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Fractional variable order anti-windup control strategy

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Abstract. In this paper, a novel anti-windup strategy is presented. It is based on using fractional variable order integrator instead of integer order one in PID controller. It is shown that among four different types of variable order derivative definitions, only one gives satisfactory results – comparable, and even slightly better than the classical back-calculation anti-windup algorithm. Results are also presented in the form of simulation plots.

Key words: fractional calculus, variable order, PID control, anti-windup.

1. Introduction

Nowadays, to control processes, PID controllers are commonly used. Due to many limitations in control signal values, modified and additional structures of PID controllers have to be applied. The most popular methods improving control process use so-called anti-windup algorithms. The idea is to oppose increasing integration signal in presence of saturated control signal. There are many types of anti-windup algorithms, with back-calculation and reset integration being the most popular [1, 2].

Despite of the variety of anti-windup structures, the development of new methods is still in progress. Fractional calculus has been recently intensively used for creating anti-windup methods [3–7].

In this paper, we propose an alternative anti-windup strategy, inspired by fractional variable order calculus.

Recently, the case when order is time-varying, begun to be studied extensively. The fractional variable order behavior can be encountered for example in chemistry when system's properties are changing due to chemical reactions. Experimental studies of an electrochemical example of physical fractional variable order system have been presented in [8]. The variable order equations have been used to describe time evolution of drag expression in [9]. Numerical implementations of fractional variable order integrators and differentiators can be found in, e.g., [10, 11]. The fractional variable order calculus can also be used to describe variable order fractional noise [12]. In [13], the variable order interpretation of the analog realization of fractional order integrators, realized as domino ladders, has been considered. Applications of variable order derivatives and integrals arise also in control [14–16].

In [17, 18], three general types of variable order derivative definitions have been given. Alternative definitions of variable order derivatives were introduced in [19, 20].

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To the best of our knowledge, the variable order operators have not been used yet for developing anti-windup algorithms. In our approach, we use changing of integration order to reduce windup phenomena. The choice of suitable variable order definition is a crucial issue.

The paper is organized as follows. At the beginning, in Sec. 2, fractional variable order derivatives are recalled, together with their discrete approximations. In Sec. 3 both classical back-calculation and the proposed novel structure is introduced. In Sec. 4 simulation results of the proposed method for different types of variable order derivatives are presented. Finally, in Sec. 5 the main results are summarized.

2. Fractional variable order operators

Below, we recall the already known different types of fractional constant and variable order derivatives and differences.

2.1. Definitions of variable order operators. The following fractional constant order difference of Grünwald-Letnikov type will be used as a base of generalization onto variable order

$$\Delta^{\alpha} x_l = \frac{1}{h^{\alpha}} \sum_{j=0}^{l} (-1)^j \binom{\alpha}{j} x_{l-j}, \tag{1}$$

where $\alpha \in \mathbb{R}$, $l = 0, \ldots, k$, and $h > 0$ is a sample time.

We will consider the following four types of fractional variable order derivatives and their discrete approximations (differences). We assume the order changes in time, i.e., $\alpha(t) \in \mathbb{R}$ for $t > 0$; and in discrete-time domain $\alpha_l \in \mathbb{R}$ for $l = 0, \ldots, k$, where $k \in \mathbb{N}$.

The $\mathscr{A}$-type variable-order derivative and its discrete approximation is given, respectively, by

$${}^{\mathscr{A}}_{0}\mathrm{D}_t^{\alpha(t)} x(t) = \lim_{h \to 0} \frac{1}{h^{\alpha(t)}} \sum_{j=0}^{\eta} (-1)^j \binom{\alpha(t)}{j} x(t - jh),$$

where $\eta = \lfloor t/h \rfloor$, and

$${}^{\mathscr{A}}\Delta^{\alpha_l} x_l = \frac{1}{h^{\alpha_l}} \sum_{j=0}^{l} (-1)^j \binom{\alpha_l}{j} x_{l-j}.$$

The $\mathscr{B}$-type variable-order derivative and its discrete approximation is given, respectively, by

$$ {}_0^{\mathscr{B}}\mathrm{D}_t^{\alpha(t)} x(t) = \lim_{h\to 0} \sum_{j=0}^{\eta} \frac{(-1)^j}{h^{\alpha(t-jh)}} \binom{\alpha(t-jh)}{j} x(t-jh) $$

and

$$ {}^{\mathscr{B}}\Delta^{\alpha_l} x_l = \sum_{j=0}^{l} \frac{(-1)^j}{h^{\alpha_{l-j}}} \binom{\alpha_{l-j}}{j} x_{l-j}. $$

The $\mathscr{D}$-type variable-order derivative and its discrete approximation is given, respectively, by

$$ {}_0^{\mathscr{D}}\mathrm{D}_t^{\alpha(t)} x(t) = \lim_{h\to 0} \left(\frac{x(t)}{h^{\alpha(t)}} - \sum_{j=1}^{\eta} (-1)^j \binom{-\alpha(t)}{j} {}_0^{\mathscr{D}}\mathrm{D}_{t-jh}^{\alpha(t)} x(t) \right) $$

and

$$ {}^{\mathscr{D}}\Delta^{\alpha_l} x_l = \frac{x_l}{h^{\alpha_l}} - \sum_{j=1}^{l} (-1)^j \binom{-\alpha_l}{j} {}^{\mathscr{D}}\Delta^{\alpha_{l-j}} x_{l-j}. $$

The $\mathscr{E}$-type variable-order derivative and its discrete approximation is given, respectively, by

$$ {}_0^{\mathscr{E}}\mathrm{D}_t^{\alpha(t)} x(t) = \lim_{h\to 0} \Big(\tfrac{x(t)}{h^{\alpha(t)}} - \textstyle\sum_{j=1}^{\eta} (-1)^j \binom{-\alpha(t-jh)}{j} \tfrac{h^{\alpha(t-jh)}}{h^{\alpha(t)}} {}_0^{\mathscr{E}}\mathrm{D}_{t-jh}^{\alpha(t)} x(t) \Big) $$

and

$$ {}^{\mathscr{E}}\Delta^{\alpha_l} x_l = \frac{x_l}{h^{\alpha_l}} - \sum_{j=1}^{l} (-1)^j \binom{-\alpha_{l-j}}{j} \frac{h^{\alpha_{l-j}}}{h^{\alpha_l}} \Delta^{\alpha_{l-j}} x_{l-j}. $$

The main motivation of considering the above definitions of fractional variable order derivatives is the fact that they are widely presented in literature and can be applied in physical systems. In [21], the $\mathscr{A}$-type of fractional variable order derivative was successfully used to design the variable order PD controller in robot arm control. In [22], the heat transfer process in specific grid-holes media whose geometry is changed in time, was modeled by a new $\mathscr{D}$-type definition. Moreover, these definitions have mutual duality properties described in [23], which can be adapted to solving the fractional variable order differential equations (see [24]).

2.2. Examples of derivatives. Let us consider the following variable (switched) order

$$ \alpha(t) = \begin{cases} \alpha_1 & \text{for } t \in [0, 10), \\ \alpha_2 & \text{for } t \in [10, 20]. \end{cases} \tag{2} $$

In Figs. 1 and 2 plots were obtained for integer orders, however, in Figs. 3 and 4 fractional order results are presented. Since, in the proposed anti-windup algorithm, orders of the integral part will be often switched from and to zero, in this subsection we will analyze different types of derivatives behavior for such switches.

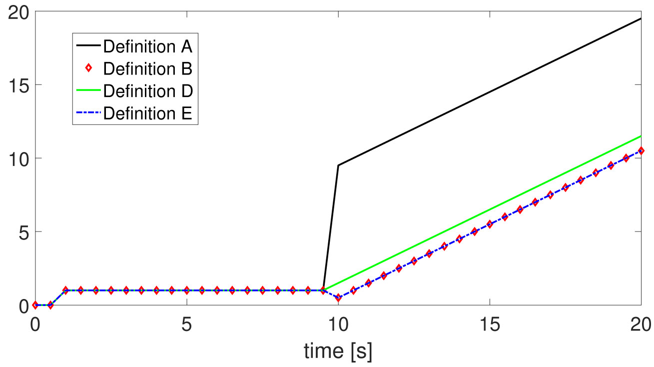

Fig. 1. Variable order derivatives of Heaviside step function for switched orders $\alpha_1 = 0$ and $\alpha_2 = -1$

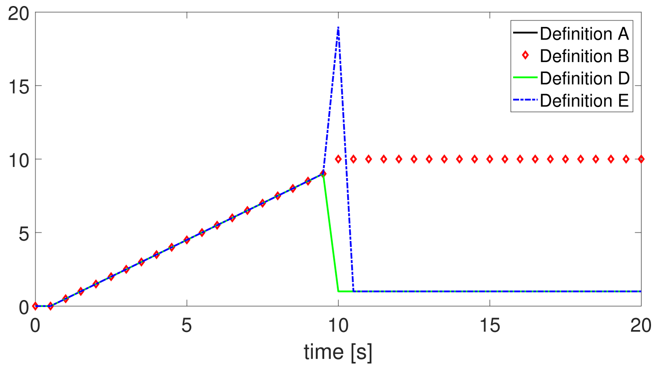

Fig. 2. Variable order derivatives of Heaviside step function for switched orders $\alpha_1 = -1$ and $\alpha_2 = 0$

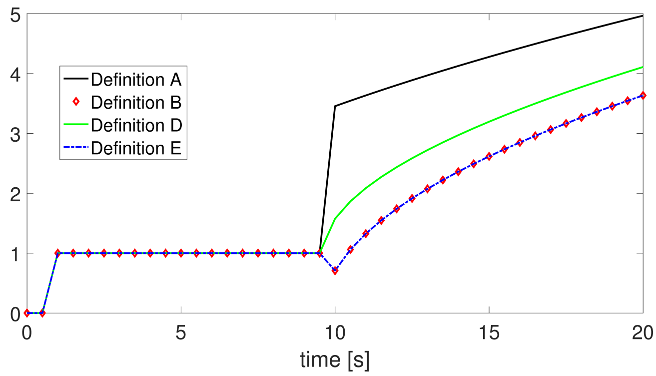

Fig. 3. Variable order derivatives of Heaviside step function for switched orders $\alpha_1 = 0$ and $\alpha_2 = -0.5$

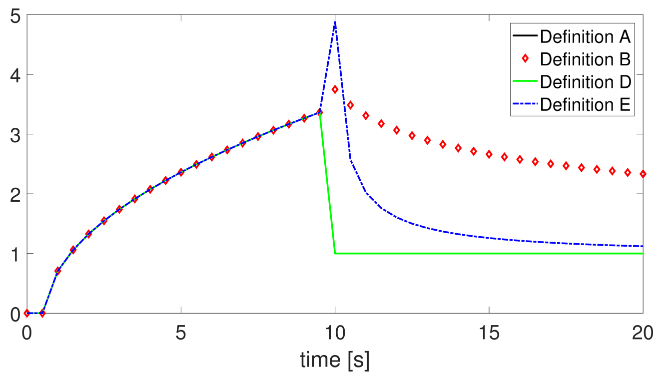

Fig. 4. Variable order derivatives of Heaviside step function for switched orders $\alpha_1 = -0.5$ and $\alpha_2 = 0$

As it can be seen in Figs. 1 and 3, the plots of $\mathscr{A}$-type definitions after order switch also switch to the values that the derivative would have for constant order integral. Such a behavior could be not efficient for the proposed algorithm, because the control signal could be switched to the same as for non-anti-windup case. It can also be seen that for a case when order is changed to zero, only $\mathscr{A}$- and $\mathscr{D}$-type definitions have a clear proportional behavior.

3. Anti-windup schemes

The windup behavior can occur for the systems with limit in control signal. It appears when the control signal passes the limit and the control error is not zero. In such a situation, the integral part of controller integrate error in order to increase the control signal (because of linear control strategy). In this way, the integrator can integrate to huge values, which will have to be reduced by opposite value of control error. This implies that for a real plant, obtained control results will contain much bigger and longer overshoot, and generally decrease control performance.

3.1. Typical anti-windup scheme for fractional order controller. In order to reduce the windup effect many additional controller structures are applied [3–7]. The most popular anti-windup scheme for integer and fractional order controllers is presented in Fig. 5. In this algorithm, an additional signal for integrator is included to avoid unnecessary integration action by decreasing the input signal of the integrator.

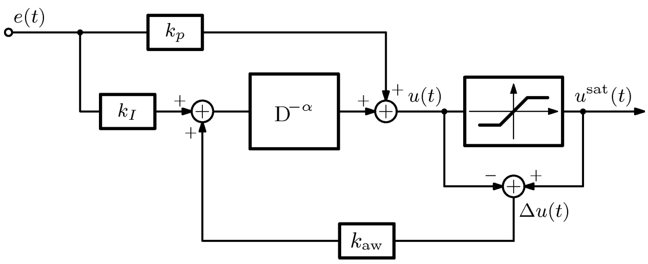

Fig. 5. Back-calculation anti-windup scheme for fractional order controller

Throughout the paper we assume that the controlled plant is given by the following transfer function:

$$G(s) = \frac{1}{s^2 + 3s + 2}, \tag{3}$$

and is governed by PI controller. There is also a limit in control signal $u(t) \in [-2.3, 2.3]$.

The controller for integer and fractional order cases have parameters: $k_p = 1$ and $k_I = 2$.

As it can be noticed in Fig. 6, for $k_{\rm aw} = 2$ the best result is obtained for this particular control problem.

In Figs. 7 and 8 the control signal after limiter and integrator signal are presented, respectively. As it can be seen, for a case without anti-windup, the integrator signal increases after reaching a maximum by control signal. This implies much higher and much longer overshoot, because these integrated values have to be reduced by control error with opposite sign.

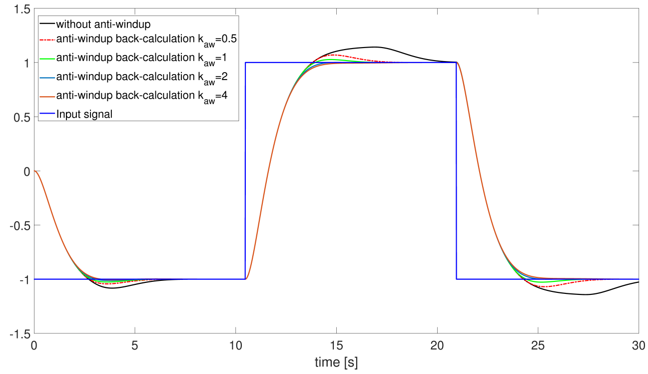

Fig. 6. Results of back-calculation anti-windup algorithm for different values of $k_{\rm aw}$ with integer order integrator

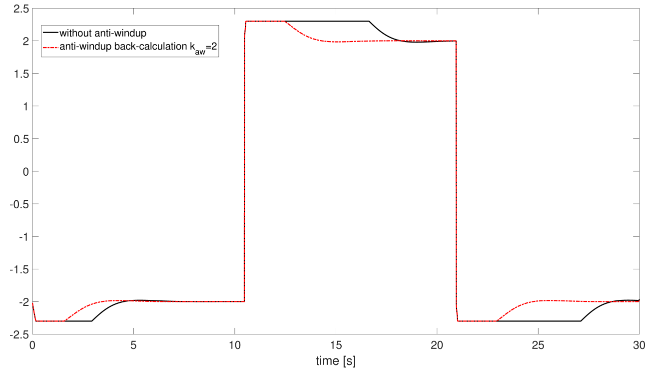

Fig. 7. Comparison of control signal $u^{\rm sat}(t)$ without anti-windup and with anti-windup for $k_{\rm aw} = 2$

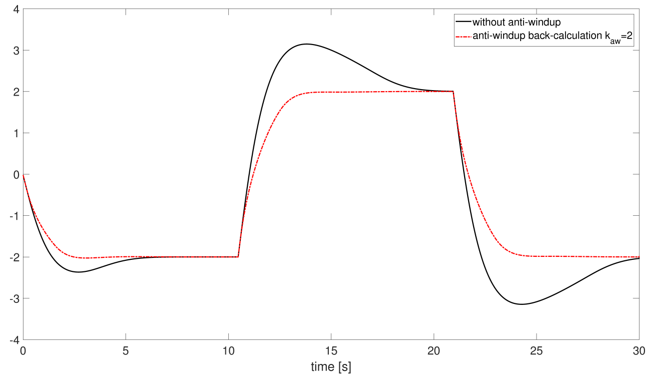

Fig. 8. Comparison of integrator signal without anti-windup and with anti-windup for $k_{\rm aw} = 2$

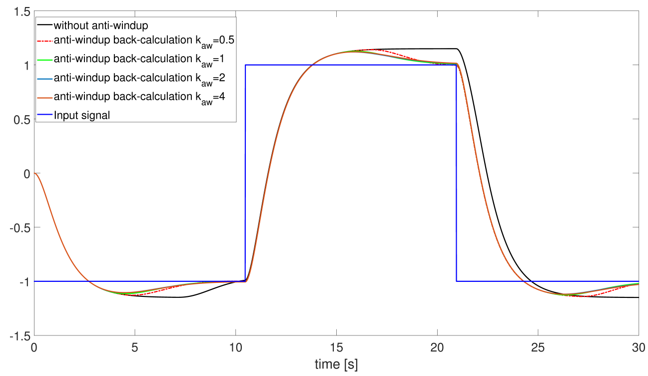

Fig. 9. Results of back-calculation anti-windup algorithm for different values of $k_{\rm aw}$ with fractional order integrator

Similar to results presented in Fig. 6, for fractional order controller the best results of anti-windup algorithm is obtained also for $k_{\text{aw}} = 2$. The order of fractional order controller was chosen as $\alpha = 1.2$.

3.2. Proposed variable order anti-windup scheme. There are numerous manners of limiting the action of the integrator in presence of control signal saturation. Inspired by variable order derivatives, in this section we propose a new, to the best of our knowledge, anti-windup algorithm. According to this method, presented in Fig. 10, the action of the integrator is governed by changing (switching) order of integration. Precisely, we can distinguish between two situations: if a control signal is not saturated, the integrator reaches some nominal order value (integer or fractional); otherwise, if control signal achieves saturation limit, the order of integration switches to the zero value, serving, in consequence, as gain action. However, the stability issue for variable order system in general is an open problem; in this case it seems to be necessary that the sum of proportional and integral gains is at least below gain margin.

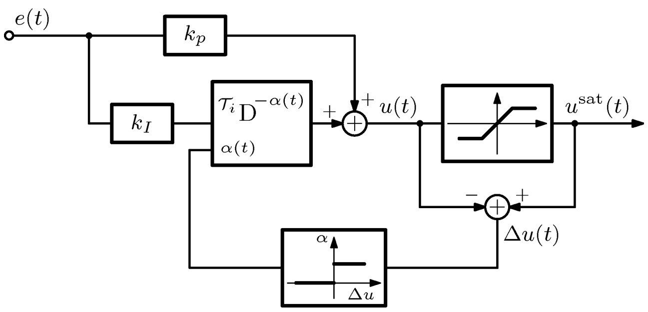

Fig. 10. Variable order anti-windup scheme

What is essential in this method is choosing an appropriate type of variable order integrator definition. In Figs. 11–14 plots of output signals are presented for different types of variable order integrator. It can be seen that only in the case of $\mathscr{D}$-type integrator the results are satisfactory, comparable with the classical back-calculation anti-windup algorithm. What is more, it seems to yield even better result, which is seen in the control signal depicted in Fig. 15. It is achieved with a pretty different integrator signal (see Fig. 16).

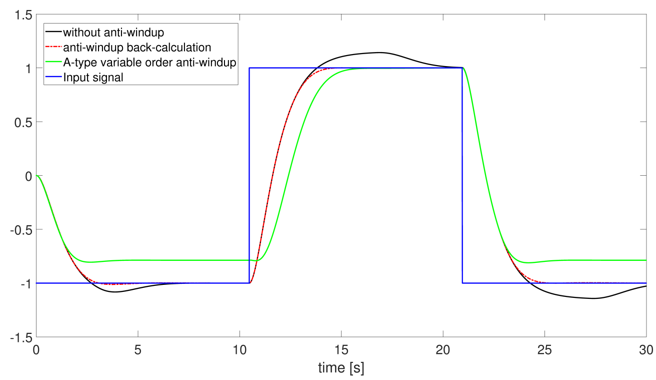

Fig. 11. Results for anti-windup fractional variable order $\mathscr{A}$-type and integer order controller

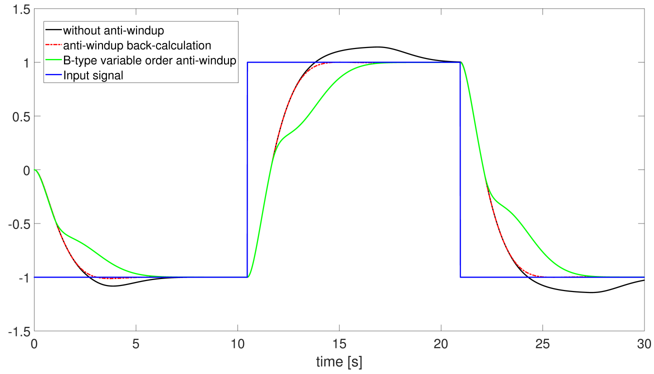

Fig. 12. Results for anti-windup fractional variable order $\mathscr{B}$-type and integer order controller

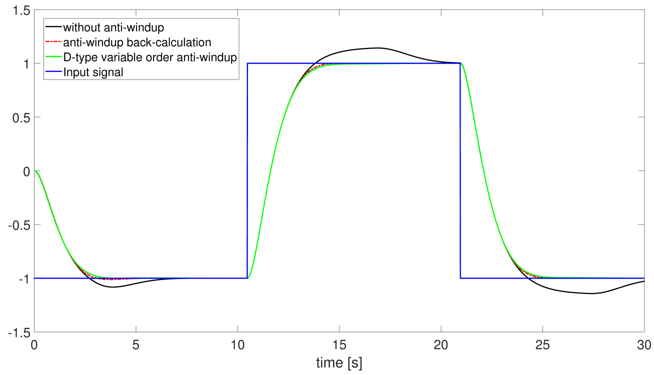

Fig. 13. Results for anti-windup fractional variable order $\mathscr{D}$-type and integer order controller

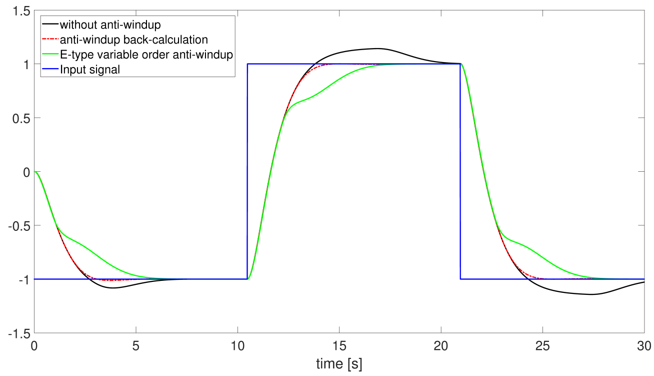

Fig. 14. Results for anti-windup fractional variable order $\mathscr{E}$-type and integer order controller

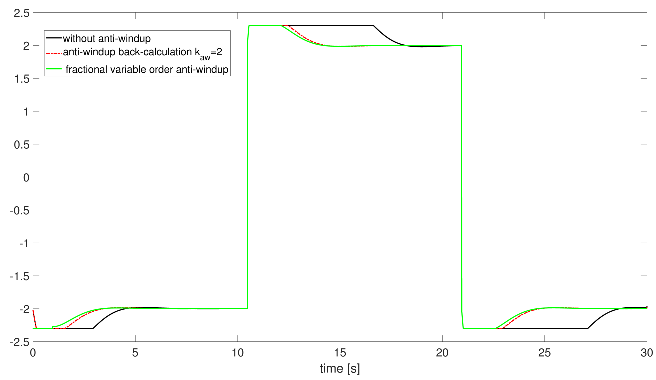

Fig. 15. Comparison of control signal $u^{\text{sat}}(t)$ without anti-windup, with anti-windup for $k_{\text{aw}} = 2$, and with proposed method

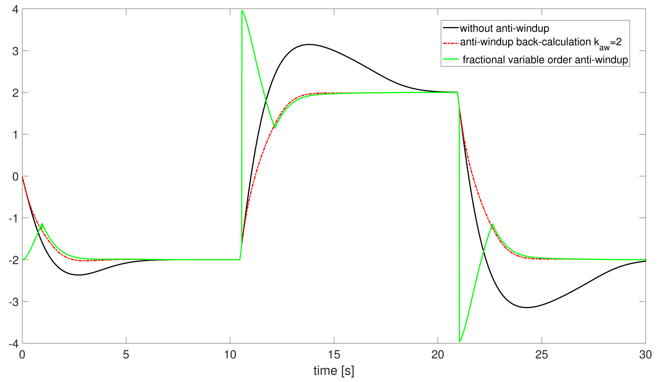

Fig. 16. Comparison of integrator signal without anti-windup, with anti-windup for $k_{\text{aw}} = 2$, and with proposed method

4. Anti-windup examples

In this section we will present and analyze results of the proposed method for different types of variable order derivative definitions. Based on these results, it will be possible to analyze which type of derivative will be the most suitable. First, we will present results for integer order controller, and then the for fractional order controller will be presented.

Example 1. Integer order controllers and anti-windup variable order algorithm for different types of variable order definitions.

In this example, the proposed variable order anti-windup algorithm will be analyzed for different types of variable order derivatives. As it was presented in Subsec. 2.2, different types of definitions have different behavior, especially for switching order to zero and from zero. That is why we can expect also different results in control applications. Results are presented in Figs. 11–14.

As it can be seen in Fig. 11, results for $\mathscr{A}$-type derivative (the most popular in literature definition) are not acceptable, because they do not provide the zero steady error. Also, the dynamics of the response is worse than in the traditional anti-windup algorithm.

Also, results for $\mathscr{B}$-type derivative, presented in Fig. 12, show that the steady error is zero, but the dynamics is much worse than for traditional algorithm.

In Fig. 13, results for $\mathscr{D}$-type derivative are presented, and it can be noticed that this version of algorithm produces very similar results to the traditional anti-windup method. Zero steady error is obtained, as well as similar dynamic behavior.

Figure 14 presents results for application of $\mathscr{E}$-type derivative, and as it can be seen, the results obtained are comparable with those based on algorithm with $\mathscr{B}$-type derivative. This means that the dynamics is much worse than in the traditional algorithm.

Example 2. Comparison of control and integrator signals for $\mathscr{D}$-type definition of the proposed variable order anti-windup algorithm.

Similar results to those shown in Figs. 7 and 8 are presented in Figs. 15 and 16, but including results for the proposed fractional variable order anti-windup algorithm. As it can be easily noticed, for this case, proposed algorithm is even slightly better than back-calculation anti-windup algorithm.

Example 3. Fractional order controllers and anti-windup fractional variable order algorithm for different types of variable order definitions.

In this example, results for fractional order controller with order $\alpha = 1.2$ will be presented. As it can be noticed in Figs. 17–20, the results for the $\mathscr{D}$-type derivative (Fig. 19) are comparable to and even better than results for typical back-calculation anti-windup method. Moreover, it can also be seen that for other types of definitions, the results obtained are not acceptable, which clearly explains that choosing variable order definitions in control application has to be done very carefully.

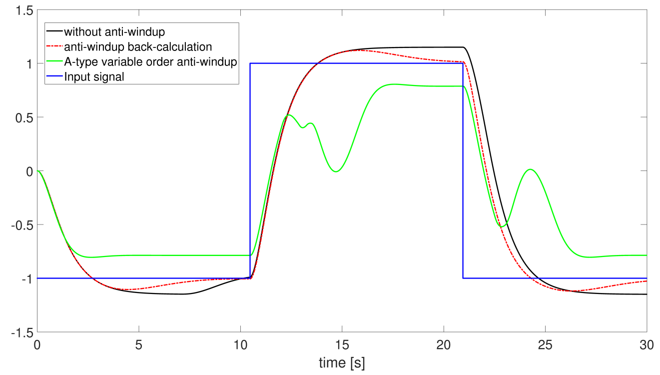

Fig. 17. Results for anti-windup fractional variable order $\mathscr{A}$-type and fractional order controller

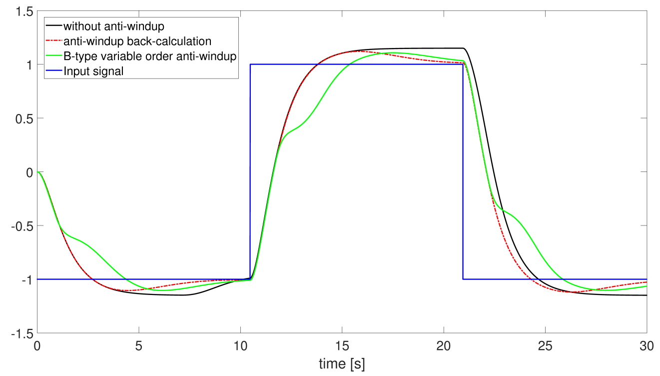

Fig. 18. Results for anti-windup fractional variable order $\mathscr{B}$-type and fractional order controller

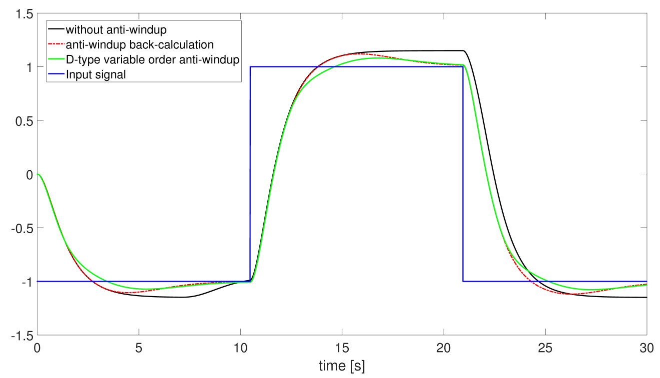

Fig. 19. Results for anti-windup fractional variable order $\mathscr{D}$-type and fractional order controller

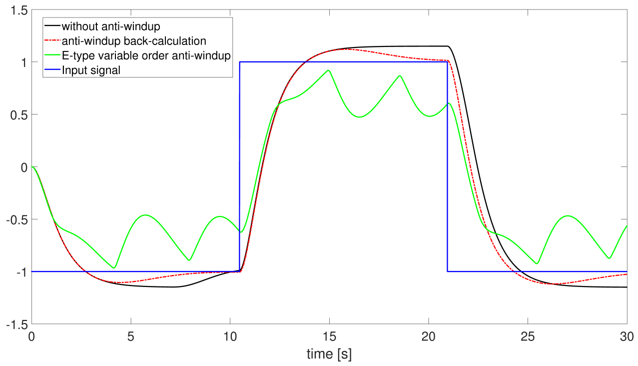

Fig. 20. Results for anti-windup fractional variable order $\mathscr{E}$-type and fractional order controller

5. Conclusions

In the paper, a new anti-windup strategy has been proposed. The novelty of the proposed method is based on applying variable order integrator in a PID controller. It has been shown that using recursive $\mathscr{D}$-type integrator and changing order between some nominal value and zero, yields similar, or even slightly better, results than using typical back-calculation anti-windup method. Moreover, the proposed method has an advantage over the classic one – it does not require any parameters to be adjusted, as in the back-calculation method. The analysis also presents important difference between different types of variable order integrators in control applications; results obtained for other types definitions than $\mathscr{D}$-type are significantly worse. The analysis presented in this paper also includes an issue that is worth further investigations – the stability of variable order systems.

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REFERENCES

[1] K. Aström, *Advanced PID Control*, ISA, Research Triangle Park, NC, 2005.
[2] A. Visioli, *Practical PID Control*, Springer-Verlag London, 2006.
[3] F. Ikeda, R. Toyama, and S. Toyama, "Anti-windup controller design by fractional calculus for linear control systems with input saturation", in *2009 ICCAS-SICE*, pp. 3317–3320, 2009.
[4] F. Padula, A. Visioli, and M. Pagnoni, "On the anti-windup schemes for fractional-order pid controllers", in *Proc. of 2012 IEEE 17th International Conference on Emerging Technologies Factory Automation (ETFA)*, pp. 1–4, Sept 2012.
[5] S. Pandey, P. Dwivedi, and A. Junghare, "Anti-windup fractional order PI^{λ}-PD^{μ} controller design for unstable process: A magnetic levitation study case under actuator saturation", *Arabian Journal for Science and Engineering*, Apr 2017.
[6] S. Pandey, P. Dwivedi, and A. Junghare, "A novel 2-dof fractional-order PI^{α}-D^{μ} controller with inherent anti-windup capability for a magnetic levitation system", *AEU – International Journal of Electronics and Communications* (79), 158–171 (2017).
[7] S. Pandey, N.K. Soni, and R.K. Pandey, "Fractional order integral and derivative (foid) controller with anti-windup for temperature profile control", in *2015 2nd International Conference on Computing for Sustainable Global Development (INDIACom)*, pp. 1567–1573, March 2015.
[8] H. Sheng, H. Sun, C. Coopmans, Y. Chen, and G.W. Bohannan, "Physical experimental study of variable-order fractional integrator and differentiator", in *Proc. of the 4th IFAC Workshop Fractional Differentiation and its Applications FDA*, 2010.
[9] L. Ramirez and C. Coimbra, "On the variable order dynamics of the nonlinear wake caused by a sedimenting particle", *Physica D-Nonlinear Phenomena* 240(13), 1111–1118 (2011).
[10] C.-C. Tseng, "Design and application of variable fractional order differentiator", in *Proc. of the 2004 IEEE Asia-Pacific Conference on Circuits and Systems* 1, 405–408 (2004).
[11] C.-C. Tseng and S.-L. Lee, "Design of variable fractional order differentiator using infinite product expansion", in *Proc. of 20th European Conference on Circuit Theory and Design (ECCTD)*, pp. 17–20, 2011.
[12] H. Sheng, H. Sun, Y. Chen, and T. Qiu, "Synthesis of multifractional gaussian noises based on variable-order fractional operators", *Signal Processing* 91(7), 1645–1650 (2011).
[13] D. Sierociuk, I. Podlubny, and I. Petras, "Experimental evidence of variable-order behavior of ladders and nested ladders", *Control Systems Technology, IEEE Transactions on* 21(2), 459–466 (2013).
[14] P. Ostalczyk, "Variable-, fractional-order discrete PID controllers", in *Proc. of the IEEE/IFAC 17th International Conference on Methods and Models in Automation and Robotics (MMAR)*, pp. 534–539, Międzyzdroje, Poland, 2012.
[15] P. Ostalczyk and P. Duch, "Closed-loop system synthesis with the variable-, fractional- order PID controller", in *Proc. of the IEEE/IFAC 17th International Conference on Methods and Models in Automation and Robotics (MMAR)*, pp. 589–594, Międzyzdroje, Poland, 2012.
[16] P. Ostalczyk and T. Rybicki, "Variable-fractional-order deadbeat control of an electromagnetic servo", *Journal of Vibration and Control* 14(9–10), 1457–1471 (2008).
[17] C. Lorenzo and T. Hartley, "Variable order and distributed order fractional operators", *Nonlinear Dynamics* 29(1–4), 57–98 (2002).
[18] D. Valerio and J.S. da Costa, "Variable-order fractional derivatives and their numerical approximations", *Signal Processing* 91(3, SI), 470–483 (2011).
[19] M. Macias and D. Sierociuk, "An alternative recursive fractional variable-order derivative definition and its analog validation", in *Proc. of International Conference on Fractional Differentiation and its Applications*, Catania, Italy, 2014.
[20] D. Sierociuk, W. Malesza, and M. Macias, "On a new definition of fractional variable-order derivative", in *Proc. of the 14th International Carpathian Control Conference (ICCC)*, Rytro, Poland, pp. 340–345, 2013.
[21] P. Ostalczyk, D.W. Brzezinski, P. Duch, M. Łaski, and D. Sankowski, "The variable, fractional-order discrete-time pd controller in the IISv1.3 robot arm control", *Central European Journal of Physics* 11(6), 750–759 (2013).
[22] P. Sakrajda and D. Sierociuk, *Modeling Heat Transfer Process in Grid-Holes Structure Changed in Time Using Fractional Variable Order Calculus*, Springer International Publishing, Cham, pp. 297–306, 2017.
[23] D. Sierociuk and M. Twardy, "Duality of variable fractional order difference operators and its application to identification", *Bull. Pol. Ac.: Tech.* 62(4), 809–815 (2014).
[24] W. Malesza, D. Sierociuk, and M. Macias, *Solution of fractional variable order differential equation*, American Control Conference, Chicago, IL, USA, pp. 1537–1542, 2015.