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Active-controlled two-terminal network implementing a convolution-type immittance operator

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Abstract. Commonly known DC-AC switching converters are commonly used in compensator branches. One example of this is a static synchronous compensator (STATCOM). It consists of a voltage source converter (VSC) and acts as an inverter with a capacitor as a DC power source. These compensators use the PWM switching scheme or space vector modulation (SVM) method. Both methods require the desired signal to be generated. In some cases, as during the synthesis of self-excited systems or active energy-compensators, it is necessary to perform the desired branch immittance, e.g. negative capacitance, inductance, resistance or irrational impedance. In such cases, it is necessary to control the universal branch on the basis of a formula. This article presents the implementation method for the convolutional type impedance operators.

Key words: immittance operators, convolution, control, digital filters.

1. Introduction

Commonly used methods for controlling the active power filter / inverter branches use are well-known in automatics and provided feedback with PI controllers designed for a specific load.

In the article, for the first time, the method of control execution of a linear time-invariant convolution-type two-terminal circuit, over discrete time and using mathematical feedback (from the formula), is presented. Mathematical feedback implements a specific formula that controls the actuator, i.e. the aforementioned voltage source or an actually switched-on constant-voltage source. This approach is simpler and more versatile than using matched analog / digital regulators [1–6]. The authors did not find such a solution in the literature.

2. Universal-controlled voltage source

In an actively controlled two-terminal network, the universal voltage-source is realized by switching a constant electromotive force E within a period of time τ. Duty cycle τ_n/τ plays a key role here.

Averaging the rectangular pulses in a period τ results in the value of the n-th voltage-source sample:

$$e_n = \vartheta_n E - (1-\vartheta_n)E = (2\vartheta_n - 1)E \tag{1}$$

where $\vartheta_n = \dfrac{\tau_n}{\tau} \in [0, 1]$ is a duty cycle for the n-th voltage sample e_n.

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The formula reciprocal to (1) has the following form:

$$\vartheta_n = \frac{1}{2}\left(1 + \frac{e_n}{E}\right) \tag{2}$$

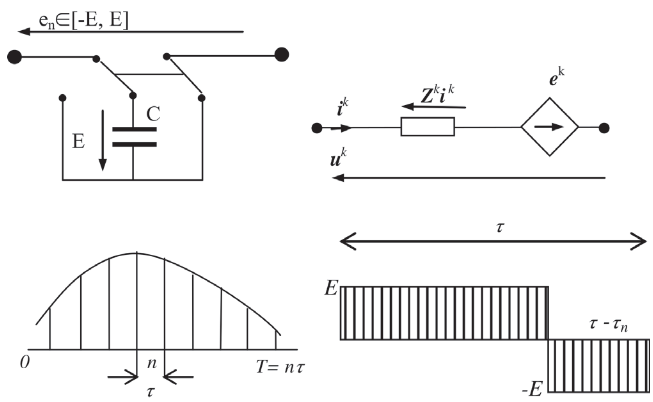

Fig. 1. Scheme of 2-level $(-E, E)$ PWM control of a switched voltage source in k-th branch, and its generated averaged voltage waveform in the n-th sample

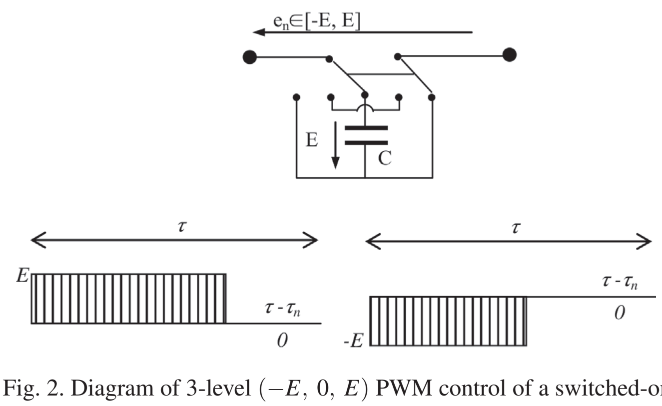

Fig. 2. Diagram of 3-level $(-E, 0, E)$ PWM control of a switched-on voltage source

whereas for 3-level control (Fig. 2):

$$\vartheta_n = sign(e_n)\frac{e_n}{E}. \tag{3}$$

The constant voltage is drawn from the energy storage amounting to $\frac{1}{2}CE^2$, where C is the capacity.

3. Implementation of negative resistance and negative conductance

The negative resistance or negative conductance are used during the synthesis of self-excited systems or active energy-compensators.

Figure 3 shows diagrams that are used to define two-terminal negative resistance and negative conductance.

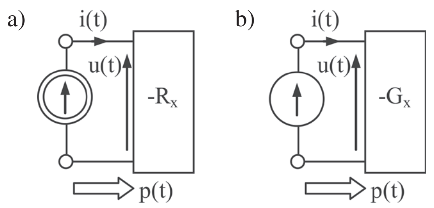

Fig. 3. Negative resistance (a) and negative conductance (b)

Negative resistance $-R_x$ is implemented by the current-controlled branch:

$$u(t) = -R_x i(t) \qquad R_x > 0 \tag{4}$$

causing the reverse flow of energy:

$$p(t) = u(t)i(t) = -R_x\,[i(t)]^2 < 0.$$

Negative conductance $-G_x$ is carried out by the voltage-controlled branch:

$$i(t) = -G_x u(t) \qquad G_x > 0 \tag{5}$$

with the reverse flow of energy:

$$p(t) = u(t)i(t) = -G_x\,[u(t)]^2 < 0$$

where $p(t)$ – instantaneous power.

Implementation of negative resistance and negative conductance is shown in Fig. 4. The negative resistance operator is implemented by a universal CCVS controlled by branch current according to the equation below:

$$(-R_x): \qquad e(t) = (R - (-R_x))\,i(t) + L\frac{\mathrm{d}i}{\mathrm{d}t}. \tag{6}$$

Whereas the negative conductance operator is implemented using a universal VCVS controlled by a branch voltage according to the following equation:

$$(-G_x): \qquad e(t) = -\,(1 + G_x R)\,u(t) - G_x L\frac{\mathrm{d}u}{\mathrm{d}t}. \tag{7}$$

The above equations are obtained from solving the equation:

$$u(t) + e(t) = Ri(t) + L\frac{\mathrm{d}i}{\mathrm{d}t} \tag{8}$$

when taking account of (4) and (5).

After sampling in the time domain $t \to n\tau$, Eqs. (6) and (7) take the form of:

$$\begin{aligned} e_n &= (R + R_x)i_n + L\frac{1}{\tau}(i_n - i_{n-1}) \\ &= (R + R_x + R_L)\,i_n - R_L i_{n-1}, \end{aligned} \tag{9}$$

$$e_n = -\,(1 + G_x(R + R_L))\,u_n - G_x R_L u_{n-1} \tag{10}$$

where $R_L = L\frac{1}{\tau}$ is the so-called inductive-resistance of the inductor.

The RL circuit, shown in Fig. 4, serves to smooth the current during switching on of voltage E.

Examples of current and voltage waveforms are shown in Fig. 5.

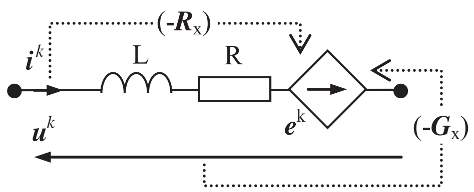

Fig. 4. Diagram of the k-th branch implementation of negative resistance $-R_x$ (using current-controlled voltage source) or negative conductance $-G_x$ (using voltage-controlled voltage source)

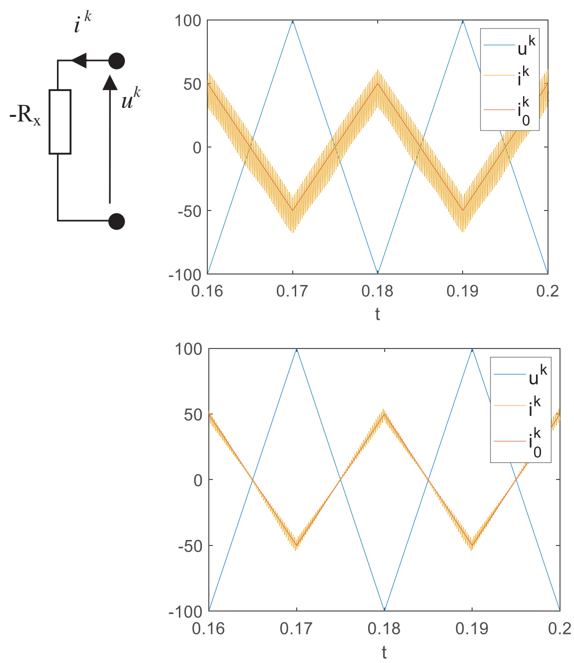

Fig. 5. Exemplary waveforms of branch currents and voltages of negative resistance, for 2- and 3-level control, for identical circuit parameters

4. Power flow analysis of a -R and -G two-terminal network

The instantaneous power $p_e(t)$ of the source $e(t)$ (Fig. 4) for negative resistance $-R_x$ is given by the following formula:

$$p_e(t) = e(t)i(t) = (R+R_x)\,[i(t)]^2 + Li(t)\frac{\mathrm{d}i}{\mathrm{d}t}. \tag{11}$$

Hence, for periodic currents:

$$P_e = \frac{1}{T}\int_0^T e(t)i(t)\,\mathrm{d}t = (R+R_x)\,\|i(t)\|^2 \tag{12}$$

where $\|i(t)\|$ – the current's norm.

The conditions $R_x > 0$ and $R > 0$ mean that energy from the two-terminal network is supplied to the system continuously.

An analogous power balance is obtained for a $-G_x$ branch:

$$p_e(t) = (1+RG_x)\,G_x\,[u(t)]^2 + LG_x^2 i(t)\frac{\mathrm{d}i}{\mathrm{d}t}. \tag{13}$$

Hence, after averaging the T-periodic signals, an analogous power formula is obtained:

$$P_e = (1+RG_x)\,G_x\,\|u(t)\|^2 \geq 0. \tag{14}$$

5. Implementation of two-terminal network immittance

A diagram of a two-terminal network realizing a given linear impedance Z_x or admittance Y_x operator, involving a CCVS$e(i(t))$ or a VCVS$e(u(t))$ is presented in Fig. 6.

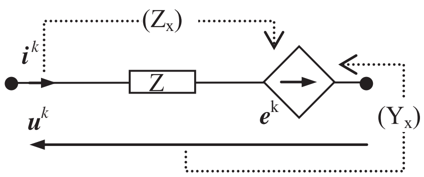

Fig. 6. Implementation of two-terminal impedance Z (using a CCVS) or admittance Y (using a VCVS)

Figure 6 shows a universal switched-voltage-source with a passive two-terminal network, of a given impedance operator Z, in series.

From the equation of the universal active branch:

$$u(t) + e(t) = Zi(t) \tag{15}$$

where $Zi(t)$ – convolution $Z(t)$ with $i(t)$.

The equation for VS control for a given impedance Z_x is obtained:

$$e(t) = (Z - Z_x)\,i(t) \tag{16}$$

or for a given admittance Y_x it is:

$$e(t) = -(1 - ZY_x)\,u^k(t). \tag{17}$$

In particular, it can be considered negative inductance $Z_x = -sL_x$ and negative capacitance $Y_x = -sC_x$, where L_x, C_x are positive values.

For $Z(s) = R + sL$, the negative inductance and negative capacitance are realized by means of the following VS control formulas:

$$(-L_x): \qquad e(t) = (R + s(L+L_x))\,i(t), \tag{18}$$

$$(-C_x): \qquad e(t) = -\left(1 - RC_x s - LC_x s^2\right) u(t) \tag{19}$$

where $s = \mathrm{d}/\mathrm{d}t$.

Examples of current and voltage waveforms of negative L and C are shown in Fig. 7.

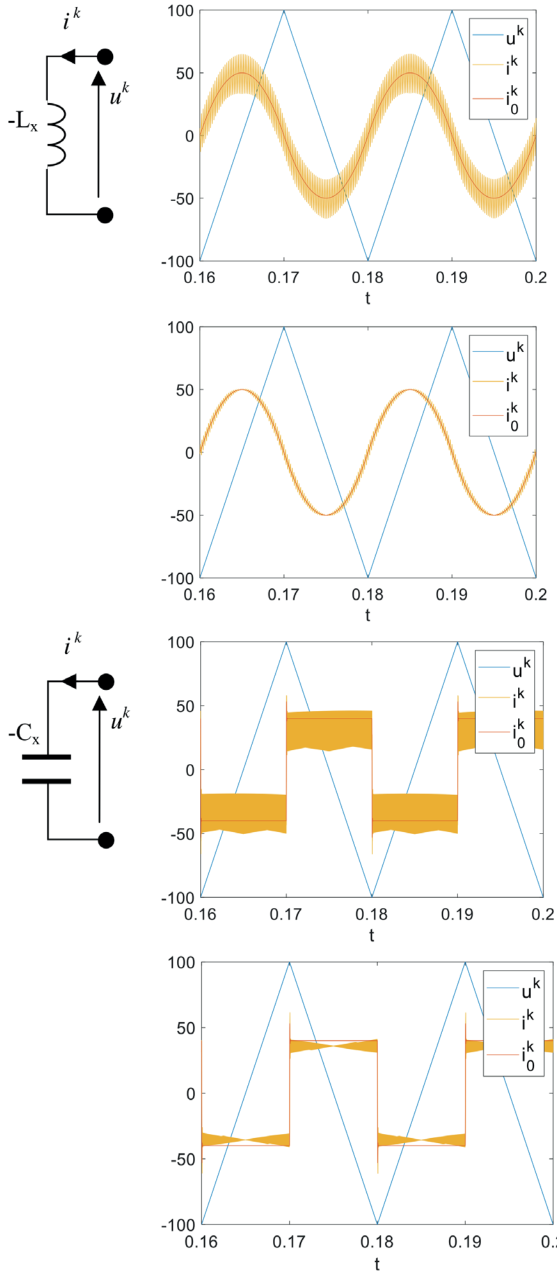

Fig. 7. Exemplary waveforms of branch current and voltages of negative inductance and negative capacitance, for 2- and 3-level control, for identical circuit parameters

6. Implementation of any immittance operator in discrete time domain

To realize a non-periodic convolutional operator for a given impedance Z_x or admittance Y_x in the discrete time domain:

$$u_n = \sum_{m=0}^{\infty} (Z_x)_m i_{n-m}\,, \tag{20}$$

$$i_n = \sum_{m=0}^{\infty} (Y_x)_m u_{n-m} \tag{21}$$

it is necessary to control the electromotive force $\{e_n\}$ by changing duty cycle $\{\vartheta_n\}$ of the switched VS according to the following formulas:

$$e_n = \sum_{m=0}^{\infty} (Z - Z_x)_m\, i_{n-m}\,, \tag{22}$$

$$e_n = -u_n + \sum_{m=0}^{\infty} (ZY_x)_m\, u_{n-m}\,, \tag{23}$$

$$\vartheta_n = \begin{cases} \frac{1}{2}\left(1+\frac{e_n}{E}\right) = \frac{1}{2}(1+\bar{e}_n) & \text{for } |\bar{e}_n| \le 1 \\ 0 & \text{for } \bar{e}_n < -1 \\ 1 & \text{for } \bar{e}_n > 1 \end{cases} \tag{24}$$

where

$$(ZY_x)_n = \sum_{m=0}^{\infty} (Y_x)_m Z_{n-m} \quad \text{for} \quad n \ge 0.$$

Analogous formulas for periodic signals, which use cyclic convolution, are given below:

$$u_n = \sum_{m=0}^{N-1} (Z_x)_{n\Theta m}\, i_m\,, \tag{25}$$

$$i_n = \sum_{m=0}^{N-1} (Y_x)_{n\Theta m}\, u_m \tag{26}$$

voltage-source samples:

$$e_n = \sum_{m=0}^{N-1} (Z - Z_x)_m\, i_{n\Theta m}\,, \tag{27}$$

$$e_n = -u_n + \sum_{m=0}^{N-1} (ZY_x)_m\, u_{n\Theta m} \tag{28}$$

duty cycle samples:

$$\vartheta_n = \frac{1}{2}(1+\bar{e}_n) \quad \text{for} \quad |\bar{e}_n| \le 1 \tag{29}$$

where:

$$(ZY_x)_n = \sum_{m=0}^{N-1} (Y_x)_m Z_{n\Theta m}\,,$$

$n \in \{0,1,\ldots,N-1\}$, N – number of samples in one period, Θ – modulo N subtraction.

7. Example

As an example, the case of a finite cable line with given characteristic impedance $Z_C = c\sqrt{\frac{(b-z)}{(a-z)}}$ matched to the load can be considered.

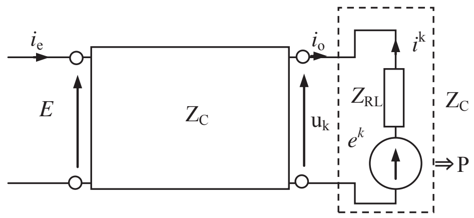

Fig. 8. Matched cable line

The generic impulse response formula of $Y_C = \frac{1}{Z_C}$ [7–9] is:

$$\left(\sqrt{\frac{(a-z)}{(b-z)}}\right)_n = \sqrt{\frac{a}{b}}\,a^{-n} \sum_{m=0}^{n} \left(\frac{a}{b}\right)^m d_{n-m} i_m$$

where:

$$d_n = \left[(1-z)^{1/2}\right]_n = -\frac{1}{2}\,\frac{1}{4}\,\frac{3}{6}\,\frac{5}{8}\cdots\frac{2n-3}{2n} = \left(\frac{2n-3}{2n}\right)!!; \qquad d_0 = 1$$

differential

$$i_n = \left[(1-z)^{-1/2}\right]_n = \frac{1}{2}\,\frac{3}{4}\,\frac{5}{6}\,\frac{7}{8}\cdots\frac{2n-1}{2n} = \left(\frac{2n-1}{2n}\right)!!; \qquad i_0 = 1$$

integral
!! – double factorial.

A fragment of the Yc impulse responses for $R = 0.72$, $C = 0.4\mathrm{e}{-6}$, $L = 0.19\mathrm{e}{-3}$ is shown below.

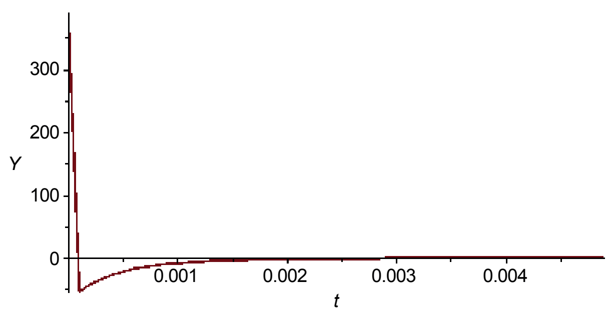

Fig. 9. Fragment of Yc impulse response

For a given u_n, we use (23) to calculate e_n with the help of $(ZY_x)_n = \sum_{m=0}^{\infty} (Y_x)_m Z_{n-m}$. Assuming $Z = 0.1 + s * 1\mathrm{e}{-3}$, we get $(ZY_x)_n$, depicted in Fig. 10.

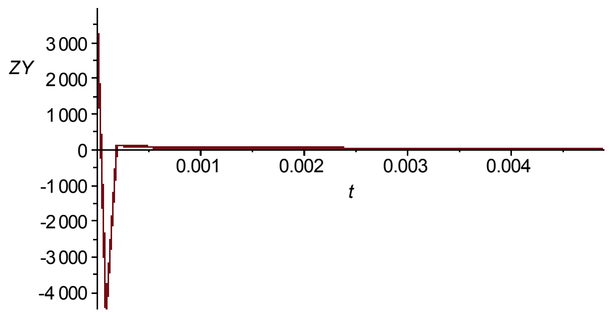

Fig. 10. Fragment of (ZY_x) impulse responses

Finally, using (28), we can calculate ϑ_n which controls the branch's voltage source.

The above solution is the only one that is valid in the whole frequency range and thanks to matched load we can draw energy from the whole frequency range as well.

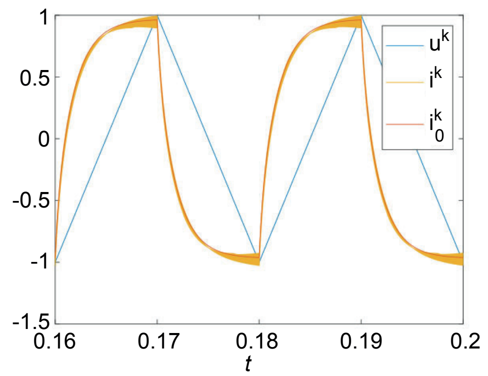

Fig. 11. Sample waveforms of current and scaled voltage (x0.01) of a characteristic impedance of a cable

8. Summary

The key element of the active-branch system realizing the given immittance operator is an ideal voltage source $\{\bar{e}_n\}$, which is controlled according to a duty cycle $\vartheta_n = \dfrac{1}{2}(1+\bar{e}_n)$. The active-branch implementing a given impedance operator is controlled by the current signal, while implementation of the admittance operator needs to be controlled by a voltage signal.

In both cases, for on-line implementation, the control takes place via a non-recursive digital filter with theoretically infinite memory. In practice, however, for the sake of stability, the filter memory is reduced to finite length.

For periodic signals, the appropriate digital filters are operating according to the cyclic convolution rule.

This work is limited in scope to the implementation of convolution-type immittance operators. However, the method described herein can also be quite easily extended to implement linear time-dependent operators [10].

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