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TRANSMITTANCE ESTIMATION FOR ANY ELEMENT OF VOLUMETRIC COMPRESSOR MANIFOLD USING CFD SIMULATION

Pressure pulsations occurring in volumetric compressors manifold are still one of the most important problems in design and operation of compressor plants. The resulting vibrations may cause fatigue cracks and noise. Accuracy of the contemporary method is not sufficient in many cases. The methods for calculating pressure pulsation propagation in volumetric compressors manifolds are based on one-dimensional models. In one-dimensional models, the assumption is made that any installation element may be simplified and modeled as a straight pipe with given diameter and length or as a lumped volume. This simplification is usually sufficient in the case of small elements and long waves. In general, the geometry of the element shall be also considered. This may be done using two ways: experimental measurements of pressure pulsations, which lead to transmittance approximation for the investigated element, or CFD analysis and simulation for the acoustic manifold element. In this paper, a new method based on Computational Fluid Dynamics (CFD) simulation is presented. The main idea is to use CFD simulation instead of experimental measurements. The impulse flow excitation is introduced as a source. The results of simulation are averaged in the inlet and outlet cross sections, so time only dependent functions at the inlet and outlet of the simulated element are determined. The transmittances of special form are introduced. On the basis of introduced transmittances, the generalized four pole matrix elements and impedance matrix elements may be calculated. The method has been verified on the basis of experimental measurements.

NOMENCLATURE

b – flow damping coefficient,

c – sound velocity,

$\dot{m}$ – mass flow rate,

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L	–	length,
p	–	pressure,
S	–	cross section area,
w	–	velocity,
τ	–	time,
ω	–	frequency,
Complex values		
$j = \sqrt{-1}$	–	imaginary unit,
P	–	complex pressure (after FFT),
M	–	complex mass flow rate (after FFT),
T	–	transmittance,
$\mathbf{A} = \{a_{ij}\}$	–	four pole matrix,
$\mathbf{Z} = \{z_{ij}\}$	–	impedance matrix.

1. Introduction

The periodicity of volumetric compressor operation is a source of pressure pulsations in its manifold. This problem concerns mainly piston-type compressors but appears also in any other type of the volumetric compressor manifold. An example of pressure pulsations registered in the screw compressor is shown on the Fig 1.

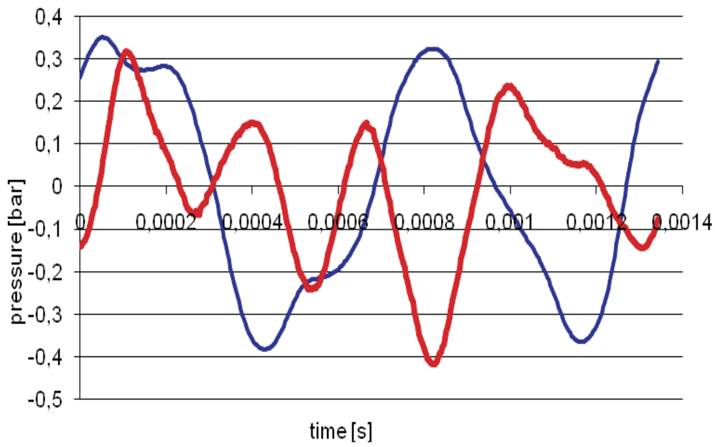

Fig. 1. Pressure pulsations in the discharge manifold of the screw compressor recorded in two points of its manifold (blue line – close to the compressor discharge, red dashed line – after the collector)

The analysis of pressure pulsations in volumetric compressor manifolds is important due to several reasons:

– they have a direct influence on the amount of energy needed for compression of a medium due to such phenomena as dynamic supercharging or just the opposite: dynamic attenuation of the suction and pumping processes;
– they excite mechanical vibrations inside the compressed gas installation leading sometimes to fatigue cracks;
– they are a source of aerodynamic and mechanical noise;
– they affect performance of operating valves in valve compressors, which results in dynamic leaks and premature failure;
– they influence heat transfer in the manifolds.

Theoretical methods for analysis of the interaction between a manifold component and the propagation of a pressure pulsation wave can be divided into three groups:

– Helmholtz analysis and a solution of the so-called telegraphic equation in the complex domain;
– one-dimensional time-space domain solution (differential, characteristics)
– multi-dimensional simulation methods CFD (Computational Fluid Dynamics – FEM, BEM, FVM, FDM).

The advantage of CFD methods is that they make it possible to model any geometry and that they take into account non-linearity of the propagation of finite disturbances. It is also possible to introduce a real gas model or two-phase fluid flow. These methods have been successfully applied in the modeling of exhaust silencers for reciprocating engines. However, compressor manifolds are much more complex. In natural gas pumping stations, where pressure pulsations are serious problem, the manifolds can be composed of many different elements and their total length easily exceeds several hundred meters. Modeling the entire geometry of such an installation using CFD preprocessor would be very laborious itself, and the calculation time is unrealistically long. In addition, any modification of the installation geometry, for example due to the change of a collector or an oil separator, would require new simulation of the entire manifold. Therefore CFD method alone seems to be difficult for application in the case of compressor manifolds.

On the other hand, one-dimensional methods, applied in commercial programs, contain numerous simplifying assumptions. Each element of the manifold is replaced by a straight section of a pipeline having a known length and diameter or by a lumped volume. In the case of oil separators, pressure pulsation mufflers of special design, and even for collectors, such simplifications may lead to serious errors when calculating resonance frequencies. Additional difficulty is related to a medium that not always can be treated as an ideal gas. An example of calculations resulting from the Helmholtz model is shown on the Fig. 2.

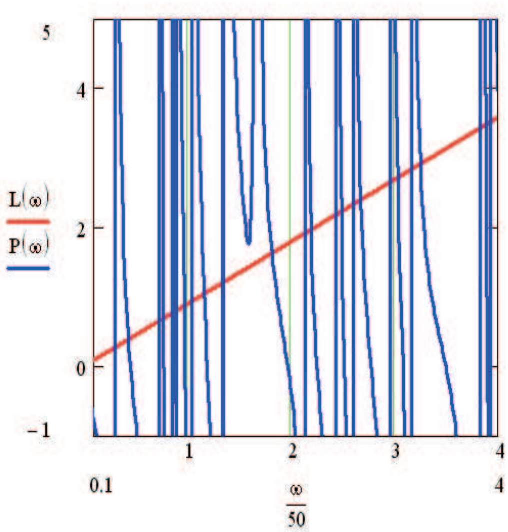

Fig. 2. The result of Helmholtz-model calculation for an example manifold resonance. L, P means left and right side of the characteristic equation. The intersection of L and P lines means resonant state for ω

It may be noticed that the model accuracy is critical in this case because a small change in the model may cause that the "neighborhood" resonance is reached.

There are also experimental methods to determine the acoustical characteristics of an element [4],[7]. They have been developed based on the pressure pulsations measurements at several, specially determined locations of a manifold. They require that the element physically exists, so it is not possible to determine those characteristics in the design stage.

The theoretical method for the determination of parameters of a given element of the manifold already at design stage is still a challenge.

The aim of this paper is to show a new method, developed by the author, for the identification of the manifold element i.e. the method for the identification of the appropriate complex four pole or transmittance matrix elements using CFD simulation.

2. The Helmholtz model

Classic complex domain approach, called here the Helmholtz model, is most used particularly in the USA. The main advantage of this method is the possibility of composing the model of a vast manifold with many branches by simple multiplication of the matrices.

The classic Helmholtz model is based on a solution of the wave equation of the form (1), derived for a straight pipeline. Solution result in complex domain is a four-pole matrix presented by equations (2) and (3). Elements of this matrix $\{a_{ij}\}$ are determined strictly for a segment of a pipeline. Concurrently with a four-pole matrix, a complex impedance matrix $\mathbf{Z}$ having the elements $\{z_{ij}\}$ defined by the relation (4) may be used.

$$\begin{cases} -\dfrac{\partial p}{\partial x} = \dfrac{1}{S}\dfrac{\partial \dot{m}}{\partial \tau} + \dfrac{b}{S}\cdot \dot{m} \\ -\dfrac{\partial \dot{m}}{\partial x} = \dfrac{S}{c^2}\dfrac{\partial p}{\partial \tau} + c_m S\left(p - p_o\right) \end{cases} \tag{1}$$

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} \cosh \gamma L & Z_f \sinh \gamma L \\ \dfrac{1}{Z_f}\sinh \gamma L & \cosh \gamma L \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{2}$$

$$\begin{bmatrix} P_1 \\ M_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \cdot \begin{bmatrix} P_2 \\ M_2 \end{bmatrix} \tag{3}$$

$$\begin{bmatrix} P_1 \\ P_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \cdot \begin{bmatrix} M_1 \\ M_2 \end{bmatrix} \tag{4}$$

Where:

$$\left.\begin{aligned} \gamma &= \sqrt{(b + j\omega)\left(\frac{j\omega}{c^2}\right)} \\ Z_f &= \frac{1}{S}\sqrt{\frac{(b + j\omega)}{\frac{j\omega}{c^2}}} \end{aligned}\right\} \tag{5}$$

In order to generalize the model for any geometry, it has been assumed that the forms of matrices $\{a_{ij}\}$ and $\{z_{ij}\}$ will not be based on equation (2), but that they will have a completely free form. This assumption leads to the statement that for a unique determination of the interaction of any acoustical element in a manifold it is sufficient to identify four elements of the four-pole or impedance matrix. The aim of the present work was to develop a method for the identification of these matrix elements.

The concept of the method developed here is the following: for a considered element of a manifold, a full multi-dimensional CFD non-linear simulation is carried out, solving the Navier-Stokes set of equations numerically together with the necessary closing models, i.e. gas state model, turbulence model, and boundary conditions. The results obtained are averaged at the inlet and outlet of the analyzed element, and then a complex transformation of the results is carried out, so that the transmittances consistent with the generalized form of matrices $\{a_{ij}\}$ and $\{z_{ij}\}$ can be calculated. In this way, the advantages of both methods can be combined: the Helmholtz model's possibility of analysis of geometrically complex installations, and the possibility of introducing real geometry of any element, without "a priori" simplifications.

3. The transmittance approach

The simple complex transmittance of the manifold element may be defined as follows:

$$T_{PM} = \frac{P_2}{M_1}; \qquad T_M = \frac{M_2}{M_1} \tag{6}$$

The input mass flow M_1 is, in both cases, the excitation, and the output pressure P_2 (or mass flow M_2) is the response. For an unsymmetrical element, the "forward" and "backward" transmittances are different.

The manifold element works as a part of the system, so in the system the transmittances (6) depend on the whole manifold reaction.

The CFD simulations have to be prepared independently from the rest of the manifold, so that the unique elements of four-pole matrices could be calculated on this basis.

When using the CFD, it is convenient to put as a boundary conditions the closed end with the closing impedance $Z_k = \propto$ and $M_2 = 0$ or an open end with $Z_k = 0$ and $P_2 = 0$ (Fig. 3).

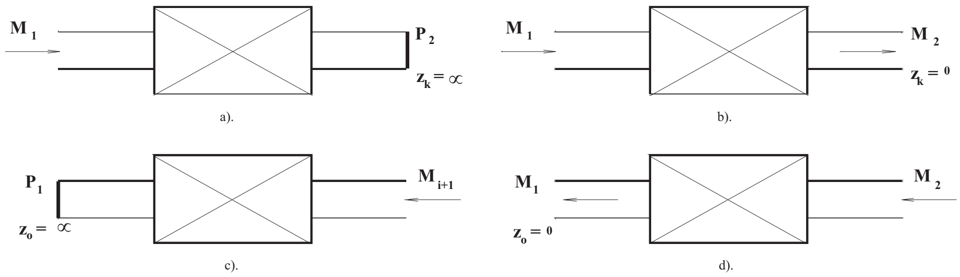

Fig. 3. Four cases for non-symmetrical element with boundary conditions for CFD simulation

For the a) and b) boundary conditions with "forward" flow, after rearrangements of (1) and (2) for $Z_k = \propto$ we get:

a) $$T_{PM} = \frac{P_2}{M_1} = z_{21} = \frac{1}{a_{21}} \tag{7}$$

for $Z_k = 0$:

b) $$T_M = \frac{M_2}{M_1} = \frac{1}{a_{22}} = -\frac{z_{21}}{z_{22}} \tag{8}$$

If the element is symmetrical for the "backward" flow, the same results are obtained. In the case of unsymmetrical elements we get for backward flow (c), (d):
for $Z_k = \propto$:

c) $$T'_{PM} = \frac{P_1}{M_2} = \frac{-\det\{a_{ij}\}}{a_{21}} = z_{12} \tag{9}$$

for $Z_k = 0$:

d) $$T'_M = \frac{M_1}{M_2} = -\frac{\det\{a_{ij}\}}{a_{11}} = -\frac{z_{12}}{z_{11}} \tag{10}$$

Since the set of four equations, i.e. (7), (8), (9), (10), has in each case four unknowns $\{a_{ij}\}$ or $\{z_{ij}\}$ – its solution is unique. This means that once we have all complex transmittances T_M, T_{PM}, T'_M and T'_{PM}, the four pole $\{a_{ij}\}$ and impedance $\{z_{ij}\}$ matrices may be uniquely calculated.

4. Estimation of transmittances

The derivation of transmittances $T(i\omega)$ for each of the four cases (a),(b),(c) and (d) and for a dozen or so significant harmonics is very time consuming. Therefore, it is better to use a system response for an impulse excitation.

Impulse excitation is defined as follows:

$$\delta(\tau) = \begin{cases} 0 & \text{for} \quad \tau \neq 0 \\ 1 & \text{for} \quad \tau = 0 \end{cases} \tag{11}$$

$$\ell\{\delta(\tau)\} = 1 \tag{12}$$

where ℓ – means Laplace transform,

which means that, based on a system response to an impulse δ function in each of the four cases a, b, c, d, one can determine characteristics of the studied element.

Transmittance can be determined analytically only for a straight section of a pipe having constant cross-section S and length l:

$$\left.\begin{aligned} T_a = -T_c &= \frac{Z_f}{\sinh(\gamma L)} \\ T_b = -T_d &= \frac{1}{\cosh(\gamma L)} \end{aligned}\right\} \tag{13}$$

The method for the transmittance estimation is based on the nonlinear CFD modeling with full set of the Navier-Stokes equations solved with eddy modeling, equation of state for gas etc.

Since the physical phenomena of traveling wave have the form of oscillating movement, the transmittances may be modeled as such.

a) First-order transmittance (only damping and time lag) has the form:

$$T(s) = \frac{K}{1 + s \cdot \zeta} \cdot e^{-s\Delta\tau} \tag{14}$$

b) Second-order transmittance has the form:

$$T(s) = \frac{K \cdot \omega_o^2}{s^2 + 2\zeta\omega_o s + \omega_o^2} \cdot e^{-s\Delta\tau} \tag{15}$$

In practice it occurs that the transmittance function have general form of damped oscillation response with multiple free frequencies ω_{oi}:

$$T(s) = e^{-s\tau_o} \sum_{i=1}^{n} \frac{K_i \omega_{oi}^2}{s^2 + 2\zeta_i \omega_{oi} s + \omega_{oi}^2} \tag{16}$$

where:

τ_o – response delay time, ω_{oi} – free oscillation frequency, K_i – amplification factor for i-th frequency, ζ_i – damping factor for i-th frequency.

The transmittance of the form (14)-(16) may be obtained as the result of simulation using an impulse function for excitation. In the case of numerical calculations using CFD, the duration time for the impulse will equal to one single time-step $\Delta\tau$.

The excitation u_o may be an impulse of the mass flow or velocity at the inlet. At the outlet, the response of the system is the pressure or mass flow function. In both cases, the formula (16) is valid. As the result, the p(x,y,z,τ) or $\dot{m}$ (x,y,z,τ) function is calculated, therefore it has to be averaged within the pipe cross section at the outlet to obtain pure p(τ) or $\dot{m}$ (τ) functions. The oscillation response for each individual frequency ω_{oi} has the form shown in Fig. 4. The parameters K_i, ω_{oi}, ζ_i, τ_o in equations (14)-(16) could be easily calculated from the function results, with the following additional relationships:

$$\zeta_i = B\sqrt{\frac{1}{\left(\frac{2\Pi}{\tau_o}\right)^2 + B^2}}; \quad K_i = \frac{A_i\sqrt{1-\zeta_i^2}}{u_o\omega_{oi}\Delta\tau}; \quad \omega_{0i} = \frac{2\pi}{\tau_{ci}\sqrt{1-\zeta_i^2}}; \tag{17}$$

The mathematical interpretation of the values B, τ_c, A, τ_o is shown in Fig. 4, $\Delta\tau$ is the duration of the single impulse excitation of the amplitude u_0.

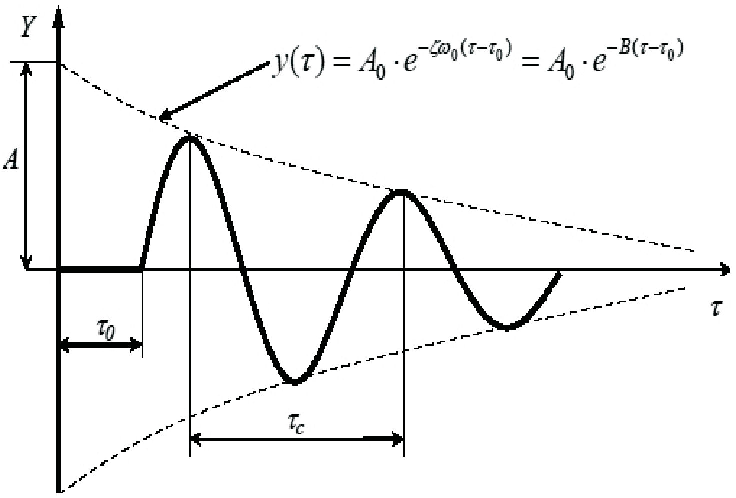

Fig. 4. Damped oscillating function and graphic presentation of its parameters

5. CFD identification applied to the muffler of special design

The method developed in this work was applied to the pressure pulsation muffler shown in Fig. 5.

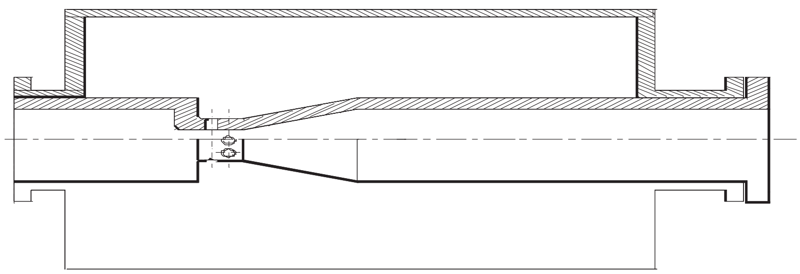

Fig. 5. Specially designed muffler used for experimental verification of the model

A simulation model of the muffler was created in a cylindrical coordinate system, using axial symmetry, which reduced the case to a two-dimensional one. A geometric model was introduced into the PHOENICS program on a PC computer. Along the radius, the grid contained 39 unequal elements resulting from geometry and 86 along the symmetry axis. The case of unsteady flow was considered, both at step and impulse excitations, with velocity amplitude of 10 [m/s]. It was decided to consider a time interval of 0÷2 [s] divided unevenly (more densely at the beginning) into 131 parts. An ideal gas model and k-ε turbulence model were used for simulation. Figs 6-8 present examples of the simulation results.

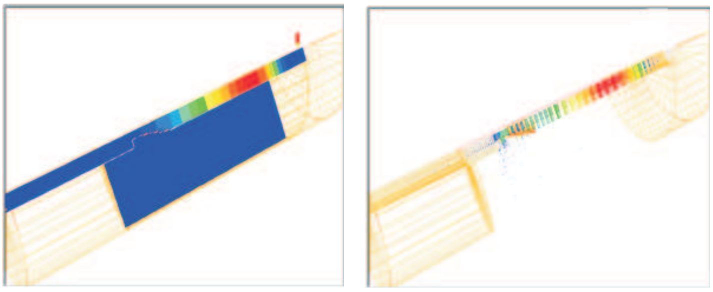

Fig. 6. Pressure distribution after 0.001 s. Velocity vectors after 0.001 s

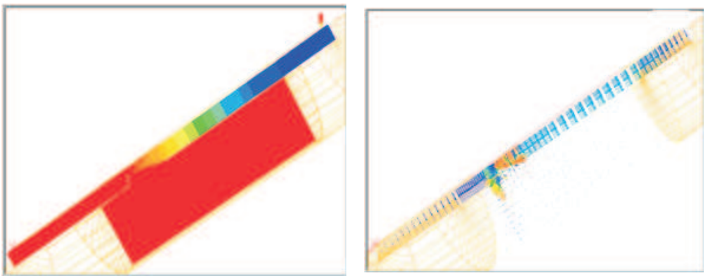

Fig. 7. Pressure distribution after 0.003 s. Velocity vectors after 0.003 s

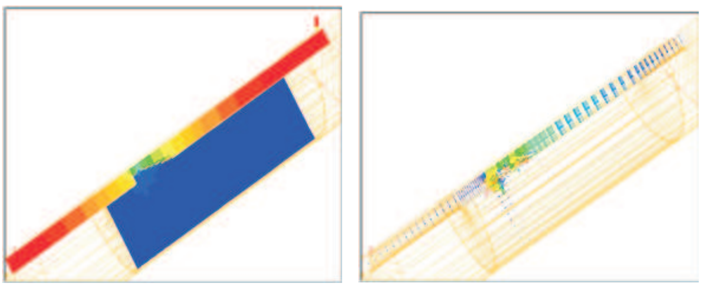

Fig. 8. Pressure distribution after 0.01 s. Velocity vectors after 0.01 s

In order to use the CFD calculation results in the Helmholtz model, the values of pressure function and mass flow rate were averaged at the inlet and outlet cross section.

Based on the diagrams obtained for a step excitation, the parameters of the first order transmittance (14) were determined.

The transform contains only damping and time lag, and the values of the coefficients are given in Table 1.

The coefficients K, ζ, ω_o, $\Delta\tau_1$ for equation (15) were determined from the time response diagrams of the m(τ) and p(τ) functions. It has been found that the values of K, ζ and $\Delta\tau_1$ correspond to the values obtained for the first order transmittance according to the relation (14). The frequency ω_o was determined graphically by measuring the time between subsequent crests of wave.

Table 1.

Values of coefficients in the (14) and (15) formulas

Transmittance	K	ω_o	ζ	$\Delta\tau$
T_a	28154	22	0.053	0.187
T_b	−0.8	35	0.095	0.186
T_c	46154	22	0.060	0.190
T_d	−1.12	35	0.130	0.185

6. Experimental verification of the developed method

In order to verify experimentally the computed matrices, an experimental set-up was constructed based on air compressor S2P216 operating with a possibility of variable control of rotational velocity (Fig. 9). In this test stand, the studied muffler was assembled on the suction line of a compressor. A measurement system consisted of capacity transducers coupled through a converter and amplifier to a transient recorder, and recorded on a PC computer. The measured P_2 curves were compared with the calculated values.

The calculated values of transmittances were used to evaluate pressure pulsations at the P_2 point based on the measured curves at the P_3 point. In this way, the interaction of the studied muffler was simulated on the basis of its computed characteristics. The method of a pressure wave decomposition into a traveling and backward component, which was applied here, had been earlier described by the author in [6] and [7]. Therefore, it will not be presented here.

Figs 10 and 11 present a comparison between experimental results and pressure curves (Fig. 10), and harmonic analysis of the pressure, obtained at

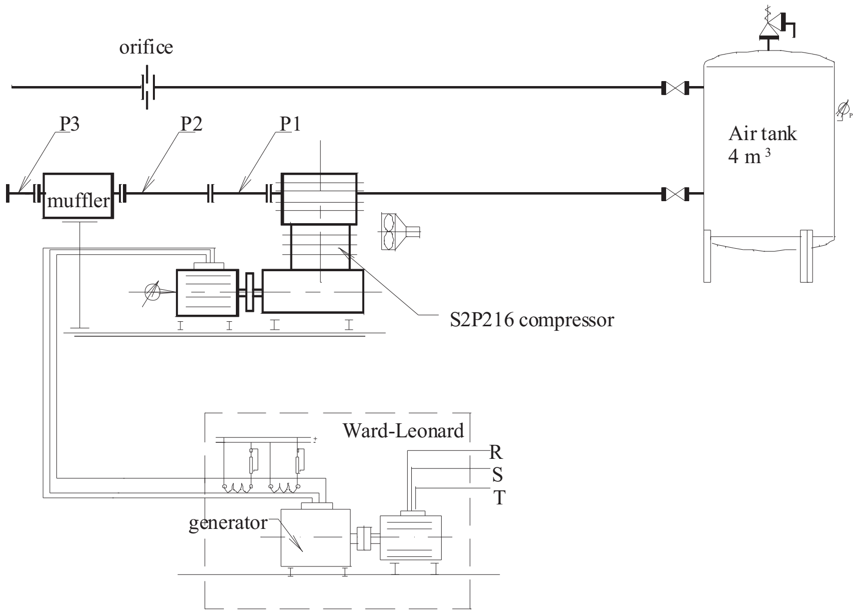

Fig. 9. Test stand for the identification of the manifold element

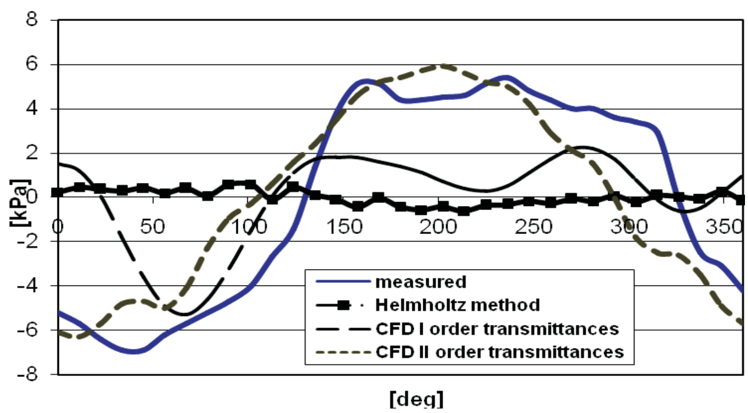

Fig. 10. Comparison of pressure pulsation curves before the damper (P_2) obtained by four methods

the P_2 point using the first-order and second-order transmittance for significant harmonics (equation 14, 15). As it can be seen, even first-order transmittance calculated on the basis of CFD simulation is a better approximation of the actual pressure pulsations than the classic Helmholtz model.

The results of identification obtained with the use of CFD modeling are very promising. Even the simple first-order model gives a better response compared to the classic Helmholtz model.

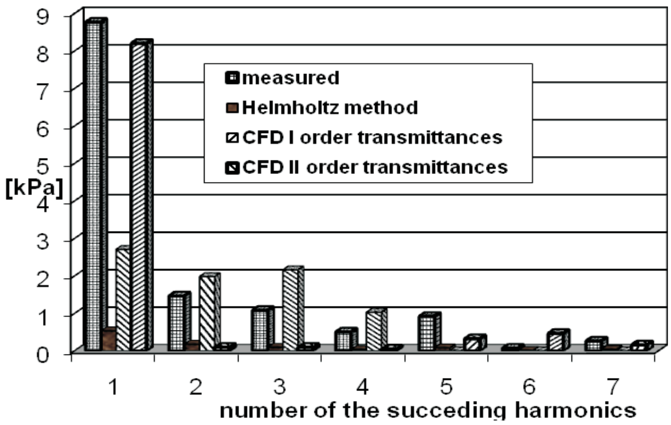

Fig. 11. Comparison of harmonic analysis results for the curves from Fig. 10

7. Summary and conclusions

The most important conclusion of this work is that in the manifold the identification of acoustic element parameters based on multi-dimensional simulation model (CFD) is feasible. The author obtained much better results from the developed method than those yielded by the classic Helmholtz model.

In the present work, mixed transmittance matrices T_{MP}, T_P were introduced apart from a matrix of wave decomposition and pressure transmittance. This is a new approach which makes it possible to use the CFD modeling for identification of parameters for a generalized Helmholtz model.

Direct application of CFD method for the entire manifold modeling is problematic not only because of computational time requirement, but also because introducing the whole geometry of a vast manifold system into the program is a very time consuming task. Therefore, it is more convenient to perform the identification of parameters i.e. of mixed transmittances T_{MP} and T_P, and on this basis estimate the four-pole or impedance matrices, and use the matrix multiplication product as a manifold model.

The method is based on an "a posteriori" linearization of the nonlinear CFD simulation results, which is the reason for higher accuracy of the method. The conclusions resulting from the application of this method are the following:

a) CFD simulation based method of the identification of objects with high damping gives better results for the pressure pulsations amplitude than the classic Helmholtz model, even when a first-order transmittance containing only damping and time lag is used. However, for simple geometries (pipe, tank) one should expect that the Helmholtz model would be better than the first-order transmittance.

b) The assumption of a second-order transmittance with four parameters improves the simulation results, and gives a better agreement with the experimental results.

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Obliczanie transmitancji dowolnego elementu instalacji sprężarki wyporowej za pomocą symulacji CFD

S t r e s z c z e n i e

Pulsacje ciśnienia powstające w instalacjach sprężarek wyporowych są wciąż jednym z najważniejszych problemów przy projektowaniu i eksploatacji stacji sprężarkowych. Wynikające z nich drgania mogą powodować pęknięcia zmęczeniowe i hałas. Dokładność współczesnych metod obliczeniowych jest w wielu przypadkach niewystarczająca. Metody obliczające pulsacje ciśnienia w instalacji bazują na modelach jednowymiarowych. W tych metodach czynione jest założenie że każdy element instalacji można zastąpić rurą o stałym przekroju i określonej długości lub zbiornikiem jako parametrem skupionym. To założenie jest zwykle wystarczające w przypadku małych elementów i długich fal akustycznych. Ogólnie jednak geometria elementu powinna być również brana pod uwagę. To można zrobić dwoma metodami: poprzez pomiary eksperymentalne pulsacji ciśnienia co prowadzi do estymacji transmitancji dla badanego elementu lub przez analizę CFD i symulację elementu akustycznego instalacji. W niniejszej pracy zaprezentowano nową metodę opartą

na symulacji CFD (Computational Fluid Dynamics). Główną ideą jest zastosowanie CFD zamiast badań eksperymentalnych. Wprowadzone jest przepływowe wymuszenie impulsowe jako źródło. Wyniki symulacji są uśredniane w przekroju wlotowym i wylotowym, w ten sposób otrzymuje się funkcje zależne wyłącznie od czasu dla badanego elementu. Na podstawie obliczonych transmitancji oblicza się macierz impedancyjną i cztero-biegunową. Metoda została zweryfikowana w oparciu o badania eksperymentalne.