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Stability state evaluation of composite three-layered annular plates with asymmetrical damage

The paper presents the response of a three-layered annular plate with damaged laminated facings to the loads acting in their planes. The presented problem concerns the analysis of the combination of global plate failure in the form of buckling with the local micro defects, like fibre or matrix cracks, located in the laminas. The plate structure consists of thin laminated, fibre-reinforced composite facings and a thicker foam core. The matrix and fibre cracks of facings laminas can be transversally symmetrically or asymmetrically located in plate structure. Critical static and dynamic stability analyses were carried out solving the problem numerically and analytically. The numerical results show the static and dynamic stability state of the composite plate with different combinations of damages. The final results are compared with those for undamaged structure of the plate and treated as quasi-isotropic ones. The analysed problem makes it possible to evaluate the use of the non-ideal composite plate structure in practical applications.

1. Introduction

The propagation process of various damages during the use of construction elements, especially made of laminated composites is difficult to predict. One of the situations, in which the element is particularly exposed to defects, is the case of rapid change in structure form associated with the buckling phenomenon. The loss of plate stability is a form of the global failure of element. Connecting this form of plate failure with the possible local defects, like micro cracks, is a complex problem of important practical meaning. The cracks of fibres or matrices of fibrous composite are one of the basic forms of failure. However, a partial or even total degradation of composite structure in the above form does not necessarily

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eliminates it from further work, particularly if the composite element is a part of a layered structure, like, for example, the popular three-layered, sandwich one. Composite laminates, owing to their material, mechanical and strength properties are used in various engineering fields, like: aircraft industry, automotive industry, civil engineering.

The evaluation of the possibilities of use of a structure with defects has been undertaken in this paper. Numerically examined was a three-layered annular plate with foam core and laminated, fibre-reinforced composite facings. The damages of plate facings change the structure rigidity and generate the disturbance in the symmetry of the plate original structure. The responses of the analysed plate structure to static and dynamic loads were evaluated considering different combinations of facing damages caused by the failures of the laminate fibres or matrix.

The examinations of layered structures are still topical and are widely undertaken. The area of the possible applications of the annular plates is extensive, for example in: aerospace industry, mechanical and nuclear engineering, civil engineering or miniature mechanical systems. The axisymmetric, dynamic stability of additionally rotating sandwich plates with viscoelastic core under periodic radial stress is presented in papers [1, 2]. The dynamic problem of vibrations of an annular plate with a characteristic microstructure and functionally graded properties is presented in work [3]. The problem of axisymmetric buckling of laminated composite circular and annular plates is presented in works [4, 5]. Three-dimensional axisymmetric buckling of laminated annular plates, which consist of transversely isotropic layers, has been analysed in that study.

The three-dimensional theory of elasticity was also used in the analysis of axisymmetric deformation of a laminated transversely isotropic annular plate. The exact solution corresponding to specified boundary conditions and plate dimensions was presented in paper [6]. The axisymmetric vibrations of annular sandwich plates with isotropic core and composite facings studied using the harmonic quadrature method are presented in paper [7]. The effect of the shear deformation and rotatory inertia in the core has been taken into account there.

The quasi-isotropic composite circular plate under quasi-static lateral load and low-velocity impact tests is presented in paper [8]. The analysis was performed with the use of non-linear approximation method and the large deflection plate theory. Analytical and finite-element results are compared with results of measurements. The results show that the low-velocity impact responses are close to the quasi-static behaviour of the plate. The fibre damage image, along with the damage propagation from the centre of plate to the edge, are presented for plates of different thicknesses.

The thin-walled sandwich rectangular plates with composite faces axially compressed are studied in [9]. Local failure damage of sandwich structure is presented using the finite element analysis. The transverse full symmetry in a plate of sandwich structure, which is composed of two multilayered FRP (fibre reinforces plastic) faces is examined.

The bifurcational instability presented for rectangular plates made of fibrous composite materials with reinforcement subjected to long-term damage is formulated and solved in paper [10]. The problem of matrix cracking and delamination in laminated composites is presented in work [11]. A model for prediction of the propagation process of transverse cracks in polymer matrix composite laminates is proposed. Different crack patterns are analysed. The failure model for simulation of change in laminated composite plates is presented in [12]. Plates are subjected to dynamic loading. Matrix cracking propagation is analysed numerically. Impact test on FRP laminated plates confirms the effectiveness of the presented model. The mathematical formulation for the modelling of damage in laminated composite plates and shells is presented in works [13, 14]. The micromechanical model for predicting the impact damage of composite laminas is proposed in [15]. The model is based on the laminate microstructure and different failure models like: matrix cracking, fibre breakage and delamination.

Numerical procedures to simulate the Lamb wave propagation in damaged CFRP laminate are presented in [16]. The examined composite plate is under low-velocity impact. The non-destructive testing method to characterize composite material local damage is presented in [17].

Paper [18] presents the use of the finite element analysis procedure, which is developed to predict the initiation and propagation of damages in laminated composite plates. A second-order damage tensor represents damage of each lamina. Constitutive relation takes the form where the elasticity tensor isn't constant but depends on the additional damage tensor. The obtained theoretical results in the form of force-time history for an impact agree well with the experimental ones. Approximate solutions for free vibration of asymmetrically laminated annular and circular plates are presented in [19]. The plates have asymmetry due to either hybridization and lay-up. The strength of laminates can be improved by high compressive-strength fibres located in the regions where compressive failure is predicted. The authors noticed the necessity to study the dynamic mechanical properties of that group of composites with mid-plane asymmetry.

Presented in this paper examinations widen the analyses of the problems, which concern the stability responses of the sandwich plates with undamaged and damaged laminated, fibre-reinforced composite facings undertaken in papers [20–22]. Static and dynamic responses of three-layered annular plate with undamaged facings are presented in paper [20]. Examples of damaged facings in plate structures in the forms of fibre or matrix cracks are analysed in papers [21, 22] for plates statically and dynamically loaded, respectively. Examined plate cases are chosen for these ones, whose structure is transversally symmetrical. It means that micro defects located in facings are in the same places in both facings. In real structure, the defects of the lamina could be distributed randomly and differently in the two plate facings. An attempt to create the full image of plate stability responses is shown in this paper. The results presented there concern the models with variously damaged structures. Transversally asymmetrical forms are analysed with special attention. Some results are presented in [23]. The way of description of the damage of laminate facings accepted in the analysis and also presented in this paper, uses the mathematical formulae proposed in work [24], which modify the elements stiffness matrix of composite structure.

According to the author knowledge, the problem of stability analysis of laminated composite sandwich plates with both healthy and damaged laminas of facings has not been sufficiently analysed. The analysed problem is theoretically interesting and practically important.

2. Problem formulation

The presented composite, three-layered annular plate consists of thin, laminated facings and a thicker foam core. Micro fibre or matrix cracks located in facing laminas create the symmetrical or asymmetrical transversal geometry of plate cross-structure. The plate is loaded in the plane of facings with static and dynamic radially compressive forces uniformly distributed on its inner or outer perimeter. In dynamic stability problem, the acting load quickly increases in time according to the formula:

$$p = st, \tag{1}$$

where: p – compressive stress, s – rate of loading growth, t – time.

The character of accepted loading is not a kind of impact, which would be the case for loads acting within the time of $10^{-4} \div 10^{-6}$ s. Then, the inertial forces in the middle plane of the plate have not been taken into account.

The analysed plate examples are for plate model with slideably clamped edges. The scheme of the analysed plate is presented in Fig. 1.

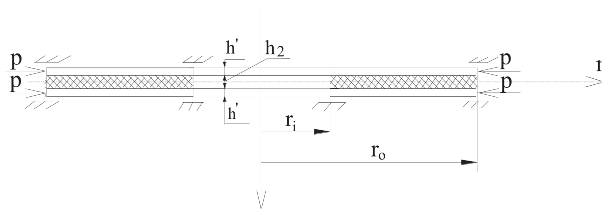


Fig. 1. Scheme of analysed plate

The evaluation of the dynamic critical parameters requires acceptation of the plate stability criterion. As a criterion, the conditions presented by Volmir in work [25] were adopted. According to this criterion, the loss of plate stability occurs at the moment of time t_{cr}, when the speed of the plate point of maximum deflection w_{dcr} reaches the first maximum value. The critical time t_{cr}, determined after the calculation operation shown in Eq. (1), expresses the value of the critical dynamic load p_{crdyn}.

Plate facings are composed of four laminas with fibres arranged according to the code [0/−45/45/90]. The configuration of laminas fulfils the conditions of quasi-isotropic composite allowing for the evaluation and the comparison between the analysed structures of plate models.

The plate model, which is built using the finite elements, or for plates with quasi-isotopic facings, is the result of the analytical and numerical solution, which uses the finite difference method.

3. Model of fibrous composite and composite degradation

The stiffness matrix of laminate facings is expressed using the mechanical relations for classical lamination theory [24, 26, 27]:

$$A_{ij} = \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k (z_k - z_{k-1}), \tag{2}$$

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right), \tag{3}$$

$$D_{ij} = \frac{1}{3} \sum_{k=1}^{N} \left(\overline{Q}_{ij}\right)_k \left(z_k^3 - z_{k-1}^3\right), \tag{4}$$

where: A_{ij}, B_{ij}, D_{ij} – extensional, coupling and bending stiffness, respectively, $\overline{Q}_{ij}$ – transformed reduced stiffness matrix of lamina, N – number of layers, z_k and z_{k-1} – coordinates in cross-section laminate of the outer surfaces of layer numbered as k and $k-1$ with thickness equal to t_k, respectively.

The elastic, engineering constants E, G, ν for configuration of quasi-isotropic composite are expressed by the following formulae [26]:

$$E = 2\frac{A_{66}}{t}\left(1 + \frac{A_{12}}{A_{11}}\right), \qquad G = \frac{A_{66}}{t}, \qquad \nu = \frac{A_{12}}{A_{11}}, \tag{5}$$

where: E – Young's modulus, G – Kirchhoff's modulus, ν – Poisson's ratio, t – thickness of the laminate, A_{11}, A_{12}, A_{66} – extensional stiffness A_{ij} (i, $j = 1, 2, 6$).

The fibre or matrix cracks in plate facings change mechanical properties of the laminate and the plate structure rigidity. The accepted model of the composite degradation is based on the theory of correction parameter method presented in work [24]. The mathematical essence of this method is based on the modification of the stiffness matrix, whose form for undamaged lamina is expressed by the following elements:

$$\begin{gathered} Q_{11} = \frac{E_1}{(1-\nu_{12}\nu_{21})}, \qquad Q_{22} = \frac{E_2}{(1-\nu_{12}\nu_{21})}, \\ Q_{12} = \frac{E_1\nu_{21}}{(1-\nu_{12}\nu_{21})}, \qquad Q_{21} = \frac{E_2\nu_{12}}{(1-\nu_{12}\nu_{21})} \qquad Q_{66} = G_{12}\,. \end{gathered} \tag{6}$$

It is assumed that the matrix crack eliminates the rigidity in the direction transverse to the fibres. It is expressed by the correction parameter η. For the lamina with matrix crack, the elements $\mathbf{Q}_{11}$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{22}$ take the following new values:

$$\mathbf{Q}_{11} = \eta \cdot Q_{11}, \qquad \mathbf{Q}_{12} = \mathbf{Q}_{22} = 0. \tag{7}$$

When the fibre crack occurs, the stiffness matrix modification is limited to replacing the elements $\mathbf{Q}_{11}$ by Q_{22} [24]:

$$\mathbf{Q}_{11} = Q_{22}. \tag{8}$$

The analysed problem of plates with quasi-isotropic composite facings was solved analytically an numerically using the orthogonalization method and the finite difference method (FDM), and only numerically using the finite element method (FEM). FEM method makes it possible to observe the plates with quasi-isotropic composite facings and exactly composite facings with damages. The examinations were conducted for plate models with damage of facings in the forms of fibre or matrix cracks of a single lamina or all laminas together.

4. Problem solution

The Finite Element Method (FEM) was the main method of the solution to the problem. FEM plate models make it possible to observe the static and dynamic stability behaviour of each of the examined structures: a composite without any defects, treated as quasi-isotropic one, and a composite having fibre or matrix cracks in selected facing laminas.

The proposed and presented in detail in works [20, 28–30] analytical and numerical solution to the problem of static and dynamic plate stability has been used to calculate the critical values of time, deflection and load for quasi-isotropic example of the examined plate model. The observations were carried out both for undamaged facings and for a complete damage in the form of fibre or matrix cracks while the cross-section symmetry remains unchanged.

4.1. Using the Finite Element Method

The calculations were carried out using the ABAQUS system at the Academic Computer Center CYFRONET-CRACOW (KBN/SGI_ORIGIN_2000/Płódzka/030/1999). The full annulus plate model was built of 9-node shell elements and 27-node solid elements creating the mesh of the facing and the solid, respectively. The outer surfaces of facings and core mesh elements are tied using the program option expressed as surface contact interaction. The options of the programme: Buckle and Dynamic were used for examining the static and dynamic stability problem. The structural stiffness of plate facings was expressed by the elements of matrixes A_{ij}, B_{ij}, D_{ij} (2)÷(4), which were calculated separately and introduced in the shell option of the ABAQUS system. When calculating the elements A_{ij}, B_{ij}, D_{ij}, the lamina parameters were modified according to the analysed form of facing failure.

4.2. Using the analytical and numerical problem solution

The solution is based on the classical theory of sandwich plates. The broken line hypothesis and the division of stresses into normal loading for the plate facings and shear load for the core have been accepted. Generally, the solution to the problem of plate dynamic deflections uses: the dynamic equilibrium equations, the linear physical relations, non-linear equations for facing geometry expressed by the Kármán's equations, the introduced stress function, the initial loading and boundary conditions and the supported conditions. The basic differential equation of plate deflections has the following form:

$$\begin{aligned} & k_1 w_{d,rrrr} + \frac{2k_1}{r} w_{d,rrr} - \frac{k_1}{r^2} w_{d,rr} + \frac{k_1}{r^3} w_{d,r} \\ & \quad + \frac{k_1}{r^4} w_{d,\theta\theta\theta\theta} + \frac{2(k_1+k_2)}{r^4} w_{d,\theta\theta} + \frac{2k_2}{r^2} w_{d,rr\theta\theta} - \frac{2k_2}{r^3} w_{d,r\theta\theta} \\ & \quad - G_2 \frac{H'}{h_2} \frac{1}{r} \left(\gamma_{,\theta} + \delta + r\delta_{,r} + H' \frac{1}{r} w_{d,\theta\theta} + H' w_{d,r} + rH' w_{d,rr} \right) \\ & \quad = \frac{2h'}{r} \left(\frac{2}{r^2} \Phi_{,\theta} w_{,r\theta} - \frac{2}{r} \Phi_{,\theta r} w_{,\theta r} + \frac{2}{r^2} w_{,\theta} \Phi_{,\theta r} - \frac{2}{r^3} \Phi_{,\theta} w_{,\theta} \right. \\ & \quad \left. + w_{,r} \Phi_{,rr} + \Phi_{,r} w_{,rr} + \frac{1}{r} \Phi_{,\theta\theta} w_{,rr} + \frac{1}{r} \Phi_{,rr} w_{,\theta\theta} \right) - M w_{d,tt}, \end{aligned} \tag{9}$$

where: $H' = h' + h_2$, $k_1 = 2D$, $k_2 = 4D_{r\theta} + \nu k_1$, $D = \dfrac{Eh'^3}{12(1-\nu^2)}$, $D_{r\theta} = \dfrac{Gh'^3}{12}$ – flexural rigidity of the outer layers, $M = 2h'\mu + h_2\mu_2$, E, G, ν – Young's and Kirchhoff's moduli and Poisson's ratio of the facings material, respectively, G_2 – core Kirchhoff's modulus, μ, μ_2 – facing and core mass density, respectively, h', h_2 – thickness of facing and core thickness, respectively, w, w_d – total and additional plate deflection, respectively, Φ – shape function.

In the solution, in order to obtain the basic differential system of equations of plate deflections, we applied the shape functions of the additional plate deflections, preliminary deflections and stress function. Plate model has a preliminary deflection expressed by the function fulfilling the conditions of the clamped edges. Some of the dimensionless quantities, expressions and shape functions, are the following:

$$\zeta = \frac{w}{h}, \qquad \zeta_1 = \frac{w_d}{h}, \qquad F = \frac{\Phi}{Eh^2}, \qquad \rho = \frac{r}{r_o}, \qquad t^* = t\frac{s}{p_{cr}}, \tag{10}$$

$$\zeta_1(\rho,\theta,t) = X_1(\rho,t)\cos(m\theta), \tag{11a}$$

$$\zeta_o(\rho,\theta) = \xi_1\eta_o(\rho) + \xi_2\eta_o(\rho)\cos(m\theta), \tag{11b}$$

$$F(\rho,\theta,t) = F_a(\rho,t) + F_b(\rho,t)\cos(m\theta) + F_c(\rho,t)\cos(2m\theta), \tag{11c}$$

where: $\zeta_1(\rho,\theta,t)$ – shape function of the additional plate deflection, $\zeta_o(\rho,\theta)$ – shape function of the preliminary plate deflection, $F(\rho,\theta,t)$ – shape function of the stress function, $h = h_1 + h_2 + h_3$ – total thickness of the plate, m – the number of the circumferential waves corresponding to the form of plate buckling, ξ_1, ξ_2 – calibrating numbers, $\eta_o(\rho) = \rho^4 + A_1\rho^2 + A_2\rho^2\ln\rho + A_3\ln\rho + A_4$, A_i – quantities fulfilling the conditions of the clamped edges by the function $\eta_o(\rho)$, $i = 1, 2, 3, 4$, p_{cr} – static, critical load. r_o – outer radius of the annular plate, t^* – dimensionless time.

Then, approximation methods: ortogonalization and the finite differences (FDM) were used for obtaining the following system of differential equations:

$$\mathbf{PU} + \mathbf{Q} = K\cdot\ddot{\mathbf{U}}, \tag{12}$$

$$\mathbf{M}_{\mathrm{Y(V,Z)}}\mathbf{Y}(\mathbf{V},\mathbf{Z}) = \mathbf{Q}_{\mathrm{Y(V,Z)}}, \tag{13}$$

$$\mathbf{M}_{\mathrm{D}}\mathbf{D} = \mathbf{M}_{\mathrm{U}}\mathbf{U} + \mathbf{M}_{\mathrm{G}}\mathbf{G}, \tag{14}$$

$$\mathbf{M}_{\mathrm{GG}}\mathbf{G} = \mathbf{M}_{\mathrm{GU}}\mathbf{U} + \mathbf{M}_{\mathrm{GD}}\mathbf{D}, \tag{15}$$

where: K – coefficient dependent on plate geometry, core material and loading parameters, $\mathbf{U}$, $\mathbf{Y}$, $\mathbf{V}$, $\mathbf{Z}$ – vectors of plate additional deflections and components F_a, F_b, F_c of the stress function $F_{a,\rho} = y$, $F_b = v$, $F_c = z$, respectively, $\mathbf{Q}$, $\mathbf{Q}_{\mathrm{Y}}$, $\mathbf{Q}_{\mathrm{V}}$, $\mathbf{Q}_{\mathrm{Z}}$ – vectors of expressions composed of the initial and additional deflections, geometric and material parameters, components of the stress function, dimensionless radius, quantity b (b – length of the interval in the finite difference method), coefficients δ, γ (δ, γ – differences of radial and circumferential displacements of the points in middle surfaces of facings) and number m (m – number of circumferential buckling waves), $\mathbf{P}$, $\mathbf{M}_{\mathrm{D}}$, $\mathbf{M}_{\mathrm{U}}$, $\mathbf{M}_{\mathrm{G}}$, $\mathbf{M}_{\mathrm{GG}}$, $\mathbf{M}_{\mathrm{GU}}$, $\mathbf{M}_{\mathrm{GD}}$, $\mathbf{M}_{\mathrm{Y}}$, $\mathbf{M}_{\mathrm{V}}$, $\mathbf{M}_{\mathrm{Z}}$ – matrices of elements composed of plate parameters, quantity b and number m, $\mathbf{D}$, $\mathbf{G}$ – vectors of expressions composed of coefficients δ and γ, respectively.

The system of Eqs. (12)÷(15) was solved using the Runge-Kutta-Gill's integration method for the initial state of the plate. Critical, static stress p_{cr} has been calculated solving the eigenproblem for the disk state task after neglecting the inertial components and nonlinear expressions.

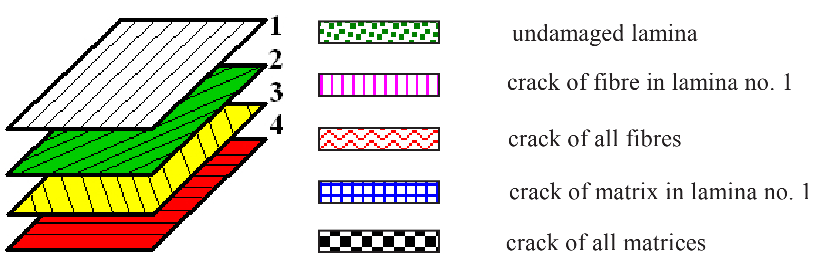
The engineering constants E, G, ν describing the quasi-isotropic parameters of composite facings, expressed by the Eqs. (5), were calculated separately using the conditions and relations applied in lamination theory.

5. Example analyses

The observations of the values of critical loads and plate buckling modes are presented for the static stability problem and the dynamic one, separately. Similar, accepted combinations of failures in the form of fibre or matrix cracks of facing laminas have been analysed for plate models statically and dynamically loaded. The results shown below create the image of the composite plate respond on the character of the acting loads.

5.1. Plate and loading parameters

Geometry of the analysed three-layered, annular plate is determined by the following parameters: inner radius $r_i = 0.2$ m, outer radius $r_o = 0.5$ m, total thickness of laminate facing $h' = 0.5$ mm, thickness of the core middle layer $h_2 = 5$ mm. Each of the laminas has the thickness equal to $h'' = 0.125$ mm and creates the facing structure, which consists of the four laminas of fibrous laminate [0/−45/45/90] (see, Fig. 2a). Material parameters of plate layers are the following: glass/epoxy composite as a material of the facing lamina: $E_1 = 53.781$ GPa, $E_2 = 17.927$ GPa, $G_{12} = 8.964$ GPa, $\nu_{12} = 0.25$, $\mu = 2900$ kg/m^3 [31], polyurethane foam with the values of Kirchhoff's modulus $G_2 = 5$ MPa, Poisson's ratio $\nu_2 = 0.3$ and mass density $\mu_2 = 64$ kg/m^3 as the core material. The quasi-isotropic material of plate facings is characterized by the calculated engineering constants E, G, ν equal to: $E = 31.1$ GPa, $G = 12.5$ GPa, $\nu = 0.245$. The correction parameter η (see, Eq. (7)) in the accepted damage theory of composite facings is equal to $\eta = 0.1$ [24]. Fig. 2b shows the analysed examples of facing laminate structure. There were taken into account the following cases: facing with all laminas undamaged, facing with damaged lamina no. 1 (see, Fig. 2a) and facing with damages in all laminas in the form of crack of fibre or matrix. In dynamic analysis, the plate model is loaded with the rate of loading growth s (1) equal to: $s = 4346$ MPa/s for plates loaded on inner edge and $s = 931$ MPa/s for plates loaded on outer edge.



(a) (b)

Fig. 2. The configuration of laminate [0/–45/45/90] (a); the legend to graphic description of laminate failure (b)

Dynamic stability problem is focused on the observation of the plate models with the number m of circumferential buckling waves equal to $m = 0$ and $m = 7$. The values obtained for FDM plate model with quasi-isotropic composite facings show the elementary axisymmetrical $m = 0$ mode and asymmetrical, circumferentially waved $m = 7$ one. Mode $m = 7$ corresponds to the minimal values of critical dynamic loads p_{crdyn} of plates radially compressed on outer edge, respectively. The values are presented in Table 1. The FDM calculations were carried out for number $N = 14$ of discrete points.

Table 1.
Values of critical, dynamic loads p_{crdyn} with corresponding modes m of FDM plate model loaded on outer edge

plate mode m	0	1	2	3	4	5	6	7	8	9
p_{crdyn} [MPa]	36.33	34.47	30.28	26.08	21.43	20.03	19.10	18.63	19.10	20.03

5.2. Static stability analysis

The values of critical static loads p_{cr} of plate models loaded on inner or outer edge are presented in Table 2. The chosen results are for the plates with the cracks of matrix or fibres located in lamina 1 (see Fig. 2a) or located in two laminas numbered 1 and 4 and for plates with all laminas damaged. The values of loads p_{cr} of plate models with different failures can be compared with those of the undamaged plates. The presented results consider only transversally symmetrical structures of plates. It has been assumed that the analysed examples of facings

Table 2.
The values of critical static loads p_{cr} and buckling modes m for plates with symmetrically damaged structure

static stability		p_{cr} [MPa] / m			
damaged combination		crack of matrix		crack of fibre	
		1	1 + 4	1	1 + 4
edge loading	inner	34.28 / 0	32.15 / 0	35.92 / 0	35.05 / 0
	outer	12.90 / ≈ 2	12.70 / 5	13.30 / ≈ 2	13.86 / 6
damaged combination		crack of all matrices		crack of all fibres	
		FDM model	FEM model	FDM model	FEM model
edge loading	inner	33.64 / 0	27.54 / 0	37.77 / 0	33.46 / 0
	outer	9.66 / 5	8.04 / 5	15.20 / 5	12.74 / 5
undamaged facings		FDM model		FEM model	
edge loading	inner	38.89 / 0		36.84 / 0	
	outer	17.39 / 6		14.63 / 6	

defects are symmetrically arranged relative to the middle plate plane. The observed buckling modes have the global, quasi-Eulerian form: axisymmetrical $m = 0$, with $m = 5$, $m = 6$ numbers of circumferential waves or rotational irregular marked in Table 2 as $m \approx 2$ (see, Fig. 3).

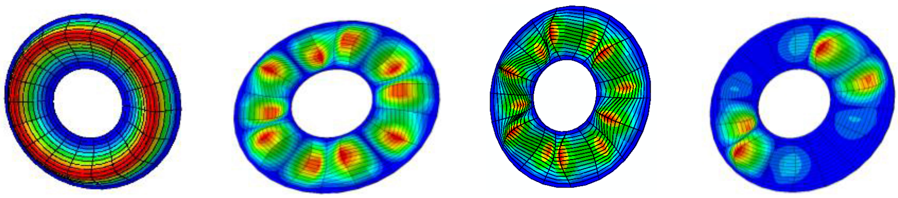


Fig. 3. Forms of plate buckling modes

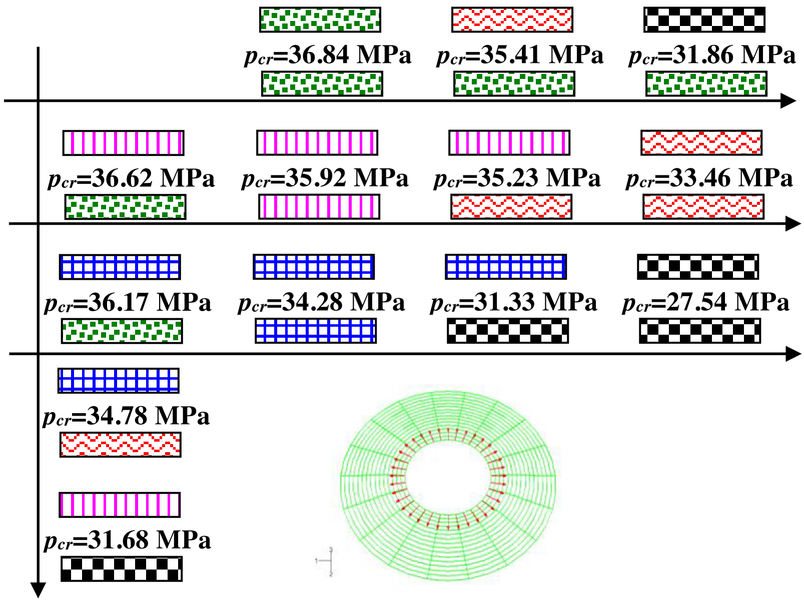
Table 2 includes the values of critical static loads p_{cr} obtained for the FDM plate model with facings treated as a quasi-isotropic composite. The comparison of values p_{cr} calculated for FEM and FDM models shows that it is possible to use the quasi-isotropic composite material model in evaluation of the static critical state of a plate completely damaged.

The minimal values of critical, static load p_{cr} are observed for the plate with damaged laminate facings in the form of matrix crack. The values of loads p_{cr} are significantly lower than the values calculated for the plates without any defects.

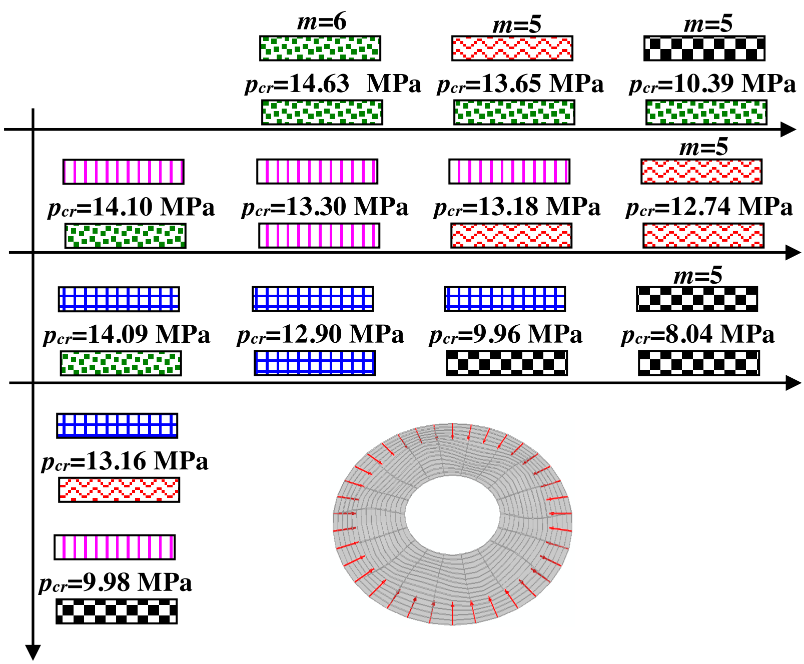
Fig. 4 shows the distribution of the static, critical loads p_{cr} of FEM plate models dependent on the combination of damages in laminated facings. The accepted pattern expresses the case of the examined damage (see Fig. 2b).

The presented results show the influence of the damaged laminas arrangement on the plate buckling parameters. The examined FEM plate models are loaded on inner or outer edge – see Fig. 4a and 4b, respectively. The graphical form of results presentation makes it possible to observe the tendency in values of critical loads p_{cr} dependent on the different forms of damage and increasing rate of failure. The presented changes are regular in directions indicated by lines. The results show asymmetric form of damage as the one which can be intermediate in failure process and also the other which does not correspond to the minimal value of critical load p_{cr}. The form of deflection of plate models loaded on inner edge is axisymmetric, $m = 0$. The plate models compressed on outer edge with defects located in lamina 1 lose static stability in the form that is rotationally irregular, denoted as $m \approx 2$, which is shown in Fig. 3. The circumferentially regular forms of the loss of plate stability with the number $m = 5$ or $m = 6$ are shown in Fig. 4b. The minimal value is observed for the plate with facings whose matrices are damaged in each of the laminas. The damaged structure is symmetrical.

Shortly, the change in values of loads p_{cr} is graphically presented in Fig. 5 for plates loaded on inner or outer edge. These are extreme examples of plates



(a)



(b)

Fig. 4. Distribution of values of critical, static loads p_{cr} dependent on the form of structure failure for plate loaded on inner edge (a), outer edge (b)

with undamaged structure and completely destroyed one that have symmetrical cross-section. The form of buckling is global.

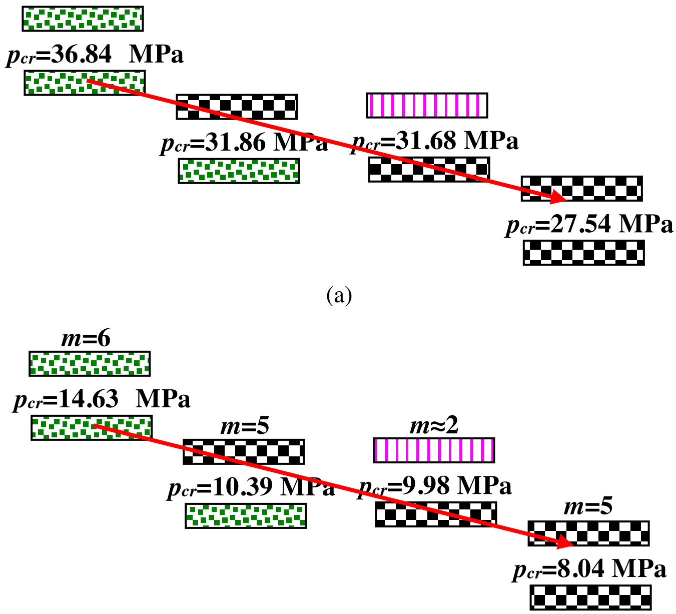
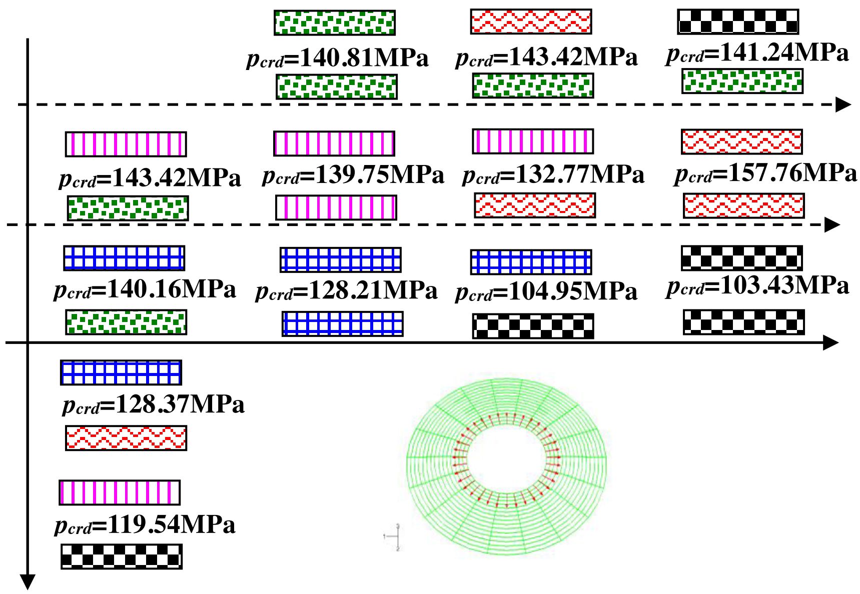


Fig. 5. The graphical presentation of the values of critical, static loads p_{cr} for plates loaded on inner edge (a), outer edge (b)

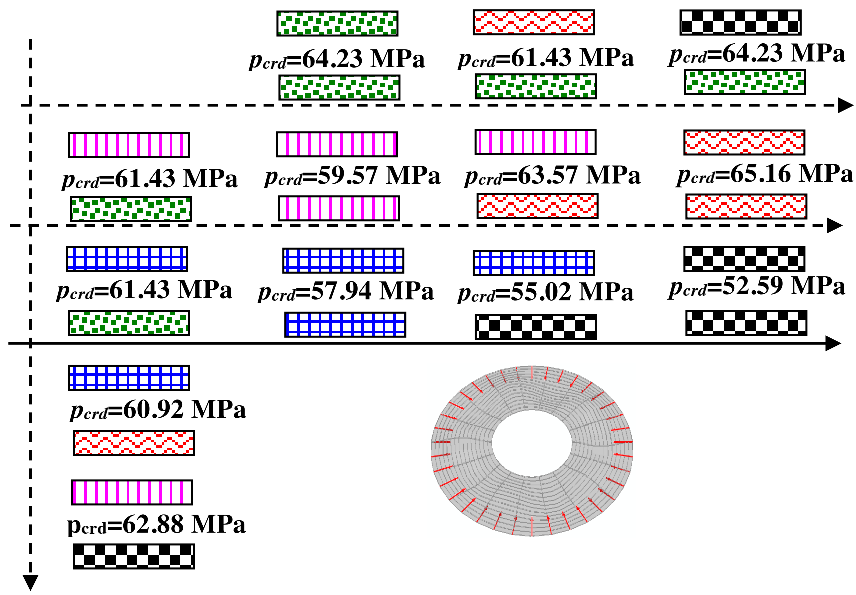
The difference between the maximum one and minimum values of loads p_{cr} is in the range of several MPa. The sandwich structure of the analysed plate allows for further working of the plate, but with a lower buckling capacity.

5.3. Dynamic stability analysis

The schemes of changes in values of dynamic, critical loads are presented in Fig. 6 for the plates loaded on inner or outer edges. The examined plate examples concern the healthy structure without any defects and the damaged structure with the transversal symmetry or with transversal asymmetry of plate structure. The cases of fibre or matrix cracks are considered, too. Similarly, like for the static analysis, the results are arranged in the way which allows for observation of the process of the plate facings failure. The examined example of the plates with load quickly changing in time does not entirely confirm the regularity in changes of values of critical loads, which has been observed for the plates statically compressed in radial direction. The full lines shown in Fig. 6 confirm such changes, but the results represented by dashed lines do not verify this regularity. Both for plates loaded on inner or outer edge, the discussed regularity of changes in load values is observed for plates with matrix cracks. The examplary plate with facings completely destroyed by matrix failure is the one, whose value of dynamic critical load is minimal.



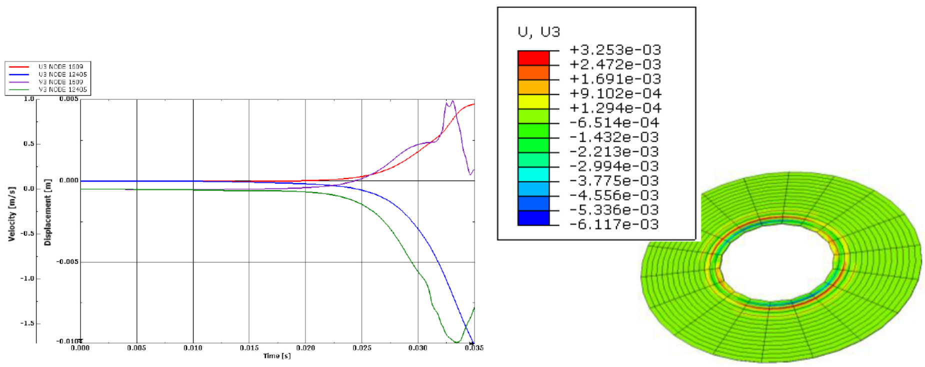
(a)



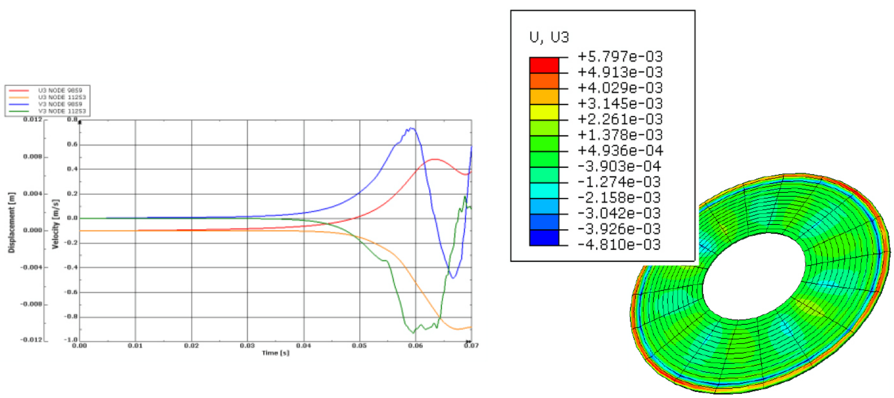
(b)

Fig. 6. The distribution of values of critical, dynamic loads p_{crd} dependent on the form of structure failure for plate loaded on inner edge (a), outer edge (b)

The forms of plate bucking are not global. They are characterized by strong local deformations localized close to the loaded edge. Fig. 7 shows the exemplary, characteristic forms of critical deformations of damaged plates loaded on inner or outer edge, respectively. The time histories of displacements and velocity of displacements for the plate points with maximum deflections in opposite directions are presented, too. The buckling form shown in Fig. 7a concerns the plate, whose value of the critical dynamic load p_{crdyn} is equal to p_{crdyn} = 141.23 MPa. The plate asymmetrical structure is composed of the upper facing fully damaged by matrix cracks and the undamaged bottom facing. Whereas, Fig. 7b presents the dynamic response of the plate compressed on outer edge, whose facings also have defects in the form of matrix cracks located in lamina 1 (see, Fig. 2a) belonging to the upper composite facing and in all, four laminas of bottom facing. The characteristic, global



(a)

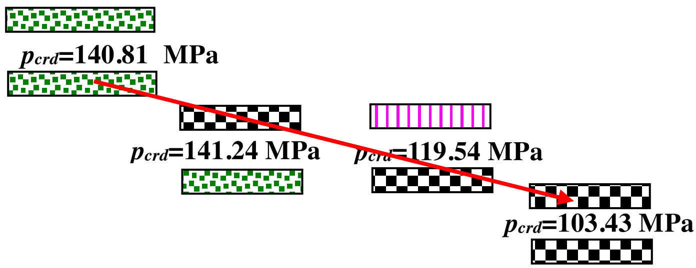


(b)

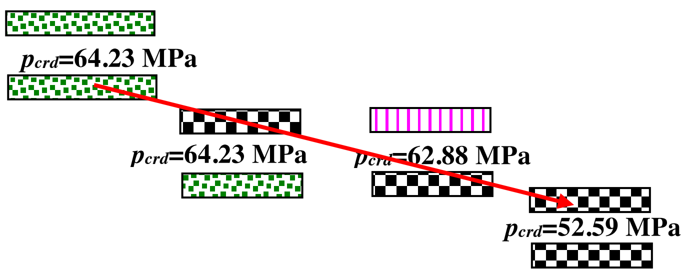
Fig. 7. Time histories of deflection and velocity of deflection and buckling deformation for plate: loaded on inner edge with damaged upper facing by matrix cracks and undamaged bottom facing (a), loaded on outer edge with matrix cracks in lamina 1 of upper facing and damaged bottom facing by matrix cracks (b)

form of buckling for this plate loaded on outer edge, which is determined by the expected number $m = 7$ of circumferential waves, is not observed. The plate loses dynamic stability for the value of the critical load equal to: $p_{crdyn} = 55.02$ MPa.

Similarly, like in the static analysis, Fig. 8 shows the range of changes in values of critical loads for plates with undamaged structure and damaged one, for whom the value of load p_{crdyn} is minimal. Additionally, two intermediate plate cases have been chosen, too. The difference in values of dynamic, critical loads is in the range of a dozen or so MPa for plates loaded on inner edge and anywhere from ten to twenty MPa for plates compressed on outer edge.



(a)



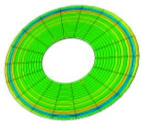
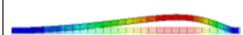
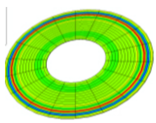
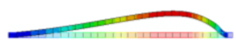
(b)

Fig. 8. The graphical presentation of the change of critical, dynamic loads p_{crd} for plates loaded on inner edge (a), outer edge (b)

5.4. Comparison of numerical models

The evaluation of the correctness of the numerical calculations has been performed through the comparison analysis of four plate models. The exemplary results are presented in Table 3. They pertain to the axisymmetrical plate compressed on outer edge, whose facings are not damaged or the plate with facings completely destroyed by the fibre cracks. The FEM basic model has been compared with a simplistic one. The FEM simplistic model has been built of axisymmetrical elements: 3-node shell elements and 8-node solid ones creating the mesh of facing and solid, respectively. This model allows for the numerical analysis of problems formulated for axisymmetrical plates. The comparisons of values of dynamic, critical loads and forms of critical deformations, which show the strong deflection zone close to loaded edge, present the correspondence between dynamic behaviours of both plate models.

Table 3.
The critical state parameters of different axisymmetrical plate models loaded on outer edge with undamaged facings and damaged ones by fibre cracks

plate structure	FEM basic model	FEM simplistic model	FEM simplistic model with quasi-isotropic composite facings	FDM model with quasi-isotropic composite facings
	p_{crdyn} [MPa]			
	71.21	77.26	35.40	36.33
undamaged facings				
	p_{crdyn} [MPa]			
	69.81	71.67	34.00	31.67
damaged all fibres				

Additionally, an interesting evaluation can be made by the comparison of the above results with the ones obtained for plate models whose facings material is characterized by the parameters of quasi-isotropic composite. It concerns the FEM simplistic model and FDM plate model obtained through the numerical and analytical solution. The quasi-isotropic material parameters of plate facings with damages in the form of fibre cracks are expressed by the following engineering constants E, G, ν equal to: $E = 18.71$ GPa, $G = 7.9$ GPa, $\nu = 0.182$. These examples show the global form of plate dynamic buckling as the one for which the corresponding critical load is much lower than that calculated for the plate with facings modelled as a full fibrous composite. The good consistence of values of critical loads of both plate FEM and FDM models indicates the possibility of the practical and effective use of the proposed, approximated analytical and numerical solution. Such a solution can be applied for transversally symmetrical plate structure with undamaged facings or fully destroyed ones. The values of critical loads about two times lower are calculated for FEM and FDM plate models with quasi-isotropic composite facings. Within the scope of basic examination of buckling problem these values can be treated as safe and useful to evaluate the buckling sensitivity of analysed plate structure, effectively. The presented comparison is also meaningful in modelling process of similar, layered plate structures.

6. Conclusions

This paper presents an approach to the evaluation of the critical state of sandwich, annular plates with defective laminated fibre-reinforced composite facings. The presented results and the behaviour images of the examined plates enable one to evaluate the responses of non-ideal structures to acting loads. The stability behaviour of the composite plates with damaged layers creates the problem, for which it is difficult to formulate general conclusions. Plates responses are ambiguous, so the meaning of both the effective solution and experimental investigations is important. The decrease in values of the critical static and dynamic loads is observed for plates with asymmetric failure of plate structure with facing damage having the form of matrix cracks, however, minimal load values are observed for both facings symmetrically destroyed. The forms of the loss of plate stability can not be global. Particularly, local deformations close to the loaded edge are observed for dynamically loaded plates with quickly changing load. The values of the critical dynamic loads are higher than those calculated in static analysis. Quasi-isotropic simplistic description of the composite plate facings does not show the real plate behaviour in dynamic conditions. However, the values of the critical static loads of plates whose all fibres or matrices are damaged show that they can be approximately calculated for the plate models with facings treated as quasi-isotropic ones.

The presented results showing the critical responses of plates with different parameters of transverse structure have significant, practical importance in the evaluation of the structure capacity. They show the ability of the three-layered plate to work despite significant decrease in facings rigidity. The results presented graphically with the graphical distribution of changes in values of static and dynamic critical loads illustrate the process of the damaging of structure during its work. On account of the possible wide range of applications of composite annular plates that work in the conditions of variable loads in: automotive industry, aircraft industry, civil engineering, the effective evaluation of the work capacity of non-ideal plate structure has a practical meaning. It should be noticed that the presented analyses should be confirmed by appropriate experimental investigations. Such investigations could be performed for a simpler element, for example, the composite beam with laminas having selected damages in the form of matrix cracking or fibre breakage.

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References

[1] Y.R. Chen, L.W. Chen and C.C. Wang. Axisymmetric dynamic instability of rotating polar orthotropic sandwich annular plates with a constrained damping layer. *Composite Structures*, 73(1):290–302, 2006. doi: 10.1016/j.compstruct.2005.01.039.

[2] H.J. Wang, L.W. Chen. Axisymmetric dynamic stability of rotating sandwich circular plates. *Journal of Vibration and Acoustics*, 126(2):407–415, 2004. doi: 10.1115/1.1688765.

[3] A. Wirowski. *Tolerance modelling of dynamics of microheterogeneous annular plates*. Monograph of the Technical University of Łódź, Łódź, 2016 (in Polish).

[4] J. Je. Axisymmetric buckling analysis of homogeneous and laminated annular plates. *International Journal of Pressure Vessels and Piping*, 62(1):153–159, 1995. doi: 10.1016/0308-0161(94)00004-3.

[5] J. Ye. *Laminated composite plates and shells*. Springer-Verlag, London, 2003.

[6] H.J. Ding and R.Q. Xu. Exact solution for axisymmetric deformation of laminated transversely isotropic annular plates. *Acta Mechanica*, 153(1-2):169-182, 2002. doi: 10.1007/BF01177450.

[7] R. Lal and R. Rani. Axisymmetric vibrations of composite annular sandwich plates of quadratically varying thickness by harmonic differential quadrature method. *Acta Mechanica*, 226(5):1993-2012, 2015. doi: 10.1007/s00707-014-1284-0.

[8] J. Lee and C. Soutis. Prediction of impact-induced fibre damage in circular composite plates. *Applied Composite Materials*, 12(1):109–131, 2005. doi: 10.1007/s10443-004-7767-8.

[9] A. Muc and P. Zuchara. Buckling and failure analysis of FRP faced sandwich plates. *Composite Structures*, 48(1-3):145–150, 2000. doi: 10.1016/S0263-8223(99)00087-2.

[10] L.P. Khoroshun and D.V. Babich. Stability of plates made of fibrous composite with components subject to long-term damage. *International Applied Mechanics*, 46(4):573–579, 2010. doi: 10.1007/s10778-010-0343-z.

[11] P. Maimi, P.P. Camanho, J.A. Mayugo, and A. Turon. Matrix cracking and delamination in laminated composites. Part II: Evaluation of crack density and delamination. *Mechanics of Materials*, 43(3):194–211, 2011. doi: 10.1016/j.mechmat.2011.01.002.

[12] A. Ahmed and L.J. Sluys: Computational modelling of impact damage in laminated composite plates. *ECCM-16-th European Conference on Composite Materials*, Seville, Spain, 22–26 June, 2014.

[13] F. Tornabene, N. Fantuzzi, M. Bacciocchi, and E.Viola. Mechanical behaviour of damaged laminated composites plates and shells: Higher-order Shear Deformation Theories. *Composite Structures*, 189:304–329, 2018. doi: 10.1016/j.compstruct.2018.01.073.

[14] F. Tornabene, N. Fantuzzi, and M. Bacciocchi. Linear static behaviour of damaged laminated composite plates and shells. *Materials*, 10(7):811, 2017. doi: 10.3390/ma10070811.

[15] Q. Meng and Z. Wang. Micromechanical modeling of impact damage mechanisms un unidirectional composite laminates. *Applied Composite Materials*, 23(5):1099-1116, 2016. doi: 10.1007/s10443-016-9502-7.

[16] A. De Luca, F. Caputo, Z. Sharif Khodaei, and M.H. Aliabadi. Damage characterization of composite plates under low velocity impact using ultrasonic guided waves. *Composites Part B: Engineering*, 138:168–180, 2018. doi: 10.1016/j.compositesb.2017.11.042.

[17] S.T. Rokotonarivo, C. Payan, J. Moysan, and C. Hochard. Local damage evaluation of a laminate composite plate using ultrasonic birefringence of shear wave. *Composites Part B: Engineering*, 142:287–292, 2018. doi: 10.1016/j.compositesb.2018.01.006.

[18] A. Ghosh and P.K. Sinha. Dynamic and impact response of damaged laminated composite plates. *Aircraft Engineering and Aerospace Technology*, 7(1):29–37, 2004. doi: 10.1108/00022660410514982.

[19] K.S. Sivakumaran. Free vibration of annular and circular asymmetric composite laminates. *Composite Structures*, 11(2):205–226, 1989. doi: 10.1016/0263-8223(89)90059-7.

[20] D. Pawlus. Stability of three-layered annular plate with composite facings. *Applied Composite Materials*, 24(1):141–158, 2017. doi: 10.1007/s10443-016-9518-z.

[21] D. Pawlus. Evaluation of critical static loads of three-layered annular plates with damaged composite facings. *Engineering Transactions*, 64(3):613–619, 2016.

[22] D. Pawlus. Dynamic response of three-layer annular plate with damaged composite facings. *Archive of Mechanical Engineerig*, 65(1):1: 83–105, 2018. doi: 10.24425/119411.

[23] D. Pawlus. Critical state evaluation of three-layered annular plates with symmetry and asymmetry damaged composite structure. *Mechcomp 3 – 3rd International Conference on Mechanics of Composites,* Bologna, Italy, 4–7 July, 2017.

[24] A. Muc. *Mechanics of Fibrous Composites*. Księgarnia Akademicka, Kraków, 2003 (in Polish).

[25] C. Volmir. *Nonlinear Dynamic of Plates and Shells*. Science, Moskwa, 1972 (in Russian).

[26] J. German. *Fundamentals of Mechanics of Fibrous Composites*. Politechnika Krakowska, Kraków, 1996 (in Polish).

[27] R.M. Jones. *Mechanics of Composite Materials*. Scripta Book Company, Washington D.C., 1975.

[28] D. Pawlus. *Dynamic Stability of Three-Layered Annular Plates with Viscoelastic Core*. Scientific Bulletin of the Technical University of Łódź, 1075, Łódź, 2010. (in Polish).

[29] D. Pawlus. Dynamic stability of three-layered annular plates with wavy forms of buckling. *Acta Mechanica*, 216(1-4):123–138, 2011. doi: 10.1007/s00707-010-0352-3.

[30] D. Pawlus. Solution to the problem of axisymmetric and asymmetric dynamic instability of three-layered annular plates. *Thin-Walled Structures*, 49(4):660–668, 2011. doi: 10.1016/j.tws.2010.09.013.

[31] *Dynamic Stability of Composite Plate Construction*, K. Kowal-Michalska, editor. WNT, Warszawa, 2007 (in Polish).