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THE INFLUENCE OF AXIAL LOAD ON ELASTIC BUCKLING OF SHELLS OF REVOLUTION

In this paper, the authors consider the influence of axial load on the stability of shells of revolution subjected to external pressure. Shells of different geometry are investigated with emphasis to barrelled shells. The variable quantities are length L and meridional radius of curvature R1 of a shell. The constant parameters are: thickness of the shell h, mass ms and reference radius r0. The material of shells is steel. Numerical calculations were performed in the ABAQUS system. All the shells considered in this paper were subjected to axial compression to determine the force corresponding to the loss of stability in such conditions. A part of this force is then used to preload shell before the buckling analysis in the conditions of external pressure is started. The buckling shapes for shells of different geometry are presented with and without the influence of axial load. The ability of controlling the buckling strength and shape is discussed.

Nomenclature

E	–	Young's modulus
F	–	axial force
h	–	thickness of a shell,
L	–	length of a shell
m_s	–	mass of a shell
n	–	natural number
p	–	external pressure
$R_1; R_2$	–	principal radii of curvature of a shell
r_0	–	radius of a reference cylindrical shell
ν	–	Poisson's ratio

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θ, θ_1 – angle coordinate
ρ_s – mass density

1. Introduction

The main factor that influences the stability of a shell structure is its geometry. Among the shells of revolution, the most favourable shape, in the case of external pressure conditions, is a sphere. Obviously a shell with this kind of geometry is neither easy to manufacture nor practical from the viewpoint of usability; for example, it is impractical in the case of storage or transport. Cylindrical shells do not have these disadvantages, but they are not able to transmit very high load in the direction normal to the midsurface. An alternative solution that combines advantages of spherical and cylindrical shells are barrelled shells whose buckling strength is up to 40 times higher than that of equivalent cylindrical shells when subjected to external pressure. That was proved by Błachut and Wang in [3] and by Błachut in [4]. There are many other works that confirm the importance of geometry in the field of shell structures design. An example can be the paper by Banichuk [2] in which the author is looking for an optimal shape of a shell of revolution with minimum mass as a criterion. Other authors, like Pontov and Dinkler [7] or Gupta and Venkatesh [5], investigate the influence of geometrical parameters of the existing shells on theirs strength and stability.

In this paper, the authors consider a barrelled shell subjected to external pressure only. An additional axial load is applied to show that this way one may increase the buckling strength of the shell. The investigation consists of three steps, described in the subsection 3.1, in which the whole family of shells of constant mass is subjected to different load conditions: axial compression (case 1), external pressure (case 2) and combination of both above-mentioned ones. The differences in buckling shapes for different load conditions are shown. The influence of initial axial compression on the stability of shell is presented in the graph.

All numerical calculations are made in the ABAQUS system. Models with perfect geometry are considered. The first mode from the buckling analysis is taken into account as the one that gives the critical value of the load. Numerical results obtained form the ABAQUS system are compared with the Shirshov's theory of the local stability of shells presented in [8].

2. Experimental model

The investigation was carried out on a family of shells of revolution. The geometry of a half of a shell is presented in Fig. 1. The parameter used to control the geometry is θ_1, which influences the length L and the radius R_1 of the shell, which means that with decreasing the value of θ_1 both parameters, L and R_1, also decrease. Variable θ_1 is an angle coordinate that varies in the interval ($\theta_1 \geq \theta \geq \pi/2$) and is related to L and R_1 according to the expression $L = 2R\cos\theta_1$. The geometric relation that characterizes the principal radii of curvature of a shell of revolution is:

$$\frac{\partial}{\partial\theta}\left(R_2 \sin\theta\right) = R_1 \cos\theta. \tag{1}$$

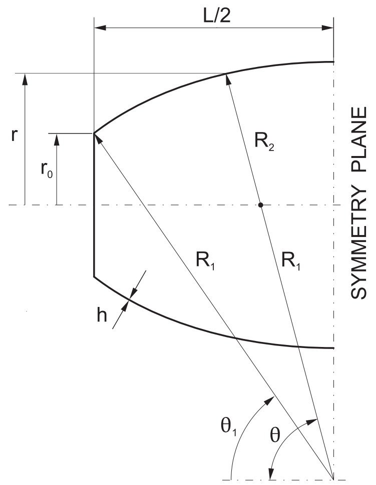

Fig. 1. Geometry of a barrelled shell

The radius r is expressed by:

$$r = r_0 + R_1\left(\sin\theta - \sin\theta_1\right). \tag{2}$$

Mass of the shell is:

$$m_s = 4\pi R_1^2 t\rho_s\left[\left(\frac{\pi}{2} - \theta_1\right)\left(\frac{r_0}{R_1} - \sin\theta_1\right) + \cos\theta_1\right], \tag{3}$$

where: ρ_s – mass density. Other denotations as in Fig. 1.
The constant parameters are: thickness h, mass m_s and radius r_0, which is the radius of the cylindrical shell that was taken as a reference shell. The parameters characterizing this reference shell are: length L = 3000 mm, radius r_0 = 500 mm, thickness h = 1.5 mm and mass m_s = 111 kg. In this case $\theta_1 = \pi/2$.

The following are the parameters of steel taken as the material of shells: Young's modulus $E = 2.05 \cdot 10^5$ MPa, Poisson's ratio $\nu = 0.3$, mass density $\rho_s = 7.85$ g/cm^3.

Numerical calculations were made in the ABAQUS system. The shell was supported at both ends along the edge, like in Fig. 2, where longitudinal movement and rotation about an axis tangent to the shell edge are allowed. Additionally, the movement along the axis of revolution is taken away in the midlength of the shell. During eigenvalue extraction analysis, one used a S4R5 shell element with four nodes and five degrees of freedom in each node: 80 elements along the model and 200 elements on a circumference.

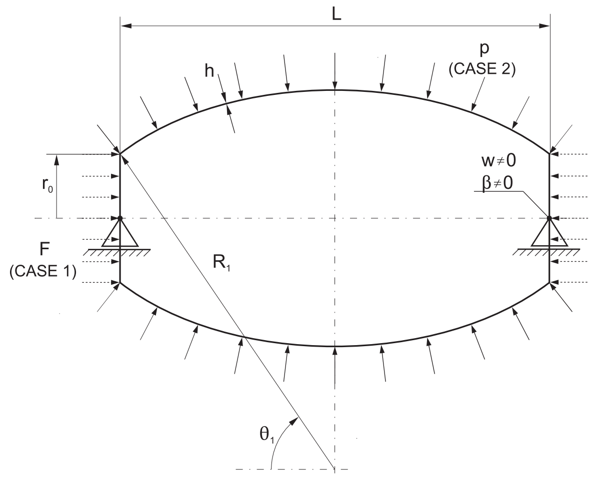

Fig. 2. Boundary conditions

3. Results of investigation

3.1 Procedure

The goal of the investigation was to present the influence of axial preload of barrelled shell under external pressure on its stability. So, in the first step, each shell was subjected to axial compression to determine the value of the buckling load F_{cr} in such conditions. After that, a curve was created that presents the influence of the geometry of the shell on the value of critical load p_{cr} in external pressure conditions. As the critical load, one assumed the value corresponding to the first buckling mode. In the last step a part of the value of the force F_{cr} (10% and 30%) was used to preload the structure and after that the buckling analysis was started in the conditions of external pressure. A similar curve as that in the previous step was created for preloaded shells.

3.2 Results

As it was mentioned above, in the first step it was necessary to determine, for each shell, the value of critical load in the conditions of axial compression. The boundary conditions in the numerical model were as shown in Fig. 2 case 1. The eigenvalue buckling prediction analysis was carried out and the load corresponding to the lowest eigenvalue was taken as the critical load F_{cr}.

In the next step, one investigated the influence of the radius of curvature R_1 on the critical external pressure. Curve 1 in Fig. 3. shows the relationship between these two quantities. The result is consistent with the intuitive notion that the smaller radius R_1, the higher critical load p_{cr}. Extreme points of the curve correspond to the spherical and cylindrical shell, respectively.

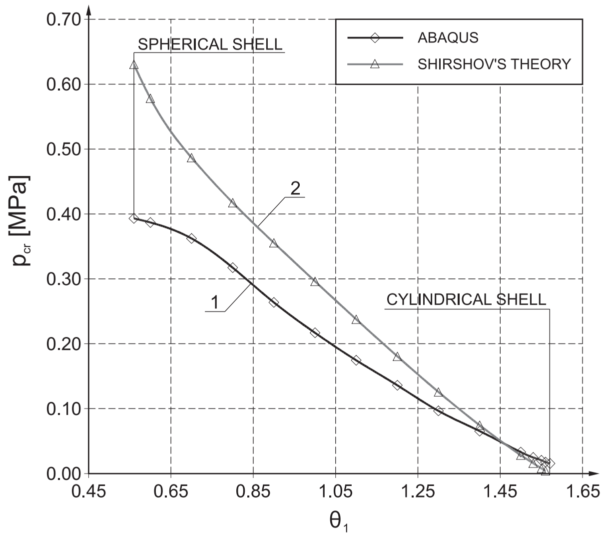

Fig. 3. Relationship between the geometry of a barrelled shell and the critical external pressure

Similar curve can be obtained (curve 2 in Fig. 3) using Shirshov's expression for critical load for closed shells of revolution, which is derived on the basis of local stability of shells:

$$p_{cr} = \frac{2E}{\sqrt{3(1-\nu^2)}} \frac{h^2}{R_2(2R_1 - R_2)}. \tag{4}$$

This expression is valid for shells with $R_1 > R_2$. If one excludes spherical and cylindrical shells, the difference between the results obtained form the ABAQUS system and those from Shirshov's theory is about 30%.

The last step was to check how the axial preload influences the stability of shells of revolution subjected to external pressure. In this analysis, one could distinguish two cases. In the first case, a part of critical force F_{cr} (10% and 30%) was applied to develop initial stresses in a shell by compression conditions. After that, buckling analysis was started with external pressure load. The results can be seen in Fig. 4. The initial compression of the shell with 10% of the critical force F_{cr} increases the critical pressure p_{cr} by about 7% in most cases. Similarly, 30% of the critical force increases the critical pressure by about 20%. The situation is different when barrels with geometry close to a cylindrical one are considered. Here, the preload force decreases the critical pressure. Of course, there is a point (see: view "A" on Fig. 4) in which the axial load does not influence the critical pressure.

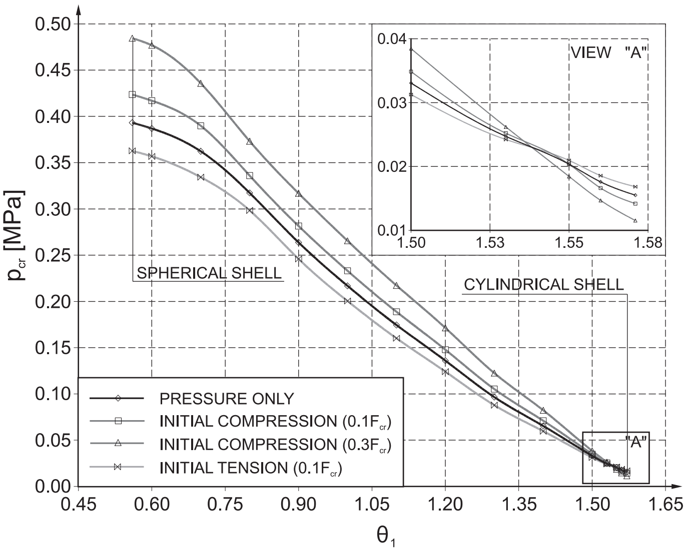

Fig. 4. The influence of the initial preload on the critical external pressure

In the second case, a similar analysis was carried out, but this time the shell was initially subjected to tension conditions using 10% of the F_{cr}. As it can be seen from Fig. 4, the result is exactly opposite to that obtained in the case of initial compression.

Since the initial preloading force influences the value of the critical pressure, it also influences the buckling shape. Two examples will be considered now. For the values of parameter θ_1 higher than 0.77, the buckling shape is like that in Fig. 5. The waves appear mainly in the central part of the shell. The application of an initial axial compression does not change the buckling shape, but circumferential waves are concentrated more closely in the middle part of the shell. The number of waves n in the first case equals 30 and in the second $n = 31$.

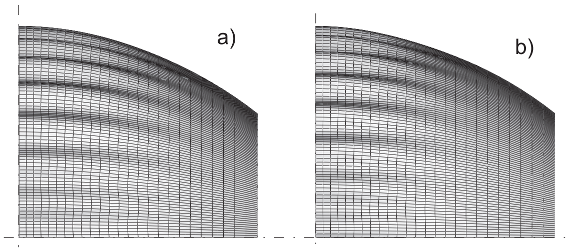

Fig. 5. Buckling shape for $\theta_1 = 0.9$; (a) external pressure only, (b) external pressure with initial preload 0.3 F_{cr}

When the parameter θ_1 takes a value lower than 0.77, that is for shells with geometry close to the spherical one ($R_1 \rightarrow R_2$), and a proper spherical shell, the waves appeared not in the centre of the shell, but on the edges of it. In this situation, the initial preload is more significant. As it can be seen in Fig. 6, the waves that appeared on the edges of the shell move to the middle of it when an adequate axial force is applied. The situation is explained in Fig. 6 by von Mises stress distribution across the meridian of the shell. One can see that, in the external pressure conditions without axial load, the highest stresses are near the edge of the shell. The preloading force changes this distribution by moving the maximum stresses to the center and decreasing its value.

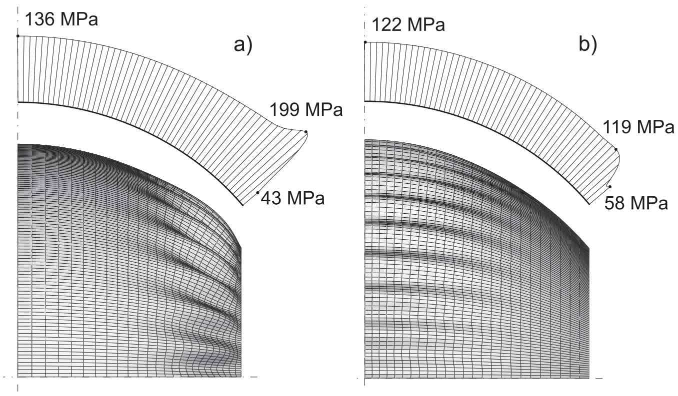

Fig. 6. Buckling shape and von Mises stresses for $\theta_1 = 0.7$; (a) external pressure only, (b) external pressure with initial preload 0.3 F_{cr}

4. Conclusion

As it was demonstrated in this paper, one can influence the buckling strength of the barrelled shells subjected to external pressure by applying an axial load. This influence can be either positive or negative, depending on the direction of this load (compression or tension) and on the geometry of the shell, as it can be seen in Fig. 4. The critical load p_{cr} may be increased by about 20% by applying the axial force of magnitude of $0.3F_{cr}$, where F_{cr} is the axial force for which the shell looses its stability. In the case of shells with geometry close to the spherical one, by applying an axial load one can not only increase the critical pressure, but also change the buckling shape (see Fig. 6).

The importance of the investigation on shell's stability is confirmed by a number of publications listed in review works by Teng [9] and Arbocz [1]. It is then reasonable to extend this investigations also onto the area of imperfect model, which for spherical shells was made by Pedersen *et al*. [6], and post-buckling analysis like that in the work by Teng and Hong [10], in which the authors showed that more than one buckling mode should be considered to obtain accurate results.

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Wpływ siły wzdłużnej na sprężyste wyboczenie powłok obrotowych

Streszczenie

W pracy niniejszej rozważany jest wpływ siły osiowej na stateczność powłok obrotowych poddanych działaniu ciśnienia zewnętrznego. Badaniu poddane zostały powłoki o różnej geometrii z naciskiem na powłoki baryłkowe. Wielkościami zmiennymi są długość L i południkowy promień krzywizny R_1 powłoki. Wielkości stałe to: grubość powłoki h, masa m_s i promień odniesienia r_0. Materiałem powłok jest stal. Badania numeryczne przeprowadzone zostały w systemie ABAQUS. Wszystkie rozważane powłoki zostały poddane osiowemu ściskaniu dla określenia siły odpowiadającej utracie stateczności w tych warunkach. Część tej siły została następnie użyta jako obciążenie wstępne przed przystąpieniem do analizy stateczności w warunkach działania ciśnienia zewnętrznego.

Jako rezultat badań przedstawione zostały postacie wyboczenia powłok o różnym kształcie, powstałe przy uwzględnieniu działania sił wzdłużnych oraz bez ich uwzględniania. Przedyskutowana została możliwość sterowania odpornością konstrukcji na wyboczenie i kształtem wyboczenia.