ -\dfrac{2B\rho_0}{c_m}\displaystyle\int \mathrm{d}x \\ -\dfrac{1}{c_m^2} \\ \dfrac{c^2}{c_m^2(c_m^2 - c^2)} + \dfrac{L_\rho}{(c_m^2 - c^2)^2 L_p} \end{pmatrix}^{\mathrm{T}}. \tag{6}$$

Projectors are evaluated with accuracy up to terms proportional to the first powers of L_p and L_ρ, as well as links given by Eqs. (5). The lower limit of integration in Eqs. (5) and (6) depends on the physical context of a flow, and the upper limit equals x. When P_3, P_4 apply at the linearised Eqs. (2), they reduce all therms containing non-specific perturbations and yield the linear dynamic equations which describe the dynamics of ρ_3 and ρ_4, respectively. These equations contain the first order partial derivatives with respect to time. In the following sections, we consider the non-linear dynamics of wave MHD perturbations and their interaction with the non-wave modes in a planar flow.

Equations (5) and (6) in their parts which refer to the third mode, are valid in the case $L_p \neq 0$. We consider only this case. It may be readily established that in the case $L_p = 0$ the links which specify the third mode take the forms:

$$v_3(x,t) = \rho_3(x,t) = 0, \qquad h_3(x,t) = -p_3(x,t).$$

That has impact on all evaluations which follow but is of minor interest.

3. Nonlinear dynamics of wave MHD perturbations

Application of P_3 and P_4 at the system (2), which includes quadratic nonlinear terms, yields weakly nonlinear evolutionary equations for the excess densities specifying the correspondent mode. Quadratic nonlinear terms become distributed between individual dynamic equations in the proper manner. The pure magnetoacoustic terms are of major importance in acoustic applications. This case implies large perturbations in MHD waves as compared with those of the non-wave modes at some temporal and spacial domains. For definiteness, we will consider the first MHD mode which propagates in the positive direction of axis Ox. It is determined by ω_1 from Eqs. (3). Since the acoustic source consists in the leading order of quadratic MHD perturbations, the linear relations for sound should be corrected. They should be supplemented by terms which make MHD wave isentropic in the leading order (Perelomova, 2016b). The corrected links are as follows:

$$\begin{pmatrix} \rho_1 \\ p_1 \\ h_1 \end{pmatrix} = \begin{pmatrix} \dfrac{\rho_0}{c_m}\left(1 + B\displaystyle\int \mathrm{d}x\right) \\ \dfrac{\rho_0}{c_m}\left(c^2 + (c^2 - 2c_m^2)B\displaystyle\int \mathrm{d}x\right) \\ \dfrac{\rho_0(c_m^2 - c^2)}{c_m}\left(1 + B\displaystyle\int \mathrm{d}x\right) \end{pmatrix} v_1 + \begin{pmatrix} \dfrac{c_m^2 - c^2(\gamma - 2)}{4c_m^4}\rho_0 \\ \dfrac{c^2(c_m^2(2\gamma - 1) - c^2(\gamma - 2))}{4c_m^4}\rho_0 \\ \dfrac{(c_m^2 - c^2)(3c_m^2 - c^2(\gamma - 2))}{4c_m^2}\rho_0 \end{pmatrix} v_1^2. \tag{7}$$

The nonlinear corrections do not depend on attenuation or amplification of MHD wave. These corrections represent the well-known terms which make the progressive Riemann's wave isentropic when $c_m = c$ (Rudenko, Soluyan, 1977). The equation governing velocity in the first magnetoacoustic planar wave, takes the form:

$$\frac{\partial v_1}{\partial t} + c_m \frac{\partial v_1}{\partial x} - c_m B v_1 + \epsilon v_1 \frac{\partial v_1}{\partial x} = 0, \tag{8}$$

where

$$\epsilon = \frac{3c_m^2 + c^2(\gamma - 2)}{2c_m^2}$$

is the parameter of nonlinearity in the MHD flow. Equation (8) coincides with that derived by Chin *et al.* (2010). It represents the particular case of Eq. (15) from (Chin *et al.*, 2010) with $\theta = \pi/2$ (that is, perpendicular magnetic strength and velocity of a plasma), zero thermal conduction and zero nonlinear term which associates with the heating-cooling function. Equation (8) recalls dynamic equations which describe wave perturbations in other flows which may be acoustically active (Osipov, Uvarov, 1992). Equation (8) was first derived and analysed in the context of propagation of a sawtooth MHD impulse by Sharma and co-authors (Sharma *et al.*, 1987) for the case $B = 0$. Equation (8) may be readily transformed into the leading-order pure nonlinear equation, if $B \neq 0$:

$$\frac{\partial V_1}{\partial X} - \frac{\epsilon}{c_m^2} V_1 \frac{\partial V_1}{\partial \tau} = 0, \tag{9}$$

by means of new quantities

$$V_1 = v_1 \exp(-Bx), \qquad X = \frac{\exp(Bx) - 1}{B},$$

$$\tau = t - x/c_m.$$

X is positive at positive distances from a transducer, x, for any non-zero B. By the way, Eq. (9) may be solved by the method of characteristics. Its graphic solution for a sawtooth impulse at a transducer is shown in Fig. 1. T denotes the period of an impulse. A transducer is situated at the plane $x = 0$, that is, $X = 0$. It may be concluded from the Fig. 1 that the magnitude of a sawtooth impulse equals

$$v_m = V_m \exp(Bx) = V_0 \frac{\exp(Bx)}{1 + K(\exp(Bx) - 1)}, \qquad (10)$$

where

$$K = \frac{2\epsilon V_0}{BTc_m^2}.$$

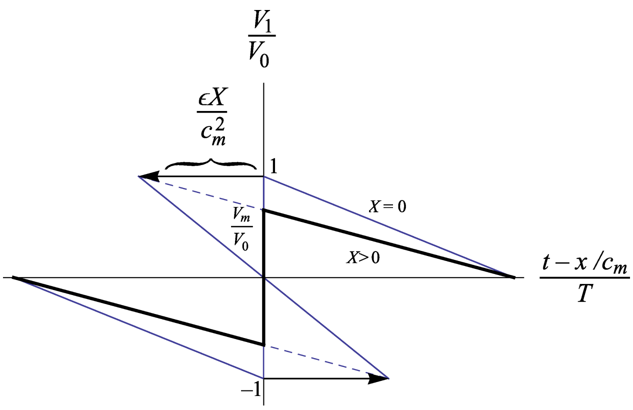

Fig. 1. Propagation of a sawtooth impulse.

Hence, a sawtooth impulse may enhance or weaken in dependence on K. The case $0 < K < 1$ reflects dominance of the energy inflow over nonlinear attenuation. In the case of $K > 1$, the nonlinear dissipation at the front of the shock wave is strong and the magnitude of an impulse gets smaller at larger distances from a transducer. The particular case $K = 1$ corresponds to the equilibrium between nonlinear attenuation and energy inflow. In this case, the magnitude of a sawtooth wave is constant and its shape is stable. Negative K reflects the case when damping on the shock front enhances damping due to loss in energy. The magnitude of an impulse quickly decreases.

In the case of very weak nonlinearity (i.e. conditioned by low magnitudes of MHD perturbations), a periodic solution of the linearised version of Eq. (8) takes the form

$$v_1(x,t) = V_0 \exp(Bx) \sin(\omega(t - x/c_m)). \qquad (11)$$

The magnitude of velocity increases or decreases in the course of wave propagation in dependence on the sign of B.

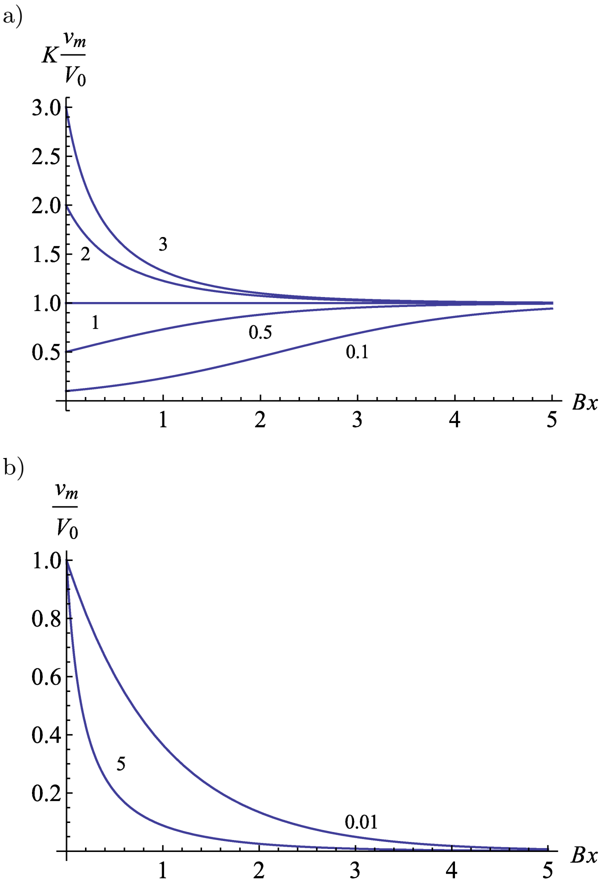

Fig. 2. Magnitude of a sawtooth impulse in the course of propagation as a function of dimensionless distance from a transducer. Cases of positive (a) and negative (b) B. The dimensionless quantity $K = \frac{2\epsilon V_0}{BTc_m^2}$ which relates to every line stands by the corespondent curve.

4. Nonlinear excitation of modes by intense magnetosonic wave

Equations (1) split into equations which describe dynamics of non-wave perturbations ρ_3 or ρ_4. That may be implemented by means of projecting rows P_3 and P_4. The nonlinear terms, that is, cross products of perturbations of different modes, represent the forces which excite the non-wave modes. Only magnetoacoustic terms belonging to the first mode will be kept among them. In turn, velocity of a fluid's elements in MHD wave should satisfy Eq. (8).

4.1. Excitation of the Alfvén mode

The equation which governs an excess density in the Alfvén mode, is the result of application of P_3:

$$\frac{\partial \rho_3}{\partial t} = \frac{\rho_0}{c_m^2 - c^2}\left(c_m B - \frac{(\gamma-1)c^4 L_p}{2c_m^4}\right) \cdot \left(v_1^2 + \frac{\partial v_1}{\partial x}\int v_1 \, \mathrm{d}x\right). \qquad (12)$$

For a periodic in time function with a period T (in the case of perturbation in the form of Eq. (11), $T = 2\pi/\omega$) in the form $\phi(t - x/c_m, Bx) = \phi(\tau, Bx)$, where $|B|c_m/\omega \ll 1$, one arrives at

$$\overline{\frac{\partial \phi}{\partial x}\int \phi \, \mathrm{d}x} = \frac{1}{T}\int\limits_{t-x/c_m}^{t-x/c_m+T}\left(\frac{\partial \phi}{\partial \tau}\int^{\tau} \phi(\tau', Bx)\,\mathrm{d}\tau'\right)\mathrm{d}\tau + O(B^2)$$

$$= \frac{1}{T}\left(\phi\int^{\tau}\phi(\tau', Bx)\,\mathrm{d}\tau' - \int^{\tau}\phi^2\,\mathrm{d}\tau'\right)\Bigg|_{t-x/c_m}^{t-x/c_m+T},$$

where top line denotes the temporal average over period of acoustic wave. For zero on average ϕ, the first term in the last line equals zero and we obtain the leading-order equality

$$\overline{\phi^2} + \overline{\frac{\partial \phi}{\partial x}\int \phi\,\mathrm{d}x} = O(B^2),$$

which does not depend on the kind of periodic (or nearly periodic) function ϕ. This is valid for v_1 in the forms of sawtooth and harmonic waves, Eq. (11). The sawtooth solution is not continuous but may be approximated with any accuracy by the continuous periodic function which is zero on average. We make use of the periodic magnetoacoustic velocity in the form Eq. (11) and of the averaged term:

$$\overline{v_1^2} + \overline{\frac{\partial v_1}{\partial x}\int v_1\,\mathrm{d}x} = \frac{B^2 c_m^2 V_0^2 e^{2Bx}}{B^2 c_m^2 + \omega^2} \approx \frac{B^2 c_m^2}{\omega^2} V_0^2 e^{2Bx}.$$

In the case of negative B, the acoustic source of the Alfvén mode is of order B^3 and we may conclude that the nonlinear coupling is very weak. As for the positive B, the acoustic force constantly enlarges with the distance from the transducer. Therefore, in spite of the factor of order B^3 standing by $\exp(2Bx)$, it might achieve significant values at large distances. This forms a new background for propagation of the MHD perturbations. Enhancement of the Alfvén mode may be slowed down due to other nonlinear interactions. The sign of acoustic force coincides with the sign of specific excess density ρ_3 (for initial zero ρ_3) and is determined by the sign of $B^2\left(c_m B - \frac{(\gamma-1)c^4 L_p}{2c_m^4}\right)$. Thus ρ_3 may take negative values for positive B and *vice versa*, depending on L_p. Anyway, $|\rho_3|$ decreases with distance from the transducer if $B < 0$ and increases otherwise.

4.2. Magnetoacoustic heating

The magnetoacoustic heating is the most important among all possible nonlinear interactions. When P_4 applies at the system Eqs. (2), we arrive at the equation which describes magnetoacoustic heating. We reproduce its averaged over the sound period form in the case of periodic (or nearly periodic) magnetoacoustic perturbations:

$$\overline{\frac{\partial \rho_4}{\partial t}} + (2c_m B - (\gamma-1)L_p)\,\overline{\rho_4} = \frac{\rho_0(\gamma-1)}{4c_m^6}$$
$$\cdot\left[c^4(3(\gamma-2)L_p - 2c_m^2\rho_0 L_{pp})\right.$$
$$+ c_m^2(L_\rho + 4\gamma L_\rho - 2\rho_0 L_{\rho\rho})$$
$$\left.+ c^2(3(\gamma-2)L_\rho + c_m^2(3L_p + 2\gamma L_p - 4\rho_0 L_{p\rho}))\right]\overline{v_1^2}, \quad (13)$$

which rearranges in the case of weak magnetic strength (i.e., approximately equal c_m and c) into the following equation:

$$\overline{\frac{\partial \rho_4}{\partial t}} + 2c\widetilde{B}\,\overline{\rho_4} = D\overline{v_1^2}, \quad (14)$$

where

$$\widetilde{B} = B - (\gamma-1)L_p/2c_m \approx (\gamma-1)L_\rho/c^3,$$

$$D = \frac{\rho_0(\gamma-1)}{4c^4}\left[c^2(5\gamma-3)L_p + (7\gamma-5)L_\rho\right.$$
$$\left.- 2c^4\rho_0 L_{pp} - 4c^2\rho_0 L_{p\rho} - 2\rho_0 L_{\rho\rho}\right].$$

It is readily integrated with the result (if $\widetilde{B} \neq 0$)

$$\overline{\rho_4} = \frac{D}{2c\widetilde{B}}\left(1 - \exp(-2c\widetilde{B}t)\right)\overline{v_1^2}.$$

If $\widetilde{B} = 0$, the result is

$$\overline{\rho_4} = D\overline{v_1^2}t.$$

These solutions correspond to zero initial ρ_4. In the case of the periodic solution (11),

$$\overline{v_1^2} = \frac{V_0^2}{2}\exp(2Bx), \quad (15)$$

and in the case of the sawtooth waveform,

$$\overline{v_1^2} = \frac{1}{3}\overline{v_m^2} = \frac{V_0^2\exp(2Bx)}{3\left[1 + K(\exp(Bx) - 1)\right]^2}. \quad (16)$$

The module of excess density $|\rho_4|$ varies differently with the distance from the transducer: it enlarges if $B > 0$ and decreases if $B < 0$. As for the temporal behaviour, it depends on $\widetilde{B}$, not on B, and grows infinitely in time if $\widetilde{B} < 0$. This reflects the fact that conditions of thermal and acoustic instability do not overlap (Field, 1965). The particular case $K = 1$ provides the stable shock waveform with constant magnitude independent of distance from the transducer. In this case, $\overline{v_1^2}$ is constant but $\overline{\rho_4}$ varies with time depending on the sign of $\widetilde{B}$ due to inflow of energy into the system. The external energy is expended on the nonlinear attenuation at the shock front which counterbalance enhancement of magnetosonic perturbations and on variation of thermodynamic perturbations which specify the entropy mode. The entropy mode is not isobaric in a flow of magnetic fluid in accordance to links Eqs. (5). The positive variation in density gives rise the negative variation in pressure since $c_m > c$. This means that the correspondent variation in temperature

$$\frac{T_4}{T_0} = \frac{p_4}{p_0} - \frac{\rho_4}{\rho_0} = -\frac{\rho_4}{\rho_0}\left(1+\gamma\frac{c_m^2-c^2}{c^2}\right) \tag{17}$$

is negative if ρ_4 is positive and it is positive otherwise, where T_0 denotes the equilibrium temperature of a gas.

5. Nonlinear interactions when the heating-cooling function depends exclusively on temperature

The meaningful particular case is the heating-cooling function which depends exclusively on temperature, $L(T)$. Its partial derivatives with respect to pressure and density may be expressed in partial derivatives with respect to temperature, assessed at the equilibrium temperature,

$$L_T = \frac{\mathrm{d}L}{\mathrm{d}T}, \qquad L_{TT} = \frac{\mathrm{d}^2L}{\mathrm{d}T^2}.$$

If $L_T > 0$, a gas is acoustically active. That follows from the acoustic dispersion relations, Eqs. (3):

$$\omega_{1,2} = \pm c_m k - ic_m B = \pm c_m k - \frac{i(\gamma-1)c^2}{2C_V\gamma\rho_0 c_m^2}L_T,$$

where C_V is the specific heat at constant volume. One may rearrange Eq. (12) into the dynamic equation

$$\frac{\partial\rho_3}{\partial t} = -\frac{c^2(-c_m^2(\gamma-1)+\gamma c^2)L_T}{2\gamma c_m^4(c_m^2-c^2)C_V}\left(v_1^2+\frac{\partial v_1}{\partial x}\int v_1\,\mathrm{d}x\right), \tag{18}$$

which equals approximately zero on average for nearly periodic sound. Variation in temperature T_3 which specifies the Alfvén mode equals zero:

$$\frac{T_3}{T_0} = \frac{p_3}{p_0} - \frac{\rho_3}{\rho_0} = 0$$

in accordance to relations of ρ_3 and p_3 established by Eqs. (5).

As for equation which governs the entropy mode, it follows from making use of P_4. We reproduce its averaged form for the periodic fast MHD perturbations and weak magnetic strength, $c_m \approx c$:

$$\overline{\frac{\partial\rho_4}{\partial t}} - \frac{L_T}{\gamma c^2 C_V\rho_0}\overline{\rho_4} = \widetilde{D}\overline{v_1^2}, \tag{19}$$

where

$$\widetilde{D} = \frac{\gamma-1}{4\gamma^2c^2C_V^2}\left(C_V(5\gamma-1)\gamma L_T - 2c^2L_{TT}\right).$$

The second-order derivative is not of importance, if

$$|L_{TT}| \ll \frac{(5\gamma-1)\gamma C_V}{2c^2}|L_T| = \frac{\gamma^2(5\gamma-1)}{2(\gamma-1)T_0}|L_T|.$$

If so, $\widetilde{D}$ and L_T are of the same sign. Equation (19) is integrated with the result

$$\overline{\rho_4} = \frac{\widetilde{D}\gamma c^2C_V\rho_0}{L_T}\left(\exp\left(\frac{L_T}{\gamma c^2C_V\rho_0}t\right)-1\right)\overline{v_1^2},$$

if $L_T \neq 0$, and with the result

$$\overline{\rho_4} = \widetilde{D}\overline{v_1^2}t,$$

if $L_T = 0$. These solutions correspond to zero initial $\overline{\rho_4}$. The conditions of acoustical activity and thermal instability coincide ($L_T > 0$). In acoustically active gas, $\overline{\rho_4}$ is positive and enlarges with time and with distance from the transducer. An excess density and temperature are related in accordance to Eq. (17): T_4 equals approximately $-T_0\frac{\rho_4}{\rho_0}$ in the case of weak magnetic strength. Hence, production of negative excess temperature specifying the entropy mode occurs in acoustically active gases.

6. Concluding remarks

We considered excitation of the non-wave modes in the field of intense sound in a magnetic fluid in the particular case when the vector of magnetic strength is perpendicular to the velocity of a gas. The system is open, that is, there exist inflow or/and loss of energy, which is described by the heating-cooling function L. It depends in general on pressure and density of a gas. Newtonian attenuation and thermal conduction of a gas are left of account, since their influence on the nonlinear motion of a gas is well-studied.

The linear features of flows in open systems are well-understood. This concerns different physical conditions of flows which become acoustically active or/and thermally unstable under some conditions which depend on the kind of L. Wave perturbations in acoustically active media may enhance in the course of propagation. Also, the nonlinear interactions may occur unusually in open systems. Magnetoacoustic heating or cooling leads to the non-uniformity of the background parameters of a plasma, that is, thermal lenses and bulk flows which follow attenuation or amplification of sound. This has impact on the sound propagation and may be of especial interest in the plasma's applications. By the way, waveguides and thermal lenses may play a crucial role.

We have considered nonlinear excitation of the non-wave modes in the particular cases of the heating-cooling function:

1) L is a function of p and ρ $\left(L_p = \frac{\partial L(p,\rho)}{\partial p} \neq 0\right)$;

2) L depends exclusively on temperature.

Weakly nonlinear dynamic equations which govern excitation of the non-wave modes in the field of intense fast MHD perturbations are derived. They are valid for periodic and aperiodic magnetoacoustic disturbances independently on their spectrum. They are instantaneous as well. Equations (12) and (13) are the main results of the study. They determine dynamics of an excess density in the Alfvén and entropy modes. As for the Alfvén mode, its perturbations are fairly small, at least when excited by periodic or nearly periodic magnetoacoustic disturbances on a transducer.

Acoustical activity implies $c^2 L_p + L_\rho > 0$, whereas condition of thermal instability implies $L_\rho < 0$ at weak magnetic strengths. These conditions, responsible for the temporal behavior of perturbations, do not certainly overlap (Field, 1965). If we consider the power dependence of L on p and ρ, proportional to $p^\beta \rho^{-\alpha}$, where α and β are positive, the conformity of acoustic and thermal instabilities implies $\gamma\beta > \alpha$. This is, among other, the case $\alpha = \beta = 1$ which corresponds to $L(T)$. The particular case $L(T)$ is considered in Sec. 5.

There are no restrictions concerning the strength of the magnetic field in this study. The only limitations are: weak nonlinearity of a flow, that is, smallness of the Mach number, and smallness of attenuation or amplification of wave perturbations during its period. The results of this study may be addressed to a hot atomic plasma with temperature greater than 10^4 K and a cold molecular gas with temperature less than 10^3 K and to different kinds of the function $L(p, \rho)$. The radiation function may also contribute in L. Various heating models for the coronal radiative losses and deposition of mechanical energy in the solar corona, are discussed in (Rosner, Tucker, 1978). This is in fact a comprehensive review of many models which were confirmed experimentally. Among them, there are no examples of L which depends exclusively on ρ, that is, $L_p = 0$. That's why this particular case is of minor importance. In the case of heating due to coronal current dissipation, L is proportional to p and does not depend on ρ (Nakariakov *et al.*, 2000). This is the case of acoustical activity of a plasma, since $L_p > 0$ for a small magnetic strength. This case corresponds to zero $\widetilde{B}$, that is, to zero diffusion coefficient in Eq. (13). Molevich *et al.* (2011) consider the heating-cooling function which depends on temperature and density and make use of astrophysical examples of heating due to photoelectric emission and the radiative cooling rate due to transitions between the electron levels. The results of the study may be helpful in the inverse problems. They point a way to establish the kind of heating-cooling function and intensity of the fast MHD perturbations in a plasma by means of remote measurements of slow perturbations.

References

1. Chin R., Verwichte E., Rowlands G., Nakariakov V.M. (2010), *Self-organisation of magnetoacoustic waves in a thermal unstable environment*, Physics of Plasmas, **17**, 32, 107–118.
2. De Moortel I., Hood A.W. (2004), *The damping of slow MHD waves in solar coronal magnetic fields. II. The effect of gravitational stratification and field line divergence*, Astronomy and Astrophysics, **415**, 2, 705–715, http://dx.doi.org/10.1051/0004-6361:20034233.
3. Field G.B. (1965), *Thermal instability*, The Astrophysical Journal, **142**, 531–567.
4. Hamilton M., Morfey C. (1998), *Model equations*, [in:] *Nonlinear acoustics*, Hamilton M., Blackstock D. [Eds.], pp. 41–63, Academic Press, New York.
5. Kelly A., Nakariakov V.M. (2004), *Coronal seismology by MHD autowaves*, Proceedings of SOHO 13 Waves, Oscillations and Small-Scale Transients Events in the Solar Atmosphere: a joint view from SOHO and TRACE, Lacoste H. [Ed.], **547**, pp. 483–488.
6. Liu W., Ofman L. (2014), *Advances in observing various coronal EUV waves in the SDO era and their seismological applications (invited review)*, Solar Physics, **289**, 9, 3233–3277, http://dx.doi.org/10.1007/s11207-014-0528-4.
7. Molevich N.E. (2001), *Sound amplification in inhomogeneous flows of nonequilibrium gas*, Acoustical Physics, **47**, 1, 102–105, doi: 10.1134/1.1340086.
8. Molevich N.E., Zavershinsky D.I., Galimov R.N., Makaryan V.G. (2011), *Traveling self-sustained structures in interstellar clouds with the isentropic instability*, Astrophysics and Space Science, **334**, 35–44, doi: 10.1007/s10509-011-0683-0.
9. Nakariakov V.M., Mendoza-Briceño C.A., Ibáñez M.H. (2000), *Magnetoacoustic waves of small amplitude in optically thin quasi-isentropic plasmas*, Astrophysical Journal, **528**, 2, 767-775, http://dx.doi.org/10.1086/308195.
10. Osipov A.I., Uvarov A.V. (1992), *Kinetic and gasdynamic processes in nonequilibrium molecular physics*, Soviet Physics Uspekhi, **35**, 11, 903–923.
11. Perelomova A. (2016a), *On the nonlinear effects of magnetoacoustic perturbations in a perfectly conducting viscous and thermoconducting gas*, Acta Physica Polonica A, **130**, 3, 727–733, http://dx.doi.org/10.12693/APhysPolA.130.727
12. Perelomova A. (2016b), *On the nonlinear distortions of sound and its coupling with other modes in a gaseous plasma with finite electric conductivity in a magnetic field*, Archives of Acoustics, **41**, 4, 691–699, http://dx.doi.org/10.1515/aoa-2016-0066.
13. Petviashvili V.I., Pokhotelov O.A. (1992), *Solitary waves in plasmas and in the atmosphere*, Gordon and Breach, Berlin.

14. Rosner R., Tucker W.H. (1978), *Dynamics of the quiescent solar corona*, Astrophysical Journal, **220**, 643–645, doi: 10.1086/155949.

15. Rudenko O.V., Soluyan S.I. (1977), *Theoretical foundations of nonlinear acoustics*, Plenum, New York

16. Ruderman M.S. (2013), *Nonlinear damped standing slow waves in hot coronal magnetic loops*, Astronomy and Astrophysics, **553**, A23, http://dx.doi.org/10.1051/0004-6361/201321175.

17. Sagdeev R.Z., Galeev A.A. (1969), *Nonlinear plasma theory*, Benjamin, New York.

18. Sharma V.D., Singh L.P., Ram R. (1987), *The progressive wave approach analyzing the decay of a sawtooth profile in magnetogasdynamics*, Physics of Fluids, **30**, 5, 1572–1574, https://doi.org/10.1063/1.866222.