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BURDEN OPTIMISATION OF LUMP CHARGE MATERIALS FOR FOUNDRY FURNACES

Calculation of economically optimum charge burden for foundry furnaces depends on a solution that can be found in the field of mathematical linear programming. A traditional linear optimisation method means looking for continuous variables that can define portion of every charge component.

Sometimes, it can be observed that a calculated portion of charge material with a defined lump mass is rounded up, which is wrong because the charge burden prepared in such a way may either show a mistaken chemical composition, or it can entail a significant cost. With charge materials characterised by defined mass lumps, it is recommended to use a modified method of integer linear optimisation.

This article contains definitions of a goal function and a system of constraints considering the charge materials as stated above. In addition, a sample calculation has been included hereto to show such optimisation for a selected process of metal melting in a casting furnace.

Keywords: burden of lump charged materials

1. Introduction

The usual procedure applied during optimisation of the cheapest charge burden for foundry furnaces depends on establishing a precise proportion of every charge material x_j , so that the goal function as given below [1-3]

$$\sum_{j=1}^N c_j x_j \quad (1)$$

can reach its minimum while the following constraints are imposed:

$$\left\{ \begin{array}{l} \sum_{j=1}^N A_{ij} x_j \geq A_i^d m_w \\ \sum_{j=1}^N A_{ij} x_j \leq A_i^g m_w \\ 0 \leq x_j^d \leq x_j \leq x_j^g \leq m_w \\ \sum_{j=1}^N x_j = m_w \\ i = 1, 2, \dots, M \end{array} \right. \quad (2)$$

where:

- N – number of charge materials,
- c_j – unit price of the j -th charge component, for example €/kg,

- x_j – mass fraction of the j -th charge material, kg,
- A_{ij} – content of i -th element in the j -th charge component, %,
- A_i^d, A_i^g – lower and upper level of i -th element content in the calculated charge material, %,
- m_w – weight of the calculated charge material, kg,
- x_j^d, x_j^g – lower and upper limit of the content of the j -th component in the charge material, kg,
- M – number of chemical elements.

Considering that a goal function and a system of constraints are exclusively expressed as linear functions, the task can be solved by adopting a selected method of linear programming like, for example, method simplex [4]. Variables x_j are continuous variables in this example. Calculation of charge burden using initial data as given in Table 1, can be an example task including such variables.

In this example, linear programming helps to calculate quality proportions x_j ($j = 1, 2, \dots, 8$) in such a way that a goal function as given here:

$$0,40x_1 + 0,27x_2 + 0,25x_3 + 0,45x_4 + 1,30x_5 + 0,98x_6 + 0,49x_7 + 5,00x_8 \quad (3)$$

can reach its minimum value under the constraints:

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Chemical composition, cost, constraints and final weight of the charge materials

Material		Chemical composition						Cost €/kg	Limit kg
		C	Si	Mn	P	S	Cu		
x_1	Pig iron	4,30	0,75	0,43	0,03	0,02		0,40	≥ 200
x_2	Iron scrap	2,80	1,60	0,50	0,04	0,03	0,35	0,27	≤ 1200
x_3	Steel scrap	0,35	0,20	0,20	0,01	0,03		0,25	
x_4	Sphero scrap	3,50	2,40	0,75	0,04	0,02		0,45	≥ 250
x_5	FeSi	0,12	68,00					1,30	
x_6	FeMn	6,40	1,00	77,00				0,98	
x_7	SiC	29,00	59,00					0,49	
x_8	Cu						99,00	5,00	
Final charge		3,10÷3,30	1,65÷1,85	0,65÷0,80	max. 0,10	max. 0,08	0,50÷0,70	$m_w = 3200$ kg	

$$\begin{cases}
 (4,3x_1 + 2,8x_2 + 0,35x_3 + 3,5x_4 + 0,12x_5 + \\
 + 6,4x_6 + 29x_7 \geq 3,1 \cdot 3200 \\
 4,3x_1 + 2,8x_2 + 0,35x_3 + 3,5x_4 + 0,12x_5 + \\
 + 6,4x_6 + 29x_7 \leq 3,3 \cdot 3200 \\
 0,75x_1 + 1,6x_2 + 0,2x_3 + 2,4x_4 + 68x_5 + x_6 + \\
 + 59x_7 \geq 1,65 \cdot 3200 \\
 0,75x_1 + 1,6x_2 + 0,2x_3 + 2,4x_4 + 68x_5 + x_6 + \\
 + 59x_7 \leq 1,85 \cdot 3200 \\
 0,43x_1 + 0,5x_2 + 0,2x_3 + 0,75x_4 + 77x_6 \geq 0,65 \cdot 3200 \\
 0,43x_1 + 0,5x_2 + 0,2x_3 + 0,75x_4 + 77x_6 \leq 0,8 \cdot 3200 \\
 0,03x_1 + 0,04x_2 + 0,01x_3 + 0,04x_4 \leq 0,1 \cdot 3200 \\
 0,02x_1 + 0,03x_2 + 0,03x_3 + 0,02x_4 \leq 0,08 \cdot 3200 \\
 0,35x_2 + 99x_8 \geq 0,5 \cdot 3200 \\
 0,35x_2 + 99x_8 \leq 0,7 \cdot 3200 \\
 x_1 \geq 200 \\
 x_2 \leq 1200 \\
 x_3 \leq 1300 \\
 x_4 \geq 250 \\
 x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 3200 \\
 x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0)
 \end{cases} \quad (4)$$

The solution of linear optimisation task (3)-(4) is provided in Table 2.

TABLE 2

Charge burden for input data included in Table 1

Material	Weight, kg	Chemical Constitution
x_1	Pig iron	C = 3,10%, Si = 1,85%, Mn = 0,65%, P = 0,03%, S = 0,03%, Cu = 0,50%
x_2	Iron scrap	
x_3	Steel scrap	
x_4	Sphero scrap	
x_5	FeSi	
x_6	FeMn	
x_7	SiC	
x_8	Cu	Cost = 0,34 €/kg

The charge burden as presented in Table 2 will constitute a basis for further calculations and comparisons.

2. Algorithm for charge burden optimisation when using defined lump mass materials

In foundry manufacturing, some charge materials like pig iron and ferrous alloys appearing in the form of ingots, batches of steel plates or shavings briquettes are characterised by determined weight of their lumps.

It has to be underlined that rounding up of the results of continuous linear optimisation to the closest full mass iteration of a defined component lump, may lead to deviations like inappropriate chemical composition of the charge burden or higher cost as compared to another material that fulfils all the accepted objectives.

If, for example, along with the data shown in Table 1, weight of particular bits of materials is introduced according to Table 3, that can bring a result of appropriate rounding up of calculation scores from Table 2, and, consequently, it can be possible to achieve charge burden calculation as shown in Table 4.

TABLE 3

Assumed mass of selected charge materials lumps

Material	Mass of single lump, kg	
x_1	Pig iron	15
x_3	Steel scrap	20
x_5	FeSi	1
x_6	FeMn	2
x_7	SiC (bag)	25

TABLE 4

Charge burden calculation based on the input data from Table 1 and Table 3 and rounded up portions values of selected materials

Material	Weight, kg	Chemical Constitution
x_1	Pig iron	C = 3,16%, Si = 1,98%, Mn = 0,66%, P = 0,03%, S = 0,03%, Cu = 0,47%
x_2	Iron scrap	
x_3	Steel scrap	
x_4	Sphero scrap	
x_5	FeSi	
x_6	FeMn	
x_7	SiC	
x_8	Cu	Cost = 0,34 €/kg

In the calculation shown in Table 4 the content of Si in the final charge is higher than the accepted levels as given in Table 1. Respectively, the content of Cu is lower.

Accepting index k , to mark a number of a charge component ($k \leq N$) for which one lump weight will show p_k , the system of constraints can be defined as follows:

$$\left\{ \begin{array}{l} \sum_{j=1}^N A_{ij} x_j \geq A_i^d m_w \\ \sum_{j=1}^N A_{ij} x_j \leq A_i^g m_w \\ 0 \leq x_j^d \leq x_j \leq x_j^g \leq m_w \\ x_k \text{ mod } p_k = 0 \\ \sum_{j=1}^N x_j = m_w \\ i = 1, 2, \dots, M \\ 1 \leq k \leq N \end{array} \right. \quad (5)$$

The optimisation task comes down to setting down such fractions proportions x_j , that the goal function (1) is minimised under the constraints (5).

The problem can be solved by adopting a branch and bound method.

Considering that different values can be displayed in p_k , like the ones shown in Table 3, the branch and bound method requires some modification where a key factor will be an order in which variables x_k will be chosen for particular stages of division and constraints modification process.

3. Calculation example

Based on the input data contained in Tables 1 and 3, optimisation of charge burden lies in designating appropriate values x_j so that the goal function is minimum under the following constraints:

$$\left\{ \begin{array}{l} 4,3x_1 + 2,8x_2 + 0,35x_3 + 3,5x_4 + 0,12x_5 + 6,4x_6 + 29x_7 \geq 3,1 \cdot 3200 \\ 4,3x_1 + 2,8x_2 + 0,35x_3 + 3,5x_4 + 0,12x_5 + 6,4x_6 + 29x_7 \leq 3,3 \cdot 3200 \\ 0,75x_1 + 1,6x_2 + 0,2x_3 + 2,4x_4 + 68x_5 + x_6 + 59x_7 \geq 1,65 \cdot 3200 \\ 0,75x_1 + 1,6x_2 + 0,2x_3 + 2,4x_4 + 68x_5 + x_6 + 59x_7 \leq 1,85 \cdot 3200 \\ 0,43x_1 + 0,5x_2 + 0,2x_3 + 0,75x_4 + 77x_6 \geq 0,65 \cdot 3200 \\ 0,43x_1 + 0,5x_2 + 0,2x_3 + 0,75x_4 + 77x_6 \leq 0,8 \cdot 3200 \\ 0,03x_1 + 0,04x_2 + 0,01x_3 + 0,04x_4 \leq 0,1 \cdot 3200 \\ 0,02x_1 + 0,03x_2 + 0,03x_3 + 0,02x_4 \leq 0,08 \cdot 3200 \\ 0,35x_2 + 99x_8 \geq 0,5 \cdot 3200 \\ 0,35x_2 + 99x_8 \leq 0,7 \cdot 3200 \end{array} \right. \quad (6)$$

$$\left\{ \begin{array}{l} x_1 \geq 200 \\ x_2 \leq 1200 \\ x_3 \leq 1300 \\ x_4 \geq 250 \\ x_1 \text{ mod } 15 = 0 \\ x_3 \text{ mod } 20 = 0 \\ x_5 \text{ mod } 1 = 0 \\ x_6 \text{ mod } 2 = 0 \\ x_7 \text{ mod } 25 = 0 \\ x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 + x_8 = 3200 \\ x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8 \geq 0 \end{array} \right.$$

In the first calculation stage, a task is to be solved using continuous variables x_j as defined by functions (3) and (4). The results of this stage can be found in Table 2.

Using a branch and bound algorithm, along with a variable x_7 that has the biggest lump mass value (25 kg), the aim is to solve two optimisation tasks P1 and P2 which include an expanded system of constraints (6) according to Table 5, keeping the same goal function (3).

Continuous variable $x_7 = 42,8$ kg, hence in task P1 inequality $x_7 \leq 25$ has to be added, whereas in task P2, the inequality to be added is $x_7 \geq 50$.

TABLE 5

Tasks P1 and P2 – results

	Optimisation task			
	variable x_7			
	P1		P2	
System of constraints	$\left\{ \begin{array}{l} (6) \\ x_7 \leq 25 \end{array} \right.$		$\left\{ \begin{array}{l} (6) \\ x_7 \geq 50 \end{array} \right.$	
Charge burden	x_1	1090,23	x_1	1137,66
	x_2	1200,00	x_2	827,54
	x_3	608,79	x_3	911,08
	x_4	250,00	x_4	250,00
	x_5	4,94	x_5	0,00
	x_6	9,12	x_6	10,48
	x_7	25,00	x_7	50,00
	x_8	11,92	x_8	13,24
Cost of charge, €/kg	0,35070		0,35300	

As the charge cost in task P1 is lower than in task P2, according to the branch and bound algorithm, the next step should be two subsequent tasks which necessitate adding to the system of constraints from task P1 either the inequality $x_3 \leq 600$ kg (task P3), or the inequality $x_3 \geq 620$ kg (task P4). The calculation results of the tasks P3 and P4, where variable x_1 is considered, have been shown in Table 6.

The calculation results of the tasks from P7 to P10, where variables x_6 i x_5 are considered, have been shown in Table 7.

In task P10, variable x_7 is not a whole multiple of 25 kg, hence optimisation P11 has to be carried out under condition

TABLE 6

Tasks P3, P4, P5 and P6 – results

	Optimisation task							
	variable x_3				variable x_1			
	P3		P4		P5		P6	
System of constraints	$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \geq 620 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \geq 1110 \end{cases}$	
Charge burden	x_1	1099,11	x_1	1109,46	x_1	1095,00	x_1	1110,00
	x_2	1200,00	x_2	1168,90	x_2	1200,00	x_2	1200,00
	x_3	600,00	x_3	620,00	x_3	600,00	x_3	589,24
	x_4	250,00	x_4	250,00	x_4	254,24	x_4	250,00
	x_5	4,88	x_5	5,43	x_5	4,77	x_5	4,79
	x_6	9,09	x_6	9,18	x_6	9,07	x_6	9,06
	x_7	25,00	x_7	25,00	x_7	25,00	x_7	25,00
	x_8	11,92	x_8	12,03	x_8	11,92	x_8	11,92
Cost of charge, €/kg	0,35107		0,35171		0,35111		0,35153	

TABLE 7

Task P7, P8, P9 and P10 – results

	Optimisation task							
	variable x_6				variable x_5			
	P7		P8		P9		P10	
System of constraints	$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \leq 8 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \geq 10 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \geq 10 \\ x_5 \leq 4 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \geq 10 \\ x_5 \geq 5 \end{cases}$	
Charge burden	x_1	908,04	x_1	1095,00	x_1	1063,51	x_1	1095,00
	x_2	1200,00	x_2	1200,00	x_2	1200,00	x_2	1200,00
	x_3	566,80	x_3	600,00	x_3	600,00	x_3	600,00
	x_4	480,24	x_4	253,29	x_4	285,57	x_4	253,32
	x_5	0,00	x_5	4,79	x_5	4,00	x_5	5,00
	x_6	8,00	x_6	10,00	x_6	10,00	x_6	10,00
	x_7	25,00	x_7	25,00	x_7	25,00	x_7	24,76
	x_8	11,92	x_8	11,92	x_8	11,92	x_8	11,92
Cost of charge, €/kg	0,35497		0,35127		0,35160		0,35132	

$x_7 \leq 0$, and optimisation P12 under condition $x_7 \geq 25$. The results of these calculations have been presented in Table 8.

In task P12 all the required variables are full multiples of defined weight of the lumps. Therefore, it is finally the optimum charge burden with parameters as shown in Table 9.

The charge burden as presented in Table 9 is considerably different from the one (Fig. 1) achieved due to continuous linear optimisation (Table 2) apart from the price which in the first case, is slightly higher.

4. Summary

A modified model for charge burden optimisation for casting furnaces which has been presented in this article can help improve technological precision aspects of smelting processes thanks to accepting the idea of using charge materials with defined lump weight.

The modified branch and bound algorithm, known in the theory of integer linear programming, enables effective designation of charge components proportions while using different forms of charge materials (lumps, briquettes, ingots, loose or granular products).

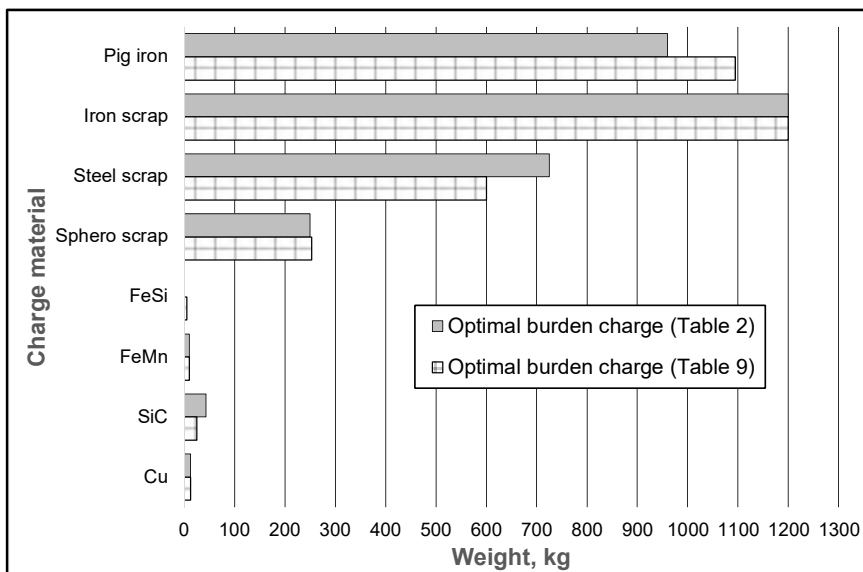


Fig. 1. Comparison of the optimization results using two different calculation methods: burden charge with continuous values (Table 2) and burden charge with lump materials (Table 9)

TABLE 8

Task P11 and P12 – results

	Optimisation task			
	variable x_7			
	P11		P12	
System of constraints	$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \geq 10 \\ x_5 \geq 5 \\ x_7 \leq 0 \end{cases}$		$\begin{cases} (6) \\ x_7 \leq 25 \\ x_3 \leq 600 \\ x_1 \leq 1095 \\ x_6 \geq 10 \\ x_5 \geq 5 \\ x_7 \geq 25 \end{cases}$	
Charge burden	x_1	1095,00	x_1	1095,00
	x_2	1200,00	x_2	1200,00
	x_3	392,91	x_3	600,00
	x_4	470,75	x_4	253,08
	x_5	19,42	x_5	5,00
	x_6	10,00	x_6	10,00
	x_7	0,00	x_7	25,00
Cost of charge, €/kg	0,36777		0,35132	

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TABLE 9

The optimum charge burden for the selected charge materials lump weight

	Material	Weight, kg	Chemical Constitution
x_1	Pig iron	1095,00	C = 3,11%, Si = 1,65%, Mn = 0,67%, P = 0,03%, S = 0,03%, Cu = 0,50%
x_2	Iron scrap	1200,00	
x_3	Steel scrap	600,00	
x_4	Sphero scrap	253,08	
x_5	FeSi	5,00	Cost = 0,35 €/kg
x_6	FeMn	10,00	
x_7	SiC	25,00	
x_8	Cu	11,92	

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