WARSAW UNIVERSITY OF TECHNOLOGY Index 351733
FACULTY OF CIVIL ENGINEERING
COMMITTEE FOR CIVIL AND WATER ENGINEERING
POLISH ACADEMY OF SCIENCES ISSN 1230-2945

DOI: 10.24425/ace.2020.134417
ARCHIVES OF CIVIL ENGINEERING
Vol. LXVI ISSUE 3 2020



# COMPARISON OF THE BENDING STRENGHT OF A STEEL PLATE–CONCRETE COMPOSITE BEAMS DEFINED EXPERIMENTALLY, THEORETICALLY AND NUMERICALLY

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The aim of the paper was to assess the bending strength of steel plate - concrete composite members, based on an experimental study performed by the authors together with theoretical and numerical analyses. The values of the mechanical parameters of the materials the beams were made from were adopted on the basis of the tests results. The proposed solutions have been verified by experiment. For this purpose the results of tests performed by the authors and other researchers have been used. The former ones are original, and the way of their presentation makes them applicable by other researchers. Following the results it can be stated that with respect to bending strengths from the experimental study the results obtained from the theoretical analysis are underestimated 6,6 % on average. The results based on the numerical analysis, on the other hand, are overestimated by - 7,5 % on average. The results of the theoretical and numerical analyses indicate that the interface slip between the steel plate and concrete part affect the bending strength of steel plate-concrete composite beams only slightly (about 2% on average).

*Keywords*: concrete, steel, composite beam, bending strength, experimental tests

## 1. INTRODUCTION

The term of composite structures covers a wide range of members composed of at least two interacting components made from different materials. The classical composite structures are of steel-concrete type, used for over a hundred years now. Also concrete-concrete structures are popular. Composite timber-concrete and steel-timber structures are much less common. Steel plate-

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concrete composite structures, in which the steel girder is replaced with a steel plate, are a version of steel-concrete composite structures. A scheme of such a member is shown in figure 1.

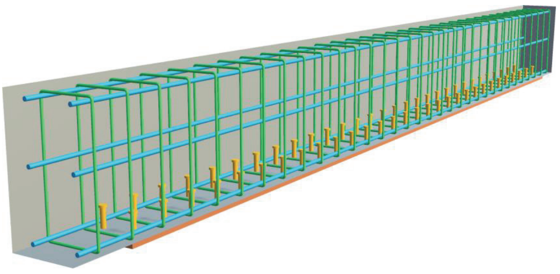

Fig. 1. A steel plate-concrete composite member

A steel plate-concrete composite member is sometimes considered to be merely a version of a reinforced concrete member in which the rebars are replaced with a steel plate of an adequate cross section. We cannot agree with such a view since when these members are under loading, qualitatively different physical phenomena take place connected with, for instance, slip, nature and effect of concrete cracking or the impact of shrinkage on concrete creep. What essentially distinguishes a typical reinforced concrete beam from a steel-plate-concrete composite beam is a different impact of concrete shrinkage. In RC beams concrete shrinkage makes the concrete press against the reinforcing bars, thus increasing natural adhesion. In steel plate-concrete composite beams, on the other hand, concrete shrinkage results in a decrease of the natural adhesion between the steel and concrete parts. In typical RC beams the concrete - rebars interaction is secured by adhesion, while in steel plate-concrete composite beams the interaction is secured by additional elements – connectors. Studies on steel plate-concrete composite structures have no long-established tradition [4, 6, 12]. On the one hand, they are modified classical steel-concrete composite structures, on the other hand, however, a continuation of Möller's structures [4], used as early as the late 19th c. There is a certain similarity between the analysed structures and RC members strengthened with steel plates or composite strips bonded with them [5]. To illustrate an up-to-date application of steel plate-concrete composite elements Wildeger's bridge [12] can be quoted. In this bridge this type of elements are used for cantilever footpath brackets. Owing to a short period in which steel plate-concrete composite beams have been used (from the time Möller's bridges were built no advanced theoretical solutions or results of experimental tests have not been preserved [4, 12]) many problems remain to be solved. The studies done so far concern mainly carrying capacity and deflections under short-term loads [12, 16]; less frequently the cracking state and the impact of concrete shrinkage and creep. In practical engineering calculations the time parameter, as well as concrete shrinkage and creep deformations have been disregarded. In the design of the analysed members the solutions valid for the classical steel-concrete composite structures cannot be applied directly despite the good knowledge in this field [10, 12, 16, 18]. This is because of completely different proportions between the concrete and steel sections in the cross section of the beam (member). The difference results, *inter alia*, from the impact of the concrete tension zone on the member's strength and rigidity, especially at the level of useful loads. The aim of the present paper is to propose solutions for the bending strength of steel plate-concrete composite members and to perform an analysis of the impact of selected parameters. The scope of the paper covers the provision of relevant formulae considering the specificity of the analysed members. The analysis of the proposed solutions includes the geometry of beams' cross section and the mechanical properties of the component materials. The proposed solutions have been verified experimentally. For this purpose the results of the studies performed by the author [12] and those performed by other researchers [16] have been used. The author's research is described in part 4 of the paper, and its results together with the results of the analysis are presented in part 5. They are original in character and the manner of the presentation of the results makes them applicable by other researchers.

## 2. GENERAL ASSUMPTION

To solve the problem of steel plate-concrete composite beams bending strength the validity of the principle of plane sections was adopted together with the principle of superposition over the entire range of the analysed load (before plasticization of steel plate). Plasticization of concrete zone in compression and plasticization of steel plate were assumed only in considering the conditions of the equilibrium of sections in the state of ultimate moment capacity. For the steel and concrete parts to interact connectors are used (flexible connectors were applied). The bending stresses between steel plate and concrete were disregarded. On the assumption of connectors flexibility and lack of bonding the possibility of slip occurrence was described in a natural way. Consequently, the bending strength of the beams in question is lower than that calculated on the assumption of rigid connection (with no slip). In solving the problem of bending strength, the operation of section identical as in the case of RC members was initially assumed. At the next stage, the specificity of steel plate-concrete composite members, including the interface slip between the steel and concrete, was taken into account. As a result, the mode of calculation was close as practicable the real design of the element. The other assumptions are given where they apply.

## 3. SOLUTION OF THE PROBLEM

At the initial design stage, in the identification of the state of strain and stress in the steel plate-concrete composite beam section disregarding the interface slip between the steel plate and concrete the general theory of reinforced concrete is applicable [3]. However, in the next stage - at higher loads levels - the interface slip between the steel plate and concrete should be included. The steel plate-concrete connection - despite the use of connectors - is flexible [2, 6, 7, 12, 14, 15, 31]. In considering the impact of the slip let us analyse a simply supported beam under two concentrated

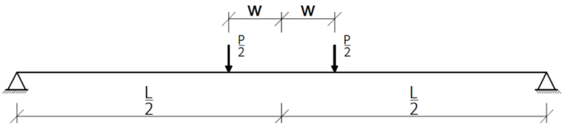


Fig. 2. Static scheme and loading of the analysed beam

loades placed symmetrically (cf. Fig. 2). Using the equations of equilibrium we obtain [12]

$$\mathrm{d}F/\mathrm{d}x = -\tau \qquad \text{Eq-1}$$

$$(\mathrm{d}M_c)/\mathrm{d}x + V_c = 0{,}5\, h_c\, \tau \qquad \text{Eq-2}$$

$$(\mathrm{d}M_s)/\mathrm{d}x + V_s = 0{,}5\, h_s\, \tau \qquad \text{Eq-3}$$

$$V_c + V_s = P/2 \qquad \text{Eq-4}$$

Where $F$ – longitudinal force,

$M_c$ – bending moment transferred through the concrete part,

$M_s$ – bending moment transferred through the steel part,

$V_c$ – transverse force in the concrete part,

$V_s$ – transverse force in the steel part,

$P$ – total force acting on the beam,

$\tau$ – transverse force per unit of length in the composition plane,

$h_c$ – height of the concrete part,

$h_s$ – height of the steel part.

Assuming that

$$d_c = \tfrac{1}{2}\,(h_c + h_s) \qquad \text{Eq-5}$$

and using equations (2), (3) and (4) we obtain

$$(\mathrm{d}M_c)/\mathrm{d}x + (\mathrm{d}M_s)/\mathrm{d}x + P/2 = d_c\,\tau \qquad \text{Eq-6}$$

The beam curvature is calculated from the formula

$$\phi = M_s\,/\,(E_s\,I_s\,) = n\,M_c\,/\,(E_s\,I_c\,) \qquad \text{Eq-7}$$

Where $\phi$ – beam curvature,

$E_s$ – elasticity modulus of steel,

$E_c$ – elasticity modulus of concrete,

$n$ – elasticity modulus of steel $E_s$ – elasticity modulus of concrete $E_c$ ($n = E_s\,/\,E_c$) ratio,

$I_s$ – moment of inertia of the steel part,

$I_c$ – moment of inertia of the concrete part.

The longitudinal strain in concrete and steel can be calculated form the formulae [10]:

$$\varepsilon_{AB} = 0{,}5\,h_c\,\phi - [n\,F/(E_s\,A_c)] \qquad \text{Eq-8}$$

$$\varepsilon_{CD} = -\,0{,}5\,h_s\,\phi + F\,/\,(E_s\,A_s) \qquad \text{Eq-9}$$

Where $A_c$ – sectional area of the concrete part,

$A_s$ – sectional area of the steel part.

Slip $\varepsilon_s$ is equal to the strain difference $\varepsilon_{AB}$ and $\varepsilon_{CD}$ (cf. Fig. 3).

Therefore

$$\varepsilon_s = \mathrm{d}_s\,/\,\mathrm{d}x = \varepsilon_{AB} - \varepsilon_{CD} = \phi\,d_c - [(F\,/\,E_s\,)\,(n\,/\,A_c + 1\,/\,A_s)] \qquad \text{Eq-10}$$

Using (6) and (7) we obtain

$$(\mathrm{d}\phi\,/\,\mathrm{d}x)\,E_s\,(I_s + I_c\,/\,n) + \mathrm{P}/2 = d_c\,\tau \qquad \text{Eq-11}$$

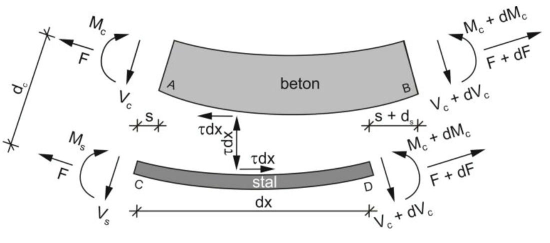


Fig. 3. Element of unitary length d$x$ and system of acting forces

The relationship between the forces acting in the connection interface, slip and connectors rigidity is described by the dependence

$$p\,\tau = k\,s \qquad \text{Eq-12}$$

Where $p$ – horizontal spacing of connectors (flexible),

$\tau$ – longitudinal force per unit of length in the connection interface,

$k$ – connector's rigidity,

$s$ – slip.

Using (11) and knowing that

$$I_o = I_s + I_c / n \qquad \text{Eq-13}$$

we obtain [12]:

$$d\phi / dx = [(k\,s\,d_c) / p - P/2] / (E_s\,I_o) \qquad \text{Eq-14}$$

Differentiating (14) and using the fact that:

$$A_o = (A_s\,A_c) / (n\,A_s + A_c) \qquad \text{Eq-15}$$

we obtain

$$(d^2 s) / (dx^2) = k\,s / (p\,E_s\,I_o)\,(d_c^2 + I_o / A_o) - [(0{,}5P\,d_c) / (E_s\,I_o)] \qquad \text{Eq-16}$$

The solution of equation (16) – while using the boundary conditions: s(0) = 0 and d$s$/d$x$ ($L$/2 = 0 – is given by the equation

$$s(\mathrm{x}) = \beta\,P\,[e^{(\alpha x\,-\,\alpha L)} + e^{(-\alpha x)})] / [2\,(1 + e^{(-\alpha L)}] + \beta\,P/2 \qquad \text{Eq-17}$$

Then strain $\varepsilon_s$ ($x$) resulting from the slip is

$$\varepsilon_s(x) = ds / dx(x) = \alpha\,\beta\,P\,[e^{\alpha x} - e^{(\alpha L\,-\,\alpha x)}] / [2\,(1 + e^{\alpha L})] \qquad \text{Eq-18}$$

Beam's additional curvature $\Delta\phi$ induced by the slip between the concrete and steel parts is

$$\Delta\phi = \varepsilon_s / h \qquad \text{Eq-19}$$

At $\Delta f$/d$x$ (0) = 0 and $\Delta f(L/2) = 0$ (point zero is assumed to be in the mid-span of the beam) we obtain

$$\Delta f = \Delta f(w) = \beta\,P\,[(L - 2w) / 4h) + (e^{\alpha w} - e^{(\alpha L\,-\,\alpha w)}] / [2\alpha\,h\,(1 + e^{\alpha L})] \qquad \text{Eq-20}$$

Using

$$f_{tot} = \alpha_k\,(M\,L^2) / 8 = \alpha_k\,(M\,L^2) / [B_o\,(1 + \xi)] \qquad \text{Eq-21}$$

and formula (20) as well as dependence (for a simply supported beam symmetrically loaded by two concentrated forces)

$$\mathrm{P} = 4\,M / (L - 2w) \qquad \text{Eq-22}$$

we obtain

$$\xi = \Delta f / [(\alpha_k\,(M\ \ L^2) / \mathrm{B_o}] \qquad \text{Eq-23}$$

Coefficient $\xi$ in formula (30) is described by the formula

$$\xi = f_{tot} / f_c - 1 = (4B_o\,\beta)/(\alpha_k\,(L - 2w)\,L^2\,)\,\Delta f \qquad \text{Eq-24}$$

or, after further transformation

$$\xi = (4B_o\,\beta) / [\alpha_k\,(L - 2w)\,L^2]\,[(L - 2w)/4h + (e^{\alpha w} - e^{(\alpha L-\alpha w)})/[2\alpha\,h\,(1 + e^{\alpha L})] \qquad \text{Eq-25}$$

Adopting the assumption that the slip induced strain changes linearly at the height of the cross section (cf. Fig. 4), additionally using the solution given in [15] the following can be written

$$\varepsilon_{ss}/h_s = \varepsilon_s/\mathrm{h} \qquad \text{Eq-26}$$

The additional longitudinal force $\Delta N_s$ created by the slip is then:

$$\Delta N_s = E_s\,A_s\,\varepsilon_{ss} = h_s\,/\,h\;\varepsilon_s\,E_s\,A_s \qquad \text{Eq-27}$$

while the bending moment:

$$\Delta M_s = \Delta N_s\,d_c = h_s\,/h\;(d_c\,\varepsilon_s\,E_s\,A_s \qquad \text{Eq-28}$$

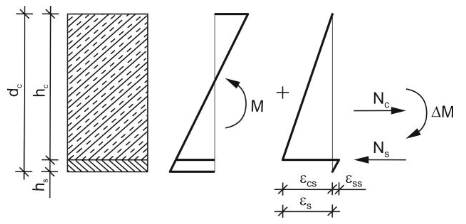


Fig. 4. Strain distribution at the height of cross section created by the slip between the steel plate and concrete

The total curvature of the beam is described by the formula:

$$\phi = M\,/\,B = M\,(1 + \xi)\,/\,B_o \qquad \text{Eq-29}$$

The slip's induced additional curvature is then:

$$\Delta\phi = M\,\xi\,/\,B_o = \varepsilon_s\,/\,h \qquad \text{Eq-30}$$

Hence:

$$\varepsilon_s = M\,h\,\xi\,/\,B_o \qquad \text{Eq-31}$$

Using (28) and (31) after transformations we obtain:

$$\Delta M_s = (h_s\,d_c\,M\,\xi\,E_s\,A_s)\,/\,B_o \qquad \text{Eq-32}$$

Hence the total bending moment $M_p$ including the slip effect is:

$$M_p = M - \Delta M_s \qquad \text{Eq-33}$$

For comparative analyses $\gamma$ ratio of the additional moment resulting from the slip to the moment calculated at the assumption of complete connection is useful. It is:

$$\gamma = (\Delta M_s)\,/\,M = (h_s\,d_c\,\xi\,E_s\,A_s)\,/\,B_o \qquad \text{Eq-34}$$

# 4. EXPERIMANTAL TESTS DONE BY THE AUTHOR

The experimental tests were performed on six beams of the total length of 5.20 m, the support span (theoretical) of 5.00 m. The dimensions of the cross section were 0.24 m (width) x 0.49 m (height). The steel plate was 10 and 16 mm thick. The length of the steel plate was 4.74 m (the plate ends did not reach the supports). The cross sections of the beams are shown in figure 2. The steel plate was bonded to the concrete (reinforced concrete) by means of connectors 13 mm in diameter. The connectors spacing varied and was between 80 mm and 200 mm. The beams' symbols, connectors' spacing and steel plate's thickness are given in table 1.

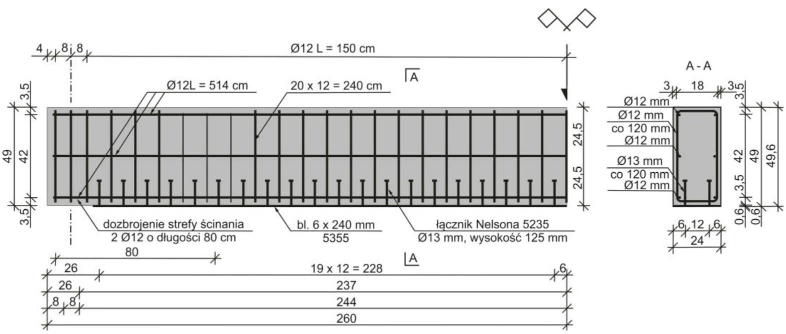


Fig. 5. Dimensions and reinforcement of beams

Table 1. Beam's reinforcement and steel plate's thickness

| No. | Beam | $s_{con}$ [mm] | Concrete part | $h_s$ [mm] | $d_{sl}$ [mm] | $d_{ss}$ [mm] | $s_{sl}$ [mm] |
|---|---|---|---|---|---|---|---|
| 1. | BZ-1 | 160 | I | 6.0 | 12 | 12 | 160 |
| 2. | BZ-2 | 200 | II | 6.0 | 12 | 12 | 160 |
| 3. | BZ-3 | 160 | I | 10.0 | 16 | 16 | 160 |
| 4. | BZ-4 | 120 | II | 10.0 | 12 | 12 | 100 |
| 5. | BZ-5 | 80 | II | 16.0 | 16 | 16 | 60 |
| 6. | BZ-6 | 160 | II | 16.0 | 16 | 16 | 120 |

$S_{con}$ – connectors' spacing; $h_s$ – steel plate's thickness; $d_{sl}$ – diameter of horizontal bars; $d_{ss}$ – diameter of stirrups; $s_{sl}$ – stirrups' spacing

The beams were reinforced with ribbed bars 12 or 16 mm in diameter. Four bars were used in the corners and two at the mid-height of the concrete section. Moreover, at the supports two additional horizontal bars were used at the bottom along the length of the beam with no steel plate cover. The reinforcement with stirrups provided carrying capacity at least 50% higher than the predicted maximum transverse force. The idea was that beams' strength should be determined by the bending moment. The manner of beams' reinforcement is shown in figure 5, and the diameters of the horizontal bars and stirrups together with their spacing are given in table 1. During the tests the beams were loaded with a concentrated force of the value ranging from zero to that corresponding to the beam's reaching its bending strength. The manner of support and loading of the beams as well as the location of the essential elements of the measuring instrumentation are shown in figure 6, the test stand is presented in figure 7. The measurements taken with mechanical, electric resistance, inductive and optical instruments covered strains and displacements. Similar measurement instrumentation was used in other studies on composite members [1, 12, 19]. In the theoretical solution the analysis was performed for a case of loading with two concentrated forces of the value of $P/2$, acting symmetrically, at the distance $w$ away from the beam mid-span, of any value in a boundary case $w = 0$. This case was adopted in the experimental studies.

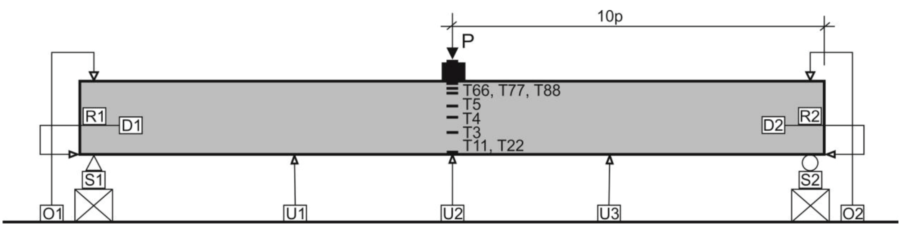


Fig. 6. Schematic of the research stand

Where V1, V2 I V3 - inductive sensors for recording displacements (measurement range of up to 200mm)

S - dynamometer (measurement range of up to 600 kN)

S1, S2 - force detectors (measurement range of up to kN)

T - electric resistance strain gauges

O1, O2 - dial indicators for measurement of support settlement

D1, D2 - points for measurement of slip on beam front

R1, R1 - inclinometers

P - measurement points in non-contact visual measurement of the slip (optimal measurements were also taken of deflections and strains and were used for drawing stress maps).

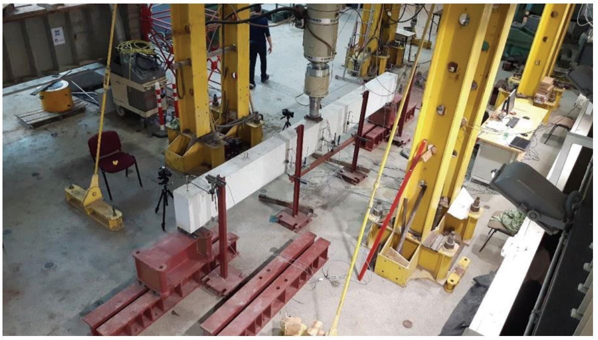

Fig. 7. View of the testing stand

The mechanical properties of the materials the beams were made from were specified each time on the basis of the tests performed on six elements. The tests were compatible with standards [19, 20, 21, 22, 23, 24, 25]. The results of the mechanical testing of concrete are given in table 2.

- concrete mean compressive strength for the first batch of concrete mix $f_c$ = 57.60 MPa; standard deviation $s_c$ = 1.16 MPa, coefficient of variation $v_c$ = 2.01 %,
- concrete mean compressive strength for the second batch of concrete mix $f_c$ = 94.37 MPa, standard deviation $s_c$ = 6.70 MPa, coefficient of variation 7.10 %,
- elasticity modulus of concrete – mean value for the first batch $E_c$ = 35.48 GPa,
- elasticity modulus of concrete – mean value for the second batch $E_c$ = 45.48 GPa.

Table 2. Mechanical properties of materials

| Property | Mean value [MPa; GPa] | Standard deviation $s$ [MPa] | Coefficient of variation ν [%] |
|---|---|---|---|
| Compressive strength of concrete for the first batch of concrete mix $f_c$ | 57.60 | 1.16 | 2.01 |
| Compressive strength of concrete for the second batch of concrete mix $f_c$ | 94.37 | 6.70 | 7.10 |
| Elasticity modulus of concrete for the first batch of concrete mix $E_c$ | 35.48 | - | - |
| Elasticity modulus of concrete for the second batch of concrete mix $E_c$ | 45.48 | - | - |

The following results were obtained for steel testing:

- strength characteristics of the steel in reinforcing bars 12 mm in diameter (steel B500SP) are given in table 3, those for bars 16 mm in diameter in table 4,
- strength characteristics of the steel of the plate 6, 10 and 16 mm thick are given in table 4,
- strength characteristics of the steel of connectors (S235J2+C470) 13 mm in diameter 125 mm high were as follows: yield point 515.0 MPa, ultimate limit strength 546.0 MPa, limit of elongability 19.0 %.

Table 3. Results of tests on reinforcing bars 12.0 mm in diameter (steel B500SP)

| Parameter | $m_E$ [GPa] | $R_{eH}$ [MPa] | $R_{eL}$ [MPa] | $R_m$ [MPa] | $A_{5,65}$ [%] |
|---|---|---|---|---|---|
| $\bar{x}$ | 205,5 | 529,1 | 525,3 | 625,2 | 18,1 |
| $s$ | 5,7 | 2,4 | 2,3 | 1,4 | 1,0 |
| $v$ [%] | 2,8 | 0,5 | 0,4 | 0,2 | 5,7 |

Table 4. Results of tests on reinforcing bars 16,0 mm in diameter (steel B500SP)

| Parameter | $m_E$ [GPa] | $R_{eH}$ [MPa] | $R_{eL}$ [MPa] | $R_m$ [MPa] | $A_{5,65}$ [%] |
|---|---|---|---|---|---|
| $\bar{x}$ | 198,5 | 559,2 | 554,1 | 679 | 136,5 |
| $s$ | 7,6 | 25,5 | 24,5 | 19,7 | 4,0 |
| $v$ [%] | 3,8 | 4,6 | 4,4 | 2,9 | 2,9 |

Tables 5. Parameters of steel S355JZ+N

| Thickness [mm] | Yield point [MPa] | Ultimate limit strength [MPa] | Elongation [%] |
|---|---|---|---|
| 6 | 445,0 | 555,0 | 30,2 |
| 10 | 424,0 | 520,0 | 25,0 |
| 12 | 420,0 | 545,0 | 31,0 |

# 5. NUMERICAL ANALYSIS

The numerical analysis was performed to confirm the results of the analytical solutions and compare them with the results of measurements performed on the studied beams. The confirmation of the correctness of the numerical model will help limit the scope of experimental tests which are time and cost consuming. The information given in papers [11, 16] was included.

The analysis covered:

- the concrete part of the composite beam (together with horizontal bars and stirrups),
- the steel plate,

- the connectors,
- beam supports and points of force application.

The geometric and calculation model was created in the PREP7 preprocessor in APDL (ANSYS Parametric Design Language). The material parameters were adopted based on the test results. The values of the parameters that were not included in the tests were taken from the certificates obtained from the producers after they have been checked against the literature data. The types of finite elements are given in table 5 [12]. They guarantee the calculations accuracy class as in other similar analyses [3, 8, 9, 12, 13, 26, 28, 30].

Table 6. Types of finite elements applied

| Type of element | Concrete beam | Steel plate | Support pads | Longitudinal reinforcement | Stirrups | Flexible connectors | Slip interface |
|---|---|---|---|---|---|---|---|
| Type of finite element | SOLID 65 | SOLID 45 | SOLID 45 | LINK 180 | LINK 180 | BEAM 188<br>COMBIN 39 | TARGET 170<br>CONTA 174 |

Modelling of the steel plate bonding with the concrete part is an essential element of the numerical model. For this purpose elements type COMBIN 39 were used, which model the operation of a spring with a non-linear dependence between the force and displacement. The dependence given in paper [1] was used in the following form

$$F_d = F_{du}\left(1 - \frac{1}{e^{\Delta u_S}}\right) \qquad \text{Eq-35}$$

Where

$$F_{du} = 1{,}3d^2\sqrt{1{,}2}f_c f_y \qquad \text{Eq-36}$$

$d$ – connector's diameter,

$f_c$ – concrete compressive strength,

$f_y$ – steel yield point of connectors,

$\Delta u_s$ – tangential displacement of a connector.

Moreover, friction between the concrete part and steel plate was included, adopting the coefficient of friction of 0,45.

## 6. RESULTS AND THEIR ANALYSIS

The results of experimental tests and those of the theoretical and numerical analyses are given in table 6. The results of the experimental tests were adopted as the reference. The results achieved lead to a conclusion that the theoretical analysis produces underestimated values of the strength of steel plate - concrete composite beams. The opposite is true when the results of the numerical analysis are compared. The comparable value was the force causing the exhaustion of strength.

Table 7. Comparison of the results of experimental tests with those of theoretical and numerical analyses

| Tests | Symbol of beam | Experimental tests [kN] | Theoretical analysis [kN] | d/c [%] | Numerical analysis [kN] | f/c [%] |
|---|---|---|---|---|---|---|
| a | b | c | d | e | f | g |
| done by the author | BZ-1 | 287,7 | 254,1 | 88,3 | 288,0 | 100,1 |
| | BZ-2 | 296,3 | 261,0 | 88,1 | 269,0 | 90,8 |
| | BZ-3 | 415,1 | 396,5 | 95,5 | 466,0 | 112,2 |
| | BZ-4 | 393,5 | 385,1 | 97,9 | 450,6 | 114,5 |
| | BZ-5 | 635,2 | 602,1 | 94,8 | 670,0 | 105,5 |
| | BZ-6 | 628,8 | 602,1 | 95,8 | 702,0 | 111,6 |
| done by other researchers | BO-2 | 433,2 | 433,2 | 100,0 | - | - |
| | BO-3 | 465,8 | 477,7 | 102,6 | - | - |
| | BO-5 | 308,6 | 261,9 | 84,7 | - | - |
| | BO-7 | 335,2 | 245,2 | 73,2 | - | - |
| | BO-8 | 325,8 | 322,7 | 99,0 | - | - |

Additional conclusions can be drawn from the tabulation given in table 7. It presents the values of forces which lead to plasticization of the steel plate ($P_y$) and reaching ultimate limit strength ($P_u$). The table shows the results of tests and theoretical analysis. In the case of force $P_y$ for five (out of six) beams the results of the theoretical analysis produce overestimated values (by 7,5 % on average), while in the case of force $P_u$ for all the six beams the values were underestimated (by 6,6 % on average) - resultant of the numerical analysis. The compared value when comparing the results of experimental research and theoretical and numerical analysis was the force causing exhaustion of the bending resistance. It resulted from the fact that in the experimental tests the set value is the force P. The bending moment M at the adopted load scheme is: M = P*l/4. In the studies the beam carrying capacity was specified on the basis of the indications from the sensors measuring deformations and displacements. It was equal to the force at which an increment of the values of these parameters was recorded with no increase of the force itself. In the finite element method carrying capacity was equated with the force at which the steel reached the yield point; similarly in the theoretical solution.

Table 8. Comparison of characteristic values of loads in tests and calculations of beams' strength

| Tests | Symbols of beams | Experimental tests | | | Theoretical analysis | | | F/C [%] | g/d [%] | h/e [%] |
|---|---|---|---|---|---|---|---|---|---|---|
| | | $P_{cr}$ [kN] | $P_v$ [kN] | $P_u$ [kN] | $P_{cr}$ [kN] | $P_v$ [kN] | $P_u$ [kN] | | | |
| a | b | c | d | e | f | g | h | i | j | k |
| done by the author | BZ-1 | - | 249,5 | 287,7 | 59,6 | 228,5 | 254,1 | - | 91,6 | 88,3 |
| | BZ-2 | - | 211,9 | 296,3 | 66,1 | 231,5 | 261,0 | - | 109,2 | 88,1 |
| | BZ-3 | - | 328,7 | 415,1 | 69,8 | 353,5 | 396,5 | - | 107,5 | 95,5 |
| | BZ-4 | - | 298,7 | 393,5 | 74,5 | 343,3 | 385,1 | - | 114,9 | 97,9 |
| | BZ-5 | - | 466,0 | 635,2 | 88,6 | 539,0 | 602,1 | - | 115,7 | 94,8 |
| | BZ-6 | - | 507,4 | 628,8 | 88,6 | 539,0 | 602,1 | - | 106,2 | 95,8 |
| done by other researchers | BO-2 | 86,0 | - | 433,2 | 73,5 | 416,9 | 438,2 | 85,5 | - | 100,0 |
| | BO-3 | 102,0 | - | 465,8 | 82,2 | 456,6 | 477,7 | 80,6 | - | 102,6 |
| | BO-5 | 40,0 | - | 308,6 | 46,1 | 250,6 | 261,9 | 115,3 | - | 84,7 |
| | BO-7 | 40,0 | - | 335,2 | 49,8 | 233,3 | 245,2 | 124,5 | - | 73,2 |

$P_u$ – the force at which the steel plate reached tensile strength, $P_{cr}$ – the force at which first cracks in concrete appeared, $P_y$ – the force at which the steel plate reached yield point.

In the paper [12] an in-depth analysis was performed on the impact of the steel plate slip on the strength of steel plate-concrete composite beams. The results of experimental tests and numerical analysis were quoted with and without the slip in this respect. On their basis it can be concluded that the slip interface between the steel plate and concrete part has no significant impact on the design value of bending strength of steel plate-concrete composite members, so in practical engineering tasks it can be disregarded. The impact of slip on the beams deflections is much more significant, the more so since the given values concern the state immediately prior to strength exhaustion. In actual reality, such members are used under lower loads, under which the slip is even less, which consequently makes its impact smaller.

## 7. REMARKS AND FINAL CONCLUSIONS

The paper is devoted to the bending strength of steel plate-concrete composite beams. The author performed experimental tests as well as theoretical and numerical analyses. The tests were done on six beams whose dimensions are given in figure 5 and in table 1. The values of mechanical parameters of the materials from which the beams were made were adopted based on the results of tests or - in exceptional cases - the values guaranteed by the producer. Following the results it can be stated that with respect to bending strength the results obtained from the theoretical analysis produce underestimated values for the six beams (by 6,6 % on average) compared with those obtained from experimental tests. The numerical analysis, on the other hand, produced overestimated values by - 7,5 % on average. The differences in the results obtained in the experimental studies and theoretical and numerical calculations are only slight considering the divergences between the mechanical features of concrete, structural steel and reinforcing bars as well as the initial assumptions for the theoretical and numerical calculations. The results of theoretical and numerical analyses indicate that the interface slip between the steel plate and concrete part affect the bending strength of steel plate-concrete composite beams only slightly (about 2% on average). This difference concerns the state immediately prior to strength exhaustion. In real life conditions the loads are significantly lower so the impact of the slip is even smaller. Bending strength is determined by the section most loaded with the bending moment, i.e. the mid-span of the beam. In the middle part of the beam, however, there is no slip, and even if there is, it is only minor. Consequently, the sections in the mid-span operate as if there were no slip, which makes its impact on the beams' strength negligible. The results of the experimental tests performed by the author were verified against those given in [12]. In both cases, that is the author's tests and those done by other researchers, similar compatibility with the results of theoretical calculations were obtained. The results of the author's tests have been tabulated, which makes them easily applicable by other researchers dealing with similar issues.

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## LIST OF FIGURES AND TABLES:

## OCENA NOŚNOŚCI NA ZGINANIE BELEK ZESPOLONYCH TYPU BLACHA STALOWA-BETON

Słowa kluczowe: beton, stal, belka zespolona, wytrzymałość na zginanie, badania eksperymentalne

**STRESZCZENIE**:

Badania konstrukcji zespolonych typu blacha stalowa-beton, nie mają długiej tradycji. Są one z jednej strony modyfikacją klasycznych konstrukcji zespolonych typu stal-beton, a z drugiej kontynuatorem konstrukcji Möllera, stosowanych już pod koniec XIX wieku. Pewne podobieństwo występuje, między analizowanymi konstrukcjami a elementami żelbetowymi wzmacnianymi zespolonymi z nimi blachami stalowymi lub taśmami kompozytowymi. Rozwiązania te stosuje się przy wzmacnianiu belek żelbetowych.

Krótki okres stosowania belek zespolonych typu blacha stalowa-beton sprawia, że wiele problemów - nawet natury podstawowej - jest jeszcze do rozwiązania. Dotyczy to także nośności przy obciążeniach doraźnych, a także ugięć i zarysowania oraz wpływu skurczu i pełzania betonu. W obliczeniach praktycznych nie uwzględnia się parametru czasu (obciążenia długotrwałe), a także odkształceń skurczowych betonu i jego pełzania.

Celem pracy jest podanie rozwiązania teoretycznego, dotyczącego nośności na zginanie elementów zespolonych typu blacha stalowa-beton oraz przeprowadzenie analizy wpływu wybranych parametrów na tę nośność. Zakres pracy obejmuje podanie stosownych wzorów z uwzględnieniem specyfiki analizowanych belek. W analizie podanych rozwiązań wzięto pod uwagę geometrię przekroju poprzecznego belek oraz cechy mechaniczne materiałów składowych.

Podane rozwiązania poddano weryfikacji doświadczalnej. Wykorzystano w tym celu wyniki badań własnych i obcych. Badania własne mają oryginalny charakter, a sposób prezentacji wyników umożliwia ich wykorzystanie przez innych badaczy. Wartości parametrów mechanicznych materiałów, z których były wykonane belki przyjęto na podstawie wyników badań. Przeprowadzono także analizę numeryczną modelu.

Analizę numeryczną przeprowadzono w celu potwierdzenia wyników rozwiązań analitycznych, a także porównania ich wyników z wynikami pomiarów wykonanych na badanych belkach. Model geometryczny i obliczeniowy został wykreowany w preprocesorze PREP7 z wykorzystaniem języka APDL (ANSYS Parametric Design Language). Parametry materiałowe, przyjęto na podstawie wyników badań. Wartości wielkości które nie były badane, wzięto z dokumentacji od producenta po skonfrontowaniu ich z danymi literaturowymi.

Istotnym elementem modelu numerycznego, było odwzorowanie MES zespolenia blachy stalowej z częścią betonową. Wykorzystano w tym celu elementy typu COMBIN 39, które modelują pracę sprężyny z nieliniową zależnością między siłą i przemieszczeniem.

Do obliczania analizowanych elementów, nie można zastosować wprost rozwiązań ważnych dla klasycznych konstrukcji zespolonych typu stal-beton, pomimo ugruntowanej wiedzy w tym zakresie. Wynika to z całkowicie odmiennych proporcji, przekroju części betonowej i stalowej w przekroju poprzecznym belki (elementu). Różnica wynika między innymi z wpływu strefy rozciąganej batonu, na nośność i sztywność elementu, szczególnie na poziomie obciążeń użytkowych i wyższym, gdy część betonu strefy rozciąganej ulega zarysowaniu.

W tym przypadku, nie można korzystać ze wzorów stosowanych w klasycznych elementach żelbetowych.

Badania doświadczalne własne, przeprowadzono na sześciu belkach o długości całkowitej 5,20 m. Rozpiętość podporowa (teoretyczna) wynosiła 5,00 m. Przekrój poprzeczny miał wymiary 0,24 m (szerokość) x 0,49 m (wysokość). Blacha stalowa miała grubość 6,10 i 16 mm. Długość blachy 4,74 m. Zespolenie blachy z częścią betonową (żelbetową), zrealizowano za pomocą łączników o średnicy 13 mm. Rozstaw łączników był zmienny i wynosił od 80 mm do 200 mm.

Belki były zbrojone prętami żebrowanymi o średnicy 12 lub 16 mm. Stosowano 4 pręty w narożach i dwa w połowie wysokości przekroju betonowego. Ponadto przy podporach stosowano u dołu dodatkowe pręty podłużne na długości belki, na której nie było blachy stalowej. Zbrojenie strzemionami, zapewniło nośność co najmniej o 50 % większą od przewidywanej maksymalnej siły poprzecznej. Chodziło o to, aby o nośności belek decydował moment zginający.

Właściwości mechaniczne materiałów wykorzystanych do wykonania belek były następujące:

– wytrzymałość średnia betonu na ściskanie, dla pierwszej partii zarobu $f_c$ = 57,60 MPa; odchylenie standardowe $s_c$ = 1,16 MPa, wskaźnik zmienności $v_c$ = 2,01 %,

– wytrzymałość średnia betonu na ściskanie, dla drugiej partii zarobu $f_c$ = 94,37 MPa, odchylenie standardowe $s_c$ = 6,70 MPa, wskaźnik zmienności $v_c$ = 7,10 %,

– moduł sprężystości betonu - wartość średnia dla pierwszej partii $E_c$ = 35,48 GPa,

– moduł sprężystości betonu - wartość średnia dla drugiej partii $E_c$ = 45,48 GPa,

– cechy wytrzymałościowe stali - prętów zbrojeniowych o średnicy 12 mm (stal B500SP) - podano w tablicy 4.13, natomiast dla prętów o średnicy 16 mm w tablicy 4.16,
– cechy wytrzymałościowe stali - blachy o grubości 6, 10 i 16 mm są podane w tablicy 4.17 (4),
– cechy wytrzymałościowe stali - łączników (S235J2+C470) o średnicy 13 mm i wysokości 125 mm były następujące: granica plastyczności 515,0 MPa, granica wytrzymałości 546,0 MPa, wydłużalność graniczna 19,0 %.

W czasie badania, belki były obciążone siłą skupioną o wartości zmieniającej się od zera do odpowiadającej osiągnięciu przez belkę nośności na zginanie. Mierzone były odkształcenia i przemieszczenia, przy wykorzystaniu przyrządów mechanicznych, elektrooporowych, indukcyjnych i optycznych.

Na podstawie uzyskanych wyników można uznać, że podane rozwiązania analityczne zostały potwierdzone wynikami badań doświadczalnych i obliczeń numerycznych oraz stwierdzić, że w stosunku do nośności na zginanie uzyskanych na podstawie wyników badań doświadczalnych, wyniki uzyskane na podstawie analizy teoretycznej są zaniżone. Zaniżenie to wynosi średnio (dla sześciu belek) 6,6 %, z kolei na podstawie analizy numerycznej uzyskano wyniki zawyżone - średnio o 7,5 %.

Wyniki analizy teoretycznej i numerycznej wskazują, że poślizg między blachą stalowa i częścią betonową w niewielkim stopniu wpływa na nośność na zginanie belek zespolonych typu blacha stalowa-beton (średnio tylko około 2 %) w partii środkowej. Większy wpływ ma poślizg na ugięcia analizowanych belek. Występowanie poślizgu pomiędzy blachą stalową i częścią żelbetową, ma miejsce praktyczne na całej długości belek - potwierdziły to pomiary optyczne wykonywane podczas badań.

Received: 14.04.2020 Revised: 20.06.2020