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SEMI-ANALYTICAL MODELLING OF MULTILAYER CONTINUOUS SYSTEMS NONLINEAR DYNAMICS

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Problems concerning structures dynamics are being one of most important subjects in recent investigations associated with railways constructions. The need of modelling of such structures and their behaviour prediction leads to necessity of seeking new approaches, mainly due to highly increasing speeds of vehicles and traffic intensity. Comparative studies carried out on experimental data, measurements and theoretical research show that models based on multi-layered approach supported by semi-analytical approximations of solutions can give new insight into undertaken analyses. More detailed consideration of roads components and their physical properties, along with application of effective estimations allowing to avoid numerical instabilities linked with extremal dynamic variations, can be important tools in obtaining new solutions both, theoretical and engineering. This paper briefly presents a number of multilayer railway track models, with special emphasis on nonlinear track properties. Existing analytical and semi-analytical solution methods are presented with main advantages of new approaches. The theoretical double-beam system with two nonlinear layers is solved and computational examples are presented along with possibility of their transition to other multilayer structures analysis.

Keywords: semi-analytical methods, multilayer structures, nonlinear properties, dynamic response, double-beam system, railway modelling.

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1. INTRODUCTION

Due to continuously increasing speeds of recent trains and construction of high speed rails, new models and approaches to their analysis are sought in order to predict dynamic behaviour of such structures. Old re-built and renovated tracks need similar studies, especially those being placed in the areas of dense urban development. Experimental studies and measurements in-situ show the necessity of consideration of all rail track components forming multi-layered structure, including their nonlinear properties. The simplest model of rail track consists of single beam resting on elastic foundation [1, 2]. The beam can be described either by the Euler-Bernoulli equation or the Timoshenko beam model. The beam foundation can be also viscoelastic with linear or nonlinear stiffness [3, 4, 5].

More advanced models of railway track are based on multi-beam structures. However, more than two beams are rarely considered. Usually the two-layer model consisting of two beams connected by elastic or viscoelastic layer and resting on elastic or viscoelastic foundation is taken into account. This model is recognized as a good enough representation of vertical vibrations generated by train passing by the observation point placed on rail. First layer in this model represents rails and the second one, considered as a rigid body, reflects behaviour of sleepers. This model was previously validated and computational results coming from parametrical analysis were compared with real measurements done on Polish railway network for several speeds of moving train [6, 7].

Inclusion of nonlinear or stochastic properties of layers connecting beams or describing foundation properties leads to significant difficulties in solving such systems. There are several numerical approaches based usually of FEM techniques, but numerical results need validation and, in addition, computations using FEM-based methods are power and time consuming. Therefore a seeking for analytical solutions is still very important for progress in railway engineering. Usually analytical methods, when one deals with nonlinearities or randomness, give closed form solution which are ineffective in parametrical analysis. These smart mathematical formulas need to be transformed to more friendly form, which means application of various approximating procedures. Analytical approximations are highly valued but, unfortunately, in many cases analytical formulas are insufficient to solve problems. Then the next step for approximation would be numerical computations. When it is possible to apply analytical approximations without discretization of domain, one can say about semi-analytical solution. When it comes to discretization and numerical approximating procedures must be applied, then one can call the method semi-numerical one.

Purely numerical approaches belong to different group of techniques which are being relatively rarely used nowadays. Significant advantages are being found in mixed techniques called hybrid approaches which obey both semi-analytical and semi-numerical modelling.

Previously published papers show interesting semi-analytical results concerned with nonlinear stiffness included in beam-foundation structures [8, 9]. One-layer and two-layer models were analysed in details under assumption of nonlinear stiffness of structure foundation, by using semi-analytical method based on wavelet approximation combined with Adomian's decomposition. Both these models are also validated by comparison with measurements done on operating railway track showing high compliance of results within vertical vibrations of the track for various sets of parameters and speeds of train [6, 7, 8].

Initial studies leading to practical results and their applications to railway dynamics study were based on the analysis of purely theoretical model of infinitely long double-beam, firstly in the linear case and after that with nonlinear foundation [9, 10]. Hence, other modifications, such as discussed nonlinear stiffness of layer between beams, which corresponds to fastening system properties, should be preceded by theoretical investigations of mechanical systems unnecessarily directly related to rail track. Therefore the case with two nonlinear layers is analysed by using the system of parameters similar to this one used before in initial studies. Recognition of mechanical properties of such dynamic system will allow to apply it to the analysis of railway track properties.

Modelling of dynamic excitations generated by train is another important subject of this study. The load produced by train has complicated structure with several terms coming from vehicle but also from track imperfections or foundation irregularities. The main terms are: the force constant in time, produced by the weight of train and responsible for stationary solution, the force changing in time, produced by several factors, including regular imperfections of wheels or rail head surface (e.g. corrugations) or the specific character of periodic structures, e.g. sleepers spacing, and finally some other irregularities difficult to describe analytically due to their randomness [6, 11-17]. Although all these terms are important for a proper description of dynamic interactions in railway track, in this paper only stationary load is considered and other cases are left for further study.

One should mention also about different class of models based on layered foundation modelling, where solid is described by equation of motion in soil. Several interesting results, including optimization of layers properties in order to minimize surface vibrations, can be found in the literature [18, 19]. It is worth to mention that semi-analytical techniques used in solving these problems can be applied to other problems based on multilayer continuous systems [20] and some initial works in this area has been already done by the Authors' research group, especially in the area of geogrids applications [21].

2. EQUATIONS AND MODELS

Because vertical vibrations of the railway track are the main factor important for its dynamic response analysis, usually beam-foundation systems are investigated. In the case of load moving along the beam, such models can be recognized as a good enough representation of main features of rail track reaction to moving train. The simplest one is based on the Euler-Bernoulli beam equation [6]:

$$EI\frac{\partial^4 w_r}{\partial x^4} + m_r\frac{\partial^2 w_r}{\partial t^2} + k_L w_r = P(x,t) \tag{1}$$

where EI [N/m^2] is a bending stiffness of rail steel, m_r [kg/m] is a rail unit mass, k_L [N/m^2] is linear foundation stiffness and $P(x,t)$ [N/m] represents force generated by moving train. w_r [m] denotes deflection of beam (rails), x [m] is a space variable along the beam and t [s] denotes the time variable.

In this equation might be included other terms describing rail track properties, such as e.g. N [N] - axial force in rail; m_r [kg/m] - rail unit mass; c_r [Ns/m^2] - viscous damping of rail foundation; k_N [N/m^4] - nonlinear part of foundation stiffness and others [8]:

$$EI\frac{\partial^4 w_r}{\partial x^4} + N\frac{\partial^2 w_r}{\partial x^2} + m_r\frac{\partial^2 w_r}{\partial t^2} + c_r\frac{\partial w_r}{\partial t} + k_L w_r + k_N {w_r}^3 = P(x,t). \tag{2}$$

This model, along with complex representation of load P consisting of two parts, constant in time and varying in time (related to conventional track with sleepers and sinusoidal signal associated with sleepers spacing), is experimentally confirmed [8]. Due to application of semi-analytical method based on wavelet analytical approximation and Adomian's decomposition, it was possible to develop effective method of parametrical analysis of the track dynamics [8, 22-25].

More convenient model of railway track uses the idea of double-beam model where two beams are connected by elastic or viscoelastic layer and the whole structure rests of foundation, again elastic or viscoelastic. The force representing moving train moves with constant speed along upper beam which represents rails. The lower layer is responsible for sleepers and it is considered as a rigid body [6, 9, 10]:

$$EI\frac{\partial^4 w_r}{\partial x^4} + m_r\frac{\partial^2 w_r}{\partial t^2} + c_r\left(\frac{\partial w_r}{\partial t} - \frac{\partial w_s}{\partial t}\right) + k_r(w_r - w_s) + k_{Nr}{w_r}^3 - k_{Ns}{w_s}^3 = P(x,t) \tag{3a}$$

$$m_s\frac{\partial^2 w_s}{\partial t^2} + c_s\frac{\partial w_s}{\partial t} + k_s w_s - c_r\left(\frac{\partial w_r}{\partial t} - \frac{\partial w_s}{\partial t}\right) - k_r(w_r - w_s) + k_{Ns}{w_s}^3 - k_{Nr}{w_r}^3 = 0. \tag{3b}$$

Indexes s and r denote sleepers and rails, respectively, w_s [m] is vertical vibration of sleepers and k_{Nr} [N/m^4], k_{Ns} [N/m^4] describe nonlinear stiffness of fastening system and foundation, respectively.

Although the system of differential equations (3a-b) can be easily solved in the case of linear properties of layers, inclusion of nonlinear part of stiffnesses makes it extremely complex. Either numerical methods must be involved or semi-analytical approximations applied with strong regime with regard to convergence of approximate solutions. The series representing the dynamic response of the system can be controlled by conditions checked already for other less complex systems. The main criterion is the stabilization of solution in relation to physical parameters within the area of the model applicability. Proper choice of the approximation order can be secured by controlling so called "error index" which allows to obtain solution with assumed level of accuracy [9, 10, 25]:

$$0 \le \frac{\left\| w_{r_{j+1}} \right\|}{\left\| w_{r_j} \right\|} < 1,\ 0 \le \frac{\left\| w_{s_{j+1}} \right\|}{\left\| w_{s_j} \right\|} < 1 \tag{4}$$

$$w_{r_n}^{er} = \frac{\left|\, \|S_n^{w_r}\| - \|S_{n-1}^{w_r}\| \,\right|}{\|S_n^{w_r}\|},\ w_{s_n}^{er} = \frac{\left|\, \|S_n^{w_s}\| - \|S_{n-1}^{w_s}\| \,\right|}{\|S_n^{w_s}\|} \tag{5}$$

for consecutive terms of Adomian series ($j=0,\ 1,\ 2,\ldots$ and $\left\| w_{r_j} \right\| \neq 0$, $\left\| w_{s_j} \right\| \neq 0$) of both layers response (w_r and w_s) with the maximum norm:

$$\left\| w_{r_j} \right\| = \max_x \left| Re\left[w_{r_j}(x) \right] \right|,\ \left\| w_{s_j} \right\| = \max_x \left| Re\left[w_{s_j}(x) \right] \right|. \tag{6}$$

$S_n^{w_r}$ and $S_n^{w_s}$ denote the n^{th} partial sum of the approximate solution in physical domain obtained for rails and sleepers, respectively.

Modelling of load generated by train is difficult issue. It is relatively easy to represent weight of train and dynamic forces produced by regular imperfections or periodic character of structure. In real scenario, some irregularities are unpredictable and in this case the load should be considered as a dynamic stochastic process. In simplified form, the load can be considered as a random function with three main terms:

$$P(x,t) = P_C(x,t) + P_D(x,t) + P_R(x,t,\gamma). \tag{7}$$

The term $P_C(x,t)$, constant in time, is generated by weight of train, the term $P_D(x,t)$ is responsible for forces produced by regular imperfections of rail track, e.g. corrugations or sleepers spacing, and the term $P_R(x,t,\gamma)$ is random function generated by other unexpected and difficult to predict changes in structure that might cause additional dynamic interactions (including vehicle dynamics). The technique leading to analytical formula describing random factor of load was shown in published papers, however its application to railway engineering modelling remains problem to solve [6, 8, 15].

Before that, one must analyse all important features of the developed model along with its computational capabilities. Although combination of random procedure with hybrid semi-analytical approximations can give new results important for railway engineering, it needs careful study due to errors that could be done by several approximations involved in computations. Additionally, one must remember that the considered system possesses relatively high sensitivity for changing physical parameters of structure due to character of semi-analytical methods applied to solve the problem [8, 9, 25]. This influences effectiveness of the procedure which is crucial for applications.

In this paper, simplified theoretical model of infinitely long double-beam dynamic response is considered:

$$EI_u \frac{\partial^4 u}{\partial x^4} + m_u \frac{\partial^2 u}{\partial t^2} + c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) + k_u(u - w) + k_{Nu}u^3 - k_{Nw}w^3 = P(x,t)\exp(i\Omega t) \quad (8a)$$

$$EI_w \frac{\partial^4 w}{\partial x^4} + m_w \frac{\partial^2 w}{\partial t^2} + c_w \frac{\partial w}{\partial t} + k_w w - c_u \left(\frac{\partial u}{\partial t} - \frac{\partial w}{\partial t}\right) - k_u(u - w) + k_{Nw}w^3 - k_{Nu}u^3 = 0 \quad (8b)$$

where the following notations are used:

u and w – vertical displacements of upper and lower beam, respectively;

EI_u, m_u – bending stiffness and mass of upper beam;

EI_w, m_w – bending stiffness and mass of lower beam;

c_u, k_u and c_w, k_w – viscous damping and linear part of stiffness of the layer between beams and the foundation layer, respectively;

k_{Nu}, k_{Nw} – nonlinear part of stiffness of the layer between beams and the foundation layer, respectively;

Because at points far enough from the point of load (moving forces) the central line deflection, its slope and curvature should tend to zero, homogeneous boundary conditions can be applied. The system is subjected to a set of two distributed forces harmonically varying in time with constant frequency $\Omega = 2\pi \cdot f_\Omega$, moving along the upper beam with constant velocity V at the constant distance between them. Each force is distributed on some interval and can be described by the following equation [7-9]:

$$P(x,t) = \frac{P}{r}\cos^2\left(\frac{\pi(x - Vt)}{2r}\right) H(r^2 - (x - Vt)^2) \quad (9)$$

where $H(.)$ is the Heaviside step function and $2r$ is the span of each force. Moving coordinate system is introduced in order to simplify computations: $\tilde{x} = x - Vt$.

Similar system was solved previously in two cases:

1) with linear foundation and nonlinear layer connecting beams [9];
2) with nonlinear foundation and linear stiffness of beams connection [10].

The system described in present paper is essential extension of these two models and, in further study, it will be studied with an assumption of load consisting of three terms mentioned above (Eq. 7). The issue of random load is however omitted in this paper. Future work will lead to transition of the developed approach to railway track analysis.

3. COMPUTATIONAL EXAMPLES

Proposed semi-analytical hybrid approach leads to effective computational procedure allowing parametrical analysis of the investigated system. Such a study must be carried out before application of the developed model to practical problems, including railway engineering and especially the dynamic response of rail track to moving train. This is of importance for detailed analysis of solution convergence, with regard to nonlinear factors appearing in equations, and determination of the range of applicability for particular sets of physical and geometrical parameters. The wavelet approximation itself does not need additional testing in terms of convergence. This can be achieved on the basis of wavelet theory and it was done in previous publications [25]. However, determination of appropriate order of approximation needs careful case-sensitive checking, as the criterion of solution stability depending on equations parameters remains the only one valid way of control.

Figures provided in this paper form a set of initial computational examples calculated by using the developed model. At the first step, it is assumed that nonlinearities in both layers are similar and the influence of the distance between two moving loads on the system response is partially checked. Other parameters are taken from past publications describing linear double-beam behaviour and similar system with one nonlinear layer, both the foundation one either the layer connecting beams [9, 10]: $P = 5 \cdot 10^5\ \mathrm{N/m}$, $k_u = 4 \cdot 10^{13} \mathrm{N/m^4}$, $k_w = 4 \cdot 10^{13} \mathrm{N/m^4}$, $EI_u = 10^7 \mathrm{Nm^2}$, $m_u = 100\ \mathrm{kg/m}$, $k_u = 4 \cdot 10^7 \mathrm{N/m^2}$, $c_u = 0.06 \cdot \sqrt{k_u \cdot m_u}$, $EI_w = 1.5 \cdot 10^9 \mathrm{Nm^2}$, $m_w = 3500\ \mathrm{kg/m}$, $k_w = 5 \cdot 10^7 \mathrm{N/m^2}$, $c_w = 0.06 \cdot \sqrt{k_w \cdot m_w}$, $f_\Omega = 10\ \mathrm{Hz}$. This is done for better possibility of comparative studies of cases with nonlinearities included in double-beam models which is left for further work.

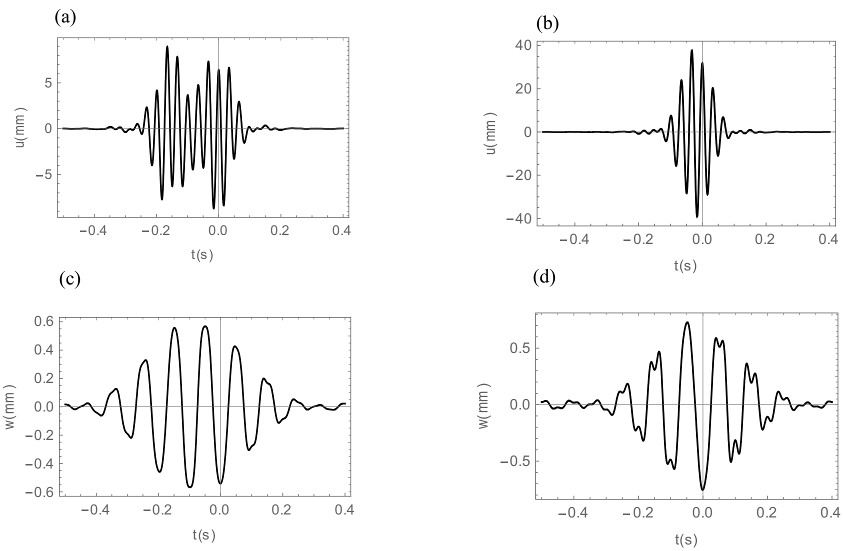

Fig. 1. Vertical vibrations of double-beam system in the case of two nonlinear layers and load speed 25 m/s: (a)-(c) two forces at a distance of 4 m; (b)-(d) two forces at a distance of 1 m.

Figure 1 shows vertical vibrations of two layers (beams) generated by two forces moving along upper beam with speed equal to 25 m/s, at a distance of 4 meters (Fig. 1 (a)-(c)) and 2 meters (Fig. 1 (b)-(d)) apart. One can observe significant growth of vibrations amplitude in the case of upper beam when the distance between forces decreases. The two loads cannot be recognized in signals as separated forces. This observation is similar to those obtained for other double-beam systems subjected to dynamic excitations. One should note that the lower beam starts to vibrate more variationally, i.e. the smooth character of the response function vanishes only due to changes of the distance between forces. This points out stronger influence of nonlinear factors on the structure vibrations when the forces are closer one to another. Vibrations amplitude of lower beam remains almost at the same level for considered set of parameters.

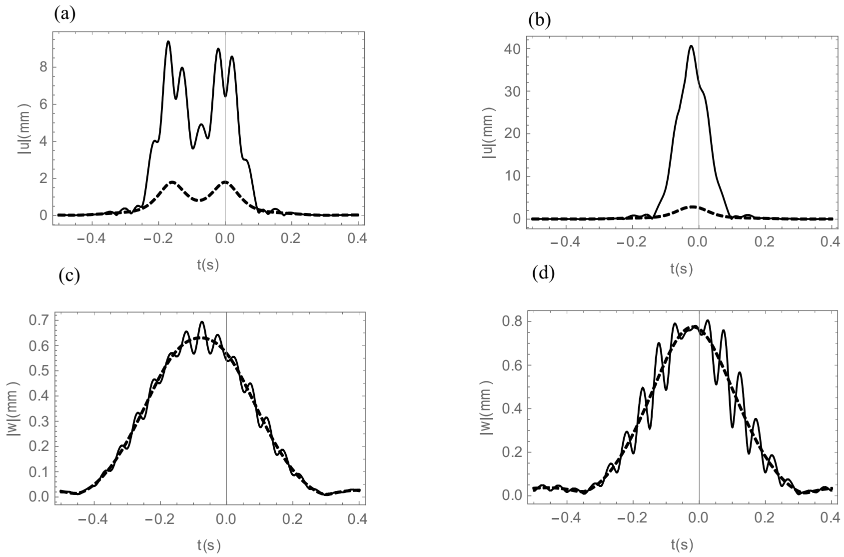

Fig. 2. The maximal response of double-beam system in the case of two nonlinear layers and load speed 25 m/s (dashed - linear; solid - nonlinear): (a)-(c) two forces at a distance of 4 m; (b)-(d) two forces at a distance of 1 m.

In the case of hardly visible changes in characteristics of vibrations (vibration history - comp. Fig. 3 (d)) it is convenient to analyse the "maximal response" represented by complex modulus of solution (Fig. 2). Usually it allows to recognize some hidden features and carry out more detailed analysis of the system sensitivity, in particular for nonlinear problems, where the real part of solution might not have clear interpretation.

Computational examples show that upper beam is more sensitive for nonlinear factor, which is in accordance with studies carried out previously. The lower beam response shows only small changes compared to linear case. One can see in Fig. 2 that the deflection line of lower beam transforms from smooth curve to ragged one, but the amplitude remains very close to linear case, independently of physical parameters of the system. However, the amplitude of vertical vibrations of the lower beam slightly grows with increasing distance between forces. One can observe accumulation of nonlinearity influence on the system for a set of forces placed in a small distance one from another, especially for the upper beam vibrations (Fig. 2 (a)-(b)).

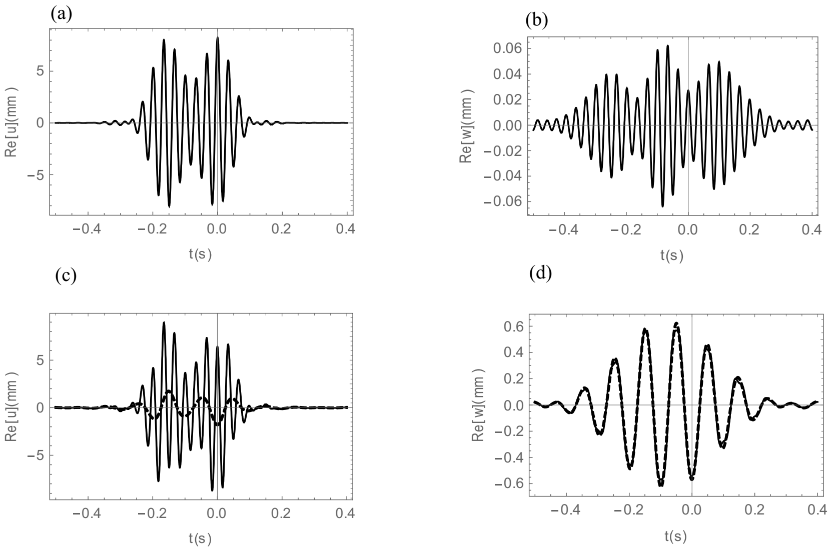

Fig. 3. Vertical vibrations of double-beam system in the case of two nonlinear layers, load speed 25 m/s and the distance between forces equal to 4 m: (a)-(b) nonlinear part of solution; (c)-(d) dashed - linear solution, solid - nonlinear solution.

Figures 3 (a)-(b) show examples of response associated with nonlinear factor only, i.e. without linear part of solution. It is computationally represented by the Adomian series approximating solution of the problem, without its first term which is found as a solution of linear system of equations [7, 8, 22]. One can observe clearly separated responses of upper beam for two moving forces, whereas the lower beam does not reflect the nature of the excitation. Figures 3 (c)-(d) present comparison of solution in linear case with approximate solution of nonlinear system (comp. Fig. 1 (a)). One can see that upper beam is much more sensitive for nonlinear factors compared to the lower one. This can suggest that behaviour of the upper beam will determine convergence of the approximate solution for the considered system with two nonlinear layers.

Computational examples prepared with a use of the developed hybrid semi-analytical method show that parametrical analysis of the investigated infinitely long continuous double-beam can be carried out. More detailed study leading to synthetic analysis and adoption of the method to railway track analysis and other multilayer systems is planned as future work.

4. CONCLUSIONS

Solution of continuous infinitely long double-beam system with nonlinear viscoelastic layer between beams and resting on nonlinear viscoelastic foundation is presented. The system is subjected to a set of two harmonically varying in time forces moving with constant velocity along upper beam and placed in a constant distance one from another. Vertical vibrations of beams are found by using Adomian's decomposition method combined with wavelet based approximation. This hybrid semi-analytical method allows for parametrical analysis of the model leading to possibility of effective study of the investigated structure. More detailed study will enable application of the proposed technique to extend validated previously railway track multilayer models with possibility of fastening system analysis with special emphasis on its nonlinear properties.

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Fig. 1. Vertical vibrations of double-beam system in the case of two nonlinear layers and load speed 25 m/s: (a)-(c) two forces at a distance of 4 m; (b)-(d) two forces at a distance of 1 m.

Rys. 1. Drgania pionowe belki podwójnej z dwoma nieliniowymi warstwami i prędkością obciążenia równej 25 m/s: (a)-(c) dwie siły w odległości 4 m od siebie; (b)-(d) dwie siły w odległości 1 m od siebie.

Fig. 2. The maximal response of double-beam system in the case of two nonlinear layers and load speed 25 m/s (dashed - linear; solid - nonlinear): (a)-(c) two forces at a distance of 4 m; (b)-(d) two forces at a distance of 1 m.

Rys. 2. Odpowiedź maksymalna belki podwójnej w przypadku dwóch nieliniowych warstw i prędkości obciążenia równej 25 m/s (linia przerywana - przypadek liniowy, linia ciągła - przypadek nieliniowy): (a)-(c) dwie siły w odległości 4 m od siebie; (b)-(d) dwie siły w odległości 1 m od siebie.

Fig. 3. Vertical vibrations of double-beam system in the case of two nonlinear layers, load speed 25 m/s and the distance between forces equal to 4 m: (a)-(b) nonlinear part of solution; (c)-(d) dashed - linear solution, solid - nonlinear solution.

Rys. 3. Drgania pionowe belki podwójnej w przypadku dwóch nieliniowych warstw, dla prędkości obciążenia równej 25 m/s i odległości między siłami równej 4 m: (a)-(b) nieliniowa część rozwiązania; (c)-(d) linia przerywana - rozwiązanie liniowe, linia ciągła - rozwiązanie nieliniowe

SEMI-ANALITYCZNE MODELOWANIE NIELINIOWEJ DYNAMIKI WIELOWARSTWOWYCH UKŁADÓW CIĄGŁYCH

Słowa kluczowe: metody semi-analityczne, struktury wielowarstwowe, właściwości nieliniowe, odpowiedź dynamiczna, układ belki podwójnej, modelowanie drogi kolejowej.

STRESZCZENIE:

Problemy dynamiki konstrukcji stanowią jeden z najbardziej istotnych działów współczesnych badań związanych z konstrukcjami kolejowymi. Potrzeba modelowania takich konstrukcji oraz przewidywania ich dynamicznych oddziaływań prowadzi do konieczności poszukiwania nowych rozwiązań, głównie z powodu rosnących prędkości pojazdów i intensywności ruchu. Studia porównawcze prowadzone zarówno w ramach prac eksperymentalnych, pomiarowych, jak i teoretycznych rozważań pokazują, że modele wielowarstwowe w połączeniu z metodami semi-analitycznymi mogą dać nowe spojrzenie na wyniki dotychczasowych analiz. Dokładniejsza analiza elementów drogi oraz ich fizycznych właściwości, razem z zastosowaniem efektywnych estymacji i aproksymacji pozwalających uniknąć numerycznych niestabilności spowodowanych ekstremalnymi dynamicznymi zmianami układów, mogą być ważnymi narzędziami służącymi otrzymaniu nowych rozwiązań, zarówno teoretycznych jak i inżynierskich. Artykuł ten prezentuje w skrócony sposób wybrane modele wielowarstwowe drogi kolejowej, ze szczególnym uwzględnieniem jej nieliniowych właściwości. Zostały omówione istniejące rozwiązania analityczne i semi-analityczne, ze wróceniem uwagi na korzyści płynące z ich zastosowania. Teoretyczny model belki podwójnej został rozwiązany. Pokazano przykłady obliczeniowe wraz z omówieniem możliwości ich adaptacji do analizy innych struktur wielowarstwowych.

Dotychczas opublikowane prace zawierają interesujące wyniki semi-analityczne związane z założeniem nieliniowej sztywności elementów konstrukcji typu belka-podłoże. Modele jednowarstwowy i dwuwarstwowy zostały szczegółowo przeanalizowane przy założeniu nieliniowej sztywności podłoża, przy użyciu semi-analitycznej metody opartej na aproksymacjach falkowych połączonych z dekompozycją Adomiana. Modele te zostały również poddane walidacji poprzez porównanie wyników z pomiarami wykonanymi na rzeczywistym torze kolejowym, pokazujące wysoką zgodność rezultatów w zakresie drgań pionowych drogi kolejowej dla różnych zbiorów parametrów i prędkości pojazdów.

Wstępne badania prowadzące do praktycznych wyników oraz ich zastosowań do analizy dynamiki drogi kolejowej są oparte na analizie czysto teoretycznego modelu nieskończenie długiej belki podwójnej, najpierw w przypadku liniowym, a następnie przy założeniu nieliniowej sztywności podłoża. Stąd inne modyfikacje, jak założenie nieliniowej sztywności warstwy pomiędzy belkami, co odpowiada nieliniowej sztywności systemu przytwierdzeń, powinny być poprzedzone teoretycznymi badaniami układów mechanicznych niekoniecznie bezpośrednio powiązanych z drogą kolejową. Dlatego badany jest model z dwiema warstwami nieliniowymi, przy założeniu układu parametrów analogicznego do użytego w poprzednio rozważanych modelach. Rozpoznanie mechanicznych właściwości takiego układu dynamicznego pozwoli na jego zastosowanie w modelowaniu właściwości drogi kolejowej.

Przykłady obliczeniowe przygotowane z użyciem opracowanej hybrydowej metody semi-analitycznej pokazują, że analiza parametryczna badanego układu ciągłego nieskończenie długiej belki podwójnej jest możliwa. Bardziej

szczegółowe badania prowadzące do syntetycznej analizy i przystosowanie opracowanej metody do analizy drogi kolejowej oraz innych struktur wielowarstwowych są planowane jako przyszłe prace.

Wnioski opisane poniżej zostały sformułowane w oparciu o analizy opisane w artykule.

Zaprezentowano rozwiązanie dla dynamicznej odpowiedzi nieskończenie długiej belki podwójnej z nieliniową warstwą lepkosprężystą pomiędzy belkami, spoczywającej na nieliniowym podłożu lepkosprężystym. Układ został poddany obciążeniu dwóch harmonicznie zmiennych w czasie sił poruszających się ze stałą prędkością wzdłuż górnej belki w stałej odległości od siebie. Drgania pionowe belek zostały opisane przy użyciu dekompozycji Adomiana połączonej z aproksymacją falkową. Ta hybrydowa semi-analityczna metoda pozwala na parametryczną analizę modelu, dając możliwość efektywnego przebadania rozważanego układu. Bardziej szczegółowa analiza pozwoli na zastosowanie zaproponowanej metody do rozbudowy zweryfikowanych poprzednio modeli wielowarstwowych drogi kolejowej, z możliwością analizy systemu przytwierdzeń i szczególnym zwróceniem uwagi na jego nieliniowe właściwości.

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