Two-Dimensional Drone Base Station Placement in Cellular Networks Using MINLP Model

Mina Taghavi, Jamshid Abouei

Abstract—Utilization of drones is going to become predominated in cellular networks as aerial base stations in order to temporary cover areas where stationary base stations cannot serve the users. Detecting optimal location and efficient number of drone-Base Stations (DBSs) are the targets we tackle in this paper. Toward this goal, we first model the problem using mixed integer non-linear programming. The output of the proposed method is the number and the optimal location of DBSs in a two-dimension area, and the object is to maximize the number of covered users. In the second step, since the proposed method is not solvable using conventional methods, we use a proposed method to solve the optimization problem. Simulation results illustrate that the proposed method has achieved its goals.

Keywords—5G cellular networks, drone base stations, mobile base stations, node placement

I. INTRODUCTION

NOWADAYS, because of scientific progress in robotics, artificial intelligent, controls and computers, the unmanned terrestrial, marine and aerial vehicles are paid more attention. The Unmanned Aerial Vehicles (UAVs) are considerably developed and applied in many militaries, urban and commercial fields. One of the most important applications of drones is the temporary coverage of users in mobile cellular networks. When some temporary events happen such as broken Base Stations (BSs), bad weather conditions, natural disasters and data transmission errors [1, 2], drones are not able to service all users because of overcrowding. In this case, drones equipped with miniature cellular BSs, namely Drone Base Stations (DBSs), are immediately sent to a target area and essential communication links are built without predefined infrastructures to cover the area.

Comparing with traditional terrestrial wireless communications, the DBS-based communication has several advantages. Firstly, they are slightly affected by channel disorders such as shadowing and fading. Since there is a high probability of direct link between DBSs and ground users, they also have more reliable air-to-ground channels. Secondly, they can freely fly and flexibly deploy in 3D spaces. Thirdly, in order to enhance the desired communication links or to prevent interfering by designing an efficient path, the mobility of DBSs can be properly under control. Besides these benefits, using DBSs has many open challenges such as assigning deployment and trajectory design, optimal number of DBSs, resource allocation, multiple access, backhaul links and user association

[3, 4]. The DBS deployment problem has devoted the attention of researchers as this issue has significant influence on the air-to-ground link reliabilities. The main goal is to optimize the location, altitude and the number of DBSs for maximizing the coverage area in a network. To have a better understanding, the related works for the above issue are categorized as follows.

Optimal Drone Deployment: The drone deployment in optimal locations has several applications in wireless networks. In [5], the optimal 3D location and the minimum number of DBSs to cover all users are calculated by a heuristic algorithm where a DBS provides the coverage of a desired area by changing its altitude. It means that in high dense areas, it decreases its altitude to suppress the effect of interferes of farther users served by itself, while in low dense areas, it increases its altitude to cover a larger area and serve more users. The authors in [6] and [7] employ numerical methods to detect the optimal 3D location of the DBS in order to maximize the number of covered users.

The authors in [8] present a proactive drone-cell deployment method to decrease the overland caused by the flash crowd traffic in 5G networks. The drone-cell deployment problem is considered as a clustering problem so that users assigned to a drone-cell are regarded as a cluster. The deployment of a drone-cell in the center of a cluster ensures that the drone-cell has a minimum summation of the square of the distance to all the clusters' members. Thus, it uses a constant bisecting k-means for the drone-cell deployment problem. In [9], a drone-cell deployment is proposed for cache-enabled drones to improve the Quality of Experience (QoE) of users. In this work, a drone caches the required information based on a predicted model. Such a cache can decrease the delay of data transmission. Reference [10] studies the multiple drones deployment problem where a mapping drone method is developed to the area with a high traffic demand with a cost function based on neural networks. The authors in [11] provide a method to detect the optimal 3D location of users equipped with directed antennas. The detection is performed using a circle packing theory so that the total coverage of the area is maximized. Reference [12] presents an analytical model to find the optimal altitude of a UAV whose goal is to maximize the coverage area. In [13], finding the optimal cell boundaries and the locations of multiple non-interfering UAV deployment are considered in order to minimize the total transmit power of DBSs. In [14], the optimal location of UAVs using the Brute Force search is found to prevent disasters and improve the security of public

The authors are with Department of Electrical Engineering, Yazd University, Iran (e-mail: minataghavi@stu.yazd.ac.ir; abouei@yazd.ac.ir, corresponding author).

communications. Authors in [15] discuss about finding an optimal 3D location of a drone-cell so that the number of users who have acceptable Signal to Noise Ratios (SNRs) is maximized.

Mobile Drones: The dynamic movements of DBSs have recently been considered instead of their deployments in optimal locations. In [16, 17], controlling the movement of a single UAV is discussed. They provide the minimal spanning tree model for controlling the UAV movement to improve the connectivity of mobile ground users in an ad hoc manner. In reference [18], the probability of the end-to-end link between ground users through multiple UAV links is investigated.

In [19], the chain motion of a UAV to improve the link capacity between two mobile nodes is investigated. The authors employ the Artificial Potential Field model to control the angular speed and the direction of UAVs. The throughput maximization for communication between a fixed source and destination of a UAV is studied in [20]. They present the trajectory optimization of the fixed power allocation as a non-convex optimization problem. Adapting the location of a UAV working as a relay, to collect data from mobile users and forward them to the next base station is also studied in [21]. It is assumed that a UAV can predict the location of users through utilizing any location prediction algorithm. The objective is to optimize the accessible uplink rate of users [21]. In [22], the dynamic mobility control is applied for a DBS to maximize the spectral efficiency of a downloaded link. In [23], DBSs have non-stop, steady and constant movements at the top of their cells to serve users. Each user follows the traffic model proposed by the 3rd Generation Partnership Project (3GPP) where inner-cell users move according to the Random Way Point (RWP) mobility model. In this work, the game theory is used to decide about DBSs movements.

Through comparing the above related works, we see that the DBS deployment is one of the main issues in mobile cellular networks and is greatly affected by the energy consumption and the interferer produced by DBSs. However, as seen by the aforementioned literature, a few papers point out the connection between the DBS deployments and other network efficiency parameters (e.g., the network coverage). In addition, most of the previous works do not consider the deployment with the minimum number of DBSs. Furthermore, the main problem of the existing methods for the DBS deployment is that they provide more DBSs to cover users. In this regard, and as it will be presented in Section 3, we aim to answer these questions: "what is the optimum number of DBSs and where they should be deployed so that the maximum number of users is covered?" To answer these questions and at the first step, we model the problem in a way that the number of covered users in the network is maximized. Since the problem is nonlinear, it is not solvable by non-heuristic algorithms. To solve the optimization problem and determine the minimum number and location of DBSs, we propose an algorithm to solve the problem in order to cover the maximum number of users in the network.

The contribution of the paper is as follows, 1) a presentation of an optimization model by considering input and output parameters. The output of the Optimization Problem (OP) is the number of DBS and their locations considering the bandwidth limitation in DBSs. 2) A presentation of a proposed method to solve the nonlinear optimization problem. We proposed a heuristic method to find the best solution. 3) Simultaneous determining the number and location of DBSs. Studies in the literature do it in separate phases.

The rest of the paper is organized as follows. In Section II, the system model is described and the main assumptions required for our optimization are introduced. In Section III, we first present the DBS deployment problem and the way we can compute the optimum number of DBSs. A novel intelligent search to solve the problem will be proposed in Section IV. Simulation and numerical results are given in Section V. Finally, in Section VI, an overview of the results and some conclusion remarks are presented.

II.   SYSTEM MODEL AND ASSUMPTIONS

In this work, we consider a rectangular-shaped area with the size of X$\times$Y (meters) and NU users, located at (xu_j, yu_j), $j \in \{1, 2, \ldots, NU\}$. It is assumed that each user j is connected to the closest DBS to get services over bandwidth U_{bw_j} in the downlink mode. Moreover, there are n DBSs with coordination of (xd_i, yd_i), $i \in \{1, 2, \ldots, n\}$, which are used to provide the full coverage of users in the target area. The main goal of the paper is to determine the minimum number of DBSs and their optimal locations to cover the maximum number of users. Fig. 1 shows a possible snapshot of a target area in which only blue color users are covered by DBSs.
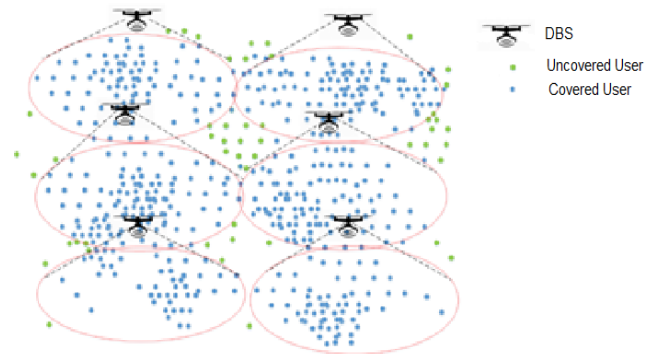

Fig. 1. A possible scenario for the DBSs distribution and their coverage.

In reality, we encounter with a three dimensional DBS placement problem, however, since the height of DBSs is assumed to be constant, it is likely to consider the problem as a two dimensional localization case. In addition, each DBS has the ability to move with a constant speed without hitting obstacles. Furthermore, it is assumed that the coverage radius of each DBS, denoted by R_c, is fixed. Moreover, we ignore the interference between the DBS and possible existing BSs through employing the orthogonal signaling [24]. The proposed algorithm can be used in two scenarios: i) when there is no small cell in the area, so that DBSs are in charge to provide services to all users, ii) when there are small cells in the area, so that DBSs can focus on providing service to users who are not covered by small cells. Since the available bandwidth of DBSs is constrained, the number of users covered by each DBS is limited. Hence, in order to take these restrictions into account, we consider both the location and required bandwidth of users to determine the optimal position of DBSs.

It should be noted that the BS deployment problem in conventional cellular networks has been well studied in the

literature (e.g., see [12]) and this issue differs from the problem of mobile DBS localization in many aspects. Firstly, besides choosing the DBS location in two dimensional space ($⟦xd⟧\_i$, $⟦yd⟧\_i$),i∈{1,···,n}, determining the DBS attitude is also very important, which introduces a three dimensional optimization problem. Secondly, the coverage area of each DBS strongly depends on its altitude which is determined ahead of the deployment process, while, the coverage area of the terrestrial BS is predefined and known. In this regard and to make a fair comparison, we use the model in [12] to determine the coverage area of the terrestrial BS. Thirdly, terrestrial BSs should wait for mobile users to move towards them, while the mobility of DBSs let them move towards any direction when there is a demand from the user. Generally, the second and third items denote the size and the location of the coverage area, respectively. In a realistic scenario, having 100% user coverage in the target area is not feasible. To provide user coverage less than 100%, we deploy parameter α to help us model the problem more effectively. As will be shown in Section 4, parameter α determines the acceptable rate of the user in our proposed optimization problem where the solution demonstrates that providing the user coverage more than α percent are valid. Of course, the value of α varies with respect to the problem characteristics.

III.   PROBLEM FORMULATION

The main goal of this section is to formulate the 3D DBS placement problem to maximize the number of covered users, where the input and output of the optimization problem and its constraints are introduced in details as follows.

Input: The input parameters of the optimization problem are given as follows: i) the target area size (X×Y), ii) distribution of users and the location of each user (xu_j, yu_j), j ∈ {1,2,···,NU}, iii) coverage radius of each DBS (R_c), iv) the acceptable rate of the user's coverage (α), v) the average required bandwidth of user j (i.e., U_{bw_j}). This parameter is derived according to the statistical traffic usage of users in cellular networks, vi) limited available bandwidth of each DBS which is derived based on the type of the backhaul link that connects the DBS to the infrastructure (DBS_{Maxbw}).

Output: Number of DBSs and their locations.

Optimization Problem: The objective function explained in this section is derived based on the characteristics of the DBS placement problem. The optimization problem aims to maximize the number of covered users while the number of DBSs is as less as possible, i.e.,:

$$max_{xd,yd} \sum_{i=1}^{NDBS} \sum_{j=1}^{NU} UCD_{i,j} \qquad (1)$$

where NDBS is the number of DBSs that is unknown in this phase and the binary variable $⟦UCD⟧\_(i,j)$ represents a user indicator function, in which, if user j is served by DBS i, then $⟦UCD⟧\_(i,j)$ is set to 1; otherwise, it is set to 0. The result of the summation in (1) is the total number of users who are covered by at least one DBS; so we try to maximize it. The constraints related to the optimization problem in (1) are explained as follows:

$$\sum_{j=1}^{NU} UCD_{i,j} \times U_{bw_j} < DBS_{Maxbw}, \quad \forall i \, \epsilon\{1,...,NDBS\}$$

$$UCD_{i,j}\left((xu_j - xd_i)^2 + (yu_j - yd_i)^2\right) \leq R_c^2$$

$$\sum_{i=1}^{NDBS} UCD_i = 1 \quad, \quad \forall j \, \epsilon\{1,...,NU\}$$

$$\left(\sum_{i=1}^{NDBS} \sum_{j=1}^{NU} UCD_{i,j}\right) / NU \geq \alpha$$

Since users have different demand rates in various applications, the total available bandwidth of each DBS (DBS_{Maxbw}) is considered as a restriction. Regarding to constraint (2) and in order to have a full cover of all users, the sum of the used bandwidth of users for each DBS should be less than the maximum bandwidth to serve all users.

Constraint (3) makes sure if user i is located inside the covered area of the corresponding DBS j, then user j will be covered by DBS i.

Constraint (4) illustrates that each user is a subset of a DBS and must be covered by it. It is less efficient if a user is covered by more than one DBS. Thus, in our optimization problem, we aim to minimize the overlap between cells.

Constraint (5) shows that the coverage must be more than α%. Here we take α=90% since having α=100% makes the cost of the solution unreasonable. In fact, in practical environments, when some users are located far from the others, it is not practical and cost efficient to cover all of them.

Since the optimization problem in (1) is a mixed integer non-linear programming (MINLP), so we cannot solve it using traditional solutions. More precisely, since in practical scenarios, NDBS in the upper border of summation in (1) is high, traditional mathematical methods increase exponentially the computational costs in solving such a problem if it is possible. To overcome this problem and make the solution feasible, in the next section we propose a heuristic method that helps us to solve the problem using mathematical methods

IV.   PROPOSED SOLUTION

In this section, we employ the proposed binary search algorithm to calculate the final solution of the optimization problem in (1) which is the number of DBSs denoted by NDBS. In the best possible scenario at least one DBS is needed to cover the target area, therefore the minimum amount of NDBS, Min_D, is one. We first compute the maximum number of DBSs denoted by Max_D and then binary search algorithm is performed in the interval {1, ···, Max_D }, i.e., the possible minimum and maximum number of DBSs.

Let assume that all DBSs are located in an area X×Y. As depicted in Fig. 2-B, each DBS with the coverage radius R_c can cover a rectangle with the side *dr*. Considering the fact that the area size is X×Y, in the worst case scenario presented in Fig. 2-A, the $⟦Max⟧\_D$ can be calculated as follows:

$$dr^2 = R_c^2 + R_c^2 \qquad (5)$$
$$dr = \sqrt{2}R_c$$

Thus, the maximum number of DBSs is obtained as:

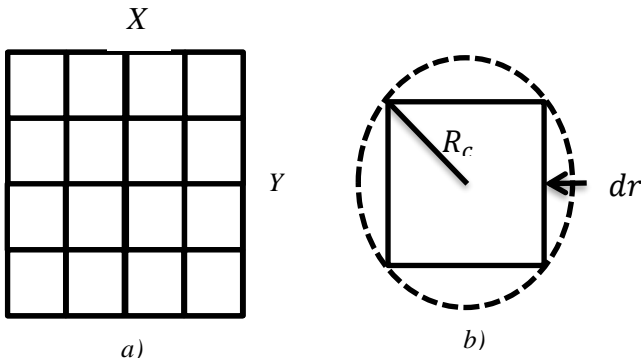$$Max_D = \frac{X}{\sqrt{2}R_c} \times \frac{Y}{\sqrt{2}R_c} \qquad (6)$$

Fig. 2. a) A rectangular-shaped area with the size of X×Y (meters);
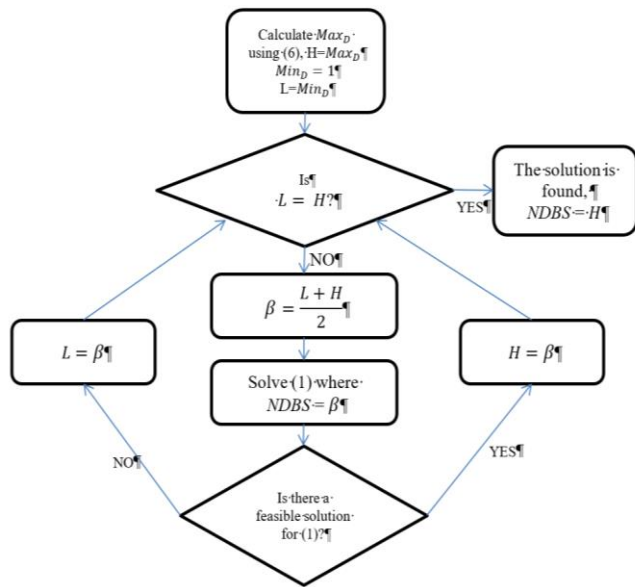b) calculate dr

Fig. 3. Flow chart of the proposed binary search

Using the proposed binary search algorithm presented in the flowchart in Fig. 3, the best number of DBSs and their locations are calculated in order to cover more than α% of users. We want to find the minimum number of the DBSs while the coverage condition is satisfied in the interval $\{Min_D, ..., Max_D\}$. According to Fig. 3, if the current value of the NDBS is bigger than its actual value, there would be a contradiction between (4) and (5) because there is no solution that could be able to cover α% of users while constraint (4) is satisfied. Since, constraints (2) and (3) determine the location of the BDSs, so when there is contradiction between (4) and (5), constraints (2) and (3) are in the shadow. Otherwise, we try to solve the (1) with NDBS smaller than its actual value which is still unknown in this step. It should be noted that $\beta = (L + H)/2$ represents a temporary parameter equal to the average of Max_D and Min_D.

V.   SIMULATION RESULTS

In this section, we evaluate the proposed DBS deployment method based on a real world dataset. We use a rectangle area of 300 × 400 meters so that the number of users, that are subsets of each DBS, is considered as inputs of different scenarios, and are randomly distributed in this area. In our simulations, we assume that the altitude and the coverage radius of DBSs are fixed and set as 50 and 100 meters, respectively. We use the IBM's Optimization Programming Language (OPL) [25] for simulation and optimization. In our simulation setup, a real dataset is used in which it is achieved from real world users (we have published the dataset online [26]). The dataset was collected by the organization's Statistics Centre for various departments in 2016. It contains coordinates of each DBS, MAC address and the number of online users covered by each DBS in different times including month, day, hours and minutes (see one scenario in Fig. 4). In Fig. 4 a plot of the execution of the proposed algorithm is illustrated where Dots are users and ovals are the coverage area of each DBS. We know the coverage area of a DBS is a circle, but because of the unbalanced scale of the horizontal and vertical axis in Fig. 4 you see it as oval. There are 10 DBSs in Fig. 4. The distribution of the users is based on the data set mentioned earlier.
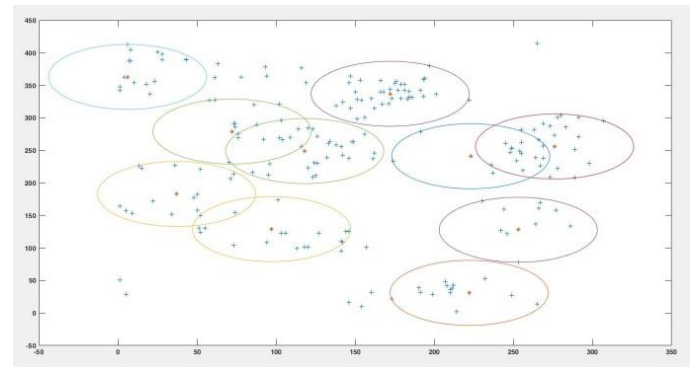

Fig. 4. A sample scenario of running the proposed algorithm

In our numerical results, we evaluate the following performance metric:
• The required number of DBSs by assuming inconstant number of users and constant coverage area.
• The percentage of users covered by DBSs with different bandwidth (10, 50, 100, 200) Mb.
• The number of DBSs with different bandwidth for a temporary cover by considering a constant number of users (i.e., 50, 100, 150, 200 users).
• Sum rate of DBSs for served users.
• Distribution and suitable number of DBSs to cover the variable number of users.

Regarding the contributions of the paper presented in section 1, Figs. 5 & 6 and Tables I & II, address the performance of the proposed algorithm with respect to the bandwidth limitation. Fig. 7 illustrates that the problem is solved successfully and both the number of DBSs and their location are achieved simultaneously.
Fig. 5 illustrates the required number of DBSs with different bandwidth to cover users temporarily. It should be noted that for this simulation results, the users' location does not change and in each time some new users are added to the location. As seen from this figure, when the number of users increases the required number of DBSs will be increased as well, in particular, for the bandwidth equal to 10 Mbps the growth for NDBS is much higher than other bandwidth. Moreover, the more bandwidth for each DBS, the less number of DBSs we need. A summary of information about Fig. 5 is given in Table I.

Fig. 5 emphasizes on the performance of the proposed algorithm with respect to the bandwidth utilization. Number of users influences the number of DBSs severely when we have limited bandwidths, but when we have enough available bandwidth (e.g. when the bandwidth is 100mbps) the influence is much less. Equation (2) is in charge of such action.
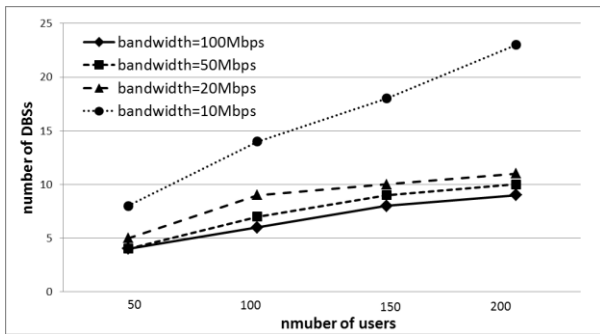

Fig. 5. The number of DBSs vs. the number of users for different bandwidth

TABLE I

THE NUMBER OF REQUIRED DBSS WITH DIFFERENT BANDWIDTH TO COVER (50, 100, 150, 200) USERS

Number of users	Number of DBSs			
	10Mbps	20Mbps	50Mbps	100Mbps
50	8	6	4	4
100	14	9	7	6
150	18	10	9	8
200	23	11	10	9

Table II presents the percentage of users covered by DBSs with different bandwidth (10, 100, 50, 200). It is seen from the results in Table II that in all cases the proposed algorithm manages to provide the required coverage which are bigger than 90%. Since we have another goal in the optimization problem which is the least number of DBSs, it is vivid when we have limitation in the maximum bandwidth and bigger number of users, the coverage is more close to 90%. This worst case comes from the fact that the condition of the problem is so harsh that puts the proposed algorithm in trouble to solve the problem.

Fig. 6 illustrates the sum of the service rate required for different number of users. Each line presents the maximum service rate for different bandwidths (50, 100, 150, 200). As an example, for 150 users covered by DBSs with maximum bandwidth 10 Mbps, it needs 315 Mbps as the summation of the total services for the whole network. It means that the backhaul link needs to be able to handle at least 315 Mbps.

TABLE II

COVERAGE PERCENTAGE FOR (50, 100, 150, 200) USERS WHERE THERE ARE DBSS WITH THE MAXIMUM BANDWIDTH (10, 50, 100, 200)

The coverage percentage				Number of users
10 Mb	20 Mb	50 Mb	100 Mb	
90%	90%	92%	100%	50
91%	93%	93%	100%	100
90.85%	91.4%	93.3%	94.8%	150
90%	91%	91%	93%	200

Fig. 6 illustrates the influence of the performance of the proposed algorithm with respect to the total bandwidth used by all users. Vertical axis in Fig. 6 is the summation of the bandwidth used by all users. As it is vivid, when the number of users is small, there are no distinctive results among cases with

different available bandwidth of DBS, but the actual result is vivid when the number of users increases. As the number of users is 150, the proposed algorithm could manage the total bandwidth more effectively.
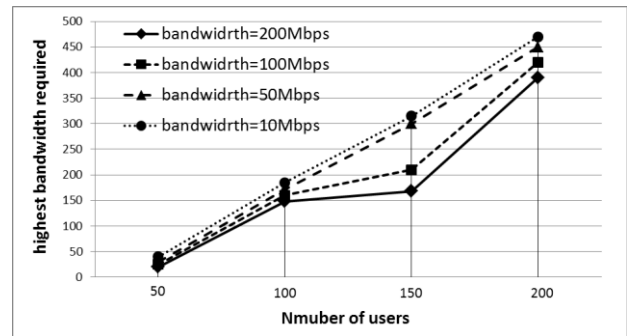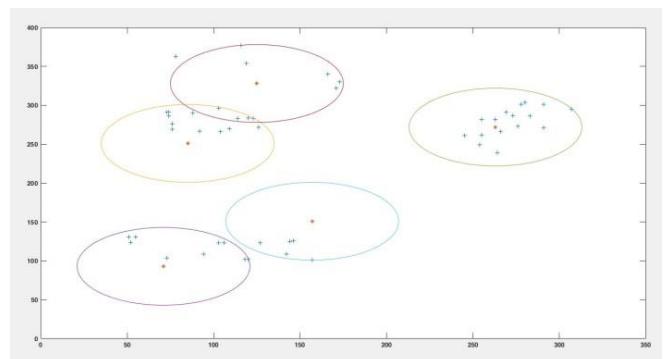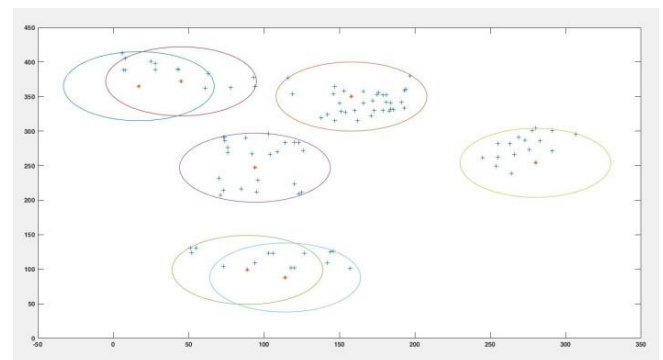

Fig 6. Number of users vs. the required bandwidth (10, 50, 100, 200).

At the rest of this section, we illustrate the performance of the optimization problem subjects. As mentioned earlier, points in Fig. 7 presents users distributed in the area based on the data set provided in [26] and circles are the covered area of each DBS. There is no stationary BS in the area and the aim is to cover the whole train using DBSs. The location of DBSs is the output of the optimization problem in (1). In Fig. 7 two scenarios are considered, 1) Fig. 7.A. depicts scenario when we have 50 users and 5 DBSs and, 2) Fig. 7.B depict scenarios when we have 100 users and 7 DBSs.

a) 50 user 5 DBSs

b) 100 user 7 DBSs

Fig. 7. Positions of users when there are DBSs with the most 50 Mb bandwidth

Fig. 7 helps us to have a better understanding on the performance of the proposed algorithm. In this figure, we want to show the integral performance of each subject in the proposed optimization model. In Fig. 7.A, we can easily see users which are not covered by DBSs. This helps us to understand the

influence of constraint (5). Moreover, at the top-left corner of Fig. 7.B we can see two DBSs located so close to each other. This happens because users must have enough bandwidth and coverage. This situation is related to influence of constraints (2) and (3). As a conclusion, it is vivid that the proposed algorithm has managed to achieve its goals where you can find the plot of the execution of the algorithm in Fig. 7.

VI. CONCLUSION

In this paper, we proposed an optimal two-dimensional DBSs placement in a cellular network. We have used a MINLP to model the problem while the goals are to find the optimal number of DBSs and their exact locations. Because of the complexity of the proposed optimization problem, we used a proposed heuristic algorithm to solve the problem. Although the area has different density of users, simulation results demonstrate the acceptable performance of the proposed algorithm. In current study, we focused on homogeneous traffic and we do not consider the terrestrial BSs. As future works, non-homogeneous traffic and full cover of users can be studied when there are some terrestrial BSs that cannot cover all users because of some events like broken terrestrial BSs, bad weather conditions, etc.

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