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Robust Estimation in VaR Modelling - Univariate Approaches using Bounded Innovation Propagation and Regression Quantiles Methodology

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Abstract

In the paper we present robust estimation methods based on bounded innovation propagation filters and quantile regression, applied to measure Value at Risk. To illustrate advantage connected with the robust methods, we compare VaR forecasts of several group of instruments in the period of high uncertainty on the financial markets with the ones modelled using traditional quasi-likelihood estimation. For comparative purpose we use three groups of tests i.e. based on Bernoulli trial models, on decision making aspect, and on the expected shortfall.

Keywords: Robust estimation, quantile regression, CAViaR, ARMA-GARCH models.

JEL Classification: C22, G17

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1 Introduction

Value at Risk is the standard tool used to measure potential loss of the instrument or portfolio that would be reached or exceeded with a given probability (usually 1%) over a fixed time horizon. It is employed for internal control and regulatory reporting. VaR is a characteristic of the distribution of the future value of instrument. In practice, this probability distribution is unknown and is replaced by statistical model. There are several VaR models but the differences between them are due to the manner in which the distribution is constructed. In linear VaR model the distribution of risk factor returns is assumed to be normal, and the portfolio is required to be linear. The historical simulation model uses a large quantity of historical data to estimate VaR, but makes minimal assumption about risk factor return distribution (only concerning stability of distribution). Monte Carlo approach may be used with great diversity of risk factor return distribution.

Recently the most popular are approaches which are considerably more flexible than the most parametric models used to this aim. One of them are GARCH models estimated with use of the QML method based on heavy tailed likelihood. Nonetheless, the significant kurtosis of the standardized residuals, although smaller than in the raw data, is still noticed (Bollerslev and Wooldridge (1992)).

According to Frances and Ghijsels (1999), this phenomenon stems from *additive outliers* in rate of returns. For the most part such outliers are not properly included in the ARMA- and-GARCH-class of models. This leads to bias of the estimator in the conditional mean and variance models. This phenomenon is observed by Andersen and Bollerslev (1988), Ledorter (1989) and Jorion (1995). Conditional mean model should be included in variance modelling, because incorrect identification of the equation for the conditional mean may affect the results of testing for conditional heteroscedasticity (Lumsdaine and Ng (1999)).

There are two methods for dealing with the occurrence of outliers. The first is to estimate the model parameters using the maximum likelihood method, and then to diagnose the innovations for the outliers identification. However, because of the possibility of masking effect of outliers, the method may be inaccurate.

The second way is to use robust estimators. This method was developed originally by Huber (1973). It limits the influence of outliers on estimators of parameters, and this in turn translates into robustness when error distribution deviates from the assumed one.

The most important estimators for ARMA and GARCH models were M-estimators, filtered M-estimators (Martin, Samarov, Vandaele (1983)), filtered S-estimators (Martin and Yohai (1996)), or filtered τ-estimators (Bianco, Garcia, Yohai (2005)). Unfortunately filtered estimators are asymptotically biased and in case of M-estimators, large outliers have still strong effect on the estimators.

Muler and Yohai (2008) and Muler, Pena and Yohai (2009) for the ARMA and GARCH model presented robust estimators which are referred to as BIP-estimators (bounded propagation innovation). In both models the first estimator is determined by minimizing the appropriately modified likelihood function, and the second is constructed similarly but has an additional mechanism to reduce the propagation of the effect of an outlier on the subsequent conditional variance estimators. The authors presented the asymptotic properties of both estimators, proved their consistency and asymptotic normality.

Another, completely different robust estimation method, is regression quantiles, described by Koenker and Bassett (1978), which yields the ordinary sample quantiles. The linear function of order statistics often exhibits desirable robustness, particularly to heavy-tailed distribution and outlying observation. The paper presents the robust approach also due to its wide applicability in risk measurement and portfolio allocation. The most popular is CaViaR approach presented by Engle and Manganelli (2004): time-varing VaR is modelled directly via autoregression.

The robust approach among the Polish researchers was developed by Zieliński (1983). Doman (2005), Kaszuba (2008), Majewska (2008), Orwat (2007), Trzpiot (2008a), Trzpiot and Majewska (2008) have analysed robust estimation methods in scope of risk management and portfolio analysis. However, the robust estimation methods were not analysed for the period of high volatility, and forecasts generated by the models are not compared. Our study focuses on such comparisons. In addition, we examine whether the models based on robust estimation accurately reflect variability clustering, presence of fat tails and skewness of the distribution.

This paper is organised as follows. In the first section we present basic notation. In section 2 we discuss foundations of robust estimation of the ARMA-GARCH class of models and regression quantiles. Then the application to risk measurement in section 3 is provided. Section 4 concludes.

2 Notations and preliminaries

Fox (1972) considered types of outliers that may occur in time series. The first class is that of innovative outliers which may be modelled using ARMA-GARCH framework. The innovation outlier first appears at the moment t and afterwards similar large values are observed close to original line of slope. In consequence there is one innovation outlier and a certain number of leverage points which can potentially improve accuracy of estimation (figure 1 - 2). The second type are additive outliers that affects a single observation. In this case the observations do not obey classical ARMA-GARCH model (figure 3 - 4). Hence large outliers at the time t can affect estimated values of μ_{t+k} and h_{t+k} up to several periods $t+k$. This is caused by the fact that outliers are not adequately included in these models and some extension of model is needed.

Rate of return of the financial instrument in presence of additive outliers can be written as ARMA(p, q)-GARCH(r, m) models with an indicator function (Hamilton

(1994), Muler, Pena and Yohai (2009)):

$$r_t^* = r_t + aI_t \tag{1}$$

$$\phi(L)(r_t - \mu) = \theta(L)u_t \tag{2}$$

where $u_t = \sqrt{h_t}z_t$, $z_t \sim N(0,1)$, h_t is a conditional variance of u_t (types of conditional variance models analysed in paper are contained in table 2), a_t corresponds to size of innovation and I_t is the indicator function that takes value 1 if the innovation occurs at time t, and 0 otherwise.

Figure 1: Innovation Outliers in time series

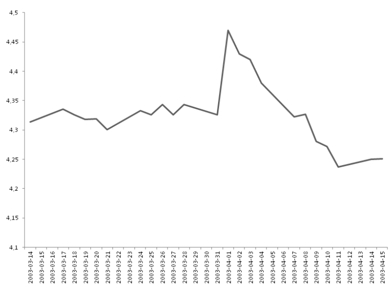


Figure 2: Example of Innovation Outliers - scatterplot

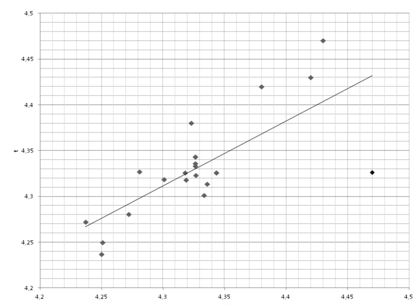


Figure 3: Additive Outliers in time series

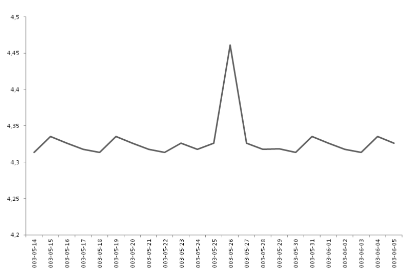
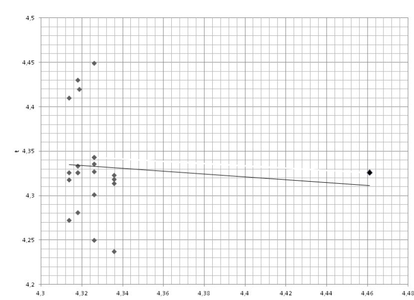


Figure 4: Additive Outliers - scatterplot



Let $\delta(L) = \phi^{-1}(L)\theta(L) = 1 + \sum_{i=1}^{\infty} \delta_i L^i$, then equation (2) can be rewritten as follows:

$$r_t = \mu_t + u_t \tag{3}$$

Table 1: Conditional variance h_t.

Model	Analytical form	Restriction
GARCH$(r,\ m)$ (*Bollerslev (1986)*)	$h_t = \alpha_0 + \sum_{i=1}^{r} \alpha_i u_{t-i}^2 + \sum_{j=1}^{m} \beta_j h_{t-j}$	$\alpha_0 > 0,\ \alpha_i \geq 0,$ $\beta_j \geq 0\ ,\ r > 0\ m \geq 0$
EGARCH$(r,\ m)$ (*Nelson (1991)*)	$\log h_t = \alpha_0 + \sum_{j=1}^{r} \beta_j \log h_{t-j} +$ $+ \sum_{i=1}^{m} \alpha_i \{\lvert\upsilon_{t-i}\rvert + E\lvert\upsilon_{t-i}\rvert + \kappa\upsilon_{t-i}\}$	$r > 0$
GJR(r,m) (*Glosten, Jagannathan, Runkle (1993)*)	$h_t = \alpha_0 + \sum_{j=1}^{r} \beta_j h_{t-j} +$ $+ \sum_{i=1}^{m} \left(\alpha_i u_{t-i}^2 + \kappa_i u_{t-i}^2 I_{t-i}\right)$	$r > 0,\ m \geq 0$

Source: Hamilton (1994)

$$\mu_t = \mu + \sum_{i=1}^{\infty} \delta_i u_{t-i} \tag{4}$$

where μ_t is the conditional mean of r_t. In case of quasi-maximum likelihood method

Table 2: Error distribution specification.

Error distribution	Density function
Normal(0,1)	$f(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}u^2},$
Student-t (*Bollerslev (1986)*)	$f(u_t, \upsilon) = \frac{\Gamma[(\upsilon+1)/2]}{\sqrt{\upsilon\pi}\Gamma[\upsilon/2]} \left(1 + \frac{u_t^2}{\upsilon}\right)^{-\frac{\upsilon+1}{2}}$ $\Gamma(\cdot)$ is the gamma function
GED (*Nelson (1991)*)	$f(u_t, \upsilon) = \frac{\upsilon exp\{-\frac{1}{2}\lvert\frac{u_t}{\lambda}\rvert^{\upsilon}\}}{\lambda\Gamma\left(\frac{1}{\upsilon}\right)} 2^{-\frac{\upsilon+1}{\upsilon}}$ where λ is constant given by $\lambda = \left[\frac{\Gamma\left(\frac{1}{\upsilon}\right)}{\Gamma\left(\frac{3}{\upsilon}\right)} 2^{-\frac{2}{\upsilon}}\right]^{\frac{1}{2}};$ $\Gamma(\cdot)$ is the gamma function.

Source: Hamilton (1994)

of estimation (QML), the general form of likelihood function is described as follows:

$$L_T(\theta) = \sum_{t=p+1}^{T} \log f\left(u_t - \log h_t(\theta)\right) \tag{5}$$

where f is assumed density function of errors (candidate functions included in our empirical application are described in table 2) and $\theta = (\alpha_0, \alpha_1, \ldots, \alpha_r, \beta_1, \ldots, \beta_m)$ is estimated vector of parameters.
The k-period-ahead forecasts of conditional mean and volatility are given by:

$$(r_{t+k|t} - \mu) = \hat{\phi}_0 + \sum_{i=1}^{p} \hat{\phi}_i (r_{t+k-i|t} - \mu) + \sum_{j=1}^{q} \hat{\theta}_j u_{t+k-j|t} \tag{6}$$

$$h_{t+k|t} = \hat{\alpha}_0 + \sum_{i=1}^{r} \hat{\alpha}_i u^2_{t+k-i|t} + \sum_{j=1}^{m} \hat{\beta}_j h_{t+k-j|t} \tag{7}$$

3 Methodology

3.1 Bounded innovation propagation methodology

Muler and Yohai (2008) and Muler, Pena and Yohai (2009) show that the QML estimators of ARMA and GARCH models are not efficient in presence of outliers. Hence, they proposed to use robust filters that reduce the propagation effect of the outliers on the estimated value of μ_t and h_t. In case of ARMA models they replace equation (3)-(4) by auxiliary models for the contaminated returns r_t^*, denoted as BIP-ARMA (*boundary innovation propagation-ARMA*):

$$r_t^* = \mu_t + u_t + aI_t \tag{8}$$

$$\mu_t = \mu + \sum_{i=1}^{\infty} \delta_j \sqrt{h_{t-i}}\, \eta \left(\frac{u_{t-i}}{\sqrt{h_{t-i}}} \right) \tag{9}$$

The same idea is used in case of GARCH model to limit the propagation effect of potentially additive outliers on the future volatility. In case of the QML estimator, the factor aI_{t-i}, which includes occurrence of outliers, has no impact on h_t, while assuming GARCH process of r_t^* it has decaying effect on volatility prediction. The Muler and Yohai (2008) present auxiliary GARCH model with imposed filters on standardised residuals. They called this model the BM-GARCH (*boundary M-GARCH*):

$$h_{t,k} = \alpha_0 + \sum_{i=1}^{r} \alpha_i h_{t-i,k}\, \omega \left(\frac{u^2_{t-i}}{h_{t-i,k}} \right) + \sum_{i=1}^{m} \beta_i h_{t-i,k} \tag{10}$$

Using the same methodology as for the GARCH models, the residuals could be also downweighted in *exponential* GARCH and GJR models to capture asymmetric effect in volatility, especially occurring in equity. The analytical form of BM-EGARCH model is as follows:

$$\log h_t = \alpha_0 + \sum_{j=1}^{r} \beta_i \log h_{t-j} + \sum_{i=1}^{m} \alpha_i \left\{ \frac{|u_{t-i}|}{\sqrt{h_{t-j}}} - E\left(\frac{|u_{t-i}|}{\sqrt{h_{t-j}}}\right) \right\} + \sum_{i=1}^{m} L_i \frac{u_{t-i}}{\sqrt{h_{t-j}}} \quad (11)$$

and BM-GJR is given by:

$$h_t = \alpha_0 + \sum_{j=1}^{r} \beta_j h_{t-j} + \sum_{i=1}^{m} \alpha_i \omega \left(\frac{u_{t-i}^2}{h_{t-j}}\right) h_{t-j} + \sum_{i=1}^{m} L_i I_{t-i} \omega \left(\frac{u_{t-i}^2}{h_{t-j}}\right) h_{t-j} \quad (12)$$

Muler and Yohai (2008) and Muler, Pena and Yohai (2009), due to inefficiency of the QML estimation of the BIP-ARMA and BM-GARCH models, propose using the M-estimator that minimizes the average value of the objective function ρ, evaluated at the log-transform of squared standardised returns, i.e.:

$$\hat{\theta} = \arg\min_{\theta} \frac{1}{T} \sum_{t=p+1}^{T} \rho\left(\log \frac{u_t^2}{h_t}\right) \quad (13)$$

where u_t is given by equation (9), h_t is one of the selected GARCH-type models (10-12) and $\rho = -\log(f)$.
The choice of ρ functions trades off robustness versus efficiency. Muler and Yohai (2008) recommended the ones associated with Normal distribution of residuals: BM1-GARCH with loss function:

$$\rho_1(z) = m(\rho_N(z)) \quad (14)$$

and BM2-GARCH with loss function:

$$\rho_2(z) = 0.8m\left(\frac{\rho_N(z)}{0.8}\right) \quad (15)$$

where

$$\rho_N(z) = \frac{1}{\sqrt{2\pi}} \exp[-\frac{1}{2}(\exp(z) - z)] \quad (16)$$

and m is smoothed version of

$$m(x) = xI(x \leqslant 4.02) + 4.02I(> 4.02) \quad (17)$$

The BM1 estimator is more similar to the QML because on large interval it is equal to identity (Muler and Yohai (2008)). Boudt, Danielsson, Laurent (2013) and Laurent, Lecourt, Palm (2011) propose loss function associated with the Student-t distribution with v degrees of freedom:

$$\rho_{t_v}(z) = \frac{(v+1)}{2}\log\left(1+\frac{\exp(z)}{v-2}\right) - z/2 \tag{18}$$

The associated loss function in case of the generalised error distribution (GED) is as follows:

$$\rho_{GED}(z,v) = -\frac{1}{2}\left|\frac{\exp(z)}{\lambda}\right|^{v} - \exp(z) \tag{19}$$

where

$$\lambda = \left[\frac{\Gamma\left(\frac{1}{v}\right)}{\Gamma\left(\frac{3}{v}\right)}2^{-\frac{2}{v}}\right]^{\frac{1}{2}}$$

The k-period-ahead forecast in the BIP-ARMA model is obtained by:

$$\hat{r}^*_{t+k|t} = \mu - \sum_{i=1}^{\infty}\delta_i u^*_{t+k-i|t} = \mu - (\theta(L)^{-1}\phi(L)-1))u^*_{t+k-i|t} \tag{20}$$

where $r^*_t = \mu$ for $t \leqslant 0$. The filtered innovation residual for period t is calculated as:

$$\hat{u}^*_{t+k|t} = r_{t+k|t} - \hat{r}^*_{t+k|t}$$

and cleaned value of r^*_{t+k}:

$$r^*_{t+k|t} = \hat{r}^*_{t+k|t} + \sqrt{\hat{h}_{t+k|t}}\eta\left(\frac{\hat{u}^*_{t+k|t}}{\sqrt{\hat{h}_{t+k|t}}}\right)$$

The k-period-ahead forecast in the BIP-ARMA model can be also described as:

$$\hat{r}^*_{t+k|t} = \mu + \sum_{i=1}^{p}\hat{\phi}_i(r^*_{t+k-i|t}-\mu) + \sum_{j=1}^{q}\hat{\theta}_j\sqrt{\hat{h}_{t+k-j|t}}\eta\left(\frac{\hat{u}^*_{t+k-j|t}}{\sqrt{\hat{h}_{t+k-j|t}}}\right) \tag{21}$$

In the above expressions, $\hat{h}_{t+k}$ is the k-period-ahead forecast of volatility, which in case of the GARCH model is formulated as:

$$\hat{h}_{t+k|t} = \hat{\alpha}_0 + \sum_{i=1}^{r}\hat{\alpha}_i\hat{h}_{t+k-i|t}\omega\left(\frac{\hat{u}^{*2}_{t+k-i|t}}{\hat{h}_{t+k-i|t}}\right) + \sum_{j=1}^{m}\hat{\beta}_j\hat{h}_{t+k-j|t} \tag{22}$$

3.2 Quantile regression

According to many researchers, quantile regression is one of the most important achievements in the area of robust methods for linear models. Koenker and Bassett (1978) proved asymptotic normality of these estimators and showed that they are of comparable effectiveness to least squares estimator for linear Gaussian models, while significantly outperform the least squares estimator for a wide class of non-Gaussian error distributions. They generalized a simple minimization problem yielding the ordinary sample quantiles in the location model to the regression model.
Let r_t be written as a regression process:

$$r_t = x_t^{'}\beta - u_t \tag{23}$$

where $x_t^{'}\beta$ is α-quantile for r_t with given vector of explanatory variables x_t measurable relative to σ-algebra ψ_{t-1}, where ψ_{t-1} is the information set available at time $t-1$. Then k-dimensional vector of β parameters minimizing the function:

$$\min_{\beta}\left(\sum_{t|r_t \geq x_t\beta} \theta\left|y_t - x_t\beta\right| + \sum_{t|r_t < x_t\beta} (1-\theta)\left|r_t - x_t\beta\right|\right) \tag{24}$$

is a general form of the regression quantiles.

3.3 Value at Risk

Let q_t denote α-quantile of variable distribution z_t. Then one-day-ahead VaR forecast for long and short position at the significance level $\alpha = 1\%$ is:

$$VaR_{t+1}^{l}(\alpha) = -\mu_{t+1|t} - \sqrt{h_{t+1|t}}q_{\alpha} \tag{25}$$

$$VaR_{t+1}^{s}(\alpha) = \mu_{t+1|t} + \sqrt{h_{t+1|t}}q_{1-\alpha} \tag{26}$$

where $\mu_{t+1|t}$ and $h_{t+1|t}$ denote the one-day-ahead forecasts of conditional mean and volatility, respectively, determined in ARMA-GARCH class of models estimated using QML (equations (6)-(7)) or robust method based on bounded innovation propagation methodology (equations: (21)-(22)).
The CAViaR models, based on the regression quantiles framework, directly describe the quantile of the rate of return. The general form of CAViaR models is described by the equation:

$$VaR_t = f(x_t, \beta_\theta) = \beta_0 + \sum_{i=1}^{p} \beta_i VaR_{t-i} + l(\beta_{p+1}, ..., \beta_{p+q}; \psi_{t-1}) \tag{27}$$

One-period-ahead forecast of *VaR* based on CAViaR models is determined directly from models with analytical forms given in table 3.3.

Table 3: CaViaR models.

Model	Analytical form
AD-Adaptive	$VaR_t^l = VaR_{t-1}^l + \beta_0[I(y_{t-1} \leq -VaR_{t-1}^l) - \alpha]$ $VaR_t^s = VaR_{t-1}^s + \beta_0[I(y_{t-1} \geq -VaR_{t-1}^s) - \alpha]$
SAV- Symmetric Absolute Value	$VaR_t^l = \beta_0 + \beta_1 VaR_{t-1}^l + \beta_2 \|y_{t-1}\|$ $VaR_t^s = \beta_0 + \beta_1 VaR_{t-1}^s + \beta_2 \|y_{t-1}\|$
AS- Asymmetric Slope	$VaR_t = \beta_0 + \beta_1 VaR_{t-1}^l + \beta_2 (y_{t-1})^+ - \beta_3 (y_{t-1})^-$ $VaR_t = \beta_0 + \beta_1 VaR_{t-1}^s + \beta_2 (y_{t-1})^+ - \beta_3 (y_{t-1})^-$ where $(x)^+ = \max(x,0)$,$(x)^- = \min(x,0)$
IGARCH- Indirect GARCH	$VaR_t = \sqrt{\beta_1 + \beta_2 VaR_{t-1}^2 + \beta_3 y_{t-1}^2}$

Source: Engle and Manganelli (2004)

4 Application

4.1 Preliminary analysis

In this part we present the empirical application of robust estimation to *VaR*, and we compare the results with those obtained from models estimated using the QML method. All the calculations and analysis were made using MATLAB application.
As the dataset we use: FX rates of EUR/PLN (mid spot quotations since January 2, 2003 - 2347 observations), USD/PLN (mid spot quotations since January 3, 1994 - 4689 observations) and of CHF/PLN (mid spot quotations since January 3, 1996 - 4162 observations); indexes of WIG20 (closing quotations since January 2, 1995 - 4260 observations), FTSE100 (closing quotations since January 4, 2000 - 3032 observations) and of S&P 500 (closing quotations since January 3, 2000 - 3020 observations) and equities of TPSA (closing prices since January 4, 1999 - 3262 observations), PKN Orlen (closing prices since January 3, 2002 - 2513 observations), and of Pekao (closing prices since January 4, 1999 - 3262 observations). All quotations are up to December 31, 2011. The time series contain many discontinuities (figure (6) presents returns for

the EUR/PLN as an example).
The time series of the quotations, prices and rates of return were checked for the presence of following features: fatter tails than in the normal distribution (identified on the basis of the quantile-quantile plots, histograms - figure (7)-(8) for EUR/PLN is shown as an example - and the Lomnicki-Jarque-Bera test - results in table 15); stationarity (identified on the basis of an augmented Dickey-Fuller test (ADF) - results in table 16); autocorrelation of the rates of returns (identified on the basis of the Ljung-Box Q-test - results in table 17 - autocorrelation, ACF, and partial autocorrelation, PACF, figures); skewness, kurtosis of rates of return (checked using descriptive statistics - results in table 14); volatility clustering and leverage effect (both identified on the basis of rate of returns figures).
The most adequate ARMA-GARCH-class models which reflected the above identified phenomena are presented in table 4.

Table 4: ARMA(p,q)-GARCH(r,m)-class model selection.

Instrument	Model	Error distribution
EUR/PLN	ARMA(0,1)-GARCH(1,1)	Student-t
USD/PLN	ARMA(0,0)-EGARCH(2,2)	Student-t
CHF/PLN	ARMA(0,1)-EGARCH(1,1)	Student-t
WIG20	ARMA(1,0)-GJR(1,1)	normal
FTSE 100	ARMA(1,1)-EGARCH(2,2)	Student-t
S&P 500	ARMA(0,0)-GARCH(1,1)	GED
TPSA	ARMA(0,0)-GARCH(1,1)	GED
PKN Orlen	ARMA(0,0)-GARCH(1,1)	GED
Pekao	ARMA(0,0)-GARCH(1,1)	GED

4.2 Estimation

According to the Akaike and Schwarz information criteria, Student-t or GED distribution for residuals turned out to be the most adequate among the analysed forms, which is confirmed by significance of estimated degree of freedom parameter (ν). As an example, the estimation of ARMA-GARCH models for EUR/PLN based on QML and robust estimation BIP-BM1 and BIP-BM2 are presented in the table 18.
In the Symmetric Absolute Value models (SAV), parameters β_0 for all analysed type of instruments, and parameter β_1 for Pekao and PKN Orlen, β_3 for USD/PLN and Pekao are insignificant. In case of the Asymmetric Slope models (AS) and Indirect GARCH, only β_1 parameter is significant. Parameter β_0 of the Adaptive models is

insignificant for all analysed time series. As an example the results of the CAViaR models estimation for EUR/PLN are presented in table 19.
In preliminary analysis the ARCH test indicated heteroscedasticity (results in table 17). Post-estimation analysis based on the standardized residuals derived from ARMA-GARCH-class models, estimated by the QML and robust method, indicates that most of the selected models sufficiently explain the heteroscedasticity in time series. Exception is the ARMA(1,1)-EGARCH(2,2) model for FTSE100 that does not entirely explain heteroscedasticity.

4.3 Results

The aim of the empirical research is examination and comparison of *VaR* forecasts obtained on the basis of robust estimation (i.e. using bounded innovation propagation methodology in ARMA-GARCH-class models and regression quantiles in case of CaViaR models) with the ones based on volatility described by the ARMA-GARCH-class models estimated using the QML.
As a comparative criterion three groups of test/measures are used. The first group includes tests based on the Bernoulli trials, like *back-test*, the LR Test of Unconditional Coverage (LR_{UC}) proposed by Kupiec (1995), the Joint Test of Coverage and Independence (LR_{CC}), and Dynamic Quantile Test (DQ) presented by Engle and Manganelli (2004).
The second group of measures is based on the decision-making aspect and includes Binary Loss (BL), which is determined by number of exceptions in the specified period (Lopez (1998), Pipień (2006)); Regulatory Loss (RL), which measures square deviations of rate of return from the VaR forecast (Lopez (1999), Pipień (2006)); and the Firm Loss (FL) that embraces lost opportunities, associated with capital that the institution must maintain in order to protect against the risk predicted by *VaR* (Sarma, Thomas, Shah (2003), Pipień (2006)).
The third group is based on expected shortfall (ES), which indicates how much an investor may lose on average when the model fails. Test statistics and formulae of measures are given in table 5. When exceptions are checked for the sample used for parameter estimation (hereinafter *in-sample*), the fraction of exceptions (i.e. the number of times the estimated VaR was breached), in case of the ARMA-GARCH class of models estimated on the basis of QML and BIP method, is close to the expected number for the chosen confidence interval. It is indicated by *back-test*. The test statistics of LR_{UC} and of LR_{CC} are not statistically significant at the 5% confidence level, which indicates adequate level of *VaR*.
In case of the CAViaR models, the LR Test of Unconditional Coverage indicates for most models overestimation *VaR* (results in table 6). The statistics of the Joint Test of Coverage and Independence are also statistically significant at 5% confidence level, which indicates the first order autocorrelation (results in table 7) .

The DQ test informs about the autocorrelation of *VaR* in subsequent periods. For the both ARMA-GARCH-class models based on QML and BIP estimation, and

Table 5: Test and measures.

Test/Measures	Statistics/Formula
LR_{UC}	$LR_{UC} = 2\left[\ln\left\{(1-\hat{p})^{T-T_*}\,\hat{p}^{T_*}\right\} - \ln\left\{(1-p)^{T-T_*}\,p^{T_*}\right\}\right] \sim \chi^2_1$ where T_* means the total number of exceptions and $\hat{p} = \frac{T_*}{T}$
LR_{CC}	$LR_{CC} = LR_{UC} + LR_{ind} \sim \chi^2_2$ $LR_{ind} = 2\left[\ln\left\{(1-\hat{\tau}_{01})^{T_{00}}\,\hat{\tau}_{01}^{T_{01}}\,(1-\hat{\tau}_{11})^{T_{10}}\,\hat{\tau}_{11}^{T_{11}}\right\} - \right.$ $\left.\ln\left\{(1-\hat{\tau})^{(T_{00}+T_{10})}\,\hat{\tau}^{(T_{01}+T_{11})_*}\right\}\right] \sim \chi^2_2$ where $\hat{\tau}_{ij} = T_{ij}/(T_{i0}+T_{i1})$; $\hat{\tau} = (T_{01}+T_{11})/T$ for $j, i = 0, 1$, T_{ij} - number of points at time $\{t; 2 \le t \le T\}$ for which the $I_t = i$ follows $I_{t+1} = j$.
DQ	$DQ = \frac{\hat{\beta}' X' X \hat{\beta}}{p(1-p)} \sim \chi^2_k$ where $\hat{\beta}$ - the OLS estimator of linear regression: $Hit = \beta_0 + \beta_1 Hit_{t-1} + \beta_2 Hit_{t-2} + \ldots + \beta_r Hit_{t-r} +$ $+\beta_{r+1} VaR_t + \beta_{r+2} x_1 + \ldots + \beta_k x_k + v_t$ and Hit - the binary variable of $\mathbf{1}_{[r_t < VaR(\alpha)]}$ (for long position).
BL	$BL = \sum_{t=T}^{T+T'} f_t^{(i)}$ $f_t^{(i)}(r_{t+n}; q_t) = \begin{cases} 1 & for \quad r_{t+n} < VaR_t \\ 0 & for \quad r_{t+n} \ge VaR_t \end{cases}$
RL	$RL = \sum_{t=T}^{T+T'} f_t^{(i)}$ $f_t^{(i)}(r_{t+n}; q_t) = \begin{cases} 1 + (r_{t+n} - VaR_t)^2 & for \quad r_{t+n} < VaR_t \\ 0 & for \quad r_{t+n} \ge VaR_t \end{cases}$
FL	$FL = \sum_{t=T}^{T+T'} f_t^{(i)}$ $f_t^{(i)}(r_{t+n}; q_t) = \begin{cases} 1 + (r_{t+n} - VaR_t)^2 & for \quad r_{t+n} < VaR_t \\ c \cdot VaR_t & for \quad r_{t+n} \ge VaR_t \end{cases}$ where $c > 0$ - opportunity cost
ESI $ESII$	$ESI = E\left(r_{t+n} < VaR_t\right)$ $ESII = \frac{E\left(r_{t+n} < VaR_t\right)}{VaR_t}$

Table 6: The LR Test of Unconditional Coverage LR_{UC} in-sample results.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN	+	+	+	+	s(-)	s(-)	l(-)s(-)
USD/PLN	+	+	+	+	s(-)	s(-)	s(-)
CHF/PLN	+	+	+	s(-)	s(-)	s(-)	s(-)
WIG20	+	+	+	+	s(-)	+	+
FTSE 100	+	+	+	s(-)	s(-)	s(-)	+
S&P 500	+	+	+	s(-)	s(-)	+	l(-)s(-)
TPSA	+	+	+	+	+	+	+
PKN Orlen	+	+	+	+	+	s(-)	+
Pekao	+	+	+	+	s(-)	s(-)	s(-)

Notations: + VaR is not overestimated or underestimated; s(-) - test rejects null hypothesis for a short position; l(-) - test rejects null hypothesis for a long position; l(-)s(-) - test rejects null hypothesis in case of both position.

Table 7: The Joint Test of Coverage and Independence LR_{CC} in-sample results.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN				l(-)	s(-)	s(-)	l(-)
USD/PLN	+	+	+	l(-)s(-)	s(-)	s(-)	s(-)
CHF/PLN	+	+	+	s(-)	s(-)	s(-)	s(-)
WIG20	+	+	+	+	s(-)	+	+
FTSE 100				+			+
S&P 500				l(-)s(-)			-
TPSA				+			s(-)
PKN Orlen							+
Pekao	+	+	+	+	s(-)	s(-)	s(-)

Notations: + lack of first order autocorrelation; s(-) - test rejects null hypothesis for short position; l(-) - test rejects null hypothesis for long position; l(-)s(-) - test rejects null hypothesis in case of both position; empty cells mean that the case was not registered in investigated periods.

for the CAViaR models, test indicates the presence of higher-order autocorrelation of *VaR* (results in table 8).

In case of *forecast*, The LR Test of Unconditional Coverage indicates overestimation or underestimation of *VaR*, especially in CAViaR models (results in table 9). Statistics of the Joint Test of Coverage and Independence, if the case occurred in investigated periods, are also low. Summary results are shown in table 10.

The *DQ* test for both the ARMA-GARCH-class of models based on QML and BIP estimation and for CAViaR models indicates the presence of higher order autocorrelation (results in table 11). Pipień (2006) indicated weakness of the LR Test of Unconditional Coverage as a tool for comparing models. He proved it by

Table 8: The DQ test in-sample results.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN	+	+	+	l(-)s(-)	l(-)s(-)	s(-)	l(-)s(-)
USD/PLN	s(-)	s(-)	s(-)	l(-)	l(-)s(-)	s(-)	l(-)s(-)
CHF/PLN	+	+	+	l(-)s(-)	s(-)	s(-)	l(-)s(-)
WIG20	+	l(-)	l(-)s(-)	l(-)s(-)	s(-)	+	l(-)s(-)
FTSE 100	+	+	+	l(-)s(-)	s(-)	+	l(-)s(-)
S&P 500	+	+	+	l(-)s(-)	l(-)	l(-)	l(-)s(-)
TPSA	l(-)	l(-)	l(-)s(-)	+	+	+	l(-)s(-)
PKN Orlen	+	+	+	l(-)	s(-)	s(-)	l(-)
Pekao	+	+	l(-)	l(-)s(-)	s(-)	s(-)	l(-)s(-)

Notations: + lack of higher-order autocorrelation; s(-) - test rejects null hypothesis for short position; l(-) - test rejects null hypothesis for long position; l(-)s(-) - test rejects null hypothesis in case of both position.

Table 9: The LR Test of Unconditional Coverage LR_{UC} result for the forecast.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN	+	+	+	+	+	s(-)	l(-)s(-)
USD/PLN	s(-)	s(-)	s(-)	+	l(-)s(-)	s(-)	l(-)
CHF/PLN	+	+	+	+	s(-)	s(-)	+
WIG20	+	+	+	+	s(-)	s(-)	s(-)
FTSE 100	+	+	+	l(-)s(-)	+	+	l(-)s(-)
S&P 500	l(-)	l(-)	l(-)	l(-)	l(-)s(-)	l(-)	l(-)s(-)
TPSA	s(-)	l(-)	l(-)	+	+	s(-)	+
PKN Orlen	+	+	+	s(-)	+	+	+
Pekao	+	+	+	s(-)	l(-)	+	+

Notations: + VaR is not overestimated or underestimated; s(-) - test rejects null hypothesis for short position; l(-) - test rejects null hypothesis for long position; l(-)s(-) - test rejects null hypothesis in case of both position; empty cells mean that the case was not registered in investigated periods.

comparing the predictions of *VaR* obtained with use of the Bayesian methods with results of analysis performed by M. Doman and R. Doman (2004) based on the standard forecast. He indicates the difficulty in comparing the results of the specified *VaR* models and argues that the results of the test depend on the number of generated forecasts. For lower number of forecasts, the test indicates better adequacy of model, and excludes the adequacy in case of large number of forecasts. Therefore, as a second set of measures, we use tests based on decision-making aspects.
The Binary Loss (BL) provides a ranking of the specification models independent of the assumed significance level of *VaR*. It is based on the number of exceptions. The most adequate models for forecasts are presented in table 12. This criterion favours models that overestimate *VaR* and provides a low score for models that generate

Table 10: The Joint Test of Coverage and Independence LR_{CC} result for the forecast.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN							
USD/PLN	+						
CHF/PLN				+			
WIG20							
FTSE 100						+	
S&P 500							
TPSA							
PKN Orlen							
Pekao							

Notations: + lack of first order autocorrelation; s(-) - test rejects null hypothesis for short position; l(-) - test rejects null hypothesis for long position; l(-)s(-) - test rejects null hypothesis in case of both position; empty cells mean that the case was not registered in investigated periods.

Table 11: The DQ test result for the forecast.

Instrument	ARMA-GARCH class			AD	AS	IGARCH	SAV
	QML	BIP-BM1	BIP-BM2	regression quantiles			
EUR/PLN	s(-)	s(-)	s(-)	s(-)	s(-)	s(-)	+
USD/PLN	s(-)	s(-)	s(-)	s(-)	s(-)	s(-)	s(-)
CHF/PLN	+	s(-)	s(-)	s(-)	s(-)	s(-)	+
WIG20	+	+	+	l(-)	+	+	l(-)
FTSE 100	+	+	+	+	+	l(-)	+
S&P 500	l(-)	l(-)	l(-)	+	l(-)	l(-)	+
TPSA	+	+	+	+	+	+	+
PKN Orlen	l(-)	l(-)	l(-)	l(-)	+	+	l(-)
Pekao	l(-)	l(-)	l(-)	+	+	l(-)	l(-)

Notations: + lack of higher-order autocorrelation; s(-) - test rejects null hypothesis for short position; l(-) - test rejects null hypothesis for long position; l(-)s(-) - test rejects null hypothesis in case of both position.

liberal VaR forecasts. According to that criterion, in our comparison the best seem to be the CaViaR models.

While Binary Loss (BL) informs only about occurrence of exception, the loss from the regulator perspective (RL) enables also to compare for different models amount of losses received that are associated with the observed exception. According to this criterion, for most analysed instruments, the best models turn out to be the ones based on the robust estimation (table 12).

The loss from firm perspective (FL) includes the opportunity cost, i.e. the cost of capital that institution have to set aside to cover market risk. The model that generates too conservative VaR forecasts, will have lower scores, as a result of an excessive amount of capital held to protect against the risk. In this case, the best are

Table 12: The models most adequate according to the second group of measures.

Instrument	Binary Loss		Regulatory Loss		Firm Loss	
	l	s	l	s	l	s
EUR/PLN	SAV	SAV	SAV	SAV	AS	BIP-BM1
USD/PLN	AS	SAV	SAV	SAV	AS	SAV
CHF/PLN	SAV	QML	QML	QML	AS	BIP-BM1
WIG20	AS	SAV	BIP-BM1	SAV	QML	AS
FTSE100	BIP-BM1	BIP-BM1	BIP-BM2	BIP-BM2	AS	AS
S&P500	SAV	AD	SAV	AD	AD	AS
TP S.A.	SAV	IGARCH	SAV	SAV	SAV	BIP-BM2
PKN Orlen	AD	AD	BIP-BM2	AD	BIP-BM2	BIP-BM2
Pekao S.A.	AD	AD	AD	AD	AS	AS

Notations: s - short position; l - long position; e.g. AD(l) - most adequate model for long position turned out to be Adaptive model.

Table 13: The models most adequate according to the third group of measures.

Instrument	ES I		ES II	
	l	s	l	s
EUR/PLN	SAV	SAV	SAV	SAV
USD/PLN	BIP-BM1	BIP-BM1	SAV	AD
CHF/PLN	BIP-BM1	AS	AS	BIP-BM2
WIG20	BIP-BM2	SAV	SAV	SAV
FTSE100	BIP-BM2	BIP-BM2	IGARCH	QML
S&P500	SAV	AD	SAV	AD
TP S.A.	BIP-BM2	BIP-BM2	BIP-BM2	BIP-BM2
PKN Orlen S.A.	IGARCH	AD	BIP-BM2	AD
Pekao S.A.	AS	AS	AS	BM1

Notations: s- short position; l- long position; e.g. AD(l) - most adequate model for long position turned out to be Adaptive model.

the ARMA-GARCH models based on the robust estimation (results in table 12) The third group of measures is purely informational. They are similar in nature to the test based on decision-making aspects. They inform about the loss in case of exceptions. Results of *Expected Shortfall I* and *Expected Shortfall II* are shown in table 13. In terms of this criterion, the best models turn out to be also ARMA-GARCH class based on the robust estimation. Tables 20-21 contain calculations of above tests for EUR/PLN, as an example. Figures 9-22 show the projected VaR of EUR/PLN for both long and short positions received on the basis of analysed models.

Conclusions

In the paper we have briefly discussed some of robust estimation methods used to calculate risk measure. We have also proposed to broaden robust estimation method BIP to asymmetric GARCH type models that better fit to daily data of equity and equity indexes. The purpose of the study was to compare the methods with quasi maximum likelihood estimation and to present consequences of their implementation in VaR modelling.

We analysed time series of FX rates, indexes and equities, changing the length of series, to check whether this would have significant impact on the forecasts. We would like to emphasize that we applied our analysis to series in high volatility period (due to sub-prime crisis), because this has not yet been covered in the current literature concerning robust estimation method.

The analysis showed that most adequate distribution for residuals turned out to be the Student-t distribution and GED distribution; the latter especially for equities and equity indexes.

Our analysis shows also that in the period of increased volatility on the financial markets for instruments characterized by clustering of observations, fat tails, skewness distribution, autocorrelation and heteroscedasticity, the models based on bounded innovation propagation method quite well describe volatility, regardless of the length of the time series, for which the parameters are estimated.

The *VaR* forecasts derived from the ARMA-GARCH models estimated robustly are underestimated in the same cases as the ones from the models estimated using QML methodology. Nevertheless the LR Test of Unconditional Coverage indicated the significance of underestimation for models robustly estimated in only six of 36 cases considered. The *DQ* test, analysing higher-order autocorrelation, indicated its significance in case of both robustly and standard estimated ARMA-GARCH-type models, often for the same instruments. In view of the decision-making aspect concerning the number of exceptions, losses due to the Value at Risk breaches, as well as opportunity costs, the most adequate models turn out to be the ones based on BIP estimation methodology. For regulatory purposes, both models based on BIP robust estimation and QML are the most adequate.

Forecasts based on the second robust estimation metod, i.e. regression quantiles, do not perform so well in periods of increased volatility. Forecasts of *VaR* are for many instruments overestimated. In addition, past values of *VaR* significantly affect the future values, which is verified by the DQ test. However, from the point of view of the decision-making aspect, CAViaR models for some of the instruments behave much better than the ARMA-GARCH class models. Hence, it is worth to take them also into consideration by selecting the VaR methods for a particular instrument. It is especially important in case of inadequacy of forecasts based on other models.

The empirical results prove that the robust estimators used in the ARMA-GARCH class models appear to be a promising tool and competitive for standard used QML method in scope of the volatility modelling and forecasting of *VaR*. Moreover, it is shown that the models based on quantile regression can ably fill the gap in case of the instruments for which QML and BIP estimators fail. Osiewalski and Pajor (2010) have proposed Bayesian parametric approach to Value at Risk and have concluded that the hybrid MSF-SBEKK models can compete with CaViaR models. Hence, it is worth to compare in next step BIP methodology with the Bayesian approach.

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Appendix - tables and figures

Figure 5: Exchange rate EUR/PLN.

Figure 6: EUR/PLN rate of returns.

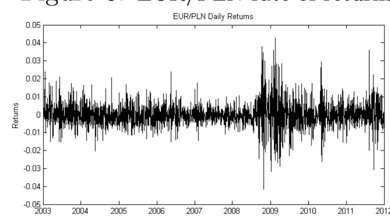

Figure 7: QQ plot of EUR/PLN rate of returns.

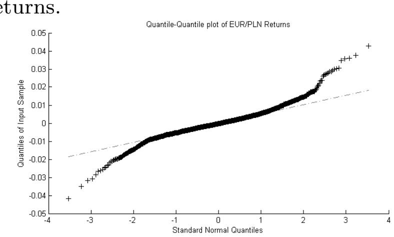

Figure 8: Histogram of EUR/PLN rate of returns.

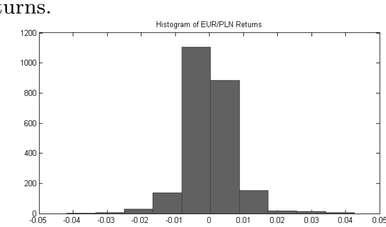

Table 14: Descriptive statistics

Instrument	Min	Max	Median	Average	Kurtosis	Skewness	Std.Dev.	Variance
EUR/PLN	-0.0418	0.0426	-0.0002	0.0000	7.7676	0.3367	0.0069	0.0000
USD/PLN	-0.0607	0.0523	0.0000	0.0001	8.7136	0.3269	0.0083	0.0001
CHF/PLN	-0.0799	0.0576	0.0000	0.0001	11.2321	-0.0332	0.0085	0.0001
WIG20	-0.1416	0.1371	0.0000	0.0002	6.5799	-0.1423	0.0187	0.0003
FTSE100	-0.0926	0.0938	0.0003	-0.0001	8.6110	-0.1345	0.0133	0.0002
S&P500	-0.0947	0.1096	0.0005	0.0000	10.0367	-0.1525	0.0139	0.0002
TPSA	-0.1202	0.1278	0.0000	0.0000	5.1223	0.0583	0.0223	0.0005
PKNOrlen	-0.1216	0.1287	0.0000	0.0002	5.1832	-0.0881	0.0223	0.0005
PekaoSA	-0.2059	0.1356	0.0000	0.0003	6.9204	-0.0336	0.0237	0.0006

Table 15: Jarque-Bera test.

Instrument	Stat.	p-value
EUR/PLN	2266.18	0.0000
USD/PLN	6460.15	0.0010
CHF/PLN	11750.05	0.0010
WIG20	2288.60	0.0010
S&P500	3985.24	0.0010
FTSE100	6240.30	0.0010
TPSA	613.87	0.0010
PKNOrlen	502.13	0.0010
PekaoSA	2088.90	0.0010

Table 16: ADF test for returns

Instrument	Stat.	p-value
EUR/PLN	-48.64	0.0010
USD/PLN	-67.24	0.0010
CHF/PLN	-65.77	0.0010
WIG20	-63.81	0.0010
SP500	-60.03	0.0010
FTSE100	-57.76	0.0010
TPSA	-58.74	0.0010
PKNOrlen	-49.49	0.0010
PekaoSA	-56.32	0.0010

Figure 9: EUR/PLN. VaR for sample estimated on the basis of ARMA(0,1)-GARCH(1,1) model.

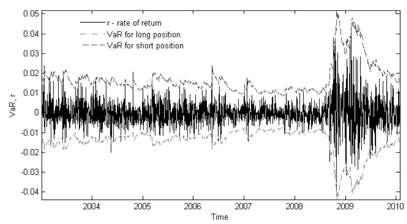

Figure 10: EUR/PLN. Forecast of VaR received on the basis of ARMA(0,1)-GARCH(1,1) model.

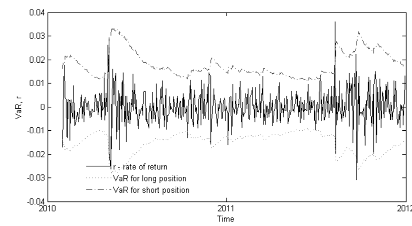

Figure 11: EUR/PLN. VaR for sample estimated on the basis of BIP-ARMA(0,1)-BM1-GARCH(1,1) model.

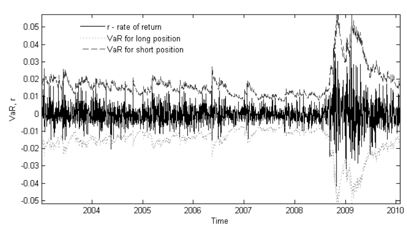

Figure 12: EUR/PLN. Forecast of VaR received on the basis of BIP-ARMA(0,1)-BM1-GARCH(1,1) model.

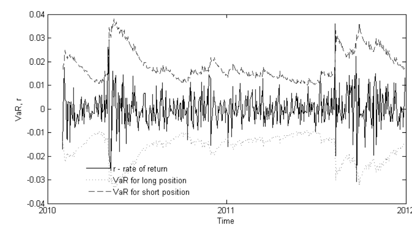

Figure 13: EUR/PLN. VaR for sample estimated on the basis of BIP-ARMA(0,1)-BM2-GARCH(1,1) model.

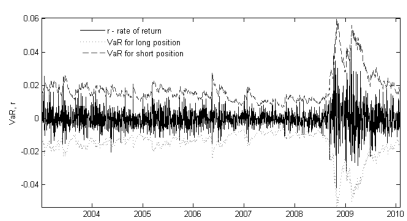

Figure 14: EUR/PLN. Forecast of VaR received on the basis of BBIP-ARMA(0,1)-BM2-GARCH(1,1) model.

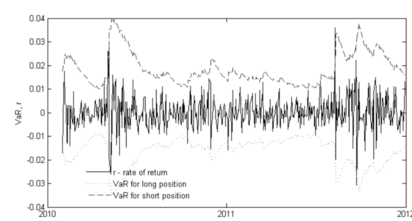

Figure 15: EUR/PLN. VaR for sample estimated on the basis of Symmetric Absolute Value model.

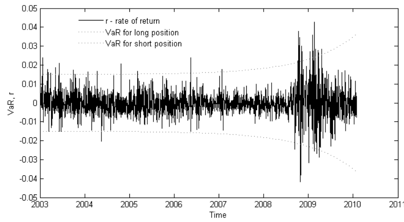

Figure 16: EUR/PLN. Forecast of VaR received on the basis of Symmetric Absolute Value model.

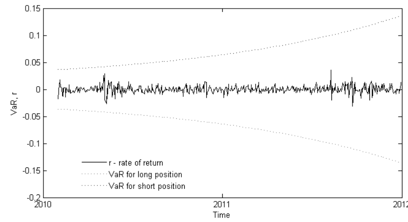

Figure 17: EUR/PLN. VaR for sample estimated on the basis of Asymmetric Slope model.

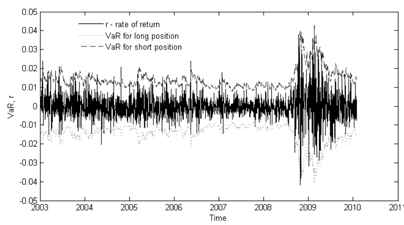

Figure 18: EUR/PLN. Forecast of VaR received on the basis of Asymmetric Slope model.

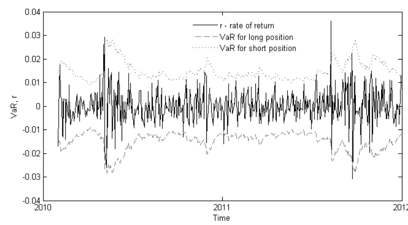

Figure 19: EUR/PLN. VaR for sample estimated on the basis of Indirect GARCH model.

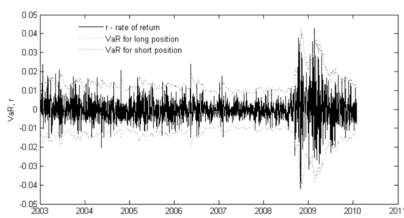

Figure 20: EUR/PLN. Forecast of VaR received on the basis of Indirect GARCH model.

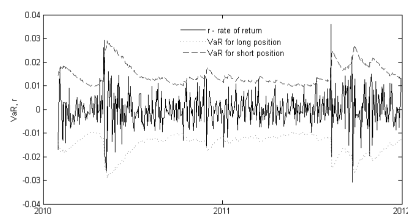

Figure 21: EUR/PLN. VaR for sample estimated on the basis of Adaptive model.

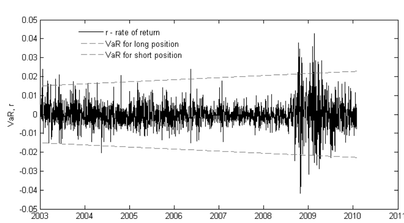

Figure 22: EUR/PLN. Forecast of VaR received on the basis of Adaptive model.

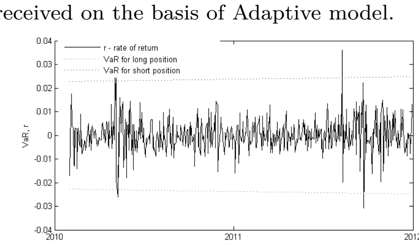

Table 17: Quantifying the correlation.

Test	Instrument	Lag5		Lag10		Lag15	
		Stat.	p-value	Stat.	p-value	Stat.	p-value
Ljung-Box	EUR/PLN	29.91	0.0009	34.40	0.0030	40.49	0.0043
	USD/PLN	35.51	0.0001	51.33	0.0000	53.80	0.0001
	CHF/PLN	37.18	0.0001	50.03	0.0000	57.93	0.0000
	WIG20	19.48	0.0346	28.07	0.0211	39.21	0.0063
	SP500	76.45	0.0000	77.93	0.0000	89.91	0.0000
	FTSE100	47.86	0.0000	67.95	0.0000	100.23	0.0000
	TPSA	17.13	0.0716	22.13	0.1044	30.93	0.0561
	PKNOrlen	28.81	0.0013	37.69	0.0010	42.22	0.0026
	PekaoSA	23.20	0.0100	26.46	0.0334	40.21	0.0047
Ljung-Box for squared returns	EUR/PLN	1176.91	1.000	1514.75	1.000	1800.48	1.000
	USD/PLN	2942.34	1.000	4007.66	1.000	4902.06	1.000
	CHF/PLN	895.45	1.000	1144.58	1.000	1635.30	1.000
	WIG20	1216.51	1.000	1471.50	1.000	1665.74	1.000
	SP500	2114.23	1.000	2792.13	1.000	3344.72	1.000
	PKNOrlen	451.80	1.000	594.42	1.000	665.22	1.000
	FTSE100	2376.64	1.000	3324.42	1.000	4092.36	1.000
	TPSA	338.23	1.000	388.53	1.000	430.86	1.000
	PekaoSA	644.12	1.000	802.54	1.000	943.22	1.000
ARCH	EUR/PLN	450.54	1.000	477.07	1.000	516.73	1.000
	USD/PLN	1020.99	1.000	1072.57	1.000	1101.59	1.000
	CHF/PLN	425.18	1.000	465.75	1.000	619.38	1.000
	WIG20	568.09	1.000	585.30	1.000	593.75	1.000
	SP500	787.72	1.000	915.66	1.000	925.95	1.000
	FTSE100	692.33	1.000	719.92	1.000	771.11	1.000
	TPSA	157.26	1.000	153.22	1.000	168.15	1.000
	PKNOrlen	227.19	1.000	240.13	1.000	242.68	1.000
	PekaoSA	297.92	1.000	320.76	1.000	334.51	1.000

Table 18: Estimation of ARMA-GARCH model for EUR/PLN.

Model	ARMA(0,1) -GARCH(1,1)		
Error distribution	Student-t	Student-t	Student-t
Estimation method → Parameters ↓	QML	BIP-BM1	BIP-BM2
$\hat{\phi}_0$	- 0,00	0,00	0,00
t	*-2,80*	*0,08*	*0,08*
$\hat{\alpha}_0$	0,00	0,00	0,00
t	*1,96*	*0,00*	*0,00*
$\hat{\delta}_1$	- 0,07	0,00	0,00
t	*-2,82*	*0,12*	*0,12*
$\hat{\alpha}_1$	0,05	0,04	0,06
t	*5,36*	*91,15*	*121,12*
$\hat{\beta}_1$	0,95	0,94	0,93
t	*102,44*	*1 436,36*	*1 485,76*
$\hat{\nu}$	6,57	10,84	6,82
t	*6,18*	*5,89*	*6,02*

Table 19: Estimation of CaViaR models for EUR/PLN.

Model → Parameters ↓	SAV	AS	IGARCH	AD
$\hat{\beta}_0$	-0.00005	0.00049	0.00000	0.00001
t	*0.00*	*0.03*	*0.00*	*0.01*
$\hat{\beta}_1$	1.00348	0.90483	0.92494	
t	*9.81*	*4.25*	*7.19*	
$\hat{\beta}_2$	2.35896	0.24778	0.30017	
t	*33.42*	*0.72*	*0.52*	
$\hat{\beta}_3$		0.12496		
t		*0.47*		
RQ	0.40	0.33	0.33	0.42

Notations: SAV-*Symmetric Absolute Value*; AS-*Asymmetric Slope*; IGARCH-*Indirect GARCH*; AD-*Adaptive*; RQ-*Regression Quantile*.

Table 20: Adequacy of VaR measure for EUR/PLN - I group of tests.

Model	Position	Number of observations	Back-test	Fraction of exceptions	LR_{CC}	*p-value*	LR_{UC}	*p-value*	*DQ*	*p-value*
sample										
ARMA(0.1)-GARCH(1.1)	*l*	1 846	20	1.08%	-	-	0.13	0.7223	7.80	0.3502
	s	1 846	16	0.87%	-	-	0.35	0.5560	1.10	0.9931
BIP-ARMA(0,1)-BM1-GARCH(1,1)	*l*	1 846	14	0.76%	-	-	1.19	0.2758	7.52	0.3771
	s	1 846	19	1.03%	-	-	0.02	0.9000	5.13	0.6437
BIP-ARMA(0,1)-BM2-GARCH(1,1)	*l*	1 846	18	0.98%	-	-	0.01	0.9140	7.65	0.3643
	s	1 846	18	0.98%	-	-	0.01	0.9140	5.36	0.6163
SAV	*l*	1846	10	0.54%	8.82	0.0121	4.70	0.0302	14.34	0.0455
	s	1846	28	1.52%	4.89	0.0869	4.30	0.0381	56.17	0.0000
AS	*l*	1846	20	1.08%	-	-	0.13	0.7223	20.37	0.0048
	s	1846	36	1.95%	13.29	0.0013	13.18	0.0003	45.46	0.0000
IGARCH	*l*	1846	18	0.98%	-	-	0.01	0.9140	0.93	0.9959
	s	1846	41	2.22%	21.66	0.0000	20.63	0.0000	50.79	0.0000
AD	*l*	1846	12	0.65%	6.01	0.0495	2.61	0.1065	23.17	0.0016
	s	1846	26	1.41%	3.53	0.1709	2.76	0.0966	83.90	0.0000
forecast										
ARMA(0.1)-GARCH(1.1)	*l*	500	7	1.40%	-	-	0.72	0.3966	1.41	0.9850
	s	500	5	1.00%	-	-	0.00	1.0000	42.96	*0.0000*
BIP-ARMA(0,1)-BM1-GARCH(1,1)	*l*	500	5	1.00%	-	-	-	1.0000	0.77	0.9977
	s	500	5	1.00%	-	-	-	1.0000	41.99	0.0000
BIP-ARMA(0,1)-BM2-GARCH(1,1)	*l*	500	5	1.00%	-	-	-	1.0000	0.75	0.9980
	s	500	5	1.00%	-	-	-	1.0000	42.48	0.0000
SAV	*l*	500	0	0.00%	-	-	-	-	5.05	0.6500
	s	500	0	0.00%	-	-	-	-	5.05	0.6500
AS	*l*	500	5	1.00%	-	-	0.00	1.0000	0.51	1.0000
	s	500	8	1.60%	-	-	1.54	0.2100	27.60	0.0000
IGARCH	*l*	500	6	1.20%	-	-	0.19	0.6600	2.85	0.9000
	s	500	10	2.00%	-	-	3.91	0.0500	24.67	0.0000
AD	*l*	500	2	0.40%	-	-	2.35	0.1300	1.84	0.9700
	s	500	3	0.60%	-	-	0.94	0.3300	38.18	0.0000

Notations: SAV-*Symmetric Absolute Value*; AS-*Asymmetric Slope*; IGARCH-*Indirect GARCH*; AD-*Adaptive*.

Table 21: Adequacy of VaR measure for EUR/PLN - second and third group of measures.

Model	Position	*BL*	*RL*	*FL*	*ESI*	*ESII*
sample						
ARMA(0,1)-GARCH(1,1)	*l*	20	23.27	49.94	1.67	1.21
	s	16	20.92	52.52	2.22	1.51
BIP-ARMA(0,1)-BM1-GARCH(1,1)	*l*	14	16.66	45.52	0.02	1.25
	s	19	23.31	55.48	1.92	1.24
BIP-ARMA(0,1)-BM2-GARCH(1,1)	*l*	18	20.52	49.28	0.01	1.20
	s	18	22.31	54.82	1.95	1.25
SAV	*l*	10	17.04	50.67	2.64	1.27
	s	28	42.66	75.95	2.47	1.28
AS	*l*	20	23.65	50.22	1.75	1.19
	s	36	46.52	72.79	2.06	1.25
IGARCH	*l*	18	22.36	48.42	1.77	1.24
	s	41	53.32	79.01	1.90	1.26
AD	*l*	12	20.22	54.90	2.72	1.28
	s	26	43.16	77.59	2.56	1.31
forecast						
ARMA(0.1)-GARCH(1.1)	*l*	7	8.02	15.52	1.89	1.19
	s	5	11.39	20.33	2.35	1.85
BIP-ARMA(0,1)-BM1-GARCH(1,1)	*l*	5	5.53	13.74	1.73	1.21
	s	5	11.13	14.17	2.35	1.54
BIP-ARMA(0,1)-BM2-GARCH(1,1)	*l*	5	5.54	13.71	1.73	1.22
	s	5	10.75	14.25	2.35	1.52
SAV	*l*	0	0.00	36.31	-	-
	s	0	0.00	36.31	-	-
AS	*l*	5	5.82	13.22	1.93	1.26
	s	8	14.96	22.33	2.13	1.42
IGARCH	*l*	6	7.27	14.52	2.04	1.25
	s	10	18.63	25.85	1.98	1.45
AD	*l*	2	2.50	14.34	2.85	1.20
	s	3	4.81	16.63	3.04	1.29

Notations: BL-*Binary Loss*; RL-*Regulatory Loss*; FL-*Firm Loss*; ESI-*Expected Shortfall I*; ESII-*Expected Shortfall II*; *l*-long position; *s*-short position; SAV-*Symmetric Absolute Value*; AS-*Asymmetric Slope*; IGARCH-*Indirect GARCH*; AD-*Adaptive*.