

# Sinusoidal Oscillator Circuits Reexamined

V. C. Prasad

**Abstract**—Singular network condition is proposed to study oscillators. It states that a circuit is a potential oscillator *if and only if* the rank of the network matrix of size  $n \times n$  is  $(n - 1)$  at the frequency of oscillations. The dual (if it exists) and adjoint circuit of an oscillator are also oscillators. Limitations of Barkhausen's approach are pointed out. It is explained that there are many ways to generate oscillations other than Barkhausen's positive feedback configuration. The new approach emphasizes that appropriate D C inputs / initial conditions are important.

**Keywords**—sinusoidal oscillators, Barkhausen's criterion, analog circuits, feedback circuits

## I. INTRODUCTION

A sinusoidal oscillator is an important building block of electronic circuits. It is so fundamental that it is widely taught to undergraduate students in basic electronics courses. At present Barkhausen's criterion [1 - 6] is widely used to understand / design oscillators. It uses the concept of positive feedback in an amplifier. Let  $A$  ( $\beta$ ) denote the complex gain of the amplifier (feedback network). The output (input) of  $A$  is input (output) of  $\beta$  network (Fig.1). According to Barkhausen's criterion such a configuration generates sinusoidal oscillations at a frequency  $\omega_0$  if  $A\beta = 1$  at  $\omega_0$ . This equation in complex numbers gives two equations in real numbers. One of them is used to calculate  $\omega_0$ . The other is used to decide the gain of the amplifier required for oscillations. This approach has the following problems. (i) This criterion is only for positive feedback configuration of  $A$  and  $\beta$  blocks. It cannot be used to decide whether oscillations are possible in an arbitrary network or not. But this ability is important to suppress oscillations in some circuits to improve stability. Further it enables us to explore new oscillator circuits. (ii) It requires identification of the amplifier and feedback blocks correctly. (iii)  $A$  and  $\beta$  values have to be calculated carefully taking appropriate loading and other effects into account. (iv) We need to ensure that the reverse flow of signal in  $A$  and  $\beta$  blocks is negligible. Thus lot of attention is required to use the method correctly. In this communication singular network condition is presented which overcomes these problems as there are no  $A$  and  $\beta$  blocks in it. Further it shows that (i) every oscillator circuit is a singular network at the frequency of oscillation, (ii) other configurations of  $A$  and  $\beta$  blocks can also generate oscillations. In fact Barkhausen's configuration can be looked upon as a parallel configuration if the direction of the signal in the  $\beta$  block is opposite to that in positive loop configuration (Fig.1) (iii) the adjoint and dual networks of an oscillator are also oscillators.

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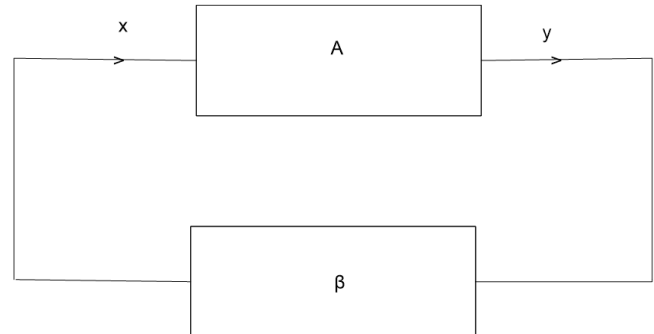


Fig. 1 Interconnection of amplifier ( $A$ ) and network block ( $\beta$ )

Singular network condition is presented in Section II. It is compared with Barkhausen's criterion in Section III. Some new configurations are also suggested in this section. Finally contributions of this paper are summarized in Section IV.

## II. SINGULAR NETWORK CONDITION

Let  $N$  be a linear time invariant network. All linear elements like  $R$ ,  $L$ ,  $C$ ,  $M$ , linear controlled sources, linear ideal operational amplifiers, independent D C voltage and current sources are allowed. Reactive elements can have initial conditions. Voltages and currents in  $N$  are functions of time. Assume that the network  $N$  is solvable. Let  $N_s$  be  $N$  in the complex frequency ( $s$ ) domain. Equations of  $N_s$  are of the form

$$\mathbf{M}(s) \mathbf{X}(s) = \frac{1}{s} \mathbf{b} \quad (1)$$

Where  $\mathbf{X}(s)$  is a vector of network variables like voltage and / or currents of  $N_s$ . They can be linear combinations of voltages and currents. They can be mesh currents (node voltages) if mesh (node) analysis is used. Any network analysis method can be used. So  $\mathbf{X}(s)$  will depend upon the method used. After deriving equations some variables can be eliminated if one thinks that there are too many equations. In other words there are no restrictions on the nature of the equations (1). Consequently elements of  $\mathbf{M}(s)$  can be ratios of polynomials in  $s$  in the general case. Let  $\mathbf{M}$  be a square matrix of order  $n$ . Then  $\mathbf{X}$  is a vector of  $n$  variables.  $\mathbf{b}$  is a vector of  $n$  real numbers. Their values depend upon the D C sources and initial conditions in  $N$ . Let  $N$  be a stable network. i.e., poles of all variables of  $\mathbf{X}$  lie in the left half of the  $s$  plane and poles on  $j\omega$  axis, if any, are simple. This implies that  $\mathbf{x}(t)$  can have sinusoidal components and / or D C components only under steady state. Since the network is solvable  $\mathbf{x}(t)$  cannot be zero unless  $\mathbf{b}$  is zero. In principle,  $\mathbf{x}(t)$  can have sinusoidal components at several frequencies. But we restrict to one frequency only. Let  $\omega_0$  be this frequency. There is no loss of generality in this assumption because the arguments of the paper can be applied to each

frequency if several frequencies exist. Thus we want to determine the sinusoidal components of  $\mathbf{x}(t)$  at  $\omega_0$ , if they exist. Let  $\Delta(s)$  denote the determinant of  $\mathbf{M}(s)$ . Then

$$\Delta(s) = \frac{P(s)}{Q(s)} \quad (2)$$

where  $P(s)$  and  $Q(s)$  are polynomials in  $s$ . Cancel any common factors of  $P(s)$  and  $Q(s)$  by using, say, continued fraction expansion technique[7]. Assume that  $\frac{dP(s)}{ds} \neq 0$  at  $s = j\omega_0$ . It is clear that  $(s^2 + \omega_0^2)$  is not a factor of  $Q(s)$ . This also means that  $\Delta(s)$  is not a constant.

Let  $N'$  be  $N$  under steady state. Consider a KCL of  $N'$ . It is of the form  $i_1(t) + i_2(t) + \dots + i_p(t) = 0$  where  $i_k(t)$  is of the form  $a_k + b_k \sin(\omega_0 t + \theta_k)$  for all  $k = 1, 2, \dots, p$ . Since a DC (sinusoidal) component cannot cancel a sinusoidal (DC) component for all  $t$ , KCL must hold for DC components and AC components separately. Similarly the DC components and AC components of voltages must satisfy KVL separately. DC voltages and DC currents satisfy DC equations of components (Inductor is a short and capacitor is open for DC,  $V = IR$  for a resistor etc). Similarly AC voltages and AC currents satisfy AC equations of the components ( $V = j\omega L I$ ,  $I = j\omega C V$ ,  $V = IR$  etc). This is so even if the AC voltage or AC current of a component is zero. i.e., an inductor (capacitor) need not be seen as a short (open) even if its AC current (AC voltage) is zero. In view of this construct a network  $N_{ac}$  from  $N$  as follows:  $N_{ac}$  and  $N$  have same elements connected in the same way in both networks except that all independent sources and all initial conditions are killed in  $N_{ac}$ . i.e., all independent current sources are open circuited and all initial currents of inductors are made zero so that they do not appear in KCL equations. Similarly all independent voltages sources are short circuited and all initial voltages of capacitors are made zero so that they do not appear in KVL equations. The sinusoidal components of voltages and currents of  $N$  are retained in  $N_{ac}$ . This is done even if some of them are zero. Therefore voltages and currents of  $N_{ac}$  satisfy KVL and KCL equations.  $L$  in  $N_{ac}$  is  $L$  in  $N$  with impedance  $j\omega L$ . Similarly  $C$  in  $N_{ac}$  is  $C$  in  $N$  with admittance  $j\omega C$ . Other components have same values as in  $N$ . The AC voltages and currents satisfy the component equations in  $N_{ac}$ . It is clear from this description that the equations of  $N_{ac}$  are

$$\mathbf{M}(j\omega)\mathbf{X}'(j\omega) = \mathbf{0} \quad (3)$$

where  $\mathbf{X}'(j\omega)$  is complex number representation of the sinusoidal components of  $\mathbf{x}(t)$ . If  $N_{ac}$  has cut vertices [8] split the network into its biconnected components [8]. In addition to that  $N_{ac}$  may itself be broken into several connected components because of open circuited elements, if any. Each connected component of  $N_{ac}$  will have an equation of the form (3).

$N$  is said to satisfy *singular network condition* if the rank of the network matrix  $\mathbf{M}(j\omega_0)$  of at least one connected component of  $N_{ac}$  derived from  $N$  is  $(n-1)$  where  $\mathbf{M}$  is of size  $n \times n$ .

Referring to equation (1)  $N$  is said to be a *potential oscillator* if there exists a vector  $\mathbf{b}$  of real numbers such that at least one component of the vector  $\mathbf{x}(t)$  has a sinusoidal component under steady state. Note that every element of the network need not have sinusoidal current and / or voltage even if  $N$  is an oscillator.

**Theorem:**  $N$  is a potential oscillator if and only if it satisfies singular network condition.

**Proof:** *Necessity:*  $N$  is a potential oscillator. i.e., appropriate initial conditions and / or independent DC inputs exist in  $N$  so that at least one component of the vector  $\mathbf{x}(t)$  has sinusoidal components at  $\omega = \omega_0$ . To simplify notation let  $N_{ac}$  denote that component in which oscillations are present. We want to prove that the rank of the matrix  $\mathbf{M}(j\omega_0)$  of this component is  $(n-1)$ .  $x_i(j\omega) = 0$  if  $x_i(t)$  has no AC component. Since all voltages and currents of  $N_{ac}$  are linear combinations of the variables of the vector  $\mathbf{X}'(j\omega)$ , they will all be zero if  $\mathbf{X}'(j\omega) = \mathbf{0}$ . Therefore  $\mathbf{X}'(j\omega)$  cannot be a null vector if  $N$  is an oscillator. So the rank of  $\mathbf{M}$  cannot be  $n$  at  $\omega = \omega_0$ .

Consider  $\mathbf{M}^{-1}(s) = \frac{1}{\Delta(s)} \text{Adj } \mathbf{M}(s)$ . Since the network is solvable, this matrix exists. Let  $m_{ij}$  be the  $(i, j)$ th element of  $\text{Adj } \mathbf{M}(s)$ . Let  $m_{ij} = \frac{P_1(s)}{Q_1(s)}$  where  $P_1(s)$  and  $Q_1(s)$  are polynomials in  $s$ . Eliminate common factors of these polynomials, if any. Then a nonzero  $(i, j)$ th element of  $\mathbf{M}^{-1}$  is  $\frac{Q(s) P_1(s)}{P(s) Q_1(s)}$ . If  $(s^2 + \omega_0^2)^{q_1}$ ,  $q_1 \geq 1$ , is a factor of  $P_1(s)$  for all elements, then  $(s^2 + \omega_0^2)^{q_1}$  of  $P(s)$  cancels off. In this case  $\mathbf{x}(t)$  cannot have sinusoidal components. This is a violation. Therefore  $(s^2 + \omega_0^2)$  cannot be a factor of  $P_1(s)$  for at least one  $i$  and one  $j$ . i.e., the rank of  $\mathbf{M}(j\omega_0)$  must be  $(n-1)$ .  $(s^2 + \omega_0^2)$  is a simple factor of  $P(s)$ . It is not a factor of  $Q(s)$  and  $P_1(s)$ . It cannot be a factor of  $Q_1(s)$  because no element of  $\mathbf{X}(s)$  can have a double pole at  $s = j\omega_0$  as the network is stable. This shows that  $(s^2 + \omega_0^2)$  cannot be a factor of numerator or denominator of every element of  $\text{Adj } \mathbf{M}(s)$ . It is a factor of  $\Delta(s)$  only.

*Sufficiency:* We are given that the rank of the matrix  $\mathbf{M}(j\omega_0)$  of a connected component of  $N_{ac}$  is  $(n-1)$ . We need to prove that  $N$  is a potential oscillator. Let  $\Delta(s) = (s^2 + \omega_0^2) p(s)$  where  $p(s)$  is a polynomial in  $s$  and  $p(j\omega_0) \neq 0$ . Since the rank of  $\mathbf{M}(j\omega_0)$  is  $(n-1)$ , at least one cofactor of  $\mathbf{M}(s)$  is nonzero for  $s = j\omega_0$ . Therefore let  $(i, j)$ th element of  $\mathbf{M}^{-1}$  be nonzero. Choose  $b_j = 1$  and  $b_i = 0$  for all  $i \neq j$ . Then  $X_i(s)$  is of the form,

$$X_i(s) = \frac{k(s+z_1)(s+z_2)\dots}{s(s^2 + \omega_0^2)p(s)} \quad (4)$$

Where  $k$  is a constant.  $s = j\omega_0$  is a simple pole as the network is stable.  $p(s)$  is the remaining polynomial of the denominator. It is one if there are no other powers of  $s$ . Using partial fractions  $X_i(s)$  can be written as

$$X_i(s) = A_0 + A_{in}s + \frac{A_1}{s} + \frac{A_2 s + A_3}{(s^2 + \omega_0^2)} + \frac{Y_i(s)}{p(s)} \quad (5)$$

where  $Y_i(s)$  is a polynomial in  $s$  which depends on  $X_i(s)$  and  $p(s)$  and  $A_0, A_{in}, A_1, A_2, A_3$  are constants. Some of them can be zero.

This gives

$$k(s + z_1)(s + z_2) \dots = [(A_0 s + A_{in} s^2) A_1] (s^2 + \omega_0^2) p(s) + (A_2 s + A_3) s p(s) + Y_1(s) s (s^2 + \omega_0^2) \quad (6)$$

The left hand side is not zero for  $s = j\omega_0$ . Therefore the right hand side cannot be zero for  $s = j\omega_0$ . This implies that at least one of  $A_2$  and  $A_3$  must be nonzero. i. e.,  $x_i(t)$  has a sinusoidal component under steady state showing that  $N$  is a potential oscillator.

**Example 1 (Wien bridge Oscillator):** Consider the Wien bridge oscillator shown in Fig.2 (a) [1, 2]. The method based on Barkhausen's criterion requires identifying  $A$  and  $\beta$  blocks, calculate their gains and use the equation  $A\beta = 1$  to determine the conditions for oscillations. We will use the singular network condition.  $N_{ac}$  network of the oscillator is shown in Fig.2 (b). Since the operational amplifier is an open circuit and a short circuit simultaneously on the input side, the equations of  $N_{ac}$  are

$$\begin{bmatrix} R_1 & -Z_2 \\ R_2 & -Z_1 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (7)$$

Where  $Z_1 = R + \frac{1}{j\omega C}$  and  $Z_2 = R \parallel \frac{1}{j\omega C}$ . Equating the determinant of the matrix to zero we get  $\omega_0 = \frac{1}{RC}$  and  $R_2 = 2R_1$ . The rank of the matrix is one. Thus the singular network condition is satisfied and the network is a potential oscillator. The equations of the Wien bridge oscillator in  $s$ -domain are

$$\begin{bmatrix} R_1 & \frac{-R}{RCs+1} \\ 2R_1 & -\left(R + \frac{1}{Cs}\right) \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \frac{1}{s} \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \quad (8)$$

Where  $b_1$  and  $b_2$  are real constants decided by the D.C. inputs and initial conditions of the network. Let  $R = R_1 = 1, R_2 = 2$ ,

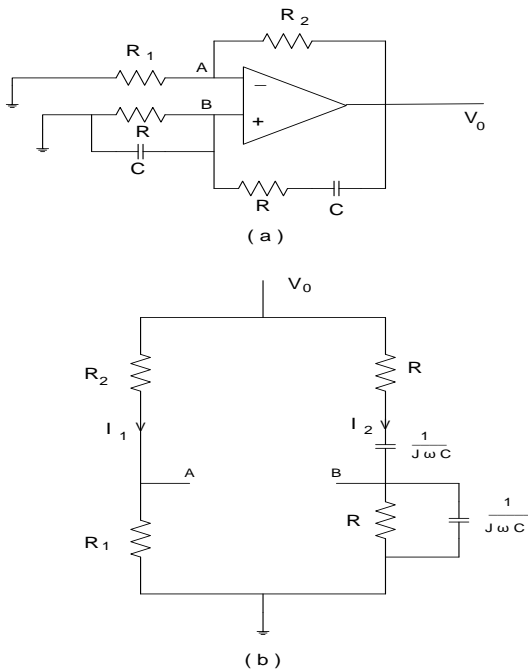


Fig. 2 (a) Wien bridge oscillator (b) Its  $N_{ac}$  network

$C = 1, b_1 = 1$  and  $b_2 = 2$ . Solving the equations we get  $I_1(s) = \frac{1}{s}$  and  $I_2(s) = 0$ . This shows that oscillations may not exist unless appropriate D.C. inputs and initial conditions are provided. But a slight change in the values of  $b_1$  and  $b_2$  will solve the problem.

**Example 2 (An arbitrary Oscillator circuit):** Consider the circuit shown in Fig.3 (a). Its  $N_{ac}$  is shown in Fig. 3(b). The equations of  $N_{ac}$  are

$$(R_1 + R_3 + j\omega L - \frac{j}{\omega C_1}) I_1 - R_3 I_2 + \frac{j}{\omega C_1} I_3 = 0 \quad (9a)$$

$$\alpha I_1 - I_2 - \alpha I_3 = 0 \quad (9b)$$

$$\frac{j}{\omega C_1} I_1 - R_2 I_2 + \left\{ R_2 - j\left(\frac{1}{\omega C_1} + \frac{1}{\omega C_2}\right) \right\} I_3 = 0 \quad (9c)$$

Choose  $R_2 = 0$  for simplicity. Equating the determinant of the matrix to zero, Real part gives

$$\omega_0 = \frac{1}{\sqrt{L(C_1 + C_2)}} \quad (10a)$$

Imaginary part gives  $\alpha = \frac{(R_1 + R_3)(C_1 + C_2)}{R_3 C_1} \quad (10b)$

The rank of the matrix is two. There are no  $A$  and  $\beta$  blocks in this network. Therefore Barkhausen's criterion cannot be used. Yet it is a potential oscillator. Thus singular network condition allows us to try new circuits for oscillations.

Singular network condition allows interaction between  $A$  and  $\beta$  blocks. For example, let  $A$  block contain a current controlled voltage source of  $K I$  volts where  $I$  is a current of  $\beta$  block.

Then  $A$  block depends on the  $\beta$  block. Barkhausen's criterion does not consider such possibilities.

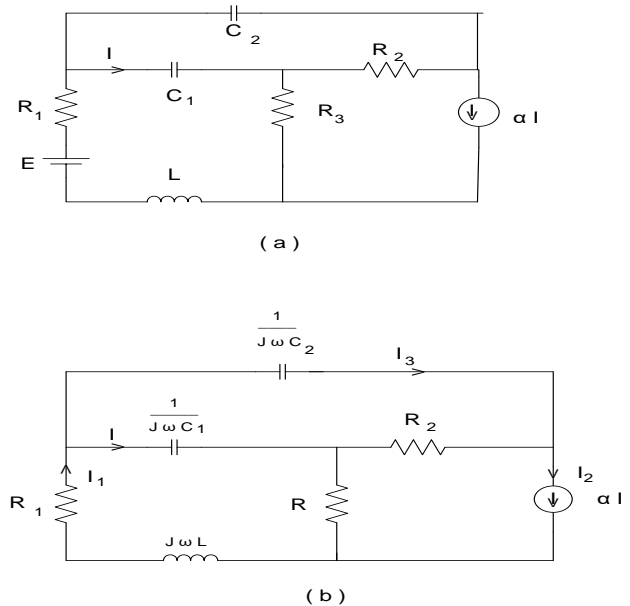


Fig. 3 (a) Network  $N$  (b) Its  $N_{ac}$  network

**Example 3 (One port network):** This example shows that a linear one port can oscillate under appropriate conditions. Consider a linear one port containing  $R, L, C, M$ , controlled sources and linear ideal operational amplifiers but no independent source. Its driving point impedance is of the form  $Z = R(\omega) + jX(\omega)$  in the frequency domain. Connect a DC voltage source  $E$ , a negative resistance  $-R_{neg}$  and the one port to form a loop. Let  $I$  be the loop current. Then the equation of its  $N_{ac}$  is  $[\{R(\omega) - R_{neg}\} + jX(\omega)]I = 0$ . Since  $n = 1$ , the theorem requires that  $R(\omega_0) = R_{neg}$  and  $X(\omega_0) = 0$ . The second equation can be used to determine the frequency of oscillation and the first equation can be used to decide the value of the negative resistance.

Consider a special case of this in which the one port is a series RLC circuit connected to a one volt DC source. Assume zero initial conditions. Then the theorem requires that  $R_{neg} = R$  and  $\omega_0 = \frac{1}{\sqrt{LC}}$ . Oscillations exist in the network at this frequency [1–3].

(i) Next consider the same circuit but with an initial voltage of -1 volt across the capacitor. The net excitation is zero and hence oscillations do not exist. This shows that appropriate initial conditions are important to establish oscillations.

(ii) Consider the same series RLC circuit with a DC current excitation and zero initial conditions. The equation of its  $N_{ac}$  is  $I = 0$ .  $\det M \neq 0$ . Therefore this network will not oscillate. It is clear that if a general one port oscillates with a DC voltage source input, it may not oscillate with a DC current source input. This shows that appropriate inputs are important to generate oscillations.

(iii) The negative resistance oscillator of series RLC elements is generally explained without using Barkhausen's criterion. But it is possible to use Barkhausen's approach. Take an external ground. Let  $R_{neg}$  and  $R$  in series constitute the  $A$  block. Let  $V_x (V_y)$  be the input (output) voltage of the  $A$  block measured with respect to ground.  $L$  and  $C$  in series constitute the  $\beta$  block. The output ( $x$ ) of the  $\beta$  block is connected to input of  $A$  block. This completes the loop of the negative resistance oscillator. If  $R = |R_{neg}|$ , the voltage gain of  $A$  block is one. Similarly if  $\omega L - \frac{1}{\omega C} = 0$  the voltage gain of  $\beta$  block is also one. Since loop current flows in one direction only, reverse signals do not exist in  $A$  and  $\beta$  blocks. This satisfies  $A\beta = 1$ .

### III. BARKHAUSEN'S CRITERION

Singular network condition is compared with Barkhausen's criterion in this section.

Barkhausen gave a simple explanation of the possibility of oscillations in a positive feedback configuration. Mathematically it can be understood as follows: Let  $x$  be the input (output) of the block  $A$  ( $\beta$ ) and  $y$  be the output (input) of the block  $A$  ( $\beta$ ) in the closed loop configuration. Let  $A$  and  $\beta$  be nonzero and finite.  $y = Ax$  and  $x = \beta y$  are the equations of the  $A$  and  $\beta$  blocks. Therefore  $x = A\beta x$  and  $y = A\beta y$ . This implies that  $A\beta = 1$  if  $x \neq 0$  and  $A\beta = 1$  if  $y \neq 0$ . If there are oscillations in the network then  $x \neq 0$  and / or  $y \neq 0$ . Therefore  $A\beta = 1$  is a necessary condition for oscillations. But  $A\beta = k$  where  $k$  is a constant or a function of frequency,  $x = 0$  and  $y = 0$  also satisfy the equations. This suggests that  $A\beta = 1$

is not a sufficient condition. (i)  $A\beta = 1$  and (ii)  $x \neq 0$  and / or  $y \neq 0$  are sufficient conditions. But we cannot assume condition (ii) because that is what we want to prove. Thus  $A\beta = 1$  is a necessary but not a sufficient condition for oscillations when  $A$  and  $\beta$  are nonzero and finite. Some recent examples of circuits also suggest this [4–6]. Let us study this using singular network condition. Let  $N_{ac}$  derived from  $N$  be a single connected component. Then the equations of  $N_{ac}$  comprising  $A$  and  $\beta$  blocks can be written as

$$\begin{bmatrix} A & -1 \\ 1 & -\beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (11)$$

follows from the theorem that  $A\beta = 1$  is both necessary and sufficient for potential oscillations. In other words sinusoidal oscillations will be observed in at least one component or a group of components of the network if (i)  $N_{ac}$  derived from  $N$  is a single connected component (ii)  $A\beta = 1$  where  $A$  and  $\beta$  are nonzero and finite and (iii) appropriate DC inputs and / or initial conditions are present in  $N$ . Often, it is a simple task to satisfy condition (iii).

Barkhausen's arguments explain the possibility of oscillations when  $A$  and  $\beta$  are nonzero and finite. But there are other possibilities. A positive feedback loop can have oscillations even when  $A\beta \neq 1$ . Consider a situation in which  $A$  is an ideal linear operational amplifier. Its gain is infinity implying that its input  $x$  is zero. i.e.,  $x = 0$  is the equation of the

$A$  block.  $y$  can be nonzero if  $\beta$  is zero in the equation  $x = \beta y$  at the frequency of oscillation. This gives

$$\begin{bmatrix} 1 & 0 \\ 1 & -\beta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (12)$$

This does not satisfy  $A\beta = 1$  condition. But singular network condition is satisfied if  $\beta = 0$  showing that the feedback loop is a potential oscillator. Similarly the feedback loop can oscillate when  $\beta$  is infinity and  $A$  is zero at the frequency of oscillation. These are only three cases of  $A$  and  $\beta$ . But there are nine possible cases in all as  $A$  and  $\beta$  can take any one of the three values  $0, \infty$  and finite. In general  $A$  and  $\beta$  are ratios of polynomials. Therefore if  $(s^2 + \omega_0^2)$  is a factor of the numerator polynomial of  $A$  or  $\beta$  then it will be zero for  $s = j\omega_0$ . If it is a factor of the denominator then it will be infinity for  $s = j\omega_0$ . It can be infinity because of a linear operational amplifier also. In all other cases  $A$  and  $\beta$  will be finite for  $s = j\omega_0$ . If  $A$  and  $\beta$  are both zero, the determinant of the matrix in equation (5) is nonzero. So oscillations are not possible. If both  $A$  and  $\beta$  are infinity,  $x = 0$  and  $y = 0$  are the equations. This gives a unit matrix. Since it is nonsingular oscillations are not possible. If  $A$  is infinity and  $\beta$  is finite the equations are  $x = 0$  and  $x - \beta y = 0$ . This gives a nonsingular matrix. So oscillations are not possible. Similarly oscillations are not possible in other cases also. Conversely let there be oscillations in the network. If  $x$  and  $y$  are both nonzero,  $A$  and  $\beta$  must be finite from the equations  $y = Ax$  and  $x = \beta y$ . Further  $A\beta = 1$  from the theorem. If  $x = 0$  and  $y$  is nonzero then  $A$  is infinity and  $\beta = 0$  from the theorem. If  $y = 0$  and  $x$  is nonzero then  $\beta$  is infinity and  $A = 0$  from the theorem. This proves the following statement. *Oscillations are possible in a positive feedback loop if and only if (a)  $A$  and  $\beta$  are nonzero and finite and  $A\beta = 1$  or (b)  $A$  is zero and  $\beta$  is*

infinity or (c)  $A$  is infinity and  $\beta$  is zero. This analysis shows that  $A\beta = 1$  is neither necessary nor sufficient if we restrict to Barkhausen's arguments.

Remarks:

1. Barkhausen's arguments cannot be used on an arbitrary network where  $A$  and  $\beta$  blocks do not exist or they cannot be easily identified. But an arbitrary network can oscillate. The capability to test a network for oscillations is useful. This allows us to think of new oscillator circuits. Further such information is important to understand the stability of circuits particularly at high frequencies where parasitic elements come into play.
2. In the general case Barkhausen's criterion is tedious even when it works. Amplifier and feedback blocks have to be determined correctly. Their gains have to be calculated carefully taking appropriate loading into account. We need to verify whether reverse flow of signal in  $A$  and  $\beta$  blocks is negligible or not. Compared to this testing singular network condition is straight forward.
3. Let  $N$  be a planar network. Let  $N_d$  be its dual. Since the mesh equations of one network are node equations of the other,  $N_d$  satisfies singular network condition if  $N$  satisfies singular network condition. Therefore the dual of an oscillator is also an oscillator. A parallel R L C circuit is one such example.
4. Let  $N_a$  be the adjoint network [9] of an oscillator network  $N$ . The node admittance matrix of  $N_a$  is transpose of the node admittance matrix of  $N$ . Therefore  $N_a$  is a potential oscillator if  $N$  is a potential oscillator.
5. It is generally believed that electrical noise plays a role in starting oscillations. But the mathematical analysis presented above does not require this point of view. Instead it emphasizes appropriate D C inputs and initial conditions.
6. Consider a two port network. All linear elements, D.C. independent sources and initial conditions are allowed inside the black box. Let its equations be of the form
 
$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} + \begin{bmatrix} k_1 \\ k_2 \end{bmatrix} \quad (14)$$
 where  $k_1$  and  $k_2$  are real constants decided by the D C sources and initial conditions. Leave the ports open. i.e.,  $I_1 = 0$  and  $I_2 = 0$ . Then the two port is a potential oscillator if  $y_{11}y_{22} - y_{12}y_{21} = 0$  at the frequency of oscillation. If the ports are shorted, the two port is a potential oscillator if the Z parameter matrix is singular. Similarly two ports characterized by other parameters like hybrid and transmission parameters can also give rise to oscillations with appropriate terminations. A series R L C circuit with R<sub>neg</sub> can be looked upon as a two port with D C voltage source connected to one port and the output taken across the capacitor. Then  $V_1 = 0$  and  $I_2 = 0$  in  $N_{ac}$ . It can be shown to be an oscillator using hybrid parameters.
7. Consider a cascade connection of two two ports. The overall parameters of the resulting two port can be determined. Hence the above arguments can be applied. This implies that a cascade of  $A$  and  $\beta$  blocks can give rise to oscillations under appropriate conditions.
8. Next consider a parallel connection of  $A$  and  $\beta$  blocks (Fig.1). If  $x$  ( $y$ ) is the input (output) of these blocks, the equations of the configuration are  $y = Ax$ ,  $y = \beta x$ .

Singular network condition tells us that the network oscillates if and only if  $A = \beta$ . As a special case let  $A$  be an ideal operational amplifier with infinite gain then  $\beta$  must have a pole at the frequency of oscillation.

9. Let  $A$  be a difference amplifier. Let  $x$  and  $z$  be its inputs and  $y$  be its output.  $x$  and  $z$  are outputs and  $y$  is input of the  $\beta$  block. Let  $y = A(x - z)$  and  $x = \beta_1 y$  and  $z = \beta_2 y$ . According to the theorem this configuration is a potential oscillator if and only if  $A(\beta_1 - \beta_2) = 1$ .
10. Let  $N$  be any network. Let  $x$  and  $y$  be any two network variables, say, voltages of two nodes. Write the equations of the network and eliminate all variables except  $x$  and  $y$ . Then equations of  $N_{ac}$  involving  $x$  and  $y$  will be of the form

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \quad (15)$$

$N$  will oscillate for all values of the complex constants  $a_{11}$ ,  $a_{12}$ ,  $a_{21}$  and  $a_{22}$  satisfying  $a_{11}a_{22} - a_{12}a_{21} = 0$ . Consider a special case of this in which  $a_{11} = A$ ,  $a_{22} = -\beta$ ,  $a_{12} = -1$  and  $a_{21} = 1$ . Positive feedback loop configuration of Barkhausen belongs to this category. Thus it is clear that positive feedback loop is one of the many possibilities to generate oscillations.

#### IV CONCLUSIONS

It is shown that a network generates sinusoidal oscillations if and only if (i) the rank of the matrix of the equations of the steady state part of the network is one less than its size at the frequency of oscillation and (ii) appropriate D C inputs / initial conditions exist. This allows us to think of oscillators without using positive feedback. Barkhausen's criterion, when applicable, is a special case of this.

#### REFERENCES

- [1] J. Millman and A. Grabel, „Microelectronics”, Second Edition, *Mc Graw – Hill*, New York, 1987
- [2] A. S. Sedra and K. C. Smith, „Microelectronic Circuits”, Seventh Edition, *Oxford University Press*, New York, 2014
- [3] R. W. Rhea, „Discrete Oscillator Design”, *Artech House*, Boston, 2010
- [4] E. Lindberg, „The Barkhausen Criterion (Observation ?)”, *Proceedings of 18<sup>th</sup> IEEE Workshop on nonlinear dynamics of Electronic systems*, Dresden, Germany, pp.15 – 18, 26 – 28 May 2010
- [5] V. Singh, „Discussion on Barkhausen and Nyquist Criteria”, *Analog Integrated Circuits and Signal Processing*, Springer, 62, pp. 327 – 332, 2010
- [6] L. Von Wangenheim, „On the Barkhausen and Nyquist stability Criteria”, *Analog Integrated Circuits and Signal Processing*, Springer, 66, pp. 139 -141, 2010
- [7] M. E. Van Valkenburg, „Introduction to Modern Network Synthesis”, *Wiley International Edition*, New York, 1966
- [8] N. Deo, „Graph theory with applications to engineering and computer science”, *Prentice – Hall*, Englewood Cliffs, New Jersey, 1974
- [9] L. O. Chua and P. M. Lin, „Computer aided analysis of electronic circuits : Algorithms and Computational Techniques” *Prentice Hall*, Englewood Cliffs, New Jersey, 1975