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CRITICAL APPRAISING OF HOPKINSON BAR TECHNIQUES FOR CALIBRATING HIGH *g* ACCELEROMETERS

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Abstract

The Hopkinson pressure bar has been developed to calibrate and assess high *g* accelerometers' capacity. The extreme caution is indispensable for performing calibration of severe characteristics, like the bearable super-high overload peak and wide duration of stress. In the paper, the Hopkinson bar calibrating system is being critically appraised. A limiting formula is deduced based on the stress wave theory. It indicates that the overload peak and duration of stress are limited by the elastic limit and wave speed of Hopkinson bar material. Both stress wave configurations in the form of linear ramp and cosine functions were designed theoretically to meet typical calibrating requirements. They were confirmed experimentally with the aid of the pulse shaping technique. Their corresponding calibration characteristics were analysed critically, and it was found that the cosine stress wave can achieve the values of acceleration peak or duration by $\pi/2$ times greater than those obtained with the linear stress wave. Finally, some suggestions are proposed for more extreme calibration requirements.

Keywords: Hopkinson bar, calibrating, high *g* accelerometer, limitation.

1. Introduction

The accelerometer calibration has already been drawing increasing attention from scientific and industrial bodies, due to higher and higher requirements in security and reliability. The accelerometers are usually confronted with various high overload situations in the dynamic environments, such as in performing collision tests for improving the passenger safety level, and needing to know the history accelerometer resistance capacity to high overloads within penetration processes. In these cases, it is of utmost significance to carry out the calibration taking into account the dynamically changing parameters.

In respect of the dynamical calibration, some works had already been performed, like the vibration by laser interferometry [1]. It has been even accepted as an international standard. Nevertheless, the achievable ranges (below 10 *g* in acceleration, and 10 KHz in frequency band) were so low and insufficient, that it could not meet the above-mentioned requirements. Sill revolutionarily introduced the Hopkinson pressure bar technique for calibrating transducers [2].

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Subsequently, Ueda *et al.* [3] and Togami *et al.* [4, 5] modified this technique and completed to calibrate overload amplitudes up to the order of 100,000 g (1 g = 9.81 m/s^2) and overload durations of tens microseconds. Moreover, the related experiments and analysis of the interface of Hopkinson bar-disk with an accelerometer, demonstrated that the disk could be approximated as rigid-body motion during the calibrating process, when the rising time of the incident stress pulse is long enough and the disk length is short enough [4]. In the progress of further modification and optimization work, Li *et al.* adopted cone-shape striker bars to produce a semi-sine incident stress wave to achieve calibration capability up to 200,000 g, and the relevant analysis results indicated that the calibration error was approximately 6% in the range of 100,000 g [6]. Moreover, laser interferometer systems [5, 7] were developed to expect an absolute measurement of the movement of the bar end, with the results indicating that strain gauge outputs were consistent with those of the laser interferometer. These calibration techniques were successfully evaluated in the Mechanical Shock Laboratory at Sandia National Laboratories, and compared with an NIST calibrated reference accelerometer [8]. Elaborate analyses of the Hopkinson bar technique were also performed with two models of the incident strain pulse and quartz stress gauge, aiming to exactly evaluate the accelerometers' performance, and the excellent agreement was found in the acceleration characteristics [9]. Based on this technique, similar works were performed to optimize it for specific engineering applications [10–12], with special attention paid to the acceleration level below 10,000 g [13]. Different fixtures were also designed and modified, to ensure the calibrating accelerometer undergo the desired acceleration history from the Hopkinson bar end [4, 5, 7, 13].

However, in the available literature there was demonstrated that both higher amplitude and wider duration cannot be achieved, and there seems to exist a limit for them while performing a more severe calibration with a higher amplitude and a wider duration than those mentioned above.

In the present work, the Hopkinson bar calibration technique is critically appraised, aiming to provide suggestions for potential calibration works. First, the one-dimension stress wave theory is introduced to analyse the calibrating principle, and the critical formula is then deduced. Then, the limitation curves are calculated and analysed for both typical calibrations with linear and cosine stress functions. At last, some suggestions are proposed for extreme calibrations of higher peaks, and wider durations.

2. Experimental method and theory

2.1. Hopkinson bar for calibration

The Hopkinson bar is widely used in studying constitutive behaviour of materials under high strain rate loading. It was originally developed by Hopkinson [14] in 1914 to measure pulse durations generated by the detonation of explosives, and later revolutionarily modified to produce the high overload history for the calibration and verification of accelerometers and transducers [2, 15, 16]. The involved techniques and methodologies were also developed for high-accuracy and extensive applications. A schematic is shown in Fig. 1. The accelerometer is fixed on the end of the incident bar. The pulse shaping technique is widely used in calibrating work, where a pulse shaper is usually attached to the impacting face of the incident bar by Vaseline grease, aiming to trimm the incident wave with desired stress wave configurations [17, 18]. A striker bar of abnormal shape, as an alternative method, is used to obtain a specific stress wave [6]. Upon the striker bar's impacting on the incident bar, an incident stress wave is generated, and it propagates

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along the incident bar to the fixed accelerometer, as shown in Fig. 1. On arriving at the interface between the incident bar and the accelerometer, the wave will be reflected nearly totally back into the incident bar, because the characteristic impedance of the bar is much larger than that of accelerometers [6, 16, 17]. Since then, the bar end undergoes an abrupt velocity increment, and a high overload field comes into being. Based on the stress wave theory, the velocity field of the interface is deduced, and so is its acceleration history [3–9]. More details on obtaining accurate interpretation of the stress wave can be found in the reference [19, 20].

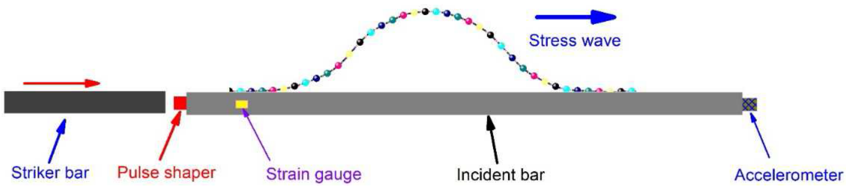

Fig. 1. A schematic of the Hopkinson bar calibration system.

$$v_{\text{int.}}(t) = 2c\varepsilon_i(t), \tag{1}$$

$$a(t) = \frac{dv_{\text{int.}}(t)}{dt} = 2c\frac{d\varepsilon_i(t)}{dt}, \tag{2}$$

where is the interface velocity of the bar, c is the wave velocity of the bar, is the incident strain wave. The high g acceleration history can be obtained theoretically based on (2). Therefore, the calibration works can be performed by comparing the results of (2) and the real output of the calibrating accelerometer. The velocity of the bar end increases to tens microseconds or several milliseconds with the aid of the pulse shaper technique. As a result, the acceleration field can be controlled in a range of several thousand g or 100,000 g. However, in order to perform the reliable and accurate calibration works, it is necessary to satisfy the following assumptions [6, 17, 18, 21]:

1. The bar deforms elastically while the stress wave is propagating, so that the one-dimension elastic stress wave theory can be applied and the experimental data can be accurately interpreted;
2. The accelerometer has a lower characteristic impedance than that of the incident bar.

The incident stress wave $\sigma(t) = \rho c v_{str.}/2$, where ρ is the bar density, and $v_{str.}$ is the impact velocity of the striker bar, which plays a decisive role in the amplitude of the incident wave [16, 17]. Thus, assumption 1 can be achieved, just by limiting the maximum velocity of the striker bar. The characteristic impedance is usually defined as the product of, where A is the interface area of the bar or accelerometer [17, 18, 22]. Considering the miniaturization and lower density and wave speed of general accelerometers, their characteristic impedance is usually lower than that of the bar, even as low as only 1/40 of the bar characteristic impedance for some general MEMS accelerometers. Thus, the reflected coefficient can be up to 95.1% (calculated based on the one-dimension stress wave theory [22]), which means that the incident wave will be nearly totally reflected. Alternatively, the Hopkinson bar with a greater diameter is preferred to reduce the characteristic impedance mismatch of bar and accelerometer [17]. However, to calibrate the bulk accelerometers [23], assumption 2 cannot be satisfied. Thus, (1) should be re-deduced theoretically by introducing the reflected wave to calculate the interface velocity $v_{\text{int.}}(t)$ [22]. The acceleration history will be accordingly transformed too. The related research is consulted with the reference [4]. For the potential general applications, the present work focuses only on critical appraising of a miniature accelerometer.

The bearable acceleration peak is an important parameter while evaluating or verifying the accelerometer capacity. Such is also the overload duration. For real application conditions, the overload duration can be anywhere in a region of tens microseconds, even to milliseconds. In addition, the high security and reliability requirements make it indispensable to know the real specifications of accelerometers under extreme overload. These requirements have required the Hopkinson bar calibrating capacity up to both 100,000 g in amplitude and hundreds of microseconds in overload duration. Does the Hopkinson bar calibrating technique satisfy them? Therefore, it is significant to carry out the critical appraisal of this calibration technique.

2.2. Limitation formula for calibration work

The theoretical work is performed based on the one-dimension stress wave theory. (2) is integrated over the rising time of the incident stress wave, and (3) is obtained as follows:

$$\frac{1}{2c}\int_{0}^{\tau_{ris.}} a(t)dt = \int_{0}^{\tau_{ris.}} d\varepsilon_i(t), \tag{3}$$

where $\tau_{ris.}$ is the rising time of the incident stress wave, and it is equivalent to the overload duration in calibrating work. The right term of (3) denotes the achieved maximum strain of the bar by the incident stress wave. According to assumption 1, the maximum strain should not exceed the elastic limit of Hopkinson bar. Therefore, it is concluded from (3) that the acceleration peak and corresponding duration are limited by the elastic strain of the bar. Thus, enough attention should be paid to calibrating experiments of which those concerning the higher acceleration and wider duration are desired.

3. Critical appraisal of calibrating techniques

The Hopkinson bar technique is preferred for calibrating accelerometers, just based on the rapid rising edge of the incident stress wave, which can be up to the rising slope of 100 MPa/μs. The rising edge duration is in the order of tens microseconds, if the incident stress wave is generated by directly impacting on the striker bar. The produced acceleration duration is about ten microseconds, which hardly meets various calibrating requirements. The incident stress wave should be optimized. Some methods are used to trim it, including the use of abnormal striker bars [6] and the pulse shaping technique [16–18]. The pulse shaping technique is more popular thanks to its simplicity. The function of pulse shaper is usually to widen the rising edge of the incident stress wave, and to extend the overload duration. In addition, it can filter out the high-frequency waves of the incident wave to greatly reduce the wave dispersion [17, 18], which also facilitates improving the calibration accuracy. With the aid of the pulse shaping technique, two typical kinds of calibration are considered and analysed: the constant acceleration calibration and the cosine acceleration calibration. The dimensions and mechanical parameters of the Hopkinson bar apparatus are listed in Table 1.

Table 1. The dimensions and mechanical parameters of the Hopkinson bar.

Bar	**Length**	**Diameter**	**Elastic modulus**	**Wave speed**	**Density**
Striker bar	0.25 m, 0.40 m	0.015 m	211,856 MPa	5,195 m/s	7,850 kg/m^3
Incident bar	1.80 m	0.015 m	211,856 MPa	5,195 m/s	7,850 kg/m^3

3.1. Constant acceleration calibration

A linear ramp stress wave is achieved with the aid of the pulse shaping technique, where the pulse shaper is made of 2A12 Al alloy with 0.002 m in diameter and 0.0015 m in thickness. The striker bar length is 0.25 m, and its impact velocity is 12.10 m/s. Then, the linear ramp stress wave is obtained as shown in Fig. 2a, where the ramp rising part is linearly fitted to a high linearity level of $R^2 = 0.9976$, and its acceleration wave is calculated and shown in Fig. 2b. It is found that the ramp stress wave produces a nearly constant acceleration plateau. For this case of calibrating constant acceleration behaviour, (4) can be obtained by integrating (3). A limitation law is obtained for the constant acceleration amplitude and overload duration:

$$a_0 \cdot \tau_{ris.} = 2c \cdot \varepsilon_e \,. \tag{4}$$

Based on (4) that for the steel Hopkinson bar with the elastic limit of bar 1% and the wave speed 5,195 m/s, the limit curve is calculated and presented in Fig. 4.

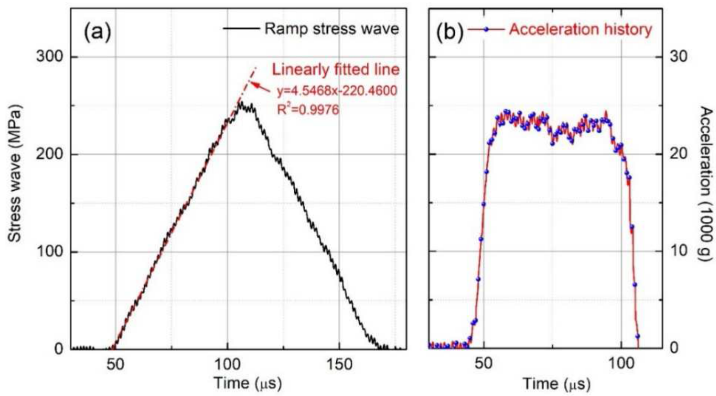

Fig. 2. A linear ramp wave and the corresponding acceleration wave.

3.2. Cosine acceleration calibration

A novel incident stress wave is proposed to calibrate potential extensive responses of accelerators, and the formula is presented as (5):

$$\sigma_i(t) = \sigma_0 \cdot [-\cos(\omega \cdot (t - t_0)) + 1]. \tag{5}$$

It is just a cosine function, which has some advantages. Firstly, the corresponding acceleration wave presents the configuration of sine function as indicated by Eq. (2), which is more close to the actual overload acceleration history [6, 7]. Secondly, it has a less steep slope than that of the ramp stress wave, and can achieve a higher acceleration peak or a wider duration than the linear ramp stress wave being calibrated with the same Hopkinson bar calibration system. At last, this incident stress wave has a small wave dispersion when it propagates along the Hopkinson bar, which is predicted based on the nature of wave propagating along a slender bar [16, 24–25]. A wave is composed of a spectrum of frequencies in nature [16], and each component has its corresponding propagating velocity [22, 24–25]. The higher-frequency components will lag behind the lower-frequency ones after traveling a distance, which consequently results in a distorted waveform, and the wave dispersion occurs. Theoretically speaking, the stress wave is a cosine or sine function, and thus it has only one stable frequency and a stable propagating velocity. So, the waveform will not change even though it propagates a long way. Thus, this stress wave can accurately measure the loading stress wave history. The corresponding acceleration wave is presented as follows:

$$a(t) = \frac{2c\sigma_0\omega}{E} \cdot \sin\left(\omega \cdot (t - t_0)\right). \tag{6}$$

To achieve this type of loading, the pulse shaping technique is employed including the pulse shaper made of 2A12 Al alloy with 0.003 m in diameter and 0.0012 m in thickness. The striker bar length is 0.40 m, and its impact velocity is 13.35 m/s. Thus, the cosine stress wave is achieved as shown in Fig. 3a, and the calculated acceleration history is obtained and shown in Fig. 3b. The experimental curves are fitted by a cosine or sine function which are shown together, respectively. The error coefficient is introduced to define the difference between the experimental and fitting curves with the formula: $\dfrac{|\sigma_{\text{exp.}} - \sigma_{\mathit{fit.}}|}{|\sigma_{\text{exp.}} + \sigma_{\mathit{fit.}}|/2} \times 100\%$, and the errors are also calculated and shown in Figs. 3c and 3d, respectively. It is found that they are in a good agreement in the main characteristics as indicated in Fig. 3c and 3d. For this specific calibrating case, (7) and (8) are obtained by integrating (3). (8) presents a limitation law for the cosine calibration.

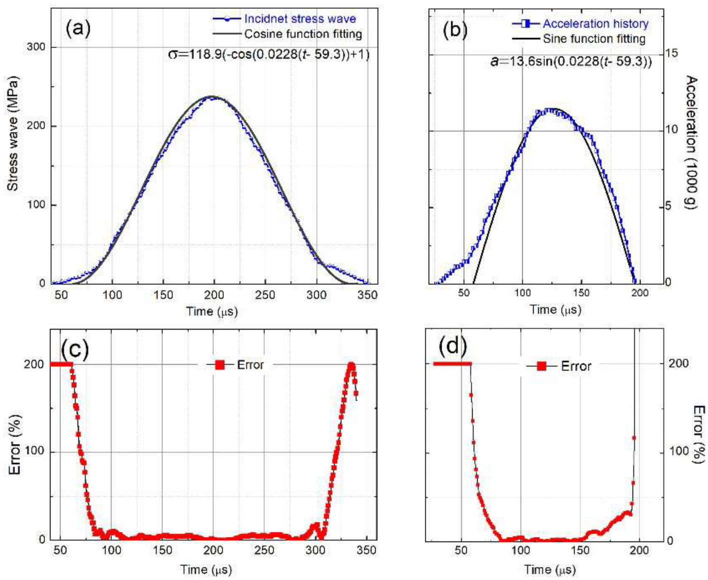

Fig. 3. Comparison and error analysis of the loading stress wave with a cosine function and the corresponding acceleration wave with a sine function, obtained from the experimental and fitting results.

$$\frac{1}{2c}\int_0^{\tau_{acc.}} a_c \sin(\omega \cdot t)dt = \int_0^{\tau_{acc.}} d\varepsilon(t), \tag{7}$$

$$a_c \cdot \tau_{acc.} = \frac{\pi}{2} \times 2c \cdot \varepsilon_e = \frac{\pi}{2} \times a_0 \cdot \tau_{acc.}, \tag{8}$$

where a_c is the peak value of acceleration, and $\tau_{acc.}$ is the duration of the sine acceleration wave, which is equal to half of the period of the cosine incident wave. The limit curve is also presented in Fig. 4.

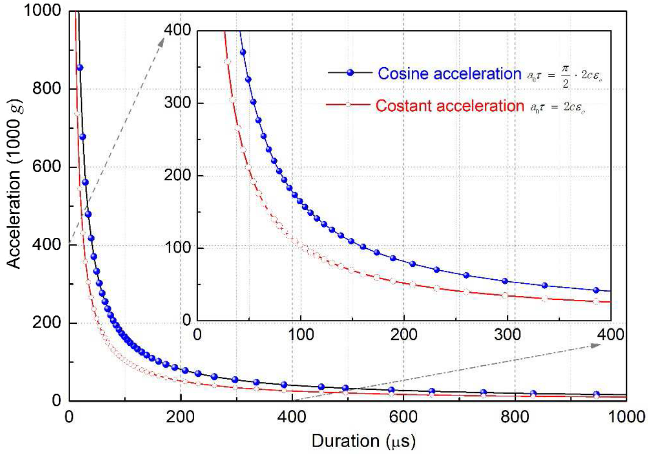

Fig. 4. Limit curves for the cosine and constant acceleration calibrations.

4. Results and discussion

Theoretically, the limitation law is derived from on the one-dimension stress wave theory [16]: for a higher amplitude or a wider duration the interface velocity of the bar should keep a continuous uptrend during calibrating. As a result, the stress or strain on the Hopkinson bar keeps a corresponding increase, and even is approaching the elastic stress/strain limit of the bar. Thus, the limitation exists as resulting from assumption 1. It is obvious from Fig. 4 that for both calibrating stress waves, the available acceleration peak decreases with the increasing duration. The cosine acceleration calibration has a distinct advantage: it enables to achieve either a higher peak acceleration at a certain duration, or a wider duration at a certain acceleration peak, both by $\pi/2$ times greater than those obtained by the constant acceleration calibration. Meanwhile, it is demonstrated that the Hopkinson bar calibration technique can provide extensive calibrations. The acceleration peak, for the constant acceleration calibration case, can range from thousands g up to more than several 100,000 g, with the corresponding overload durations ranged from a few to tens microseconds, respectively. Besides, it can calibrate all acceleration amplitudes from zero to the peak one in one calibrating session.

No matter whether it is the constant acceleration calibration or the cosine acceleration calibration, they are just the cases for obtaining different acceleration fields. For more extreme calibrating requirements, great attention should be paid to the experimental designing and bar selection. As indicated by (3), the right term value can be up to the elastic limit strain of bar, and thus (9) can be obtained from (3), aiming to offer a guidance to the more extreme calibrating. The right term is focused on the calibration designing and bar material selection. The preferred kinds of bar materials have a high wave speed and a large elastic limit. Thus, the titanium alloy is competitive, as it has a higher elastic limit strain (even up to 1.3%) and nearly the same wave speed as steel.

As shown in Fig. 4, the overload duration should be shorter for a higher acceleration peak calibration, so that the slope of the stress wave will be steep enough. Thus, the wave dispersion will be then more distinct [16]. Therefore, the Hopkinson bar with a smaller diameter is preferred, for it can reduce the dispersion effect while propagating [15]. Additionally, the incident stress waves with no-dispersion characteristics are more popular, like the above-mentioned cosine stress wave. As another more extreme calibration requirement, a wider duration calibration will be also a challenging task. Thus, a longer incident bar is necessary to propagate and record the stress wave. Also, a less steep slope of the stress wave is required. Therefore, a viscoelastic bar such as the PMMA bar will be a preferred one, for its advantage consisting in its much larger elastic limit strain (even up to 3%).

$$\int_0^\tau a(t)dt = 2c \cdot \varepsilon_e \, . \tag{9}$$

5. Conclusions

This work critically apprises the Hopkinson bar calibrating technique with focus on its calibration capacity. The limit formula is derived from the calibrating theory, and both kinds of loading stress waves are analysed for different calibration requirements. The conclusions are drawn and listed:

1. The limitation law of the Hopkinson bar calibrating technique is deduced, which indicates that the maximum acceleration peak and duration are limited by the elastic limit of Hopkinson bar material;
2. A cosine stress wave is proposed, which can achieve by $\pi/2$ times higher acceleration peak or wider duration than those obtained by the constant acceleration calibrating;
3. The pulse shaping technique is recommended to achieve different calibration stress waves and a small wave dispersion during propagation of the stress wave;
4. Titanium alloy is preferred as the material for the Hopkinson bar to achieve a super-high g acceleration calibration.

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