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Modelling and Measurement of Folk Guitar: Truss Rod and Strings in Numerical Analysis of Tone

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The study makes an attempt to model a complete vibrating guitar including its non-linear features, specifically the tension-compression of truss rod and tension of strings. The purpose of such a model is to examine the influence of design parameters on tone. Most experimental studies are flawed by uncertainties introduced by materials and assembly of an instrument. Since numerical modelling of instruments allows for deterministic control over design parameters, a detailed numerical model of folk guitar was analysed and an experimental study was performed in order to simulate the excitation and measurement of guitar vibration. The virtual guitar was set up like a real guitar in a series of geometrically non-linear analyses. Balancing of strings and truss rod tension resulted in a realistic initial state of deformation, which affected the subsequent spectral analyses carried out after dynamic simulations. Design parameters of the guitar were freely manipulated without introducing unwanted uncertainties typical for experimental studies. The study highlights the importance of acoustic medium in numerical models.

Keywords: sound quality; musical instruments; acoustic folk guitar; string tension; truss rod; non-linear modelling.

1. Introduction

The world of musical instruments is filled with various myths concerning timbre. Manufacturers, luthiers and performing musicians often boast about outstanding acoustical features of their instruments, caused by utilisation of rare and expensive materials. Some construction ideas are believed to improve guitar sound and effectively increase its price as well. These opinions remain largely unverified. While it is obvious that sound quality depends on physical properties of an instrument, attributing such a large impact to some wood types might be an exaggeration. There are other factors that influence the sound quality, such as good design and precision of assembly. These high-quality traits are usually applied to purposely better instruments in conjunction with expensive materials. To make things even more obscure, many wood genres are intuitively associated with timbre characteristics resembling visual attributes of these materials. For example, maple and spruce are said to sound bright, as opposed to dark looking and sounding mahogany or rosewood. A graphite guitar nut seems to induce a mild attack, while a steel one tends to sound harsher. Since no conclusive work has been found in this field, it is possible that many of these commonly acknowledged properties are not true and are just an excuse to raise prices.

There is a lot of research tackling the impact of materials on tone, most of which brings more questions than answers. Zoran *et al.* (2012) have recently exhibited, using replaceable top plates installed into a single guitar body, that two soundboards made of the same wood block can be as diverse in their dynamic response as two soundboards made of two different wood types. Ono and Okuda (2007); Okuda and Ono (2008) have shown that it is possible to obtain acoustic characteristics comparable with wood using polyurethane top plates. Mansour *et al.* (2015) have examined 18 copies of nominally identical guitars and have shown that despite the manufacturer and guitarists described them as sounding differently (with large diversity of opinions and general disagreement between the guitarists), it is generally hard to attribute various tone sensations to physical properties of these instruments. Fritz *et al.* (2007) and Woodhouse *et al.* (2012) have investigated psychoacoustic aspects of distinguishability of an instrument' tone by amateurs and professional musicians using synthesised and digitally processed sounds, exhibiting low importance of choice of the root instrument. Gore (2011) formulated a bold opinion – since wood types are practically indistinguishable in blind tests, and famous luthiers are successful in building guitars using non-wood materials, does the instrument material (soundboard aside) even matter in forms other than the strength of construction? Torres and Torres-Martínez (2015) further strengthened the recalled doubts by experimentally examining 12 guitars made of different materials and not stating any strong disparity between them nor even any tendency in sound. Curiously enough, Duerinck *et al.* (2014) built and tested an experimental trapezoidal violin only to find out that despite its exceptional construction there is almost no difference in the subjective sound perception as compared to traditional violins.

Since defining the impact of specific materials on an instrument sound is a distant goal, it might be more viable to investigate the influence of geometrical features on tone. Aspects such as thickness, shape and size of an instrument's part or stiffness of joints are all variables in the tone equation. One of the myths regarding construction has been recently disproved by Mottola (2007), who exhibited that the influence of neck-to-body joints on a guitar sustain is probably exactly the opposite to popular beliefs. Mottola's experiment is exceptionally reliable, because it operates on a single instrument which has its individual non-wooden parts successively replaced, thus no material randomness is introduced (e.g. random density and stiffness fields typical for wooden materials). Other than that, a comparative empirical study of different specimens might be misleading due to large sound variation between instruments of the same type. It is a common practice among musicians to try several copies of an instrument before purchase in order to choose the best sounding one, regardless of them being supposedly identical. Skrodzka *et al.* (2009) examined two violins of varying top plate thickness only to find no strong tendencies in the perceived darkness of tone (which was initially expected). However, only two specimens were tested, so stochastic approach was not involved in the study. Interestingly enough, another pair of violins was later examined (Skrodzka *et al.*, 2013) in regard to varnish, which was found to actually affect the sound of violin. Again, it is not known if the effect was a matter of coincidence or if it was reproducible. Another experiment was carried on by Skrodzka *et al.* (2011) on two guitars with varying bracing patterns in stringed and unstringed variants. Significant impact of the bracing on the lower frequency register was found, however the study included no statistical considerations and was most certainly valid only for the two guitars tested.

On the other hand, numerical methods are capable of performing a purely deterministic study, providing independent control over all parameters of a model and reducing the material and geometric randomness to zero. If executed with a sufficient precision and detail, this approach might make it possible to compare acoustic properties corresponding to different construction ideas while neglecting the material impact. Some works have already made use of advanced geometrical ideas and numerical implementations. A recent study by Lynch *et al.* (2013) have shown that even a slight change in shell shape can have a dramatic effect on the sound radiation produced by an enclosed cavity. Inácio *et al.* (2008) have executed a mixed experimental-numerical research on string-body interaction in violins and beating on the hunt after wolf notes. Issanchou *et al.* (2017) have gone as far as defining a collision effect of a vibrating string against a flat or curved surface. Kopač and Šali (1999) have investigated a surface finish influence on the wooden plates tone, which turned out to be surprisingly significant. In this context one of the most relevant and interesting parts of the study performed by Skrodzka *et al.* (2014) is that the eigenfrequency change of a violin top plate was not proportional to the change of a bass bar tension. It means that even a single-parameter modification can lead to non-linear effects not to be predicted without a separate experiment or an appropriate modelling solution.

The work presented in the paper follows the path of geometrical complexity and focuses on developing a FEM model precise enough to investigate the theoretical influence of geometrical properties on sound while keeping the material randomness under control. We can not emphasize it strong enough that we are speaking of the theoretical influence here, which means a predicted or expected effect of some geometrical application to a real guitar. Previous research has shown that it is hardly possible to actually predict any real effect neither by modelling nor by experimental measurements. It is however viable to at least theoretically justify application of some uncommon or expensive designs. It is even more worthwhile to do something opposite, i.e. show that some of these ideas are absolutely unjustified and their application should be avoided, since their only purpose is to increase an instrument's price.

This paper contributes by developing a model which treats the instrument not only as a solid vibrating body, but also as a multi-part device. Folk guitar was chosen for the research because of its availability and relative simplicity. What distinguishes this study from the others is including the active behaviour of strings and truss rod in the model. The work is based on an observation how greatly the guitar set-up affects its sound. It is a well-known fact for most guitarists that the adjustable parameters such as string action, string gauge and tension, truss rod pre-stress (neck curvature), etc. can have a decisive impact on the guitar tone. Because of that, not only the guitar geometry and materials should be taken into account when evaluating the guitar sound, but also the initial state of stress and deformation introduced by truss rod and strings, as well as relative motion of these parts during vibration. Each of these parts is a carrier of inertia (accelerating mass) as well as influences the state of equilibrium (stresses and deformation). By including these features, a typical stiff model turns into a non-linear system which can be subjected to something more than a modal analysis. A preliminary form of the study had been presented at a conference (Bielski, Kujawa, 2017).

Speaking of a well-established background, there are numerous works which focus on general physics behind instruments' acoustics. The classic book by Helmholtz (1954) provides a theoretical basis on air vibration and composition of resonance modes. Campbell and Greated (1994) and Jansson (2002) created works useful for musicians, luthiers and scientists concerning acoustics, measurements and assembly. A book by Fletcher and Rossing (2012) and Wolfe's private on-line repository (Wolfe, n.d.b.) provide a reliable basis on physics of musical instruments.

2. Materials and methods

2.1. Overview

The guitar selected for the research is LAG Tramontane T200D. It is a standard Dreadnought acoustic folk guitar. The material and construction data necessary for a proper modelling of the instrument was acquired directly form the guitar manufacturer. Most of the geometry was measured manually during the research, while inaccessible interior parts were introduced to the model using common Dreadnought construction drafts. A typical set of 0.013–0.056 inch phosphor bronze wound strings with a steel hexagonal core was used (D'Addario EJ17), totalling for 1850 N tension force on the bridge.

Classical guitars are equipped with low-tension nylon strings, while folk guitars come with high-tension steel strings. Because of that, these instruments are constructed differently, using different materials, bracing pattern, etc. One of the most interesting differences in construction of acoustic folk guitars and classical guitars is the truss rod. Increased tension of the steel strings is compensated by presence of the truss rod in the guitar neck. Truss rod is an adjustable element that counters the excessive neck curvature caused by high string tension. It is the main subject of the numerical study performed and discussed in the paper.

Both numerical analysis and experimental studies were designed to reflect the same conditions and obtain the same data type. They were targeted towards measuring accelerations and computing displacements of the selected spots of the guitar top plate. Exciting the body was executed without direct contact, allowing both the strings and the top plate to vibrate freely. Experimental conditions were rather coarse and not in line with the standard measurement methods used in acoustics, however the purpose of the experimental study is only supplementary to the numerical part.

2.2. Numerical study

2.2.1. Geometry and materials

A full 3D guitar geometry has been created in a CAD environment (Fig. 1). It has been imported to the SIMULIA Abaqus engine in separate parts. The complexity of the guitar geometry implied usage of

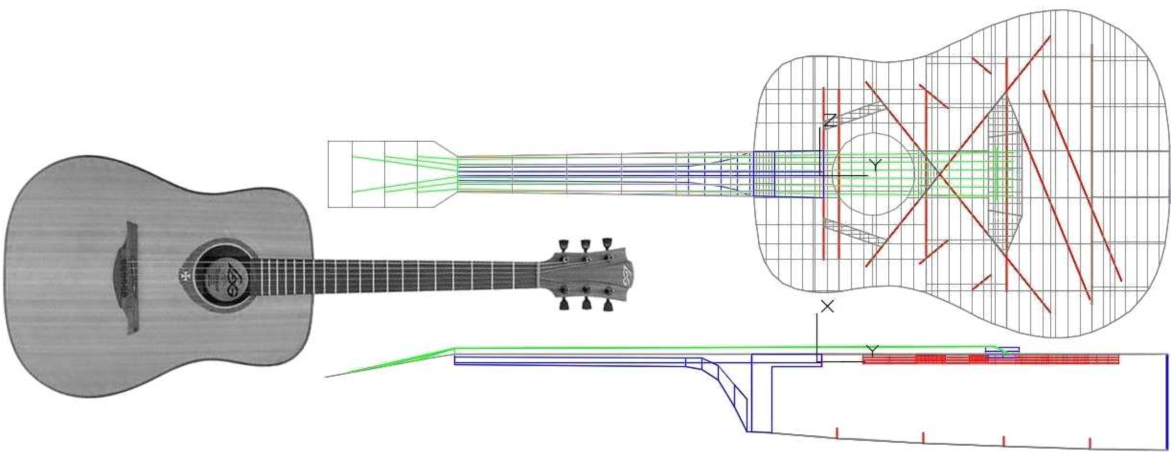

Fig. 1. Real guitar and its CAD geometry including strings and inner bracing.

solid, shell and beam elements (Zienkiewicz, Taylor, 1977). The guitar parts with their respective types have been listed in Table 1.

Most of the wood types were implemented with the orthotropic material model (Khennane *et al.*, 2014), for which the data on the material properties was obtained from USDA Forest Products Laboratory (Green *et al.*, 1999). Several parameters missing in this repository have been found in Eric Meier's Wood Database (Meier, n.d.). The only exception was Indian rosewood (*Dalbergia latifolia*), for which no information on its anisotropic properties was found even in the papers devoted specifically to this type of wood (Sproßmann *et al.*, 2017). A simple isotropic model with a single Young modulus value was used instead.

All of the values were specified for casual conditions, i.e. a room temperature and 12% moisture content. Graphite properties were hard to specify, since there are many subtypes of this material and the guitar manufacturer did not provide any detailed data about it. As the graphite parts are of marginal meaning in this model, approximate average values were used without a further investigation. The part-material pairs are listed in Table 2. The material properties can be found in Table 3.

The examined guitar was equipped with a 3.0 mm thick red cedar top plate. In later part of the study, two modifications of this property were considered – one with a 3.3 mm red cedar top, the other with a 3.0 mm spruce top. It is worth noting that 0.3 mm is a 10% increase in thickness, while spruce is roughly 10% denser than red cedar. It means that both plates were about 10% heavier than the reference plate in terms of mass, but the spruce one has significantly greater values of Young modulus.

2.2.2. Boundary conditions

Fixed boundary conditions were applied in the places where the strap buttons are located on the real guitar. They constrained the system just enough to keep it stable during the excitation. In the same time they do not affect the guitar body movement significantly, allowing it to deform and vibrate freely. All of the six degrees of freedom have been constrained on the surfaces corresponding to the strap buttons location.

Proper truss rod modelling was important to retain realistic neck stiffness. In a real guitar the truss rod is fixed on the body end and it is free to slide on the head end. The T200D guitar is equipped with

Table 1. FEM model parts sorted by element type.

Part	Element type
head, fretboard, top plate, sides, back, bracing, stiffening bars, nut, bridge	4-node shell elements with reduced integration and hourglass control (S4R)
neck, neck block	4-node tet solid elements (C3D4)
strings, truss rod bars	2-node beam elements, circular cross-section (B31)
truss rod nuts	8-node hex solid elements with reduced integration (C3D8R)

Table 2. FEM model parts sorted by material type.

Part	Material name and model
top plate	western red cedar, orthotropic
head, neck, back, sides, end block	honduran mahogany, orthotropic
fretboard	east indian rosewood, isotropic
bracing, stiffening bars	sitka spruce, orthotropic
bridge, nut	graphite, isotropic
truss rod, truss rod nuts, strings	steel, isotropic

Table 3. Mechanical properties of materials: density ρ, Young modulus E, Poisson ratio ν, shear modulus G; directions: 1 – longitudinal (axial), 2 – radial, 3 – circumferential (tangential).

Material	ρ [kg/m^3]	E_1 [MPa]	E_2	E_3	ν_{12} [–]	ν_{13}	ν_{23}	G_{12} [MPa]	G_{13}	G_{23}
cedar	320	7700	624	424	0.378	0.296	0.484	670	662	39
mahogany	450	10300	1102	659	0.314	0.533	0.600	680	886	288
spruce	360	9900	772	426	0.372	0.467	0.435	634	604	30
rosewood	750	12300	–	–	0.330	–	–	4624	–	–
graphite	2000	20000	–	–	0.200	–	–	8300	–	–
steel	7800	210 000	–	–	0.300	–	–	81 000	–	–

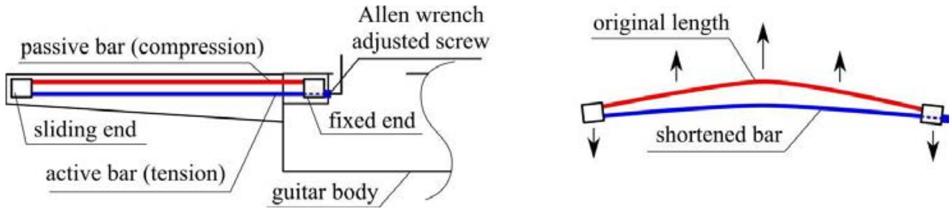

Fig. 2. Truss rod location inside guitar neck and its deformed shape acquired by shortening bottom bar.

a double action truss rod, consisting of a passive top bar (closer to the strings) and an active bottom bar (deeper in the neck), as shown in Fig. 2. By turning a hex key on the nut located on the body side, a guitar player shortens the active bar and applies a tension to it. As an effect, the truss rod bends down and pulls the neck against the strings. However, due to the sliding ability of the truss rod's opposite end, no additional compression is introduced to the guitar neck apart from the one triggered by a bending moment. This feature has been included in the model by creating a detailed contact definition between the truss rod and the neck/fretboard surfaces. Normal interaction was set to hard contact; tangential behaviour was scaled with a 0.2 friction coefficient between wood and steel (Lemoine *et al.*, 1970).

Each string has been divided into three parts – one active and two passive (Fig. 3). The active part is located between the nut and the bridge. It is the vibrating part of the string. The passive parts are located on both ends of the string. On the head end they provide a connection between the nut and the tuning pegs. On the bridge end they join the saddle with the pins. These short passive sections must not have been omitted, as they provide a counter-balance of the nut and bridge loading. Otherwise there would be too much bending applied to the saddle and the nut, resulting in a top plate warping. To avoid undesirable local effects and stress concentrations, strings have been attached to the guitar body by means of kinematic coupling distributed within a 10 mm radius of each adjoining surface. It was necessary to provide a reliable beam-shell interaction while allowing unconstrained rotations of the strings. A distance-dependent coupling has been applied with a square decay rate.

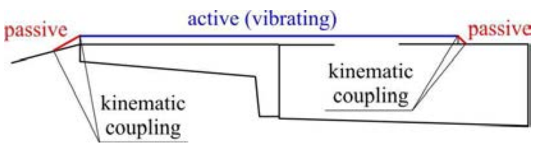

Fig. 3. Fixation of strings in numerical model.

2.2.3. Linear modal analysis

A standard modal analysis was performed in four different states. Two of them included full guitar geometry; another two consisted only of the separated top plate. In each of these pairs there was a variant with accelerometer mass added and without it. In case of the full body models, there was no strings attached and no truss rod pre-tension due to lack of non-linearity in this analysis. In the plate-only model a fixed support was applied to the outer boundary of the plate. A frequency range of up to 1000 Hz was requested, since higher frequency modes are of little use in case of modal analysis, as hinted by Zoran *et al.* (2012).

2.2.4. Tightening strings and truss rod

Setting up a real folk guitar is an iterative process. It consists of two alternately repeated steps – tightening/loosening the strings and adjusting a tension in a truss rod (Fig. 4). It is necessary to maintain a proper neck curvature when having the strings tuned to a pitch. The neck should be bowed away from the strings by a fraction of a millimetre, just enough to let the strings vibrate freely without hitting the frets. Too much of a back-bow will make the guitar uncomfortable to play, so a proper balance must be found. It is usually accomplished by trial and error by real musicians and it has been done the same way in the numerical analysis.

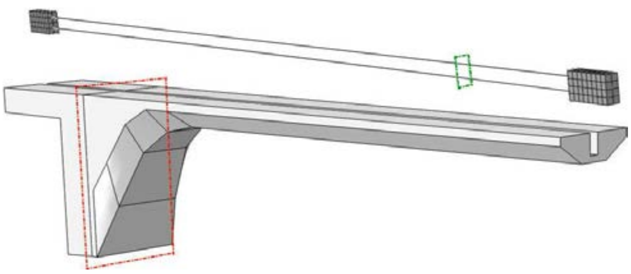

Fig. 4. Truss rod and guitar neck in FEM model.

Firstly, strings were loaded with appropriate stresses using a bolt load feature (Vegte, Makino, 2004) in order to tune them to their respective fundamental tones. The stresses were easy to calculate using common formulas for the wave speed. A guitar neck displacement on the 12th fret was measured. Then the strings have been removed completely from the model and an arbitrary bolt load was introduced to the active bar of the truss rod. The guitar neck has bent in the opposite direction and the displacement was measured again. Finally both the strings and the truss rod have been loaded. After several attempts, a desired curvature was obtained with the strings tuned to adequate fundamental tones. In the end, the active bar was shortened by 0.2 mm. Global displacement and stress fields were exported and used as a predefined beginning state for a subsequent dynamic analysis.

2.2.5. Dynamic analysis parameters

Abaqus solver offers two basic algorithms commonly used for dynamic numerical computations in time domain. Implicit analysis is carried out by Abaqus Standard, while explicit is controlled by Abaqus Explicit solver. Implicit algorithm allows for sparse discretisation of time domain, thus providing a relatively short path to the final solution by allowing large time steps. As a downside, results are written only at a small amount of fixed time points, each of which is quite costly computationally. Explicit dynamic analysis relies on a simpler algorithm, so each computational step is much faster than its implicit counterpart. If the time step is too large, however, numerical instability can occur and lead to false results. Because of that, explicit analysis is limited by a parameter called critical step, which is the largest time step Δt allowed while assuring a numerically stable solution (Arnold, 2001):

$$\Delta t \leq \frac{2}{\omega_{\max}}. \tag{1}$$

This criterion is dependent on the largest eigenvalue $\omega_{\max}$ [rad/s] of the whole system, obtained by means of modal analysis (Bathe, Wilson, 1976). A stable time increment relies heavily on FE discretisation of the model and is usually much smaller than that allowed in implicit analysis, enforcing a dense discretisation of time domain. Successive computation steps are based on inverting the mass matrix, which is much less costly to execute than manipulating the stiffness matrix typical for implicit analysis. As a result, explicit analysis is a preferred choice when a high resolution of output data is demanded.

To deal with the task effectively, a mixed implicit-explicit approach was taken. To set up the guitar, tune the strings and find the truss rod tension, the implicit algorithm was chosen due to its ability to reach equilibrium in few steps. Dynamic excitation was performed using the explicit solver, which required some import-export actions of the initial state and properties because of lack of full compatibility between these engines.

The largest eigenfrequency of the system was found to be approximately $1.25 \cdot 10^7$ rad/s by means of modal analysis, fixing the critical step at 10^{-7} seconds order of magnitude, which requires a 10 MHz sampling rate of numerical data. Since typical experimental measurements of response spectrum rarely deal with values greater than 2 kHz, let alone numerical studies, we decided to execute our analyses a 4 kHz frequency and write the output data only each $2.5 \cdot 10^{-4}$ second. It helped to reduce and manage the output files size significantly in spite of performing the computations with the required critical time step. A total time interval of 0.5 second was analysed and it resulted in a 2.5 GB output data file per analysis. The calculation time was approximately 4 hours on a moderately powerful PC with a 6-core 4.2 GHz processor and 16 GB of RAM.

2.2.6. Excitation methods

Experimental sine sweep and white noise measurements were executed without the strings attached. However, in this case the truss rod pre-stress persists, as it is not loosened together with the strings. To properly follow this procedure, the strings were removed from the model, but the truss rod was still loaded as described above. It resulted in a slight back-bow of the neck.

Since there is no acoustic medium in the model, it was not possible to precisely reproduce the speaker excitation method. A simplified method was used instead, pushing the top plate directly by applying a time-dependent pressure. White noise and 20 Hz–20 kHz sine sweep signals were generated and used as amplitude functions for the pressure, both 0.5 second long.

Strings strum excitation was divided into two steps in the numerical analysis. In the first, quasi-static step – the strings were pulled; in the second, dynamic – the strings were released and put into vibration. Function of pulling the strings consisted of 0.01 s buffer inactivity, 0.01 s linear ramp function and another 0.01 s buffer inactivity, resulting in 0.03 s total. The strings were pulled in 1/10th of their length from the bridge; the pulling amplitude was 5 mm in the direction parallel to the top plate and 2 mm in the perpendicular direction. Afterwards, the strings were released and put to vibration for 0.5 s in the second step.

2.2.7. Damping

Damping is usually introduced to dynamic analyses not only to bring a solution closer to the reference data, but also to stabilise the analysis in order to reduce the numerical noise originating from the unwanted vibration. In cases where the acoustic signal quality is not of much value, a simple mass- and stiffness-proportional Rayleigh damping (Liu, Gorman, 1995) is a likely option. In other cases, more sophisticated damping models should be introduced, often utilising variables instead of constants (Torres, 2010). However, wood materials are inconvenient enough to describe by means of static values, leave dynamic ones alone. There are several methods of measuring damping parameters of wood (Falk, Itani, 1987; Wegst, 2006), often exhibiting diverse results. Bissinger and Keiffer (2003) have stated that even though there is large variation in total damping ratios of violins, they can be roughly estimated using a common frequency-dependent curve $1/\sqrt{f}$. In a sense of being an asymptotic power function, it is similar to the mass-proportional part of Rayleigh damping:

$$\xi = \frac{\alpha}{2\omega} + \frac{\beta\omega}{2}. \tag{2}$$

It coincides with the fact that the beta coefficient responsible for damping of the higher register directly affects the stable time increment of explicit analysis mentioned before. In the equation below, ξ is the damping ratio of the highest eigenfrequency $\omega_{\max}$ of the system, thus dependent mainly on the stiffness-proportional coefficient:

$$\Delta t \leq \frac{2}{\omega_{\max}} \left(\sqrt{1+\xi^2} - \xi \right). \tag{3}$$

As a result, the critical time step tends to drop dramatically even at very small values of the beta coefficient. Since proficiency of explicit analysis suffers greatly from this factor, it is usually advised to leave out the stiffness-related component and use only the mass-proportional damping coefficient (Xiaoming *et al.*, 2015). This approach had been undertaken by Torres in his PhD thesis (2010) and it was also chosen for our study. A complicated, variable and material-dependent damping definition is not feasible. If a really accurate damping model was to be implemented, it would need to be linked to the air movement inside and outside the sound box to create a dissipative mechanism. Since acoustic medium is absent at the present stage of the modelling, a simple Rayleigh approach was chosen despite its well-known flaws and artificial nature.

The alpha coefficient was scaled to filter out vibrations below 50 Hz, which is the lowest eigenfrequency of the system (corresponding to the upwards and downwards motion of the neck). The wood damping ratio was found to equal approximately 0.02 based on various sources (Brémaud *et al.*, 2009; Brémaud, 2012). This is also the value used by Torres in his dissertation. The damping ratio of the steel strings was set to $\xi = 0.001$ (Irvine, 2004), and the truss rod to $\xi = 0.1$. Heavy damping was applied to the truss rod to reduce its disruptive effect on the output, since its mass and stiffness is much greater than that of the strings; in a real guitar, its independent high-frequency vibrations are suppressed by silicon tubes.

2.3. Experimental study

It should be noted in the very beginning that the experimental study was carried out on a specific purpose. We are well aware of the many measurement devices developed throughout the years, such as laser Doppler vibrometer, electromagnetic sensors, etc. Regretfully we did not have access to any of this equipment, since the only devices available to us are designed for civil engineering and are not very well suited for the relatively lightweight and high-frequency structures such as musical instruments. For this reason we were forced to use large piezoelectric accelerometers commonly used to measure vibration of bigger engineering structures. Still, experimental measurements were necessary to interpret the results of modal analysis and to further investigate the outcome of numerical excitations. In addition to the measurements being performed using rather massive accelerometers, the excitation was carried out by means of a guitar combo speaker. The obtained results were obviously affected by these flaws. However, the experimental study was executed primarily to compliment the numerical study – which is the main part of the research – by providing supplementary material for comparison. These result should by no means be treated as a stand-alone value of its own. We tried to compensate for the heavy accelerometers by including additional concentrated mass in the numerical model, however no rotational inertia was introduced.

When holding a guitar on stage, a musician usually mounts a strap on his instrument to prevent it from being dropped on the ground. There are two buttons on most guitars designed for installation of the strap. They are attached with long screws and they sit in the wooden body firmly. These buttons have been used as supports in the experiments (Fig. 5a). They were immobilized between jaws of two vices. Small felt discs, placed by the manufacturer between the button and the body to protect the wood, have been removed for a firmer grip.

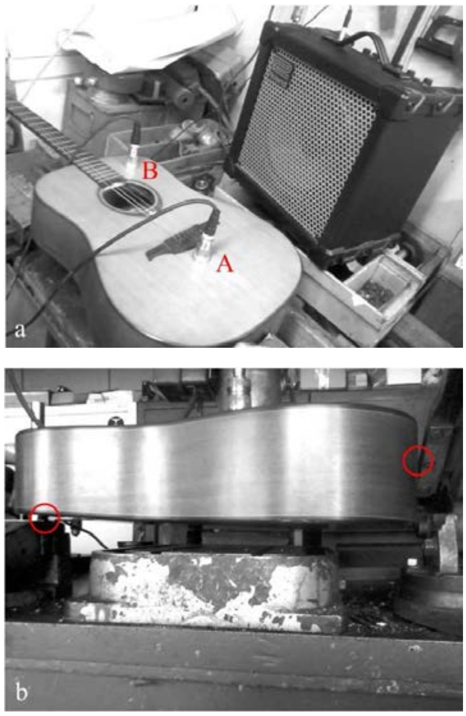

Fig. 5. Guitar support (a) and accelerometer location (b) during excitation in laboratory.

The measurement was performed with two VibraSens 500 mV/g 160 g piezo accelerometers connected to the Alitec ViMEA VE16BCA device capable of writing up to 65536 samples per second. The accelerometer A has been placed in a symmetrical position near the bridge (Fig. 5b). The accelerometer B was intended to capture different spectrum of vibrations, thus it has been installed away from the bridge, on the asymmetrical position. Both accelerometers are equipped with magnets, as they have been designed for measurement of steel structures. These magnets were used to install the accelerometers by placing small brass tiles inside the guitar body, on the opposite side of the plate (Fig. 6). A considerable force was required to dismount the accelerometers afterwards.

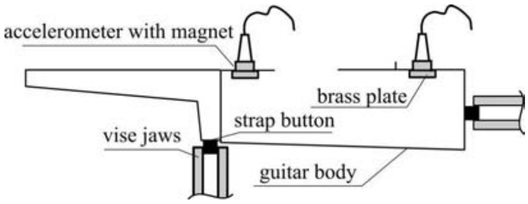

Fig. 6. Strap buttons fixed between vice jaws and accelerometers fastening using brass plates and embedded magnets.

The excitation of the guitar has been executed in the following ways:

- with the strings attached, by strumming all the strings simultaneously; no frets pressed (open strings);
- with the strings removed, using a guitar speaker to excite the top plate with an acoustic wave; a white noise sample (4 seconds exposure) and a logarithmic sine sweep through 20 Hz–20 kHz frequencies (4 seconds sample).

For technical reasons it was not possible to place the speaker directly above the top plate, in a parallel position. To ensure a proper body excitation, nearly a full power of a 40 watt Roland CUBE 40XL amplifier has been used. The data was written with a frequency of 4096 samples per second, allowing to build a harmonic spectrum up to 2048 Hz using a FFT algorithm later on. With the lowest and highest strings tuned to 82 and 330 Hz, it was possible to capture respectively 23 and 5 overtones of these notes.

3. Results and discussion

By investigating the results of the numerical modal analysis alone (Fig. 7) we can draw first conclusions. Introducing a full guitar body to the model resulted in significant changes as compared to the plate-only model. Both the shapes and eigenfrequencies were affected. Different boundary conditions (full body) of the top plate reduced the stiffness and generally shifted the resonant frequencies into a lower register. What is more, they introduced some additional modes of vibration coupled to the global body motion, which were absent in the plate-only model. Similar modes were grouped using black frames in the figure. About 2–3 new shape patterns emerged in the frequency range up to 600 Hz. The observed eigenmodes were heavily asymmetrical due to the arrangement of the top plate bracing. Hence, the choice of accelerometer A position near the middle of the bridge is reasonable in case of the examined acoustic folk guitar; contrary to the classical guitar, where it would lie on a nodal line of the even-number modes (Torres, Boullosa, 2009).

These modes, however, are purely structural, that is they are limited to the body motion and not air motion. By comparing them with the experimental results (Fig. 8) we can make further observations. Regarding Wolfe's equation for an approximate Helmholtz frequency in a guitar sound box (Wolfe, n.d.a.), the first cavity mode should occur at about 220 Hz. Since it is calculated for a constant air volume, it should be coupled with a 2nd or 3rd body mode, which means the coordinated motion of top and back plates. This peak is easily identifiable on the experimental spectrum. The first air-pumping mode, produced by inverse vibration of top and back plates (changing volume (Russell, 1998)), should be coupled with 1st body mode and is also distinguishable around 120 Hz mark. The distinctive peaks on A and B curves respectively are probably A and B accelerometers resonances (see modal analysis) around 80 Hz and 180 Hz, not coupled with neither body nor air modes (isolated vibration of top plate). Considering the mass of accelerometers we can assume that they are capable of dominating local motion of the guitar.

Examination of the numerical white noise (Fig. 9a) and sine sweep (Fig. 9b) excitations evokes some interesting, if not obscure notices. The simulation allows to identify the accelerometer peaks quite precisely, however the body modes are nearly absent. The situation gets slightly better in the upper register, above 400 Hz mark, where mass is less dominant and stiffness draws more influence on the modes. It clearly indicates that there is a problem with reflecting coupled body-air modes, which makes sense since air is not included in the model and the plate is not heavy enough (mass) to overcome the accelerometers. The first important observation can be made – acoustic medium is crucial for a proper representation of the coupled modes. Otherwise the low-frequency body modes alone are not strong enough to emerge on the spectrum.

Strumming the strings in the numerical model (Fig. 9c) resulted in a spectrum that represents a general trend line of the guitar dynamic response, with string modes only subtly marked. Harmonic peaks are not as pronounced and steep as expected by comparison to the experimental measurements. Again the situ-

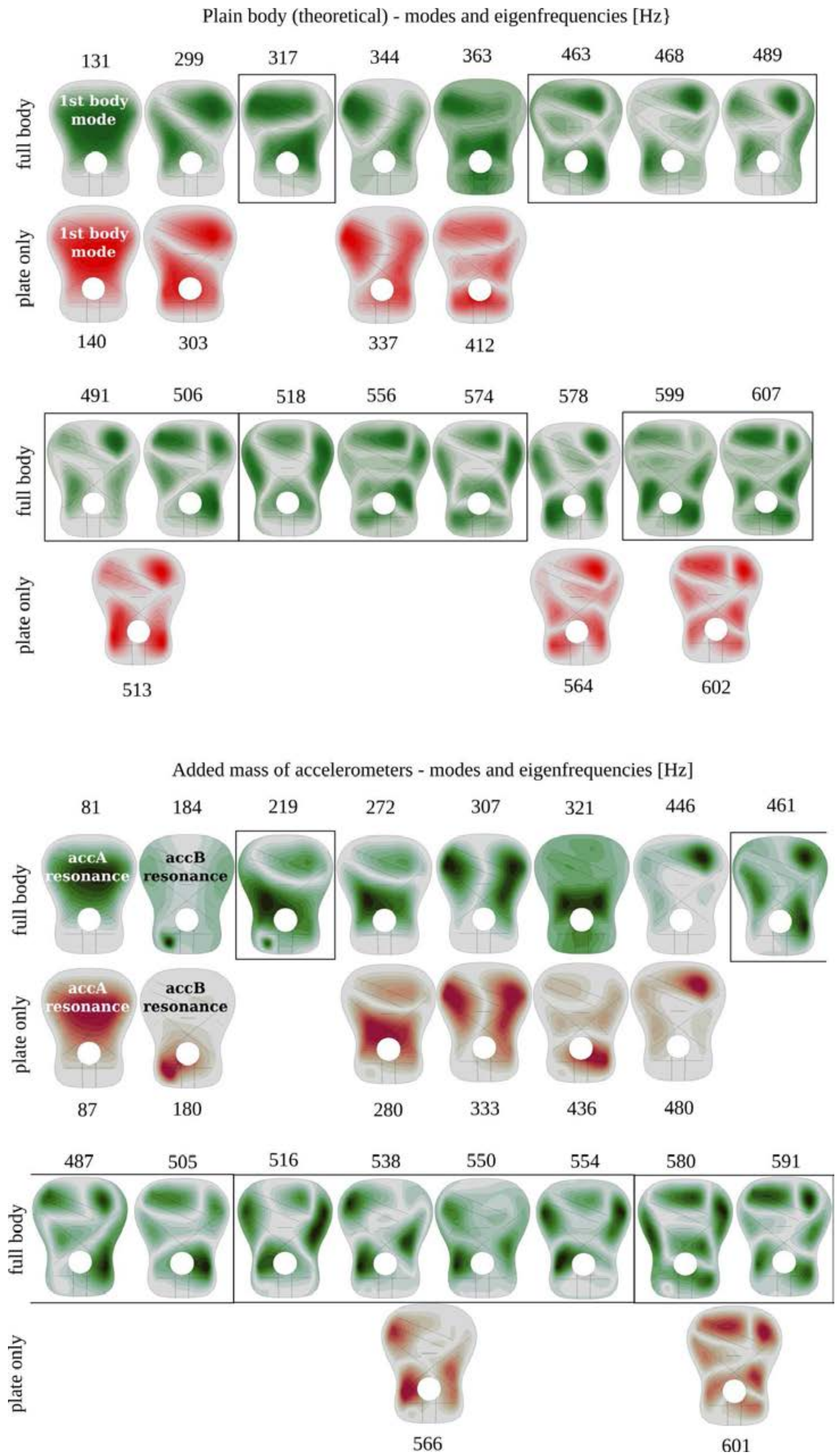

Fig. 7. Modes of vibration and corresponding eigenfrequencies [Hz] in four variants of modal analysis.

ation is slightly better in the upper register. In this case mass of the strings is probably not enough to drive the plate in a purely structural model, because the plate is not damped enough by the air inside the box and starts to resonate in its own modes. Rayleigh damping does not help much in this case, because it affects the whole

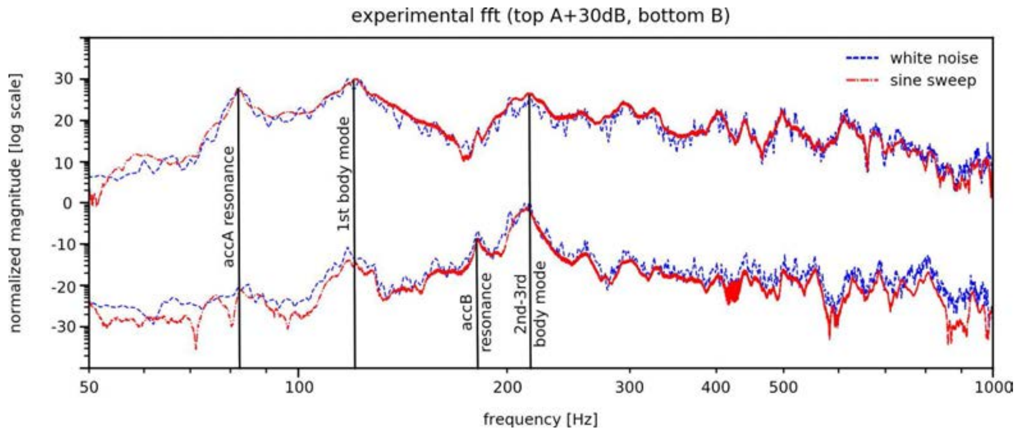

Fig. 8. Response spectra of experimental white noise and chirp excitations.

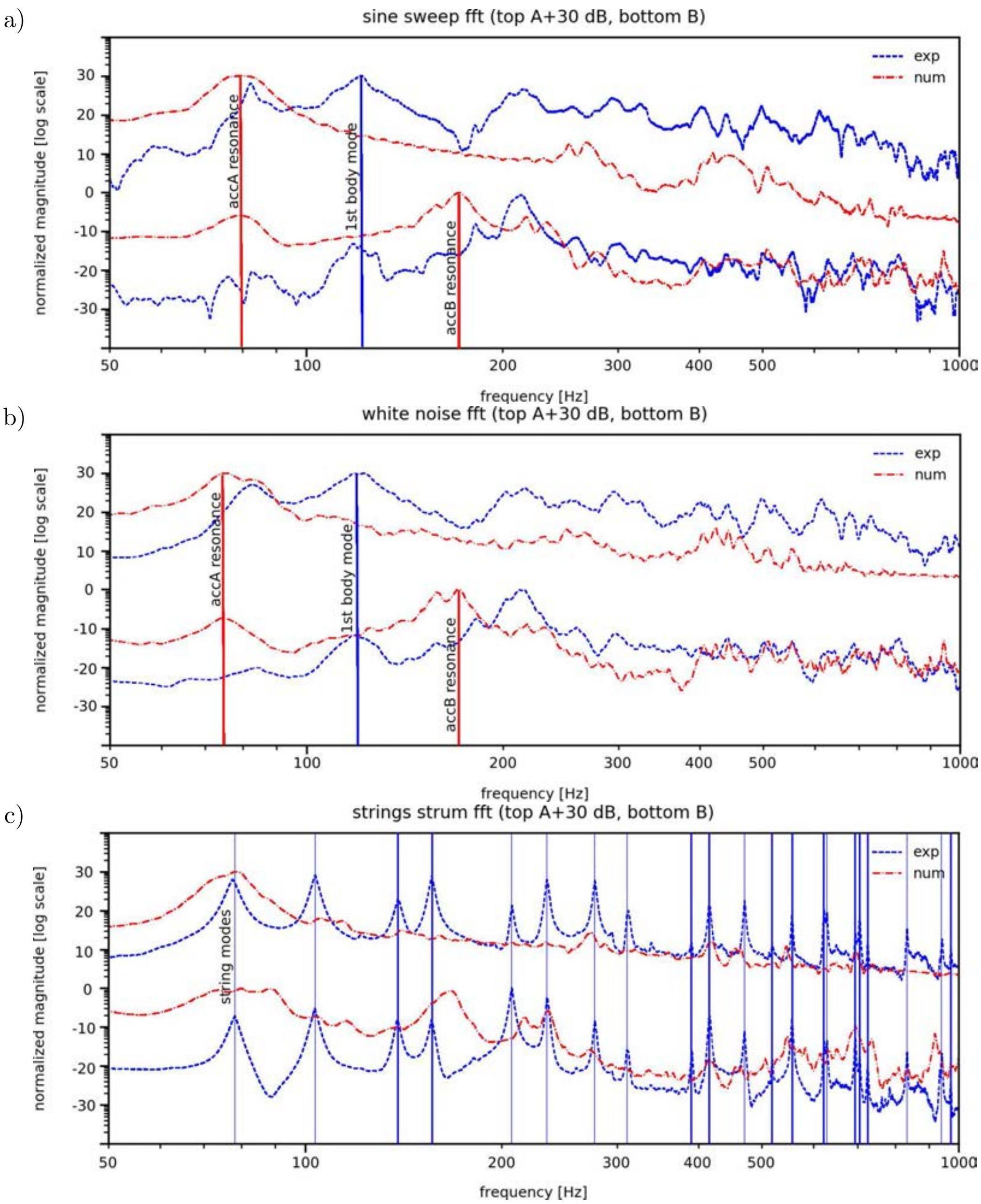

Fig. 9. Response spectra of various experimental *vs* numerical excitation variants: a) sine sweep, b) white noise, c) string strum.

model, effectively killing string harmonics. Again, air medium is necessary to provide natural damping of the body and allow the strings to vibrate freely in the same time.

Pearson correlation coefficient (Table 4) shows that sine sweep and white noise excitations gave basically the same results, however the sine sweep spectra exhibit better correlation between the numerical and experimental data, slightly outperforming the white noise excitation. Striking the strings is on the last place, nevertheless it does not stand out much, retaining resemblance to the white noise and chirp excitations (which is not desired).

Table 4. Pearson correlation coefficient r between various response spectra; E – experimental, N – numerical, sw – sine sweep, wh – white noise, str – strings strum, accelerometer position A in upper right half of the matrix, position B in bottom left half.

	Esw	Nsw	Ewh	Nwh	Estr	Nstr
Esw	1	0.757	0.973	0.727	0.560	0.687
Nsw	0.543	1	0.738	0.969	0.723	0.894
Ewh	0.949	0.545	1	0.708	0.519	0.663
Nwh	0.431	0.892	0.449	1	0.712	0.892
Estr	0.571	0.580	0.561	0.476	1	0.698
Nstr	0.276	0.690	0.237	0.676	0.452	1

The fact that the air modes have a large impact on the sound box proficiency is no mystery and it has been investigated in multiple ways. A classic research by Bissinger and Hutchins (1983) followed by Weinreich *et al.* (2000) and McLennan (2003) clearly shows that cavity modes are coupled with body modes and can be affected by a type of gas inside the sound box. In the same time, the number and size of sound holes defines not only the stiffness of the plate, but in the first place the air-body coupled modes caused by the internal pressure fluctuation. Runnemalm (1999) managed to record many forms of high-frequency resonance in a closed air cavity as well as inside a guitar box, proving that not only the fundamental cavity mode is of importance, but the whole instrument range is significantly affected by the air vibration.

Lack of air cavity resonance modes and coupled structure-air modes is apparent in the numerical solution. Regarding Wolfe's equation for an approximate Helmholtz frequency in a guitar sound box (Wolfe, n.d.a.), the first cavity mode should occur at about 220 Hz. Its presence is easily noticeable as a second (or third) large peak in the experimental spectra. Since it is absent in the numerical analyses, relatively more energy is attributed to the first body mode, resulting in its pronounced domination over the rest of the register. It should be noted that around the 800 Hz mark the numerical representation of the point A spectrum takes on a flat shelf shape, indicating low-energy structural modes with no support of cavity modes. As the air resonance does not have such a large influence on the accelerometer B position, which is (according to the modal analysis) dominated by the presence of the heavy accelerometer itself, numerical response spectra B are seemingly better aligned with the experimental data.

Air has been present in guitar numerical analyses for some time already. Efforts have been made to introduce air (Elejabarrieta *et al.*, 2002) and other types of gas (Ezcurra *et al.*, 2005) to the classical guitar analysis. Despite its proficiency in calculating air modes and providing the air-body coupling and additional mass, it was usable only in the modal analysis. Derveaux *et al.* (2003) and Bécache *et al.* (2005) have implemented a theoretically complex and exhaustive algorithm for computing a coupled sound box and air vibration. However, the method incorporated limited geometry, lacking many guitar parts, the truss rod and strings pre-tension. A more straightforward solution is to be found, such as the one suggested by Jackman *et al.* (2009). It must be capable of providing a relatively simple air-body coupling model and a natural damping mechanism by energy dissipation at the absorbing (non-reflecting) acoustic medium boundaries. Recent findings by Sauer and Luginsland (2017) regarding general FE fluid-structure surface interaction come to mind. On top of that, the model must be a proficient performer in explicit analysis.

To check the efficiency of the model two simple comparative analyses were executed. The first one checked behaviour of the model with and without the truss rod. Both models were subjected to chirp excitation and to string strum excitation. The second variant required to tighten and pull the strings, resulting in some initial deformation of the model. This deformation was compared with the same situation in the truss rod model. The picture is quite self-explanatory, as it presents the proper deformation of a guitar with truss rod (Fig. 10a, compensated straight neck) and a completely wrong and unplayable (for a guitarist) initial state in case of truss rod absence (Fig. 10b) as compared with the relaxed guitar (Fig. 10c). The resulting spectra are different, however they differ in a non-obvious way, since after truss rod removal the sine sweep variant (Fig. 11a) became duller but the string strum one (Fig. 11b) got brighter. Explanation of this effect needs more insight, however the influence of truss rod is apparent.

The other analysis introduced some design changes to the guitar model with truss rod. The reference top plate, which is a 3.0 mm thick sheet of red cedar, was changed first to a 3.3 mm plate (thickness change), then to a 3.0 mm spruce plate (material change). In each case, these changes affected both the mass distribution and the stiffness.

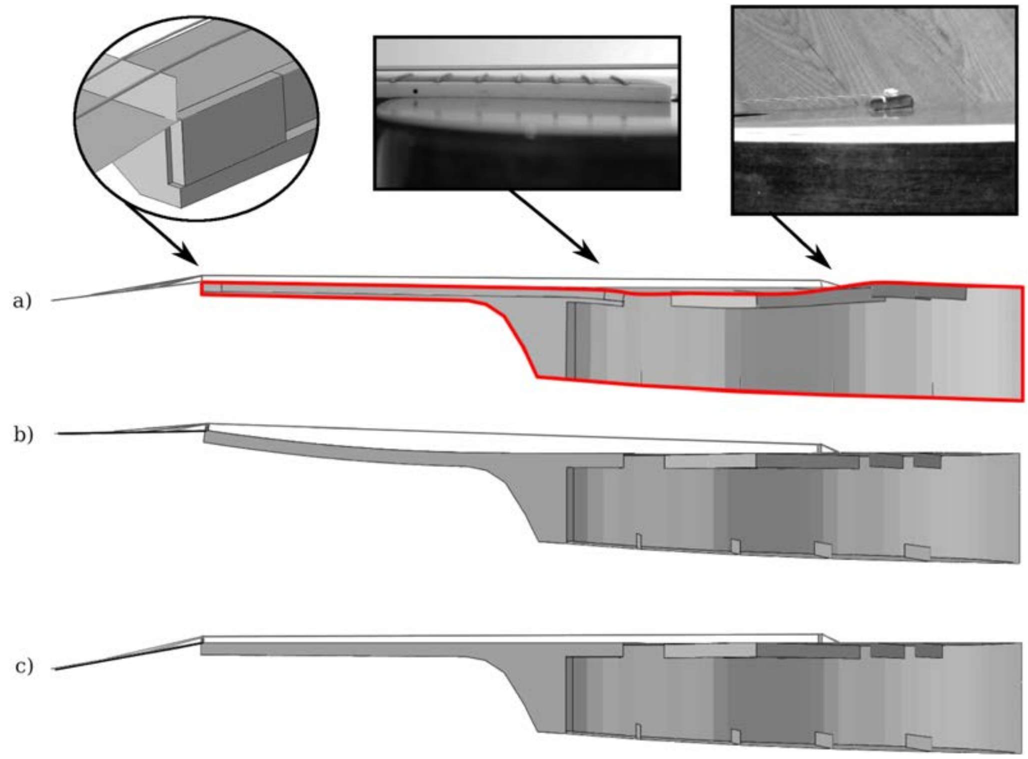

Fig. 10. a) Up-scaled deformation of FEM model in equilibrium state after performing a geometrically non-linear incremental analysis; visible resemblance to common rheological effects of old guitars and exposed ability of non-fixed truss rod end to slide along guitar neck under compression; b) deformation of guitar without truss rod; c) relaxed guitar.

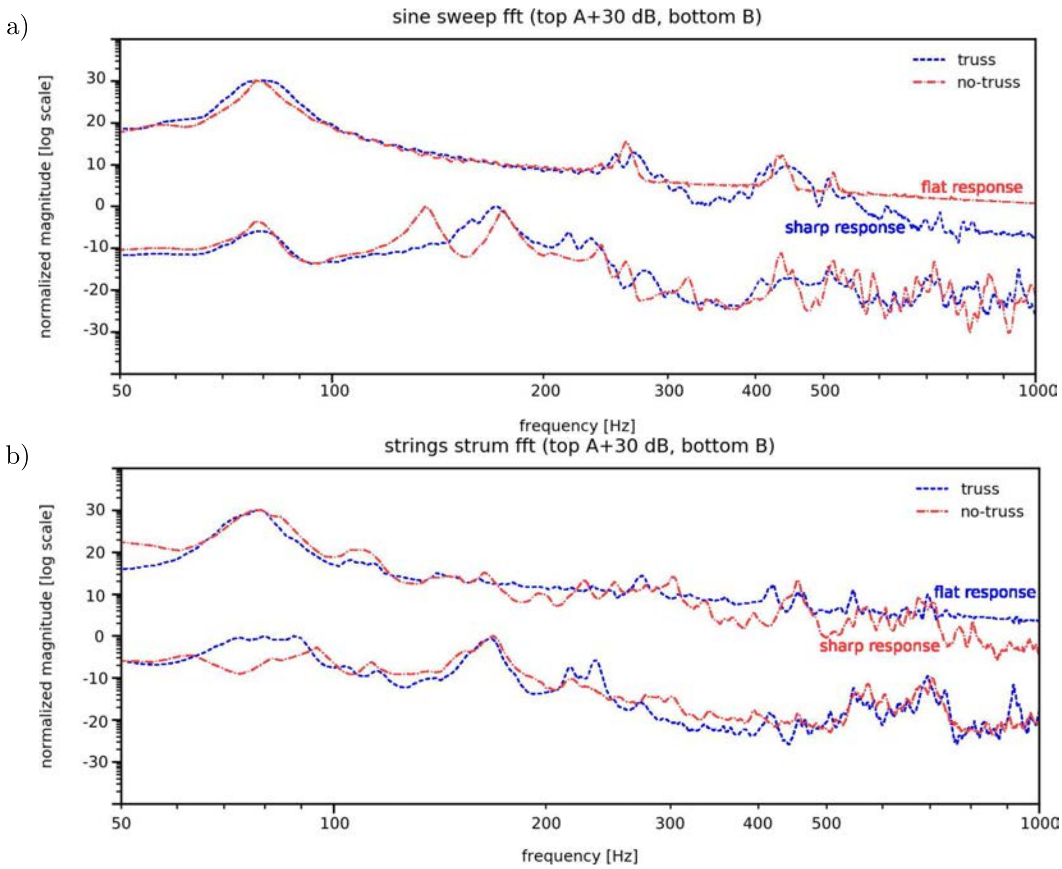

Fig. 11. Response spectra of numerical excitations with and without the truss rod: a) sine sweep and b) string strum.

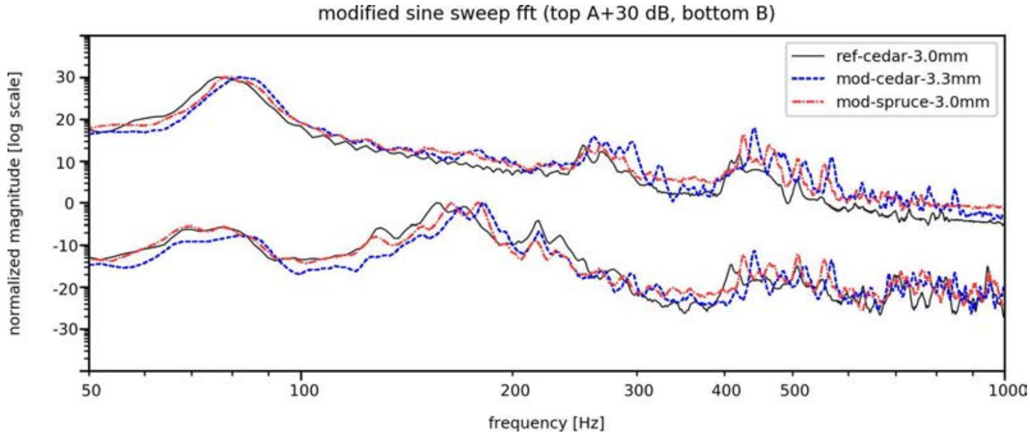

Fig. 12. Comparison of numerical response of reference guitar with red cedar top and its modifications.

Now let us consider several scenarios. It is clear that if a model includes just a single material applied to the whole body, then the material properties expression act as a constant factor in front of the governing eigenproblem equation. Therefore, in case of modal analysis of the plate-only model, we could expect proportional shifting and shrinking of the eigenvalues set – and consequently the possible response spectrum – but not a complete change of shape of the spectrum. This would be possible only if a more non-straightforward change was introduced to the stiffness matrix, i.e. if the whole body was analysed, but only the top plate was modified before. However, the observed change should not be very radical, since top plate is the main vibrating part of the modelled instrument and the rest of the body acts more like a set of interactive boundary conditions than a vibrating structure itself.

These changes start to be significant when we switch from linear modal analysis to geometrically non-linear incremental analysis. Properties of the model play their role twice in this case. First – during setting up the guitar and determining the initial state of stress and deformation. Second - during excitation, by responding differently due to both the change in properties and in the initial state. In this case we would expect the response spectrum to change its character in a heavily non-linear fashion, not just by shifting the peaks, but also by creating new ones and altering the proportions between them.

The results of the simulation are presented in Fig. 12. It is already visible that the modifications of the top plate parameters result in different shapes with new peaks emerging, rather than in modified variants of the same spectrum. For example, one can notice that more life can be brought above the 800 Hz mark by means of increased thickness (more mass and stiffness) instead of switching to the spruce top (more mass and much more stiffness). This is just a sample comparison, hard to predict intuitively, which exhibits the nature of the presented approach. We need to have more confidence in the model to draw real conclusions, however we can already tell that some non-obvious relations can be observed.

4. Conclusions

The main novelty of this paper lies in introducing the geometrically non-linear effects of truss rod and strings tension to a detailed folk guitar FEM model. The main findings can be summarised as follows:

- Modal analysis of a full guitar body leads to computation of some additional top-plate modes and shifting of the existing modes into a slightly lower register as compared to the fixed plate-only model;
- Tightening the strings in the model with an adjustable truss rod leads to a drastically more appropriate state of deformation than in a similar model with the truss replaced by a passive wooden brick;
- Response of the model with a truss rod differs significantly from that without a truss rod, however this difference is not to be obviously interpreted;
- Simulation of striking the strings will not work properly until the model is equipped with an acoustic medium, providing natural damping conditions and air-pumping modes; The same is true for any coupled modes.

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