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Dynamic Contraction Method approach to digital longitudinal aircraft flight controller design

ROMAN CZYBA and LUKASZ STAJER

This paper presents the design of digital controller for longitudinal aircraft model based on the Dynamic Contraction Method. The control task is formulated as a tracking problem of velocity and flight path angle, where decoupled output transients are accomplished in spite of incomplete information about varying parameters of the system and external disturbances. The design of digital controller based on the pseudo-continuous approach is presented, where the digital controller is the result of continuous-time controller discretization. A resulting output feedback controller has a simple form of a combination of low-order linear dynamical systems and a matrix whose entries depend nonlinearly on certain known process variables. Simulation results for an aircraft model confirm theoretical expectations.

Key words: nonlinear systems, MIMO systems, aircraft control, digital controller, singular perturbation

1. Introduction

Control of an aircraft under difficult maneuvers is a problem of both theoretical and practical interest [1, 6]. The well known approach to decoupling problem solution based on the Non-linear Inverse Dynamics (NID) method [4] may be used if the parameters of the plant model and external disturbances are exactly known. Usually, incomplete information about systems in real practical tasks takes place. In this case adaptive control methods or control systems with sliding mode [3] may be used for solving this control problem. A crucial feature of the sliding mode techniques is that in the sliding phase the motion of the system is insensitive to parameter variation and disturbances in the system. A way of the algorithmic solution of this problem under condition of incomplete information about varying parameters of the plant and unknown external disturbances is the application of the Localization Method (LM) [7], which allows to provide the desired transients for nonlinear time-varying systems. The generalization and development of LM is the Dynamic Contraction Method (DCM) [8]. The peculiarity of the DCM

The authors are with Silesian University of Technology, 16 Akademicka St., Gliwice 44-100, Poland.
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method is the application of the higher order derivatives jointly with high gain in the control law.

In general, the goal of the design of an longitudinal aircraft control system is to provide decoupling, i.e. each output should be independently controlled by a single input, and to provide desired output transients under assumption of incomplete information about varying parameters of the aircraft model and unknown external disturbances.

The paper is part of a continuing effort of analytical and experimental studies on aircraft control which were considered in [2]. The main purpose of this paper is to design a digital controller and examine aspects related to its discretization in terms of implementation on embedded systems. A solution for digital controller is presented that bases on a pseudo-continuous-time model of the control loop with pure time delay for which a linear continuous-time controller is designed based on DCM and then discretized by using the Tustin transformation. The structure of the paper is as follows. First, a mathematical description of the aircraft model is introduced. The next part includes a description of DCM method used for the control system design. The control solution along with the stages of regulators design are presented for aircraft model. Finally, the results of simulations are shown.

2. Mathematical model of the aircraft

The aircraft dynamics are described by a collection of modules (Fig. 1), each performing a specific function. The primary modules are: the equations of motion, the aerodynamics, the propulsion system, the actuator command input, and the atmospheric model. The mathematical model in detail is describe in [2], and geometrical, mass, and aerodynamic data are in agreement with the technical documentation [5]. The aircraft model is nonlinear due to the rigid body dynamic equations and to the aerodynamics represented in the data lookup tables of the force coefficients c_x, c_z and moment coefficient m_y.

Assuming that an airplane is a rigid body with three degrees of freedom. The following state and control vectors are adopted:

$$\overline{X} = [V, \alpha, \gamma, \theta, Q, x, h]^{\mathrm{T}}, \tag{1}$$

$$\overline{u} = [\delta_T, u_H]^{\mathrm{T}}, \tag{2}$$

where: V – aircraft linear velocity, α - angle of attack, γ – flight-path angle, θ – pitch angle, Q – pitch rate, x – inertial position component, h – altitude, δ_T – throttle setting, u_H – actuator commands of elevator.

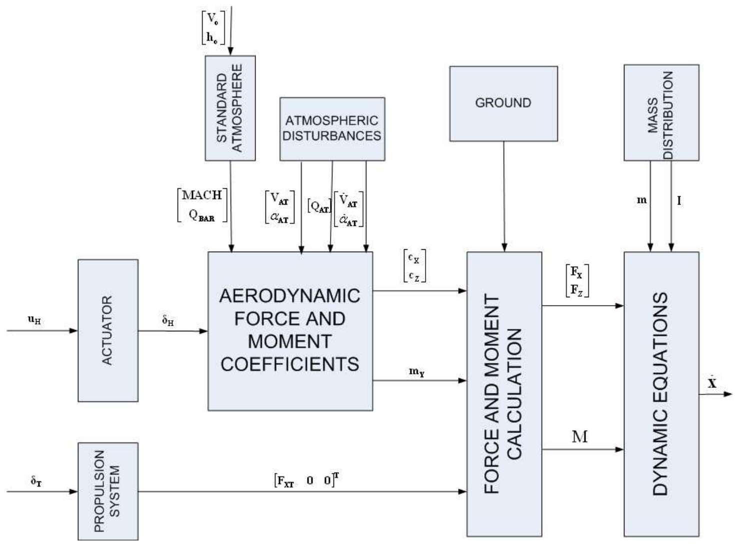

Figure 1: Block diagram of the aircraft dynamics

3. Solution of a non-linear control problem by DCM [8]

3.1. Control problem statement

Let us consider a nonlinear time-varying system in the following form:

$$\overline{x}^{(1)}(t) = \overline{h}\left(\overline{x}(t), \overline{u}(t), t\right), \qquad \overline{x}(0) = \overline{x}_0, \tag{3}$$

$$\overline{y}(t) = \overline{g}\left(t, \overline{x}(t)\right), \tag{4}$$

where: $\overline{x}(t)$ is a n-dimensional state vector, $\overline{y}(t)$ is a p-dimensional output vector, $\overline{u}(t)$ is a p-dimensional control vector.

Here, the dependence of $\overline{h}\left(\overline{x}(t), \overline{u}(t), t\right)$ and $\overline{g}\left(t, \overline{x}(t)\right)$ of time expresses the influence of external disturbances and parameters variations.

Let us assume that the first $m-1$ time derivatives of the output $\overline{y}(t)$ do not depend explicitly on the control $\overline{u}(t)$:

$$\overline{y}^{(m)}(t) = \overline{f}\left(t, \overline{x}(t)\right) + B\left(t, \overline{x}(t)\right)\overline{u}(t), \tag{5}$$

where: $\overline{y}^{(m)}(t) = \left[y_1^{(m_1)}, y_2^{(m_2)}, \ldots, y_p^{(m_p)}\right]^{\mathrm{T}}$, $\det\left(B\left(t, \overline{x}(t)\right)\right) \neq 0$, $|f_i(t,x)| \leqslant f_i^{\max}$, $i = 1, 2, \ldots, p$. The transformation of (3), (4) into the linear analytic form of (5) is required.

Assume also that a reference model for transients of $\overline{y}(t)$ is given in the form of the following vector differential equation:

$$\begin{aligned} y_{1M}^{(m_1)}(t) &= F_{1M}\left(\overline{y}_{1M}(t), r_1(t)\right), \\ &\dots\dots\dots\dots\dots\dots \\ y_{iM}^{(m_i)}(t) &= F_{iM}\left(\overline{y}_{iM}(t), r_i(t)\right), \\ &\dots\dots\dots\dots\dots\dots \\ y_{pM}^{(m_p)}(t) &= F_{pM}\left(\overline{y}_{pM}(t), r_p(t)\right), \end{aligned} \tag{6}$$

where: $\overline{y}_{iM}(t) = \left[y_{iM}, y_{iM}^{(1)}, \dots, y_{iM}^{(m_i-1)}\right]^{\mathrm{T}}$, $\overline{r}(t)$ is the reference value and $\overline{y} = \overline{r}$ at the equilibrium.

Denote the tracking error as follows:

$$\overline{\Delta}(t) = \overline{r}(t) - \overline{y}(t). \tag{7}$$

The task of a control system is stated so as to provide that

$$\overline{\Delta}(t)_{t\to\infty} = 0. \tag{8}$$

Moreover, the transients $\overline{y}(t)$ should have the desired behavior defined in (6) which does not depend either on the possibly varying parameters or on the external disturbances of equations (3), (4).

Let us denote

$$\overline{\Delta}^F = \overline{F}_M\left(\overline{y}(t), \overline{r}(t)\right) - \overline{y}^{(m)}(t). \tag{9}$$

Then equation (6) defining the desired dynamics is fulfilled if and only if holds:

$$\overline{\Delta}^F\left(\overline{x}(t), \overline{y}(t), \overline{r}(t), \overline{u}(t), t\right) = 0. \tag{10}$$

So the control action $\overline{u}(t)$ which provides the control problem solution is the root of equation (10). This expression is the insensitivity condition of the output transient performance indices with respect to external disturbances and varying parameters of the system in (3), (4).

3.2. Dynamic contraction method

The solution of the control problem (10) bases on the application of the higher order output derivatives jointly with high gain in the controller (Fig. 2). The control law in the form of a stable differential equation is constructed such that its stable equilibrium is the solution of equation (10). Such equation is called differential contraction equation and can be present in the following form:

$$\begin{aligned} \mu^q \overline{v}^{(q)} + \sum_{i=0}^{q-1} \mu^i d_i \overline{v}^{(i)} &= k\overline{\Delta}^F, \\ \overline{v}(0) &= \overline{v}_0, \end{aligned} \tag{11}$$

where: $i = 1,\ldots,p$; $\nu_i(t) = \left[\nu_i, \nu_i^{(1)}, \ldots, \nu_i^{(q_i-1)}\right]^{\mathrm{T}}$ – new output of the controller; μ_i – small positive parameter $\mu_i > 0$; k – gain; $d_{i,0}, \ldots, d_{i,q_i-1}$ – diagonal matrices.

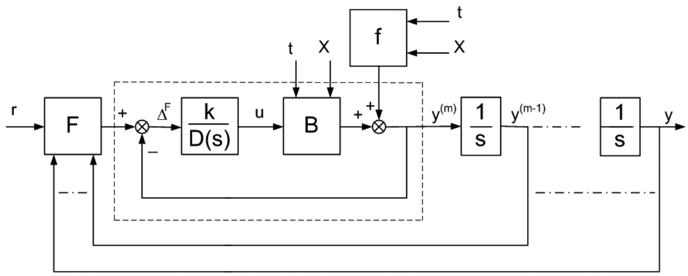

Figure 2: The control system structure

To decoupling of control channel during the fast motions let us use the following output controller equation

$$\overline{u}(t) = K_0 K_1 \overline{\nu}(t), \tag{12}$$

where: $\overline{\nu}(t)$ is a new input, $K_1 = diag\left(k_1, k_2, \ldots, k_p\right)$ is a matrix of gains, K_0 is a nonsingular matching matrix (such that BK_0 is positive definite).

Let us assume that there is a sufficient time-scale separation, represented by a small parameter μ_i, between the fast and slow modes in the closed loop system. Methods of singularly perturbed equations can then be used to analyze the closed loop system and, as a result, slow and fast motion subsystems can be analyzed separately. Following differential equation determines the fast dynamics of controller:

$$D(\mu s) = \mu^q s^q + \sum_{i=0}^{q-1} \mu^i d_i s^i. \tag{13}$$

4. Analog controller design

In the present paper the control task is stated as a tracking problem for the aerodynamic state variables:

$$\lim_{t\to\infty} [V_0(t) - V(t)] = 0, \tag{14}$$

$$\lim_{t\to\infty} [\gamma_0(t) - \gamma(t)] = 0, \tag{15}$$

where $V_0(t)$, $\gamma_0(t)$ are the desired values of the considered variables.

In addition, we require that transient processes have desired dynamic properties, are mutually independent and are independent of varying airplane parameters.

Feedback data for the regulator are two variables: V, γ. Control signals are δ_T, u_H. For the sake of controller synthesis we neglect very fast elevator actuator dynamics assuming that $u_H = \delta_H$ (elevator deflection). The inverse dynamics of (3), (4) are constructed by differentiating the individual elements of $\overline{y}$ a sufficient number of times until a term containing $\overline{u}$ appears in (5). When using the DCM method the order of the derivatives of the output $\overline{y}$ depending explicitly on the control vector $\overline{u}$ must be determined. From equations of aircraft motion [2] and relationship (5) it follows that

$$\begin{pmatrix} V^{(2)} \\ \gamma^{(3)} \end{pmatrix} = \begin{pmatrix} f_V \\ f_\gamma \end{pmatrix} + B \begin{pmatrix} \delta_T \\ u_H \end{pmatrix}, \qquad B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, \tag{16}$$

where the values of functions $f(\cdot) = f_i(V, \alpha, \gamma, \theta, Q)$, $i = V, \gamma$ and elements of matrix B are bounded.

The relationship (16) neglects the derivatives of the aerodynamic forces (but not the aerodynamic moments and thrust) with respect to the control surface deflections δ_T, δ_H. Although the system is influenced through these force effects, they are small for most aircraft configurations and are not primary paths of aerodynamic control. The principal function of the control surface deflections is to impart aerodynamic moments about the various body axes. The neglected force effects can be considered as disturbances of the system, and will be included in the numerical calculations of the object.

Let us assume that the desired dynamics are determined by a set of mutually independent differential equations:

$$\tau_V^2 V^{(2)} = -2\tau_V \alpha_V V^{(1)} - V + V_0, \tag{17}$$

$$\tau_\gamma^3 \gamma^{(3)} = -3\tau_\gamma^2 \alpha_\gamma \gamma^{(2)} - 3\tau_\gamma \alpha_\gamma^2 \gamma^{(1)} - \gamma + \gamma_0 . \tag{18}$$

Parameters τ_i and α_i $(i = V, \gamma)$ have very well known physical meaning and their particular values have to be specified by the designer.

The output controller equation from (12) is as follows:

$$\begin{pmatrix} \delta_T \\ u_H \end{pmatrix} = K_0 K_1 \begin{pmatrix} \nu_V \\ \nu_\gamma \end{pmatrix}, \tag{19}$$

where: $K_1 = diag(k_V, k_\gamma)$, assume that $K_0 = B^{-1}$ because matrix BK_0 must be positive definite.

In normal flight conditions the following requirement is satisfied $\det(B(t, x(t))) \neq 0$, which is also a sufficient condition for the existence of an inverse system model to (3), (4). However, for a certain configuration of state variables, the reversibility condition of matrix B may not be fulfilled. In case of singularity of the matrix B, a numerical protection is required to ensure the fulfillment of the above reversibility condition.

The control law from (11) has the following form

$$\mu_V^2 \nu_V^{(2)} + 2d_{V,1}\mu_V \nu_V^{(1)} + d_{V,0}\nu_V = k\left(-\tau_V^2 V^{(2)} - 2\alpha_V \tau_V V^{(1)} - V + V_0\right), \tag{20}$$

$$\mu_\gamma^3 \nu_\gamma^{(3)} + 3\mu_\gamma^2 d_{\gamma,2}\nu_\gamma^{(2)} + 3\mu_\gamma d_{\gamma,1}\nu_\gamma^{(1)} + d_{\gamma,0}\nu_\gamma = k\left(-\tau_\gamma^3 \gamma^{(3)} - 3\tau_\gamma^2 \alpha_\gamma \gamma^{(2)} - 3\tau_\gamma \alpha_\gamma^2 \gamma^{(1)} - \gamma + \gamma_0\right). \tag{21}$$

The entire closed loop system is presented in Fig. 3.

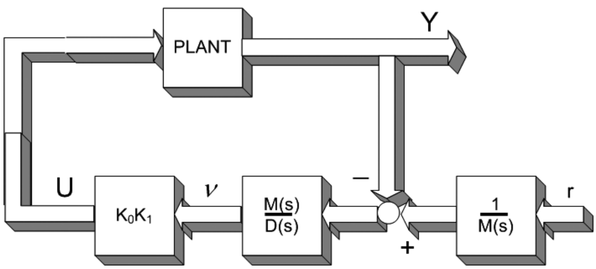

Figure 3: Block diagram of the control system

5. Digital controller design

Because the implementation of modern controllers is usually based on the use of a digital signal processor, this part is devoted to the problem of digital controller design for continuous nonlinear time-varying systems. In particular, the design of digital controller based on the pseudo-continuous approach is presented, where the digital controller is the result of continuous-time controller discretization.

5.1. Pseudo-continuous-time model with pure delay

Provided that a digital controller fitted with a ZOH (Zero-Order Hold) device is used, we assume a nonlinear pseudo-continuous-time model

$$\overline{x}^{(1)}(t) = \overline{h}\left(\overline{x}(t), \overline{u}(t-\tau), t\right), \qquad \overline{x}(0) = \overline{x}_0, \tag{22}$$

$$\overline{y}(t) = \overline{g}\left(t, \overline{x}(t)\right) \tag{23}$$

with a delay $\tau = T_s/2$ taken into account (T_s is a sampling period).

So the control problem has been reformulated as the following insensitivity condition

$$\overline{\Delta}^{F}\left(\overline{x}(t),\overline{y}(t),\overline{r}(t),\overline{u}(t-\tau),t\right)=0. \tag{24}$$

The delay τ caused by discretization alters the stability of the fast motion subsystem, and degrades the transient performance in the closed-loop system. Hence, the control law parameters should be selected to maintain quality of the control transients in the presence of quantized feedback.

Due to:

$$K_0 \approx \{B\}^{-1} \tag{25}$$

and assumptions in (11), (12), the transfer function of the i-th open-loop fast motion subsystem with time delay is given by

$$G_i^O(s)=\frac{k_i \exp(-\tau s)}{D_i(s)}, \tag{26}$$

where

$$D_i(s)=\mu_i^{q_i}s^{q_i}+\mu_i^{q_i-1}d_{i,q_i-1}s^{q_i-1}+\ldots+\mu_i d_{i,1}s+d_{i,0}\,. \tag{27}$$

Denote by $\omega_{i,c}$ the crossover frequency on the Nyquist plot of the i-th channel, such that

$$\left|G_i^O\left(j\omega_{i,c},\mu_i\right)\right|=1. \tag{28}$$

From (26) and (28) follows the relationship

$$\left|D_i\left(j\mu_i\omega_{i,c}\right)\right|=k_i\,, \tag{29}$$

which may be used to obtain the value of $\omega_{i,c}$.

Then the phase margin $\Delta\varphi_i$ of the fast motion subsystem equals:

$$\Delta\varphi_i=\pi-Arg\,D_i\left(j\mu_i\omega_{i,c}\right)-\tau\omega_{i,c}\,. \tag{30}$$

From the requirement

$$0<\Delta\varphi_i^d<\Delta\varphi_i \tag{31}$$

follows the expression

$$T_{i,s}=2\left\{\pi-\Delta\varphi_i^d-Arg\,D_i\left(j\mu_i\omega_{i,c}\right)\right\}/\omega_{i,c}\,. \tag{32}$$

The above equation may be use to determine the value of the sampling period T_s. Based on the requirement (31) it follows that:

$$T_s \leqslant \min_{i=1,\ldots,p} T_{i,s}\left(\Delta\varphi_i^d\right). \tag{33}$$

5.2. Digital realization of a continuous controller

The continuous control law (11) can be represented as the system of the decomposed linear differential equations:

$$D_i(s)\overline{v}_i(s) = -k_i\left[A_i(s)y_i(s) - B_i(s)r_i(s)\right], \tag{34}$$

where: $i = 1, \ldots, p$,

$$A_i(s) = s^{m_i} + a_{i,m_i-1}s^{m_i-1} + \ldots + a_{i,1}s - a_{i,0}, \tag{35}$$

$$B_i(s) = b_{i,0}\,. \tag{36}$$

The digital realization of the continuous-time controller (34) can be found by the Z-transform. Alternatively various approximations can be applied. In particular, the Tustin transformation, which maintains the stability conditions of the continuous-time system, can be used to (34) to obtain its digital approximation by a substitution

$$s = \frac{2}{T_s}\frac{z-1}{z+1}\,. \tag{37}$$

As a result of the transformation (37), from (34)-(36) we obtain a digital controller:

$$\overline{D}_i(z)\overline{v}_i(z) = -k_i\left[\overline{A}_i(z)y_i(z) - \overline{B}_i(z)r_i(z)\right]. \tag{38}$$

This can also be presented as the following recursive equations:

$$\overline{v}_{i,n} = \sum_{j=1}^{q_i}\overline{d}_{ij}\overline{v}_{i,n-j} + \sum_{j=1}^{q_i}\overline{a}_{ij}y_{i,n-j} + \sum_{j=1}^{q_i}\overline{b}_{ij}r_{i,n-j}\,, \tag{39}$$

where: $\overline{v}_i(t) = \overline{v}_{i,n}$, $nT_s \leqslant t < (n+1)T_s$.

5.3. Aircraft digital controller

By the Tustin transformation (37), from (20), (21) the control law in the form of the difference equation is obtained:

- for velocity controller

$$\begin{aligned} v_{V,n} = {}& \widetilde{d}_{V1}v_{V,n-1} + \widetilde{d}_{V2}v_{V,n-2} + \widetilde{a}_{V0}y_{V,n} + \widetilde{a}_{V1}y_{V,n-1} + \widetilde{a}_{V2}y_{V,n-2} \\ & + \widetilde{b}_{V0}r_{V,n} + \widetilde{b}_{V1}r_{V,n-1} + \widetilde{b}_{V2}r_{V,n-2}\,, \end{aligned} \tag{40}$$

where:

$$\widetilde{d}_{V0} = 4\mu_V^2 + 4\mu_V d_{V,1}T_s + d_{V,0}T_s^2\,, \qquad \widetilde{d}_{V1} = \left\{8\mu_V^2 - 2d_{V,0}T_s^2\right\}/\widetilde{d}_{V0}\,,$$

$$\widetilde{d}_{V2} = -\left\{4\mu_V^2 - 4\mu_V d_{V,1} T_s + d_{V,0} T_s^2\right\} / \widetilde{d}_{V0},$$

$$\widetilde{a}_{V0} = -k\left\{4\tau_V^2 + 4\tau_V \alpha_V T_s + a_{V,0} T_s^2\right\} / \widetilde{d}_{V0},$$

$$\widetilde{a}_{V1} = 2k\left\{4\tau_V^2 - a_{V,0} T_s^2\right\} / \widetilde{d}_{V0}, \qquad \widetilde{a}_{V2} = -k\left\{4\tau_V^2 - 4\tau_V \alpha_V T_s + a_{V,0} T_s^2\right\} / \widetilde{d}_{V0},$$

$$\widetilde{b}_{V0} = k\left\{b_{V,0} T_s^2\right\} / \widetilde{d}_{V0}, \qquad \widetilde{b}_{V1} = 2k b_{V,0} T_s^2 / \widetilde{d}_{V0}, \qquad \widetilde{b}_{V2} = k\left\{b_{V,0} T_s^2\right\} / \widetilde{d}_{V0}.$$

- for flight-path angle controller

$$\begin{aligned} v_{\gamma,n} = {} & \widetilde{d}_{\gamma1} v_{\gamma,n-1} + \widetilde{d}_{\gamma2} v_{\gamma,n-2} + \widetilde{d}_{\gamma3} v_{\gamma,k-3} + \widetilde{a}_{\gamma0} y_{\gamma,n} + \widetilde{a}_{\gamma1} y_{\gamma,n-1} + \widetilde{a}_{\gamma2} y_{\gamma,n-2} \\ & + \widetilde{a}_{\gamma3} y_{\gamma,n-3} + \widetilde{b}_{\gamma0} r_{\gamma,n} + \widetilde{b}_{\gamma1} r_{\gamma,n-1} + \widetilde{b}_{\gamma2} r_{\gamma,n-2} + \widetilde{b}_{\gamma3} r_{\gamma,n-3}, \end{aligned} \tag{41}$$

where:

$$\widetilde{d}_{\gamma0} = 8\mu_\gamma^3 + 12\mu_\gamma^2 d_{\gamma,2} T_s + 6\mu_\gamma d_{\gamma,1} T_s^2 + d_{\gamma,0} T_s^3,$$

$$\widetilde{d}_{\gamma1} = \left\{24\mu_\gamma^3 + 12\mu_\gamma^2 d_{\gamma,2} T_s - 6\mu_\gamma d_{\gamma,1} T_s^2 - 3d_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{d}_{\gamma2} = -\left\{24\mu_\gamma^3 - 12\mu_\gamma^2 d_{\gamma,2} T_s - 6\mu_\gamma d_{\gamma,1} T_s^2 + 3d_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{d}_{\gamma3} = \left\{8\mu_\gamma^3 - 12\mu_\gamma^2 d_{\gamma,2} T_s + 6\mu_\gamma d_{\gamma,1} T_s^2 - d_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{a}_{\gamma0} = -k\left\{8\tau_\gamma^3 + 12\tau_\gamma^2 \alpha_\gamma T_s + 6\tau_\gamma \alpha_\gamma^2 T_s^2 + a_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{a}_{\gamma1} = k\left\{24\tau_\gamma^3 + 12\tau_\gamma^2 \alpha_\gamma T_s - 6\tau_\gamma \alpha_\gamma^2 T_s^2 - 3a_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{a}_{\gamma2} = -k\left\{24\tau_\gamma^3 - 12\tau_\gamma^2 \alpha_\gamma T_s - 6\tau_\gamma \alpha_\gamma^2 T_s^2 + 3a_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{a}_{\gamma3} = k\left\{8\tau_\gamma^3 - 12\tau_\gamma^2 \alpha_\gamma T_s + 6\tau_\gamma \alpha_\gamma^2 T_s^2 - a_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{b}_{\gamma0} = k\left\{b_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0}, \qquad \widetilde{b}_{\gamma1} = k\left\{3b_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0},$$

$$\widetilde{b}_{\gamma2} = k\left\{3b_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0}, \qquad \widetilde{b}_{\gamma3} = -k\left\{b_{\gamma,0} T_s^3\right\} / \widetilde{d}_{\gamma0}.$$

6. Results of simulation

The main goal of the simulation is to provide the control of ascent and descent of flight through the velocity V and flight path angle γ control, which is treated as an indirect altitude control. The presented maneuver consisted in transition with predefined dynamics from one steady-state flight to another. The steady-state trim conditions were determined by the values of state and control variables obtained by solving the respective trimming problems.

For the chosen parameters in [4], the appropriate fast-motion subsystem has the following phase margin $\Delta\varphi_i^d$ (30), (31):

$$\Delta\varphi_V \geqslant \Delta\varphi_V^d = 1.175 \text{ [rad]}, \qquad \Delta\varphi_\gamma \geqslant \Delta\varphi_\gamma^d = 1.25 \text{ [rad]},$$

if there are the following sampling periods (32):

$$T_{V,s} = 0.2180 \text{ [s]}, \qquad T_{\gamma,s} = 0.1544 \text{ [s]}.$$

Finally, based on the requirement (33), it follows that:

$$T_s = 0.1 \text{ [s]} \leqslant \min_{i=V,\gamma} T_{i,s}\left(\Delta\varphi_i^d\right).$$

To evaluate the performance of the DCM digital controller, the simulation was performed for two values of sampling period: $T_s = 0.1$ [s], $T_s = 1$ [s].

The simulations were realized for the geometrical and mass data in agreement with the technical documentation of [5].

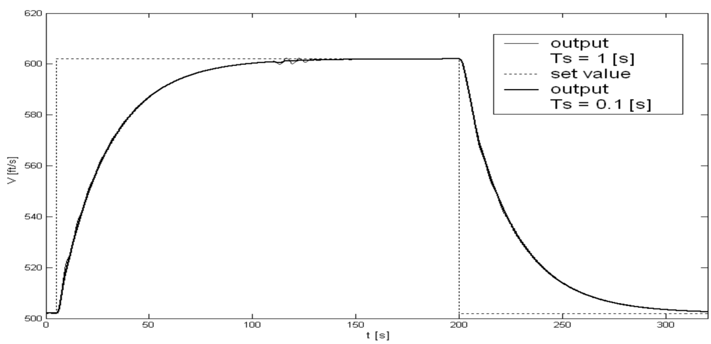

Figure 4: Dynamics of velocity V [ft/s]

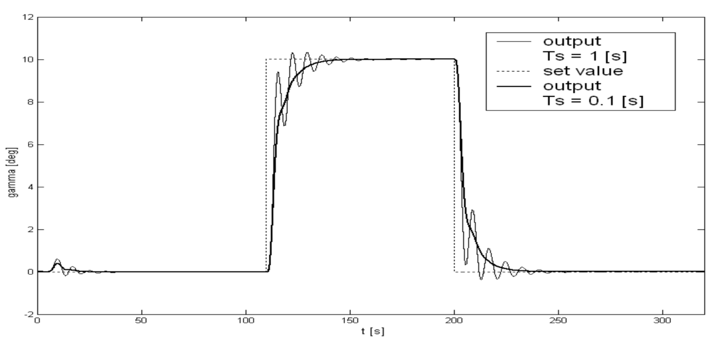

Figure 5: Dynamics of flight path angle γ [deg]

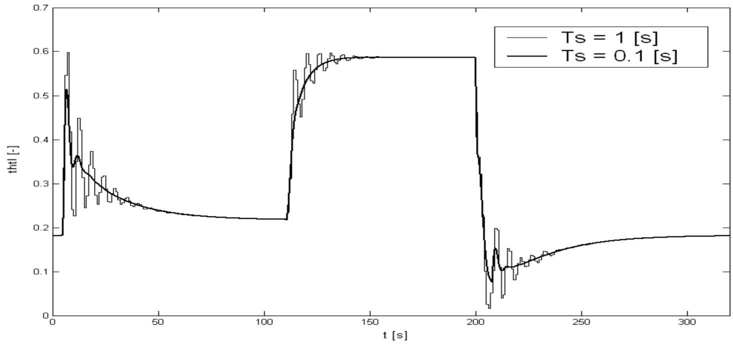

Figure 6: Dynamics of throttle $thtl \equiv \delta_T$ [–]

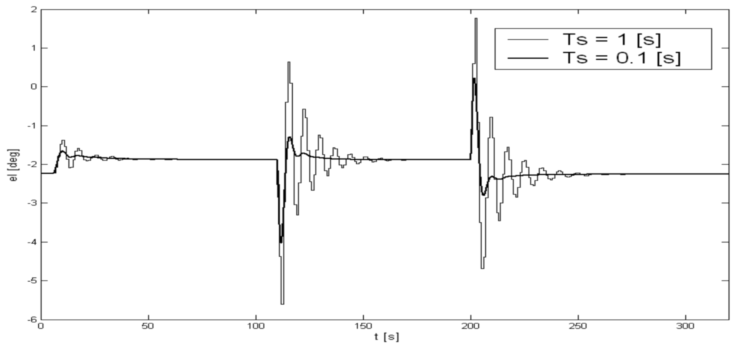

Figure 7: Dynamics of elevator $el \equiv \delta_H$ [deg]

7. Conclusion

In the paper the design of digital controller based on the pseudo-continuous approach is presented, where the digital controller is the result of continuous-time controller discretization. The applied method allows to create the expected outputs for multi-input multi-output nonlinear time-varying object, like an exemplary aircraft, and provides independent desired dynamics in control channels. The peculiarity of the propose approach is the application of the higher order derivatives jointly with high gain in the control law. This approach and structure of the control system is the implementation of the model reference control with the reference model transfer function which is equal to the inverse of the controller "dynamics". The resulting controller is a combination of a low-order linear dynamical system and a matrix whose entries depend non-linearly on some process variables. The only inconvenience of the DCM method is the output equation, which causes problems of computational and implementation nature. However, the effectiveness of the digital DCM controller demonstrated in this paper, shows the potential for its implementation on an embedded systems and its application in the control of other objects.

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