PIERRE NANCEL-PENARD*[#], ANDRÉS PARRA*, NELSON MORALES*,
CRISTOPHER DÍAZ*, ELEONORA WIDZYK-CAPEHART*

VALUE-OPTIMAL DESIGN OF RAMPS IN OPEN PIT MINING

OPTYMALIZACJA PROJEKTOWANIA RAMP W GÓRNICTWIE ODKRYWKOWYM UWZGLĘDNIAJĄCA CZYNNIKI EKONOMICZNE

The economic envelopes obtained by optimization techniques in open pit mining are transformed into operational phases that are suitable for extraction through ramp designs. This process is performed with the aid of specialized design software, which is still very manual, time consuming and highly dependent on the expertise of the planner. In this paper, we introduce a new methodology based on a mathematical model to automatically propose the design of ramps from the economic envelope of a pushback, with the resulting envelope having the maximum value. The developed model was tested against a real case scenario showing reasonable and useable solutions for the planner. Using this approach, a planner can evaluate several alternatives in a reasonable time before selecting the final design.

Keywords: Ramp design, Open pit mining, Mathematical model, Pit shell with ramps

Obwiednie opisujące czynniki ekonomiczne dla kopalni górnictwa odkrywkowego otrzymane metodą optymalizacji zostały odpowiednio przekształcone i opisane jako fazy operacyjne procesu wydobycia poprzez odpowiednie zaprojektowanie ramp. W procesie tym wykorzystywane jest specjalistyczne oprogramowanie do wspomagania projektowania, które jednak wymaga znacznego zaangażowania manualnego, a sama procedura jest czasochłonna i w dużej mierze uzależniona od doświadczenia projektanta. W niniejszej pracy zaprezentowano nowe podejście oparte o metody modelowania matematycznego, automatycznie generujące projekt rampy spośród obwiedni opisującej wielkości przeładunku, z uwzględnieniem maksymalnej wartości. Opracowany model został przetestowany poprzez jego wykorzystanie do opracowania rzeczywistego scenariusza postępowania, dając wiarygodne wyniki, użyteczne dla projektantów. Wykorzystując prezentowane podejście, projektant jest w stanie dokonać oceny kilku możliwości w rozsądnym czasie przed dokonaniem wyboru finalnego projektu.

Słowa kluczowe: projekt rampy, górnictwo odkrywkowe, modelowanie matematyczne, wyrobisko odkrywkowe, rampa

* DELPHOS MINE PLANNING LABORATORY, MINING ENGINEERING DEPARTMENT, UNIVERSITY OF CHILE; ADVANCED MINING TECHNOLOGY CENTER, UNIVERSITY OF CHILE, SANTIAGO, CHILE

Corresponding author: pierre.nancel@amtc.cl

1. Introduction

Open pit mine planning is a decision-making process that leads to a realistic and actionable plan to profitably extract mineral resources. Planning can be carried out for a wide range of periods from the very short (next shift) to the very long (life of mine) (Whittle, 2011).

The starting point of the mine planning process is a block model in which the ore body is divided into regular blocks; each block with individual attributes, such as, ore grades, recoveries, and tonnages. The block model is economically valued and the profit is assigned to each block (Bley et al., 2010). This block model together with the geotechnical constraints and the long term economic parameters (costs and commodity prices) are the basic input for open pit strategic mine planning.

A fundamental problem resides in determining the optimum pit limit of a mine in which the extraction of material will take place. The optimum pit limit of a mine is a set of blocks in the block model which, when extracted, yields the total maximum profit while satisfying the operational requirements (Cacetta & Hill, 2003). Within the ultimate pit, the deposit is divided into nested pits: from the smallest pit with the highest value in terms of profit to the largest pit with the lowest profit value (Dagdelen, 2001) for the purpose of establishing a mining sequence. Nested pits are generated by varying the price of the metals being extracted (Hustrulid et al., 2013b).

However, the ultimate pit as well as the nested pits are computed at the *block level* i.e. they are only an approximation of the actual volumes. Indeed, after obtaining the final pit and nested pits, mine designs are carried out. This is usually an iterative process between the geotechnical engineer and the mine planner (Williams et al., 2013) because mine designs must accomplish geotechnical requirements to ensure slope stability.

Mine designs break the overall pit reserve into manageable planning units called phases, which are commonly based on the sequence determined by the nested pit, taken into consideration (Hustrulid et al., 2013b):
- The probable maximum ore and waste mining rates required in a given phase.
- The size and type of the equipment to be used. This determines the required minimum operating bench width.
- Appropriate working, inter-ramp and final slope angles.

Based on these considerations, the mine planner proceeds to design phases ensuring that ramp access to each active bench is provided.

Ramps are one of the most important aspects of mine planning. Their presence should be included early in the mine planning process since they have a significant effect on the reserves (Hustrulid et al., 2013a). Roads necessarily imply the addition of waste and/or reduction in the amount of ore extracted from the optimal pit shell, which has an impact on the economic value of the mining project.

With a set of operational designs, including ramps, for each of the mine phases, the annual mining schedules based on mill feed or production requirements are determined. It may be necessary to repeat the design stage several times before an adequate mine plan is developed (Hustrulid et al., 2013b). Therefore, designing phases and building production schedules are iterative processes.

In the optimization stage, the process to obtain the final pit shell is based on mathematical models and algorithms, such as, Lerchs and Grossman (1965) that attain optimal solutions for the ultimate pit and for the nested pits. At this stage, the planner can work at a high level of ab-

straction and, therefore, his/her work consists of controlling specific parameters and criteria to generate the pits, because the mathematical framework ensures optimality of the computed pits.

On the other hand, the open pit operational design stage is carried out using specialized design software, which are tools to aid the user to make designs faster but they do not ensure the profit optimization. Thus, this stage is mostly a manual process in which the optimality depends on the user.

It thus follows that the quality of the resulting solution depends on the skills, information and time available for the design phase. While there exist some criteria to check the quality of the design (for example, measuring the difference in tonnage and value of the optimal pit shell versus the operational pit design), there is no guarantee that the results would be optimal. In addition, the process itself is very slow and time consuming.

Therefore, there are several drawbacks of the current design method. Firstly, since the task takes considerable time, there are few opportunities to consider different scenarios to check the robustness of the solutions. Secondly, even if the solution obtained satisfies the profitability, the time requirements make it a very specialized and expensive task. Finally, since the pushbacks are optimal in value but not in operational parameters, there exists a potential gap in value and tonnage between the pushbacks and the designed operational phases. Therefore, there is a need to evaluate the quality of the designs of phases.

For these reason, a mathematical model was developed to assist the planner in finding the best possible ramp design. This model determines an optimal path based on operational and economic parameters and criteria. Due to the computational speed, the model enables the planner to test different design scenarios before accepting the final solution.

The proposed mathematical model commences with a valued block model for which a pit has already been computed and then looks for a path (of blocks) that moves around the pit contour that complies with the geotechnical constraints. This model is a first step into more complex formulations that would consider multiple pushbacks and, in the long-term, will be the basis for models that would enable the automation of the entire process of strategic mine planning and open pit design.

In this paper, the review of the geometrical components of a slope and current practices of the industry in mine designs is followed by the description of the model, case study and the results of the application of the model to a real scenario.

1.1. Geometrical components of a slope

The geometrical components of a slope (Fig. 1) are in direct relationship with safety conditions of the mine since they are associated with acceptable design criteria, which allow the slope stability to be achieved.

The main geometrical components of a slope are (Hustrulid et al., 2013a):
- Bench face angle (θ): the angle between the horizontal plane and the bench wall. In most hard rock open pit mines, this parameter has values ranging between 55° and 80°.
- Bench height: the height, which typically has to adapt to the loading equipment operating in the mine.
- Berm width: the distance between the toe and crest of a bench.
- Inter-ramp angle (α): the angle between the toe of a slope where a ramp segment passes and the toe of a bench located immediately upwards.
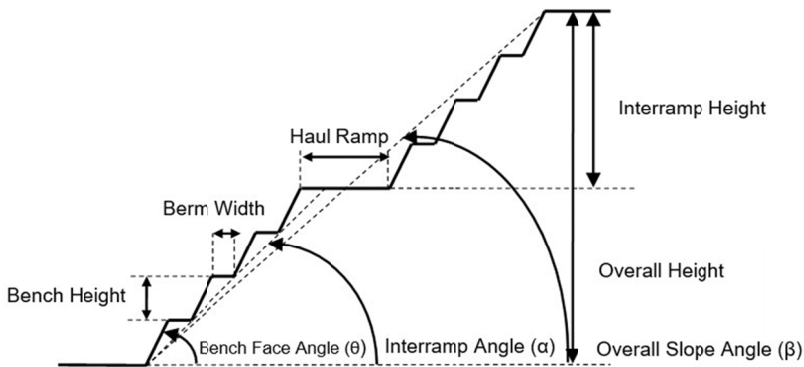
Fig. 1. Geometrical components of a slope

- Overall slope angle (β): the angle made with the horizontal of the line connecting the lowest most toe to the upper most crest.
- Global slope height: the height projected in the vertical axis between the lowest most toe and the upper most crest.
- Ramp: the road along which the trucks haul the material between a loading point and a destination.

All the desired characteristics of the components are determined using geomechanical fundamentals, which consist of defining minimal requirements towards valid design. However, as the proposed mathematical model works at the block level, the focus is to achieve a good approximation of the inter-ramp and global slope angles as well as the haul ramp width.

1.2. Review of current practices and related literature in mine designs

At the mine design stage that consists of converting volumes defined at block level into operational ones, it is necessary to smooth the final pit contour and pushbacks. Mine designs must incorporate all the geometrical components of a slope, which include hauling ramps, where trucks can access each of the phases to transport ore and waste to the final destinations.

Generally, ramps′ locations are constructed based on the criteria of the mine planner in charge of the operational mine designs of an open pit. One of the issues faced by the mine planner, which is little written about in the mining literature, is gaining initial access to the ore body (Hustrulid et al., 2013a). Some aspects, which the mine planner must consider when realizing operational designs, are:

- Minimum costs on a net present value basis for the transport of ore and waste throughout the life of mine. The preference is to use the long-life haul roads rather than short-life roads as this reduces overall road construction costs and operating costs (Atkinson, 1992).
- Roads exits from the pit wall. This is dependent upon the crusher location and the dump points (Hustrulid et al., 2013a).
- Optimum number of access points to the pit. More access points means more flexibility but the added cost could be high (Hustrulid et al., 2013a).

- Optimum number of switchbacks. It is desirable to avoid the use of switchbacks in a pit because they tend to slow traffic, cause greater tire wear and various road maintenance problems (Hustrulid et al., 2013a).
- Minimum traffic congestion (Atkinson, 1992).
- Avoidance of areas where slope stability problems could occur (Atkinson, 1992).

Therefore, the planner must deal with many criteria and considerations to generate a design, which means that the current practice does not necessarily maximizes NPV and minimizes operational costs in pit designs.

2. Problem statement

The problem addressed in this article is to determine the best path to be followed by a ramp. By *path*, we mean a guidance at a block level; and by *best*, we mean that the difference between the block-level solution and the designed-level solution is small. More specifically, the model that we propose will produce a guidance for which the overall value is maximized.

As the described path evolves over the life of the mine and its optimal design depends on many parameters as well as the different phases of the mine, in this work, the evaluation is limited to one single pit or pushback. It starts with a block model in which block i may belong (or not) to a given pit. From the borders of this pit, a certain *neighborhood* or *range* of blocks is considered. The desired path is constrained to this range. This path is allowed to move inside or outside of the pit boundaries, but is kept relatively close to it. The pit has to comply with the following limitations:

- The path starts at the top of the topography and ends at the bottom.
- The path passes once per each bench; the points where the path enters/exits the bench are called *accesses*.
- As blocks in the path are removed, some blocks in the block model must also be extracted due to slope considerations.
- An inter-ramp angle α and an overall slope angle β are considered.
- The ramp roads have a maximum grade.
- The ramp has a minimum width of r meters or a minimum of R blocks.

A mathematical model (Morales et al., 2017) determines the locations of the *accesses* to ensure that all of the considerations mentioned previously are taken into account to maximize the economic value of the removed blocks (including those in the ramps). The path being computed aims to produce a guidance for the design stage (Fig. 2).

An example of a set of all blocks B, the initial pit and the set of candidate ramps are presented in Fig. 3. This latter set is created *around* the profile of the initial pit; thus, the ramp may be constructed inside or potentially outside the pit. Therefore, the blocks outside of the set of ramp candidates are not considered in the optimization process.

The initial profile of the ramp set (Fig. 4a) consists of blocks b and b', which are considered potential accesses to the corresponding benches, with the ramp from b to b' approximated by the blocks in green. If blocks b and b' are selected as accesses, blocks in green have to be extracted as well as those in yellow (due to slope constraints). The resulting profile is presented in Fig. 4b.
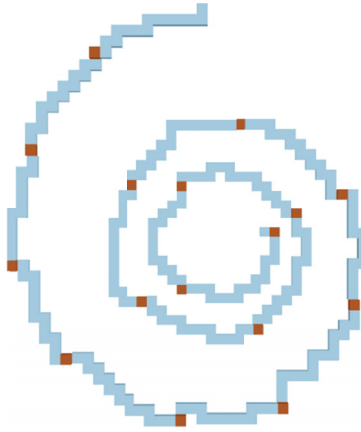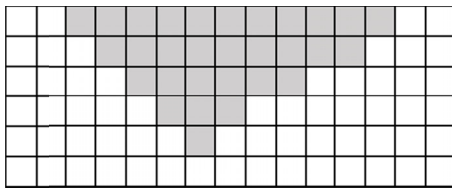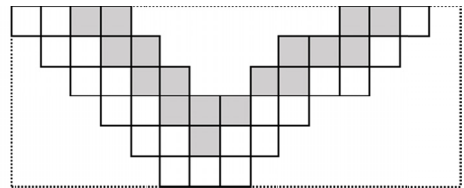
Fig. 2. Example of a computed path. Red blocks represent accesses.
Light blue blocks are blocks in the path
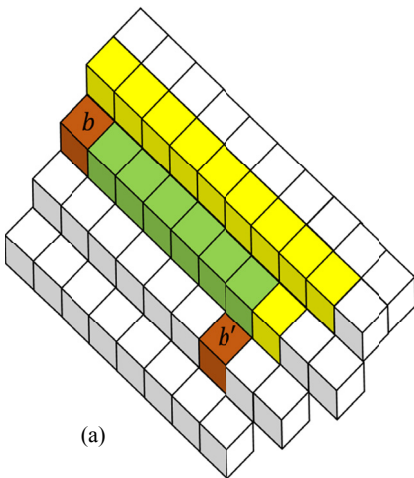

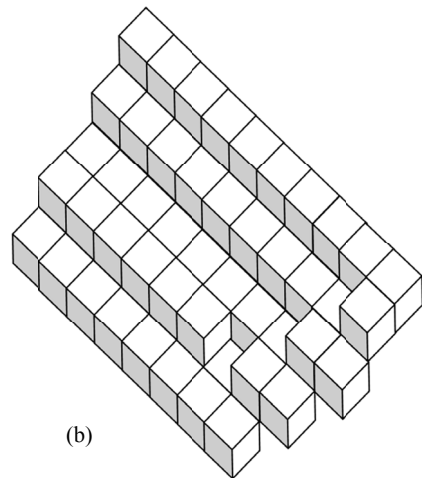(a)   (b)

Fig. 3. (a) The set of all blocks B and initial pit in gray. (b) The set of blocks candidate to be ramps.
Contour of the whole block model is also displayed for reference

(a)   (b)

Fig. 4. (a) Ramp set, blocks b and b' selected as accesses: green blocks are in the path, yellow blocks have to be
removed due to precedence constraints; (b) Resulting profile after ramp construction

In this article, an explanation of a simplified version of the model to determine the best ramp of a minimum width of one block is presented. The general model that was used in our study is presented in details in (Morales et al., 2017).

A *sub-ramp path* (green blocks and blocks b and b' in Fig. 4) for a given level k as the part of the ramp path that contains all ramp path blocks of the level k as well as the *access* of level k-1, level 0 *is* the bottom of the mine. Possible *sub-ramp paths* that respect some design aspect (the given maximum grade of roads) for each level of the mine are precomputed inside a predefined pit boundary from the surface to the bottom of the mine. The mathematical model will select which *sub-ramp paths*, connected by *accesses*, should be assembled to constitute the full ramp path respecting the design constraints and optimizing the overall value of the extracted blocks. Each sub-ramp path respect the given maximum grade of roads and the full ramp obtained will also respect this design requirement.

The linear integer mathematical model considers two different variables. The first variable, y_b, is the binary variable that decides if a block b is extracted or not. The second variable, x_k^i, corresponds to a binary variable that decide if the ith precomputed *sub-ramp path* of the level k is elected to be part of the ramp. The objective is to maximize the economic value of the extracted blocks. There is a constraint that forces all blocks of the ith precomputed *sub-ramp path* of level k to be extracted if x_k^i value is 1. The following constraints are set within the model:

1. For each extracted block, all the blocks inside the cone defined by the extracted block and the inter-ramp angle α of Fig. 1 are extracted.
2. For each extracted block, all the blocks inside the partial cone defined by the extracted block, the overall slope angle $β$ of Fig. 1, that are above a horizontal plane at the inter-ramp height distance of the extracted block are extracted.
3. The existence of an elected precomputed *sub-ramp path* of level k +1 for each elected precomputed *sub-ramp path* of level k and the connection of those path are forced.
4. The number of elected precomputed path by level cannot exceed one.
5. The existence of an elected precomputed *sub-ramp path* of level k +1 for each extracted block of level k is forced.
6. Blocks immediately below ramp blocks and blocks between the ramps and the slope cannot be extracted.

It is important to note that the constraint 5 prevents the existence of an extracted pit without ramps.

3.   Case study

3.1.   Block model and economic parameters

The case study corresponds to a real copper deposit. Due to a confidentiality agreement, the name and location of this deposit cannot be disclosed. The characteristics of the block model are as follows:

- Regular blocks of 10 meters × 10 meters × 10 meters,
- East-west length: 610 meters,
- North-south length: 800 meters,
- Depth: 280 meters.

The economic and metallurgical parameters are:
- Copper price of 4.41 USD/kg,
- Mine cost of 1.8 USD/ton,
- Processing cost of 13 USD/ton,
- Smelter and refining cost of 1.43 USD/kg,
- Metallurgical recovery of 85%.

3.2. Slope angles and geometrical components of a slope at a block level

The slopes´ angles considered for precedencies in the optimization model are the inter-ramp angles and the global slope angle. The number of ramp segments present in a profile of the slope is unknown. For this reason, the first step is to assume a number of ramp segments in a profile together with a global slope height. With these parameters, it is possible to calculate a global slope angle. The number of ramp segment present in a profile is illustrated in Fig. 5.
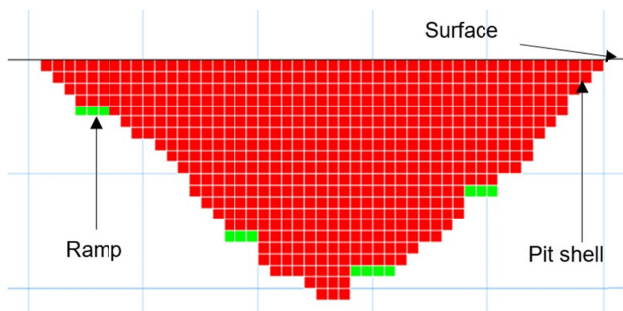

Fig. 5. Final pit shell with two ramps

Two ramp segments at the right and two ramps segments at the left are present in the profile view (green blocks in Fig. 5).

A schematic draw of a pit with blocks at a global scale with the geometrical components of a slope, such as, ramps, inter-ramp angle, global slope angle, inter-ramp height and global height is shown in Fig. 6.
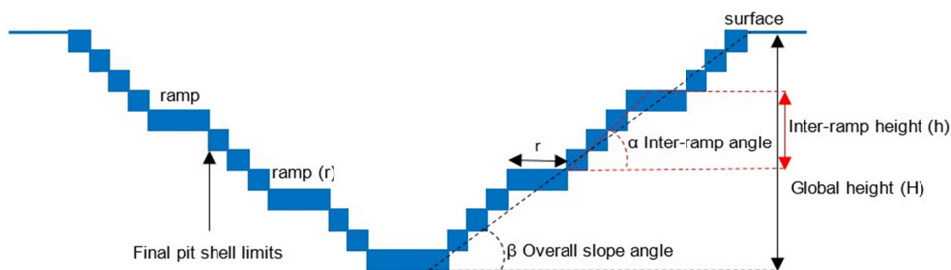

Fig. 6. Schematic draw of a pit with blocks at a global scale with the geometrical components of a slope

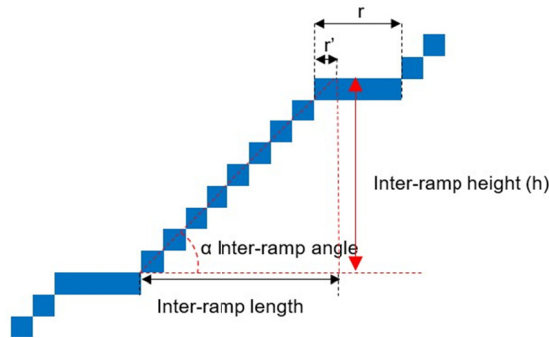A schematic draw of a pit with blocks at an inter-ramp scale is shown in Fig. 7.

Fig. 7. Pit´s schematic draw with blocks at an inter-ramp scale

Based on Fig. 6 and Fig. 7, it is possible to obtain Equation (1):

$$\tan \beta = \frac{H}{3x + N(r - r')} \tag{1}$$

where: β is the overall slope angle, H is the global slope height, N is the number of ramps present in the slope, r is the ramp's width, r' is the distance between the crest of the ramp and the projection of the inter-ramp angle from the bench below and x is the inter-ramp length measured horizontally (Fig. 7).

From Fig. 7, it is also possible to obtain Equation (2):

$$\tan \alpha = \frac{h'}{r'} \tag{2}$$

where: h' is the bench height (also block height).

By replacing Equation (2) in Equation (1), Equation (3) is obtained. This equation expresses the global slope angle as a function of the inter-ramp angle, the number of ramps present in a segment profile, the ramp's width, the overall slope height and the bench height.

$$\tan \beta = \frac{H}{\dfrac{H}{\tan \alpha} + N\left(r - \dfrac{h'}{\tan \alpha}\right)} \tag{3}$$

3.3. Design steps

The methodology for the optimization process to obtain the final pit shell with ramps as a guide to perform the final pit designs is shown in Fig. 8.

The inter-ramp angle is 53°, the ramp width is 30 m and the bench height is 10 m. Using an overall height of 230 meters, which is the maximum depth at which the ore is located, and
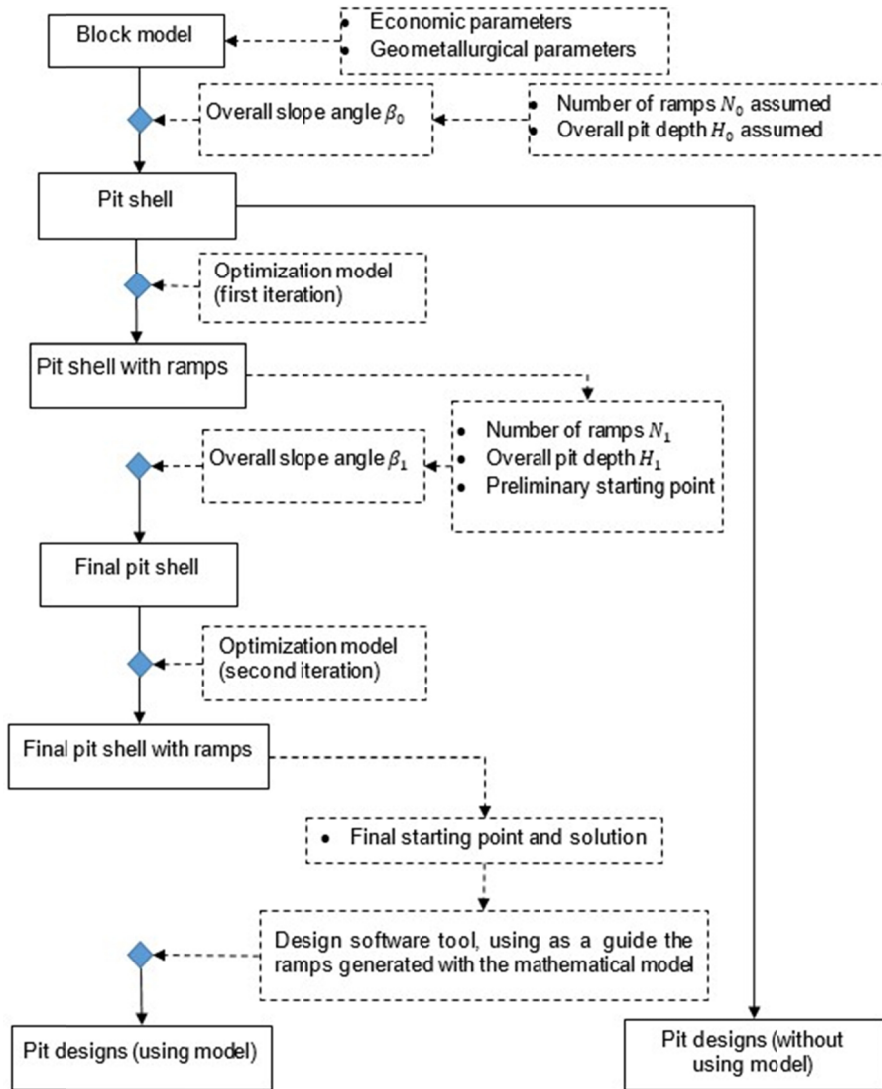
Fig. 8. Optimization process to obtain the final pit shell with ramps as a guide to do the final pit designs

assuming two ramps present in the slope, the overall slope angle β_0 is obtained as a first approximation to the optimization process. With the economic parameters, a pit shell is going to be obtained. With this first pit shell, the optimization model for ramps generation will be used to obtain a first pit shell with ramps.

To obtain a reasonable computing time for solving the optimization model, it is necessary to choose a starting point. Firstly, an economic evaluation in terms of profit, using four different starting points for the ramp's access, is carried out; the evaluation with the highest profit is selected as the point where the ramp starts going down. Subsequently, it is possible to determine

the new global slope height H_1, the number of ramps present in the slope N_1 and the new overall slope angle β_1 in the contour of the pit, as shown in Fig. 9.

The global slope angle β_1 must be calculated considering two ramps present in the slope (Fig. 9), in the plan view of the generated ramp, between an azimuths ranging from 0° to AZ_1. The global slope angle β_1' must be calculated considering three ramps in the slope, from an azimuth between AZ_1 and AZ_2. The global slope angle β_1 must be calculated considering two ramps in the slope, from an azimuth between AZ_2 and 360°. After determining the global angles, depending on the number of ramps in the slope and on the global slope height H_1, a second optimization iteration with these new parameters is carried out to obtain a more precise pit shell. Based on the second pit shell, the optimization model for ramps generation is used again to obtain a second pit shell with ramps.
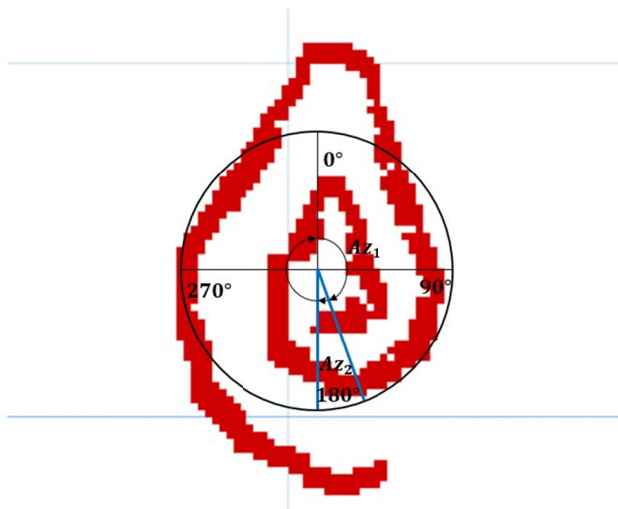

Fig. 9. Schematic view of a ramp generated by the mathematical model in the slope

In the first iteration, using the optimization model, only one external frontier layer and one internal frontier layer are considered in finding the first approximation of the solution. A pit frontier layer is a set of blocks with its internal and external limits parallel in all directions to the pit shell limits. One frontier layer is equivalent to the width of one block. In the optimization model, the frontier layers correspond to the domain where the mathematical model is going to find the solution for the location of the ramp. As the number of layers grow, the time to find the solution increases. For this reason, in the first iteration, as an approximation of the solution is sought, only one external frontier layer and one internal frontier layers is considered.

Six frontier layers are shown with layers ranging from –3 to –1 corresponding to the three internal frontier layers, and with layers ranging from 1 to 3 corresponding to the three external frontier layers (Fig. 10).

In the second iteration of the optimization process, more than one external and internal frontier layers are considered to obtain a larger domain for the location of the ramp's segments and, therefore, a more precise result is going to be found.
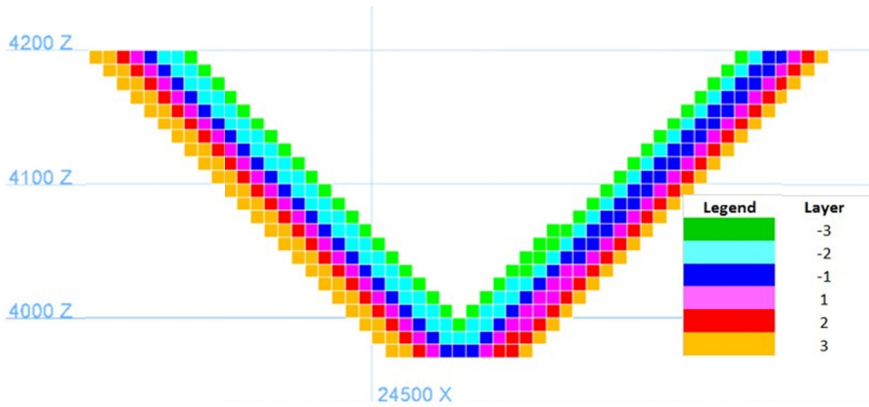
Fig. 10. Pit frontier layers of a pit shell

4. Results and analysis

For the comparison of the pit designs with and without the use of the mathematical model, the variables considered are: total material tonnage, ore tonnage, stripping ratio, copper grade, metal contained, and total profit.

A plan view at the top level of the pit shell in the first iteration with the four selected starting points is shown in Fig. 11.
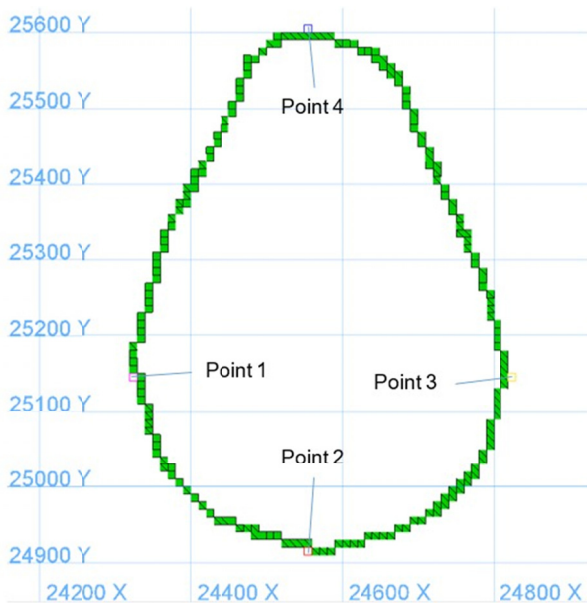

Fig. 11. Pit shell at the top level with the four selected starting points

For each of the starting points, two scenarios were considered: one by generating the ramp in a clockwise direction and the other by generating the ramp in a counterclockwise direction. The pit depth was considered to be $H_0 = 230$ m and two ramps in the slope were used to calculate the overall slope angle using Equation (3), resulting in $\beta_0 = 45.2°$. Tab. 1 provides the results for every starting point detailed in Fig. 11.

Results for the first pit shell with ramps

Starting point	Direction of ramp	Average grade of road [%]	Total tonnage [MT]	Total ore [MT]	Stripping ratio	Average Cu grade [%]	Metal contained [MT]	Profit [MUS$]
1.	Counter clockwise	9.3	58.9	44.2	0.33	0.81	0.36	222.8
2.	Counter clockwise	9.0	57.0	43.5	0.31	0.81	0.35	221.9
3.	Counter clockwise	8.9	58.1	43.9	0.32	0.81	0.35	221.2
4.	Counter clockwise	9.4	57.2	43.8	0.31	0.81	0.35	223.4
1.	Clockwise	9.0	57.9	44.0	0.32	0.81	0.36	223.7
2.	Clockwise	8.8	57.3	43.8	0.31	0.81	0.35	223.3
3.	Clockwise	9.1	57.0	43.7	0.30	0.81	0.35	223.2
4.	Clockwise	9.3	57.9	43.9	0.32	0.81	0.36	223.5

The results considering different starting points are very close (Tab. 1). The difference between the best profit and the worst profit (starting point 1 clockwise direction and starting point 3 counter clockwise direction, respectively) is calculated to be 1,1%. Taking into account that there is a gap in all optimization problems, which in this particular case was imposed to be less than 1%, then all the starting points for the ramp access have practically the same profit. For further analysis, point 1 was selected for the ramp access with a counter clockwise direction.

There are two ramps in almost all the slopes of the pit (Fig. 12). With this information, in addition to obtaining the pit depth, which is $H_1 = 190$ m, it is possible to calculate an overall slope angle using Equation (3), which is $B_1 = 44.5°$. With this new angle and taking point 1 as a reference point (selected in the first iteration of the analysis), a second iteration was carried out as detailed in the optimization process (see Fig. 8) to obtain the final pit shell with ramps as a guide to do the final pit designs.

A summary of the results for the first and the second iterations is presented in Tab. 2.

Results for the pit shell with ramps for the first iteration and the second iteration

Iteration	Average grade of road [%]	Total tonnage [MT]	Total ore [MT]	Stripping ratio	Average Cu grade [%]	Metal contained [MT]	Profit [MUS$]
1.	9.3	58.9	44.2	0.33	0.81	0.36	222.8
2.	9.1	60.9	44.9	0.36	0.82	0.37	232.7

The profit obtained in the second iteration is higher than the profit obtained in the first iteration (Tab. 2). This occurred because the pit shells considered for both iterations were different
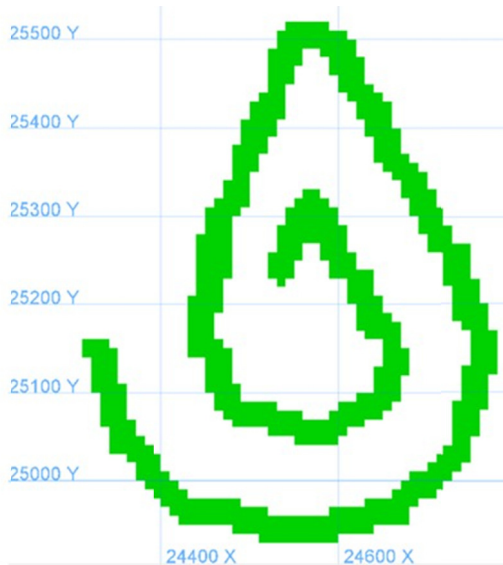
Fig. 12. Pit shell with ramps starting from point 1 in a counter clockwise direction

and, in the second iteration, three external frontier layers and three internal frontier layers were considered.

A summary of the results for the pit designs based on the pit shell with ramps from the mathematical model and the pit designs based on the planner criteria are shown in Tab. 3.

TABLE 3

Comparison between pit designs without using the mathematical model and using the mathematical model

	Total Tonnage [MT]	Total Ore [MT]	Stripping ratio	Average Cu grade [%]	Metal contained [MT]	Profit [MUS$]
Pit designs (without using model)	64.4	45.0	0.431	0.817	0.31	228.96
Pit designs (using model)	63.8	45.0	0.418	0.816	0.31	229.05
Original envelope	58.72	41.12	0.428	0.822	0.29	248.03

The mathematical model is able to produce a pit with an equivalent value to the one obtained by the standard methodology (Tab. 3). This validates the results obtained according to the proposed methodology, which means that the model can be used to aid the mine planner to test different scenarios to locate the ramp. Otherwise, using the current design practice, i.e., without using the mathematical model, the evaluation of locating the ramp entrance at different points is a time consuming task.

The results presented in this study are applicable to specific conditions; for other parameters, the difference between the current design practice and the design based on the mathematical model can vary.

5. Conclusions

The mathematical model and methodology proposed in this work for the design of ramps for open pit mines is a guide to obtain the optimal location for the ramp access in pit designs. Even though, there was no significant difference in the final results between the designs using the mathematical model and the current practice for pit designs, the most important advantage of the model is that many scenarios for locating the ramp entrance can be evaluated and, for every scenario, the model will give an optimal result in terms of profit. Contrarily, the current state of art of design cannot assure an optimal solution for maximizing the profit and the evaluation of locating the ramp entrance at different points as it takes considerable time to consider each scenario.

This work was undertaken considering a final pit with one single phase. The model is now being developed to consider cases with more than one phase (as most of the open pit projects). In addition, new heuristic methods must be introduced in the mathematical model to shorten the processing time.

Acknowledgments

References

Atkinson T., 1992. *Design and layout of haul roads*, in *SME Mining Engineering Handbook*, 2nd ed., Society for Mining, Metallurgy, and Exploration, Inc., Colorado, USA, 1334-1342.

Bley A., Boland N., Fricke C., Froyland G., 2010. *A strengthened formulation and cutting planes for the open pit mine production scheduling problem,* Computers & Operations Research **37**, 1641-1647.

Caccetta L., Hill S., 2003. *An application of branch and cut to open pit mine scheduling*, Journal of global optimization **27**, 349-365.

Dagdelen K., 2001. *Open pit optimization – strategies for improving economics of mining projects through mine planning*, International Mining Congress and Exhibition of Turkey, Turkey.

Hustrulid W., Kuchta M., Martin R., 2013a. *Geometrical considerations*, in *Open Pit Mine Planning and Design*, Taylor & Francis, eds., Leiden, The Netherlands, 290-408.

Hustrulid W., Kuchta M., Martin R., 2013b. *Production planning*, in *Open Pit Mine Planning and Design*, CRC, UK, 504-669.

Lerchs H., Grossman H.C., 1965. Optimal design of open-pit mines, Transactions C.I.M., 58, 47-54.

Morales N., Nancel-Penard P., Parra A., 2017. *An Integer Linear Programming Model for Optimizing Open Pit Ramp Design*, 38th APCOM proceedings, Session 11, 9-16.

Whittle D., 2011. *Open-pit planning and design,* in *SME Mining Engineering Handbook,* Published by Society for Mining, Metallurgy, and Exploration, Inc., 877-901.

Williams P., Floyd J., Chitombo G., Maton T., 2013. *Design implementation,* in *Open Pit Slope Design,* CSIRO publishing, Australia, 265-326.