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The sensitivity analysis of even order biquadratic elliptic filters

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Abstract. In this paper, the sensitivity analysis of the elliptic filters realized by using biquadratic structures was carried out. The influence of spread the structure parameter values on the shape of the frequency characteristic of the filter transmittance modulus was analyzed. The analysis was limited to the case of even order low-pass filter. Defining the proper class of the sensitivity coefficients, the changes influence of individual structure parameters on the deviation of basic parameter values of the characteristic was considered. The considerations were illustrated by the numerical example.

Key words: elliptic filter, biquadratic structures, frequency filter characteristic, sensitivity analysis, sensitivity coefficients.

1. Introduction

The elliptic filter from among all known types of filters distinguishes the largest selectivity [1]. Frequency characteristics of transmittance modulus have oscillations in the pass-band and the stop-band. Between those bands, it is possible to distinguish the transition-band in which there is a sudden fall of filter transmission. The characteristic of the example low-pass elliptic filter of order $r = 4$ is shown in Fig. 1.

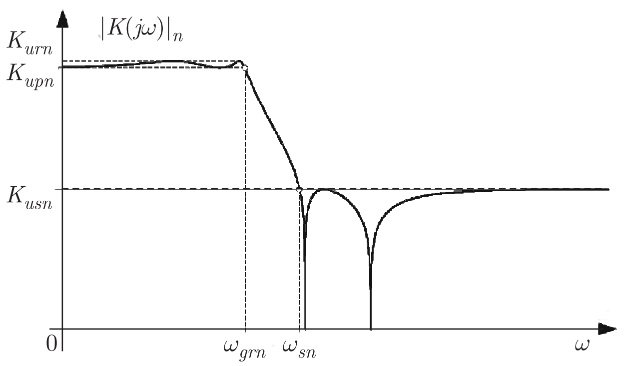

Fig. 1. The characteristic of the example low-pass elliptic filter of order $r = 4$

In this paper, the proposed sensitivity analysis of elliptic filters is limited to the case of low-pass filter of even order [2]. A transmittance of even order r filter can be expressed by the product of $\frac{r}{2}$ biquadratic factors and by the resultant fixed factor. It is possible to build elliptic filters in the class of active devices with the use of biquadratic structures that realize the individual biquadratic factors [3–5]. The fixed factor can be attainable outside biquadratic structures in the single proportional structure.

2. The nominal characteristic

The nominal frequency characteristic of transmittance modulus $|K(j\omega)|_n$ is attainable, when all structure parameters adopt the nominal values

$$|K(j\omega)|_n = K_{u0_n} \prod_{i=1}^{\frac{r}{2}} \left| \frac{-\omega^2 + \omega_{z_{in}}^2}{-\omega^2 + j2\sigma_{in}\omega + \omega_{p_{in}}^2} \right|, \qquad (1)$$

where

- $\sigma_{in}, \omega_{p_{in}}, \omega_{z_{in}}$ – the nominal parameter values of i-th biquadratic structure,
- K_{u0n} – the nominal value of the fixed factor.

In Fig. 2 general course of the nominal characteristic of low-pass elliptic filter of even order r is presented. The characteristic is clear-out described by the given nominal parameters:

- K_{urn} – the nominal maximum value of transmittance modulus in the pass-band,
- K_{upn} – the nominal minimum value of transmittance modulus in the pass-band,
- ω_{grn} – the nominal limited pulsation, above that transmittance modulus falls below K_{upn} value,
- K_{usn} – the nominal maximum value of transmittance modulus in the stop-band,
- ω_{sn} – the nominal pulsation, below that transmittance modulus grows above the maximum value of the stop-band equals K_{usn}.

In the pass-band $(0 < \omega < \omega_{grn})$ transmittance modulus has oscillation course and oscillates between the minimum value K_{upn} and the maximum value K_{urn}. The level K_{urn} is achieved by all j-th local maximums K_{ur_j} which are in the pass-band.

$$K_{urn} = K_{ur_j}, j = 1, 2, ..., \frac{r}{2}. \qquad (2)$$

The nominal value K_{upn} equals to K_{up_1} value of transmittance modulus, that is achieved for $\omega = 0$

$$K_{upn} = K_{up_1} = \lim_{\omega \to 0} |K_u(j\omega)|_n \qquad (3)$$

and all j-th local minimums K_{up_j} of the pass-band

$$K_{upn} = K_{up_j}, j = 2, 3, ..., \frac{r}{2}. \qquad (4)$$

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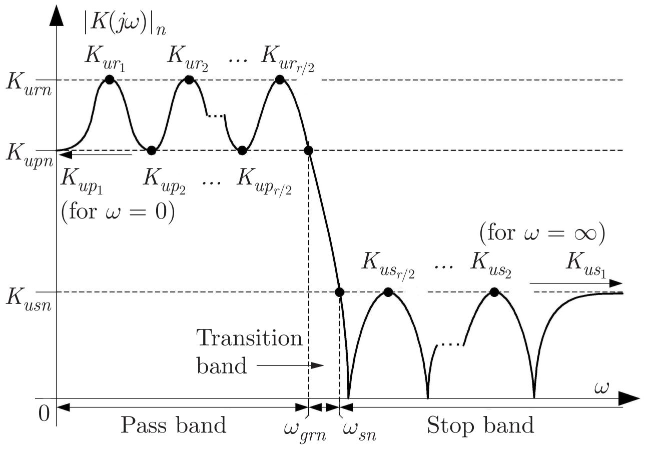

Fig. 2. The nominal characteristic of the low-pass elliptic filter of even order $r = 4$

The measure of oscillations in the pass-band is the pass-band ripple expressed in dB

$$R_{(dB)n} = 20 \log \left(\frac{K_{urn}}{K_{upn}} \right). \tag{5}$$

The maximum value K_{usn} of the stop-band equals to K_{us_1} value of transmittance modulus, that is achieved for $\omega = \infty$

$$K_{usn} = K_{us_1} = \lim_{\omega \to \infty} |K_u(j\omega)|_n \tag{6}$$

and all j-th local maximums K_{us_j} of the band

$$K_{usn} = K_{us_j}, j = 2, 3, ..., \frac{r}{2}. \tag{7}$$

The transition-band included between the pulsation ω_{grn} and ω_{sn} characterized a sudden fall of transmittance modulus value. The measure of this fall is an average slope of the characteristic $N_{dB/okt}$ expressed in dB/okt, described according to the formula

$$N_{dB/oktn} = \frac{20 \log \left(\frac{K_{upn}}{K_{usn}} \right)}{\log_2 \left(\frac{\omega_{grn}}{\omega_{sn}} \right)}. \tag{8}$$

3. The deviation characteristic

In practice, real values of structure parameters differ from nominal values, for the sake of spread of the element values, from these elements the structures are built. Deviation from the nominal value even of one parameter of the structure causes characteristic deviation from nominal case. A frequency deviation characteristic of transmittance modulus $|K(j\omega)|$ is achieved in the case when all structure parameters adopt real values

$$|K(j\omega)| = K_{u0} \prod_{i=1}^{\frac{r}{2}} \left| \frac{-\omega^2 + \omega_{z_i}^2}{-\omega^2 + j2\sigma_i\omega + \omega_{p_i}^2} \right|, \tag{9}$$

where
$\sigma_i,\ \omega_{p_i},\ \omega_{z_i}$ – the real parameter values of the i-th biquadratic structure

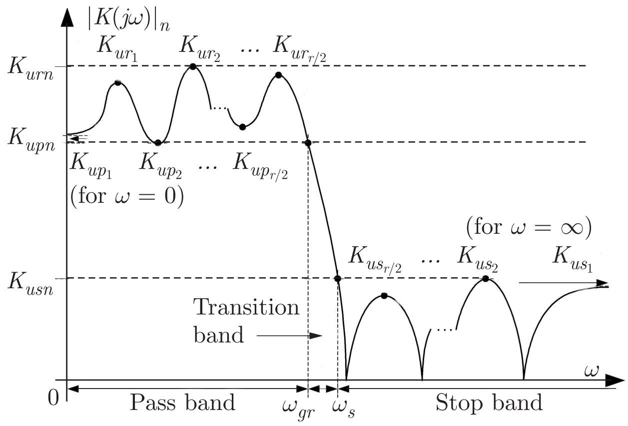

Fig. 3. The deviation characteristic of even order r low-pass elliptic filter

$$\sigma_i = \sigma_{in} \left(1 + \delta\sigma_i\right), \quad \omega_{p_i} = \omega_{p_{in}} \left(1 + \delta\omega_{p_i}\right),$$
$$\omega_{z_i} = \omega_{z_{in}} \left(1 + \delta\omega_{z_i}\right),$$

with
$\delta\sigma_i,\ \delta\omega_{p_i},\ \delta\omega_{z_i}$ – relative deviation of the parameter values: $\sigma_i,\ \omega_{p_i},\ \omega_{z_i}$ from the nominal values: $\sigma_{in},\ \omega_{p_{in}},\ \omega_{z_{in}}$, K_{u0} – the real value of the fixed factor.

In Fig. 3 there is presented a general course of the deviation characteristic of even order r low-pass elliptic filter.

The parameters describing characteristic gave in deviation from the nominal values: $K_{urn},\ K_{upn},\ \omega_{grn},\ K_{usn},\ \omega_{sn}$ to the values: $K_{ur},\ K_{up},\ \omega_{gr},\ K_{us},\ \omega_s$. They are a bit differently defined than for the nominal case.

In the pass-band $(0 < \omega < \omega_{gr})$ j-th values: K_{ur_j} and K_{up_j} aren't equal, in that case it is necessary to accept for the parameter K_{ur} the biggest value from among j-th K_{ur_j} values

$$K_{ur} = \max_{j=1}^{\frac{r}{2}} \left\{ K_{ur_j} \right\}, \tag{10}$$

however for K_{up} parameter it is necessary to accept the smallest value from among j-th K_{up_j} values

$$K_{up} = \min_{j=1}^{\frac{r}{2}} \left\{ K_{up_j} \right\}. \tag{11}$$

The limited pulsation ω_{gr} is the one above that transmittance modulus falls below the K_{up} value, however real pass-band ripple expressed in dB is characterized by the expression

$$R_{(dB)} = 20 \log \left(\frac{K_{ur}}{K_{up}} \right). \tag{12}$$

In the stop-band $(\omega_{sn} < \omega < \infty)$ j-th values K_{us_j} aren't equal, so in this case it is necessary to accept for the parameter K_{us} the biggest value from among j-th K_{us_j} values

$$K_{us} = \max_{j=1}^{\frac{r}{2}} \left\{ K_{us_j} \right\}. \tag{13}$$

The pulsation ω_s is the one below that transmittance modulus grows over the K_{us} value, however the average slope of the deviation characteristic $N_{dB/okt}$ in the transition-band expressed in dB/okt, can be described according to the formula

$$N_{dB/okt} = \frac{20\log\left(\frac{K_{up}}{K_{us}}\right)}{\log_2\left(\frac{\omega_{gr}}{\omega_s}\right)}. \tag{14}$$

4. The coefficients describing sensitivity of characteristic to changes of structure parameters

Determining proper class of sensitivity coefficients it is possible to count relative change of each characteristic parameters for any combinations relative deviations of structure parameters. The essential class of the sensitivity coefficients must determine the sensitivity very detailed characteristic parameters to changes of each structure parameter separately. Structure parameters: σ_i, ω_{p_i}, ω_{z_i} $\left(i = 1, 2, ..., \frac{r}{2}\right)$ create the vector of influential parameters $\boldsymbol{Y}$ about $\frac{3r}{2}$ elements

$$\begin{aligned} Y &= \left[Y_1, Y_2, ..., Y_k, ..., Y_{\frac{3r}{2}}\right]^T \\ &= \left[\sigma_1, \sigma_2, ..., \sigma_{\frac{r}{2}}, \omega_{p_1}, \omega_{p_2}, ..., \omega_{p_{\frac{r}{2}}}, \omega_{z_1}, \omega_{z_2}, ..., \omega_{z_{\frac{r}{2}}}\right]^T, \\ &\text{for } k = 1, 2, ... \frac{3r}{2}. \end{aligned} \tag{15}$$

In Fig. 4 there is showed general characteristic of filter, for deviation from nominal value k-th structure parameter $Y_k = Y_{kn} + \Delta Y_k$. The other structure parameters accept the nominal values. In this figure there are marked all detailed characteristic parameters that sensitivity should be described for the changes of each k-th structure parameter Y_k. There are all j-th values of the following parameters: K_{ur_j}, K_{up_j}, ω_{gr_j}, K_{us_j}, ω_{s_j} $(j = 1, 2, ..., \frac{r}{2})$. The meaning of parameters: K_{ur_j}, K_{up_j}, K_{us_j} has been explained by describing the parameters of the deviation characteristic (look at point 3).

It is necessary to explain the meaning of the parameters: ω_{gr_j}, ω_{s_j}

- j-th value of the parameter ω_{gr_j} is the pulsation, that $|K(j\omega)|$ achieves j-th value K_{up_j},
- j-th value of the parameter ω_{s_j} is the pulsation, that $|K(j\omega)|$ achieves j-th value K_{us_j}.

Within the confines of the sensitivity analysis it is necessary to determine the sensitivity coefficient of all j-th values of the characteristic parameters: K_{ur_j}, K_{up_j}, ω_{gr_j}, K_{us_j}, ω_{s_j} for the changes of each k-th structure parameter Y_k

$$\begin{aligned} &S_{Y_k}^{K_{ur_j}}, S_{Y_k}^{K_{up_j}}, S_{Y_k}^{\omega_{gr_j}}, S_{Y_k}^{K_{us_j}}, S_{Y_k}^{\omega_{s_j}}, \\ &k = 1, 2, ..., \frac{3r}{2}, \quad j = 1, 2, ..., \frac{r}{2}. \end{aligned} \tag{16}$$

The individual k-th coefficient values: $S_{Y_k}^{K_{ur_j}}$, $S_{Y_k}^{K_{up_j}}$, $S_{Y_k}^{\omega_{gr_j}}$, $S_{Y_k}^{K_{us_j}}$, $S_{Y_k}^{\omega_{s_j}}$ are possible to determine in the numerical way for each k-th structure parameter separately. In order to achieve this aim it is necessary to deviate the value of k-th structure parameter Y_k about any not large known value ΔY_k, keeping the nominal values of the rest structure parameters.

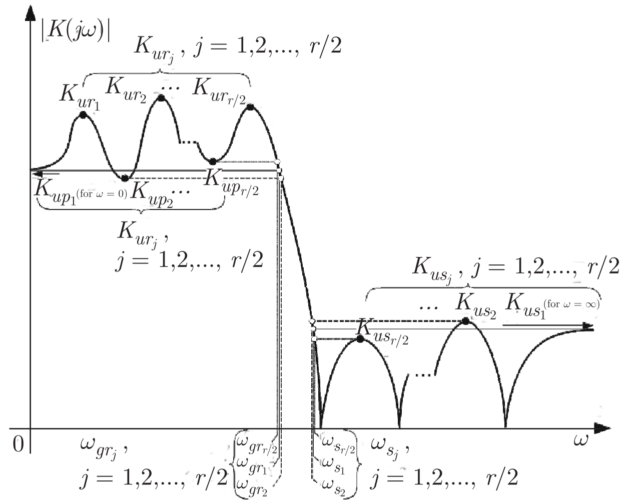

Fig. 4. General characteristic of low-pass elliptic filter for deviated from nominal value of k-th structure parameter $Y_k = Y_kn + \Delta Y_k$. The other structure parameters accept the nominal values

For such deviated k-th structure parameter it is necessary to determine, all j-th values of the structure parameters: K_{ur_j}, K_{up_j}, ω_{gr_j}, K_{us_j}, ω_{s_j}, suitable for this deviation, taking them back to the nominal values: K_{urn}, K_{upn}, ω_{grn}, K_{usn}, ω_{sn}. The coefficient values for each k-th deviated structure parameter can be counted according to the formulas

$$\begin{aligned} &S_{Y_k}^{K_{ur_j}} = \frac{K_{ur_j} - K_{urn}}{K_{urn}} \frac{Y_k}{\Delta Y_k}, \; S_{Y_k}^{K_{up_j}} = \frac{K_{up_j} - K_{upn}}{K_{upn}} \frac{Y_k}{\Delta Y_k}, \\ &S_{Y_k}^{\omega_{gr_j}} = \frac{\omega_{gr_j} - \omega_{grn}}{\omega_{grn}} \frac{Y_k}{\Delta Y_k}, \; S_{Y_k}^{K_{us_j}} = \frac{K_{us_j} - K_{usn}}{K_{us\,n}} \frac{Y_k}{\Delta Y_k}, \\ &S_{Y_k}^{\omega_{s_j}} = \frac{\omega_{s_j} - \omega_{sn}}{\omega_{sn}} \frac{Y_k}{\Delta Y_k}, \; k = 1, 2, ..., \frac{3r}{2}, \; j = 1, 2, ..., \frac{r}{2} \end{aligned} \tag{17}$$

5. Calculation of characteristic deviations

On the basis of sensitivity coefficients (17) it is possible to count the relative change of each characteristic parameter, for any deviation combination δY_k $(k = 1, 2, ..., \frac{3r}{2})$ of particular structure parameters, according to the formulas

$$\delta K_{ur} = \max_{j=1}^{\frac{r}{2}} \left\{ \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{K_{ur_j}} \delta Y_k \right) \right\}, \tag{18}$$

$$\begin{aligned} \delta K_{up} &= \min_{j=1}^{\frac{r}{2}} \left\{ \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{K_{up_j}} \delta Y_k \right) \right\} \\ &= \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{K_{up_w}} \delta Y_k \right), \quad 1 \leqslant w \leqslant \frac{r}{2}, \end{aligned} \tag{19}$$

$$\delta \omega_{gr} = \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{\omega_{gr_w}} \delta Y_k \right), \tag{20}$$

$$\delta K_{us} = \max_{j=1}^{\frac{r}{2}} \left\{ \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{K_{us_j}} \delta Y_k \right) \right\} = \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{K_{us_q}} \delta Y_k \right), \quad 1 \leqslant q \leqslant \frac{r}{2}, \tag{21}$$

$$\delta \omega_s = \sum_{k=1}^{\frac{3r}{2}} \left(S_{Y_k}^{\omega_{s_q}} \delta Y_k \right). \tag{22}$$

The sensitivity coefficients (17) on the assumption that the changes of characteristic parameters evoked by changes of structure parameters are linear. In this connection, these presented formulas to relative deviations of characteristic parameters (18), (19), (20), (21), (22) can be applied when the relative deviations of characteristic structures are not large (do not exceed 10% of the nominal value), which, however, is fulfilled in practice.

On the basis of the relative deviations of characteristic parameters it is possible to count relative values of these parameters

$$K_{ur} = K_{urn} \left(1 + \delta K_{ur}\right), \ K_{up} = K_{upn} \left(1 + \delta K_{up}\right),$$
$$\omega_{gr} = \omega_{grn} \left(1 + \delta \omega_{gr}\right), \ K_{us} = K_{usn} \left(1 + \delta K_{us}\right) \tag{23}$$
$$\omega_s = \omega_{sn} \left(1 + \delta \omega_s\right).$$

Example. To illustrate the presented considerations in this example, the sensitivity analysis of normalized ($\omega_{grn} = 1$) elliptic filter order $r = 4$ with pass-band ripple equals 3 dB was carried out. Transmittance modulus of this filter can be presented by a product of transmittance modulus and two component biquadratic structures

$$|K(j\omega)|_n = K_{u0} \left| \frac{-\omega^2 + \omega_{z_1\,n}^2}{-\omega^2 + j2\sigma_{1\,n}\omega + \omega_{p_1\,n}^2} \right| \times \left| \frac{-\omega^2 + \omega_{z_2\,n}^2}{-\omega^2 + j2\sigma_{2\,n}\omega + \omega_{p_2\,n}^2} \right|, \tag{24}$$

where

$$\sigma_{1n} = 0.21396, \ \ \omega_{p_{1n}} = 0.47121, \ \ \omega_{z_{1n}} = 4.9221,$$
$$\sigma_{2n} = 0.075257, \ \ \omega_{p_{2n}} = 0.95752, \ \ \omega_{z_{2n}} = 2.1432,$$
$$K_{u0n} = \frac{\omega_{p_{1n}}^2 \omega_{p_{2n}}^2}{\omega_{z_{1n}}^2 \omega_{z_{2n}}^2} \frac{1}{\sqrt{2}}.$$

Structure parameters create six-elemental vector of influential parameters $\boldsymbol{Y}$

$$Y = [Y_1, Y_2, Y_3, Y_4, Y_5, Y_6]^T = [\sigma_{1n}, \sigma_{2n}, \omega_{p_{1n}}, \omega_{p_{2n}}, \omega_{z_{1n}}, \omega_{z_{2n}}]^T. \tag{25}$$

A course of nominal characteristic is compatible with given in Fig. 1. The nominal values of characteristic parameters are equal to

$$K_{urn} = 1.0000, \ K_{upn} = 0.70710, \ \omega_{grn} = 1.0000,$$
$$K_{usn} = 1.2936 \cdot 10^{-3}, \ \omega_{sn} = 2.0000. \tag{26}$$

The sensitivity coefficients are determined by the formulas (17), deviating in turn the values of each structure parameters about +5% ($\Delta Y_k = 0.05$) relative to the nominal value

▼ $S_{Y_k}^{K_{ur_j}}$ $(k = 1, 2...6, \ j = 1, 2)$

$S_{Y_1}^{K_{ur_1}} = -8.5 \cdot 10^{-1}$, $S_{Y_2}^{K_{ur_1}} = -6.6 \cdot 10^{-3}$,
$S_{Y_3}^{K_{ur_1}} = -6.7 \cdot 10^{-1}$, $S_{Y_4}^{K_{ur_1}} = -2.2$, $S_{Y_5}^{K_{ur_1}} = 2.1$,
$S_{Y_6}^{K_{ur_1}} = 2.1$, $S_{Y_1}^{K_{ur_2}} = -2.8 \cdot 10^{-1}$, $S_{Y_2}^{K_{ur_2}} = -8.6 \cdot 10^{-1}$,
$S_{Y_3}^{K_{ur_2}} = 5.2 \cdot 10^{-1}$, $S_{Y_4}^{K_{ur_2}} = -3.5$, $S_{Y_5}^{K_{ur_2}} = 2.1$.
$S_{Y_6}^{K_{ur_2}} = 2.5$,

▼ $S_{Y_k}^{K_{up_j}}$ $(k = 1, 2...6, \ j = 1, 2)$

$S_{Y_1}^{K_{up_1}} = 2.8 \cdot 10^{-4}$, $S_{Y_2}^{K_{up_1}} = 2.8 \cdot 10^{-4}$, $S_{Y_3}^{K_{up_1}} = -1.9$,
$S_{Y_4}^{K_{up_1}} = -1.9$, $S_{Y_5}^{K_{up_1}} = 2.1$, $S_{Y_6}^{K_{up_1}} = 2.1$,
$S_{Y_1}^{K_{up_2}} = 4.9 \cdot 10^{-1}$, $S_{Y_2}^{K_{up_2}} = -8.0 \cdot 10^{-2}$
$S_{Y_3}^{K_{up_2}} = 7.4 \cdot 10^{-1}$, $S_{Y_4}^{K_{up_2}} = -3.9$, $S_{Y_5}^{K_{up_2}} = 2.1$
$S_{Y_6}^{K_{up_2}} = 2.3$.

▼ $S_{Y_k}^{\omega_{gr_j}}$ $(k = 1, 2...6, \ j = 1, 2)$

$S_{Y_1}^{\omega_{gr_1}} = -2.6 \cdot 10^{-2}$, $S_{Y_2}^{\omega_{gr_1}} = -8.7 \cdot 10^{-2}$,
$S_{Y_3}^{\omega_{gr_1}} = 2.5 \cdot 10^{-1}$, $S_{Y_4}^{\omega_{gr_1}} = 7.7 \cdot 10^{-1}$,
$S_{Y_5}^{\omega_{gr_1}} = 8.7 \cdot 10^{-3}$, $S_{Y_6}^{\omega_{gr_1}} = 5.5 \cdot 10^{-2}$,
$S_{Y_1}^{\omega_{gr_2}} = 2.9 \cdot 10^{-2}$, $S_{Y_2}^{\omega_{gr_2}} = -7.7 \cdot 10^{-2}$,
$S_{Y_3}^{\omega_{gr_2}} = -2.9 \cdot 10^{-2}$, $S_{Y_4}^{\omega_{gr_2}} = 1.0$
$S_{Y_5}^{\omega_{gr_2}} = 4.2 \cdot 10^{-3}$, $S_{Y_6}^{\omega_{gr_2}} = 2.9 \cdot 10^{-2}$.

▼ $S_{Y_k}^{K_{us_j}}$ $(k = 1, 2...6, \ j = 1, 2)$

$S_{Y_1}^{K_{us_1}} = -2.8 \cdot 10^{-2}$, $S_{Y_2}^{K_{us_1}} = -4.6 \cdot 10^{-3}$,
$S_{Y_3}^{K_{us_1}} = 6.0 \cdot 10^{-2}$, $S_{Y_4}^{K_{us_1}} = 2.9 \cdot 10^{-1}$, $S_{Y_5}^{K_{us_1}} = 3.0$,
$S_{Y_6}^{K_{us_1}} = -2.9$, $S_{Y_1}^{K_{us_2}} = 0.0$, $S_{Y_2}^{K_{us_2}} = 0.0$
$S_{Y_3}^{K_{us_2}} = 0.0$, $S_{Y_4}^{K_{us_2}} = 0.0$, $S_{Y_5}^{K_{us_2}} = 0.0$, $S_{Y_6}^{K_{us_2}} = 0.0$

▼ $S_{Y_k}^{\omega_{s_j}}$ $(k = 1, 2...6, \ j = 1, 2)$

$S_{Y_1}^{\omega_{s_1}} = -1.1 \cdot 10^{-3}$, $S_{Y_2}^{\omega_{s_1}} = -2.0 \cdot 10^{-4}$,
$S_{Y_3}^{\omega_{s_1}} = 3.0 \cdot 10^{-3}$, $S_{Y_4}^{\omega_{s_1}} = 1.8 \cdot 10^{-2}$,
$S_{Y_5}^{\omega_{s_1}} = -2.5 \cdot 10^{-2}$, $S_{Y_6}^{\omega_{s_1}} = 1.0$,
$S_{Y_1}^{\omega_{s_2}} = -2.7 \cdot 10^{-3}$, $S_{Y_2}^{\omega_{s_2}} = -5.0 \cdot 10^{-4}$,
$S_{Y_3}^{\omega_{s_2}} = 6.2 \cdot 10^{-3}$, $S_{Y_4}^{\omega_{s_2}} = 3.3 \cdot 10^{-2}$,
$S_{Y_5}^{\omega_{s_2}} = 1.2 \cdot 10^{-1}$, $S_{Y_6}^{\omega_{s_2}} = 8.3 \cdot 10^{-1}$.

Deviations of characteristic parameters are determined as an example deviation combination of structure parameters

$$\begin{aligned}\delta\sigma_1 &= -0.01,\ \delta\sigma_2 = -0.01,\ \delta\omega_{p_1} = +0.01,\\ \delta\omega_{p_2} &= -0.01,\ \delta\omega_{z_1} = +0.01,\ \delta\omega_{z_2} = +0.01.\end{aligned} \tag{27}$$

They are equal to

$$\begin{aligned}\delta K_{ur} &= \max_{j=1}^{2}\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{urj}}\delta Y_k\right)\right\}\\ &= \max\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{ur1}}\delta Y_k\right), \sum_{k=1}^{6}\left(S_{Y_k}^{K_{ur2}}\delta Y_k\right)\right\}\\ &= \max\left\{6.58\cdot 10^{-2}, \underline{9.76\cdot 10^{-2}}\right\}\\ &= 9.76\cdot 10^{-2} \cong +9.8\cdot 10^{-2},\end{aligned} \tag{28}$$

$$\begin{aligned}\delta K_{up} &= \min_{j=1}^{2}\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{upj}}\delta Y_k\right)\right\}\\ &= \min\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{up1}}\delta Y_k\right), \sum_{k=1}^{6}\left(S_{Y_k}^{K_{up2}}\delta Y_k\right)\right\}\\ &= \min\left\{\underset{--(w=1)--}{4.10\cdot 10^{-2}},\ 9.65\cdot 10^{-2}\right\}\\ &= 4.10\cdot 10^{-2} \cong +4.1\cdot 10^{-2},\ w = 1,\end{aligned} \tag{29}$$

$$\begin{aligned}\delta\omega_{gr} &= \sum_{k=1}^{6}\left(S_{Y_k}^{\omega_{grw}}\delta Y_k\right) = \sum_{k=1}^{6}\left(S_{Y_k}^{\omega_{gr1}}\delta Y_k\right)\\ &= -3.39\cdot 10^{-3} \cong -3.4\cdot 10^{-3},\end{aligned} \tag{30}$$

$$\begin{aligned}\delta K_{us} &= \max_{j=1}^{2}\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{usj}}\delta Y_k\right)\right\}\\ &= \max\left\{\sum_{k=1}^{6}\left(S_{Y_k}^{K_{us1}}\delta Y_k\right), \sum_{k=1}^{6}\left(S_{Y_k}^{K_{us2}}\delta Y_k\right)\right\}\\ &= \max\left\{-1.56\cdot 10^{-3},\ \underset{-(q=2)-}{0.0}\right\} = 0,\quad q = 2,\end{aligned} \tag{31}$$

$$\begin{aligned}\delta\omega_s &= \sum_{k=1}^{6}\left(S_{Y_k}^{\omega_{sq}}\delta Y_k\right) = \sum_{k=1}^{6}\left(S_{Y_k}^{\omega_{s2}}\delta Y_k\right)\\ &= 9.24\cdot 10^{-3} \cong +9.2\cdot 10^{-3}.\end{aligned} \tag{32}$$

On the basis of the formulas (23) it is possible to determine real values of characteristic parameters

$$K_{ur} = K_{urn}\left(1 + \delta K_{ur}\right) = 1.096, \tag{33}$$

$$K_{up} = K_{upn}\left(1 + \delta K_{up}\right) = 7.361\cdot 10^{-1}, \tag{34}$$

$$\omega_{gr} = \delta\omega_{grn}\left(1 + \delta\omega_{gr}\right) = 9.966\cdot 10^{-1}, \tag{35}$$

$$K_{us} = \delta K_{usn}\left(1 + \delta K_{us}\right) = 1.294\cdot 10^{-3}, \tag{36}$$

$$\omega_s = \delta\omega_{sn}\left(1 + \delta\omega_s\right) = 2.019. \tag{37}$$

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