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Comparison of algorithms for satellite attitude determination using data from visual sensors

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Abstract. The objective of the research was to investigate the efficiency of selected methods of data fusion from visual sensors used on-board satellites for attitude measurements. Data from a sun sensor, an earth sensor, and a star tracker were fused, and selected methods were applied to calculate satellite attitude. First, a direct numerical solution, a numerical and analytical solution of the Wahba problem, and the TRIAD method for attitude calculation were compared used for integrating data produced by a sun sensor and an earth sensor. Next, attitude data from the star tracker and earth/sun sensors were integrated using two methods: weighted average and Kalman filter. All algorithms were coded in the MATLAB environment and tested using simulation models of visual sensors. The results of simulations may be used as an indication for the best data fusion in real satellite systems. The algorithms developed may be extended to incorporate other attitude sensors like inertial and/or GNSS to form a complete satellite attitude system.

Key words: space; navigation; attitude determination; visual sensors.

1. INTRODUCTION

A satellite attitude is vital information for satellite navigation, stabilization, and control. In this paper, only determination of attitude is considered, by combining data from visual sensors available on-board. Sensors used for satellite attitude measurements differ by the principle of operation, measurands, and output data, so various methods of data fusion may be implemented for improving the accuracy and reliability of the attitude information. In Table 1 commonly used sensors for satellite attitude and position measurements are listed, indicating their principle of operation. The number of sensors and their mass and volume are constrained by satellite weight and size. Limited computational resources available on-board spacecraft require high efficiency of attitude calculation algorithms.

Table 1
Satellite attitude and position sensors

Sensor	Sensor type	Measured quantity
Sun sensor	visual	direction vector to the Sun
Earth sensor	visual	direction vector to the Earth
Star tracker	visual	orientation relative to stars
Magnetometer	magnetic field	components of local magnetic field
GNSS receiver	radio signal	position – single antenna, attitude – multi-antenna

The integration of visual sensors for nanosatellites was presented in several papers. In [1], the method for combining data from solar cells and a three-axis magnetometer is considered to estimate an attitude of a CubeSat by implementing an extended Kalman filter. Attitude determination, with the use of only magnetometers and a linear Kalman filter, is described in [2]. In [3], an analytical method is presented for calculating the nadir direction vector from measurements of two earth (horizon) sensors; the nadir vector is combined with the sun vector by the TRIAD method to determine the satellite attitude. In [4], data from a star tracker, GNSS and gyroscopes are integrated, using Kalman filter. In [5], data from a star tracker and an earth sensor are combined to estimate satellite position in the orbit. Visual sensor fusion is also used in fault detection algorithms. In [6], a fault diagnosis method is considered for an attitude system, based on redundant information from the sun sensor, the earth sensor, and the star tracker.

These few examples of research show the important role of visual sensors in satellite systems.

The research presented here stems from the long-term goal of developing efficient control and navigation algorithms for a nanosatellite, in which attitude calculation plays a crucial role. The study is focused on the integration of visual sensors data. The objective of the study was to combine measurements from three visual sensors: sun sensor, earth sensor, and star tracker to estimate spacecraft attitude. This was done as the step to an attitude system design, in which data available also from other sensors may be integrated in the future.

The paper provides a comparison of selected algorithms for data fusion from visual sensors to calculate satellite attitude. Such a comparison of the efficiency of various methods may be a useful indication for the selection of algorithms for the hardware.

2. ATTITUDE CALCULATION ALGORITHM

The attitude of a satellite is described here by a rotation quaternion and/or associated transformation matrix. The attitude cal-

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culation (see the flowchart in Fig. 1) consists of two major parts: attitude calculation using sun and earth sensors, and fusion of attitude data from the star tracker and sun/earth sensors.

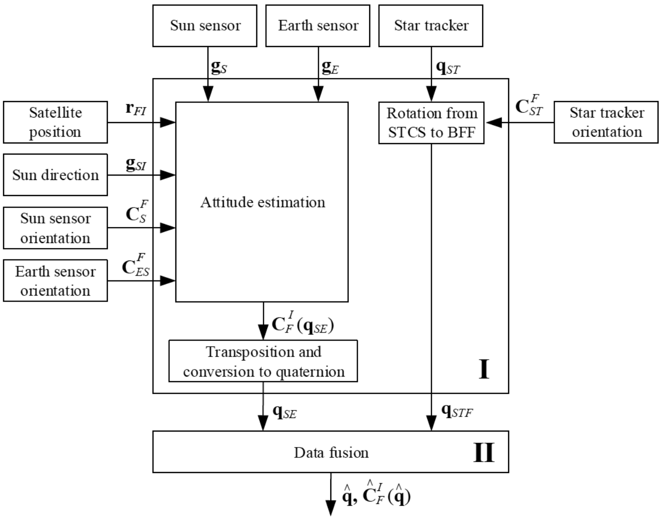

Fig. 1. Attitude calculation methodology

Such a hierarchical structure of sensor data fusion provides the flexibility of expansion for more than single sensors and the prospective implementation of sensor faults detection and elimination procedures in the future.

The sun and earth sensors alone do not provide complete data for attitude determination. The output from sun and earth sensors are direction unit vectors: $\boldsymbol{g}_S$ to the Sun and $\boldsymbol{g}_E$ to the Earth, which are combined in "attitude estimation block" to calculate a satellite attitude matrix $\boldsymbol{C}_F^I(\boldsymbol{q}_{SE})$ and an attitude quaternion $\boldsymbol{q}_{SE}$. Four methods were investigated for combining data from sun and earth sensors: the direct numerical solution, analytical and numerical solution of the Wahba problem, and the TRIAD method. The analytical solution of the Wahba problem is available only for measurements of two vectors [7], which is a case for the sensor configuration of one earth and one sun sensor here. But as the analytical solution does not allow the fusion of data from multiple sensors, so the numerical solution of the Wahba problem was included in the analysis.

A star tracker provides full information on a satellite attitude denoted here as a quaternion $\boldsymbol{q}_{STF}$.

The final satellite attitude quaternion is $\hat{q}$ estimated by combining quaternions $\boldsymbol{q}_{SR}$ and $\boldsymbol{q}_{STF}$ in "data fusion" block, which is research here for prospective fault detection and elimination subsystem. Two methods of data fusion were investigated here: weighted average and Kalman filter. Having the estimation of final attitude quaternion $\hat{q}$, final satellite attitude matrix $\hat{C}_F^I(\hat{q})$ may also be calculated.

3. SENSOR OUTPUT DATA

The coordinates systems used in this research are presented in Figs. 2, 3, and 4 illustrating sensor modelling. The $O_I x_I y_I z_I$ is the Earth-Centered Inertial (ECI) coordinates system, in which satellite attitude is determined. The $O_F x_F y_F z_F$ is the Body Frame Fixed (BFF) system of coordinates fixed to a satellite. The satellite attitude is calculated as the transformation matrix $\boldsymbol{C}_F^I(\boldsymbol{q})$ or respective quaternion $\boldsymbol{q}$ describing the rotation of the BFF relative to the ECI. Each sensor delivers data in its own $O_S x_S y_S z_S$ Sensor Coordinate System (SCS), matrices of transformation from the sensor to a satellite coordinates system are known, as determined during the design of satellite systems. The sensor models provide data of direction vectors (sun and earth sensors) or attitude quaternion (star tracker). The data noise is not included in the measurement description but is implemented in simulations.

The Sun position vector $\boldsymbol{r}_{SI}$ in the Earth-Centered Inertial (ECI) coordinate system is calculated as:

$$\boldsymbol{r}_{SI} = \boldsymbol{r}_{FI} + \boldsymbol{C}_F^I(\boldsymbol{q})\left(\boldsymbol{r}_{SS} + \boldsymbol{C}_S^F \boldsymbol{r}_S\right), \tag{1}$$

where:
- $\boldsymbol{r}_{SI}, \boldsymbol{r}_S$ – Sun position vector in ECI and SCS, respectively,
- $\boldsymbol{r}_{FI}$ – satellite position vector in ECI,
- $\boldsymbol{r}_{SS}$ – sun sensor position vector in BFF,
- $\boldsymbol{q}$ – satellite attitude quaternion (transformation from ECI to BFF),
- $\boldsymbol{C}_S^F$ – transformation matrix from SCS to BFF,
- $\boldsymbol{C}_F^I(\boldsymbol{q})$ – satellite attitude matrix (transformation matrix from BFF to ECI).

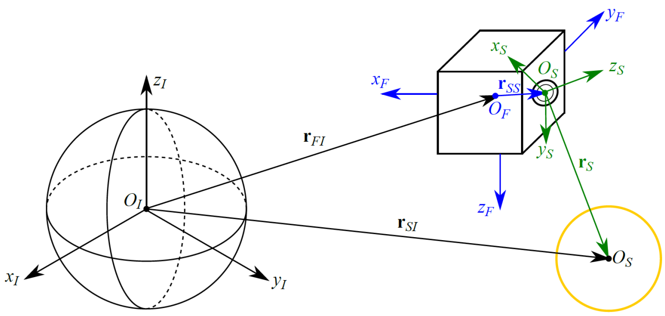

Fig. 2. Sun sensor direction vectors

The transformation matrices used in this paper are defined by the relation to quaternions and have the general form:

$$\boldsymbol{C}_F^I(\boldsymbol{q}) = \begin{bmatrix} q_0^2+q_1^2-q_2^2-q_3^2 & 2(q_1q_2-q_3q_0) & 2(q_1q_3+q_2q_0) \\ 2(q_1q_2+q_3q_0) & q_0^2-q_1^2+q_2^2-q_3^2 & 2(q_2q_3-q_1q_0) \\ 2(q_1q_3-q_2q_0) & 2(q_2q_3+q_1q_0) & q_0^2-q_1^2-q_2^2+q_3^2 \end{bmatrix}. \tag{2}$$

The vectors $\boldsymbol{r}_{SS}$ and $\boldsymbol{r}_{FI}$ are much smaller than $\boldsymbol{r}_{SI}$ and $\boldsymbol{r}_S$, therefore Eq. (1) is simplified to:

$$\boldsymbol{r}_{SI} = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_S^F\boldsymbol{r}_S. \tag{3}$$

Equation (3) is valid also for direction unit vectors, so:

$$\boldsymbol{g}_{SI} = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_S^F\boldsymbol{g}_S = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{g}_{SF}, \tag{4}$$

where:
- $\boldsymbol{g}_S$ – measured direction versor to the Sun in SCS,
- $\boldsymbol{g}_{SF}$ – Sun direction versor in BFF, $\boldsymbol{g}_{SF} = \boldsymbol{C}_S^F\boldsymbol{g}_S$,
- $\boldsymbol{g}_{SI}$ – direction versor to the Sun in ECI.

The unit vectors g_{SI} and g_{SF} are calculated for the current date and satellite position.

A similar approach is applied to the description of the earth sensor, which measures a direction unit vector to the Earth in its own Earth Sensor Coordinate System (ESCS).

The Earth Sensor Coordinate System (ESCS) location and orientation relative to the BFF may be calculated (Fig. 3) as:

$$\boldsymbol{r}_{FI} = -\boldsymbol{C}_F^I(\boldsymbol{q})\left(\boldsymbol{r}_{ES} + \boldsymbol{C}_{ES}^F \boldsymbol{r}_{EE}\right), \tag{5}$$

where:
$\boldsymbol{r}_{FI}$ – satellite position vector in ECI,
$\boldsymbol{r}_{ES}$ – earth sensor position vector in BFF,
$\boldsymbol{r}_{EE}$ – Earth position vector in ESCS,
$\boldsymbol{C}_{ES}^F$ – transformation matrix from ESCS to BFF.

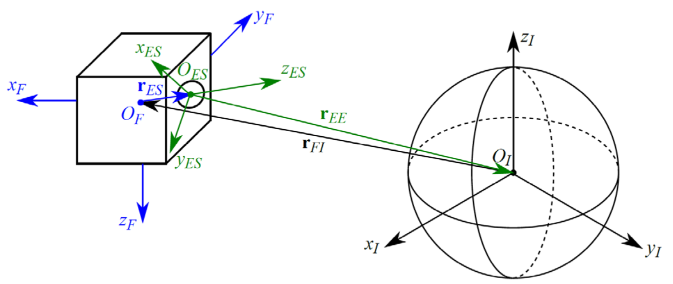

Fig. 3. Earth sensor direction vectors

The distance $\boldsymbol{r}_{ES}$ (sensor placement in the satellite) is much smaller than the $\boldsymbol{r}_{EE}$, therefore equation (5) is simplified to:

$$\boldsymbol{r}_{FI} = -\boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_{ES}^F \boldsymbol{r}_{EE}, \tag{6}$$

which, as in the sun sensor case, is also applied to direction unit vectors:

$$\boldsymbol{g}_{EI} = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_{ES}^F \boldsymbol{g}_E = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{g}_{EF}, \tag{7}$$

where:
$\boldsymbol{g}_E$ – measured direction versor to the Earth in ESCS,
$\boldsymbol{g}_{EF}$ – Earth direction unit vector in BFF, $\boldsymbol{g}_{EF} = \boldsymbol{C}_{ES}^F \boldsymbol{g}_E$,
$\boldsymbol{g}_{EI}$ – direction versor to the Earth in ECI.

The unit vectors $\boldsymbol{g}_{EF}$ and $\boldsymbol{g}_{EI}$ are calculated from known satellite position.

The detailed structure of data processing inside a star tracker is not considered in this study. A star tracker output is a measured satellite attitude quaternion $\boldsymbol{q}_{ST}$ in the Star Tracker Coordinate System (STCS) (Fig. 4).

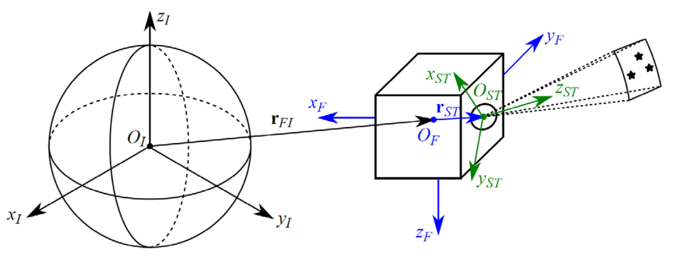

Fig. 4. Star tracker direction vectors

The STCS location and orientation relative to the BFF is described by the star tracker position vector $\boldsymbol{r}_{ST}$ and the transformation matrix $\boldsymbol{C}_{ST}^F$ from STCS to the BFF coordinate system. The transformation from STCS to the ECI coordinate system has the form:

$$\boldsymbol{r}_{StarI} = \boldsymbol{r}_{FI} + \boldsymbol{C}_F^I(\boldsymbol{q})\left(\boldsymbol{r}_{ST} + \boldsymbol{C}_{ST}^F \boldsymbol{r}_{StarST}\right), \tag{8}$$

where:
$\boldsymbol{r}_{StarI}, \boldsymbol{r}_{StarST}$ – vector towards a star expressed in ECI and STCS respectively,
$\boldsymbol{r}_{FI}$ – satellite position vector in ECI,
$\boldsymbol{r}_{ST}$ – vector of star tracker position in BFF,
$\boldsymbol{C}_{ST}^F$ – transformation matrix from STCS to BFF.

The vectors $\boldsymbol{r}_{ST}$ and $\boldsymbol{r}_{FI}$ are relatively small, so equation (8) is simplified to:

$$\boldsymbol{r}_{StarI} = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_{ST}^F \boldsymbol{r}_{StarST}. \tag{9}$$

Equation (9) is applied for versors pointing at selected star $\boldsymbol{g}_{StarST}$ – in STCS and $\boldsymbol{g}_{StarI}$ – in ECI:

$$\boldsymbol{g}_{StarI} = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_{ST}^F \boldsymbol{g}_{StarST}. \tag{10}$$

The satellite attitude quaternion q is calculated from the relation between transformation matrices:

$$\boldsymbol{C}_{ST}^I(\boldsymbol{q}_{ST}) = \boldsymbol{C}_F^I(\boldsymbol{q})\boldsymbol{C}_{ST}^F, \tag{11}$$

using the quaternion measured by the star tracker $\boldsymbol{q}_{ST}$ (which describes the rotation from ECI to STCS).

In the sensor models matrices $\boldsymbol{C}_{ST}^F$, $\boldsymbol{C}_{ES}^F$, $\boldsymbol{C}_{ST}^F$ describe sensor attitude with respect to spacecraft body. The most convenient is to define them by angles of rotations for instance Euler angles as used in aeronautics. In such a case providing yaw Ψ, pitch Θ, and roll Φ rotations sequence, the matrices will have the general form:

$$\boldsymbol{C}_{()}^F = \begin{bmatrix} c\Psi c\Theta & s\Psi c\Theta & -s\Theta \\ c\Psi s\Theta s\Phi - s\Psi c\Phi & s\Psi s\Theta s\Phi + c\Psi c\Phi & c\Theta s\Phi \\ c\Psi s\Theta c\Phi + s\Psi s\Phi & s\Psi s\Theta c\Phi - c\Psi s\Phi & c\Theta c\Phi \end{bmatrix}, \tag{12}$$

where s denotes sine and c denotes cosine functions and yaw Ψ, pitch Θ, and roll Φ rotations are defined for each specific sensor placement.

4. METHODS OF DATA FUSION

A single direction unit vector measured either by a sun sensor equation (4) or by an earth sensor equation (7) provides information only on two quantities (for instance, two angles) [7]. The satellite attitude matrix $\boldsymbol{C}_F^I(\boldsymbol{q})$ depends on three independent quantities, like three rotation angles. Therefore, at least two non-collinear direction unit vectors must be used.

The unit vectors to the Sun $\boldsymbol{g}_{SI}$ and to the Earth $\boldsymbol{g}_{EI}$ in the ECI are known for current satellite position and date. Two direction unit vectors measured in BFF by a sun sensor $\boldsymbol{g}_{SF}$ and by an earth sensor $\boldsymbol{g}_{EF}$ facilitate creating a system of equations:

$$\begin{cases} \boldsymbol{g}_{SI} = \boldsymbol{C}_F^I \boldsymbol{g}_{SF}, \\ \boldsymbol{g}_{EI} = \boldsymbol{C}_F^I \boldsymbol{g}_{EF}, \end{cases} \tag{13}$$

in which the satellite attitude matrix $\boldsymbol{C}_F^I$ is unknown. The attitude matrix is orthonormal, i.e.

$$\begin{cases} \boldsymbol{C}_F^I\left(\boldsymbol{C}_F^I\right)^T = \boldsymbol{I}, \\ \det\left(\boldsymbol{C}_F^I\right) = 1. \end{cases} \tag{14}$$

To solve equation (13) fulfilling conditions equation (14), four methods are investigated:
- The direct numerical solution,
- solutions to Wahba problem:
  - analytical,
  - numerical,
- TRIAD method.

4.1. Direct numerical solution

The system of equations (13) and orthogonality conditions (14) form an overdetermined system of 16 equations with nine unknown elements of the matrix $\boldsymbol{C}_F^I$. They are calculated by solving a system of equations in the form:

$$\boldsymbol{f}(\boldsymbol{x}) = \boldsymbol{0}, \tag{15}$$

where for description brevity vector $\boldsymbol{x}$ denotes here elements of the matrix $\boldsymbol{C}_F^I$. The subsequent components of vector $\boldsymbol{f}(\boldsymbol{x})$ are defined as:

$$\begin{cases} \boldsymbol{f}_{[1:3]}\left(\boldsymbol{C}_F^I\right) = \boldsymbol{C}_F^I \boldsymbol{g}_{SF} - \boldsymbol{g}_{SI}, \\ \boldsymbol{f}_{[4:6]}\left(\boldsymbol{C}_F^I\right) = \boldsymbol{C}_F^I \boldsymbol{g}_{EF} - \boldsymbol{g}_{EI}, \\ \boldsymbol{f}_{[7]}\left(\boldsymbol{C}_F^I\right) = \det\left(\boldsymbol{C}_F^I\right) - 1, \\ \boldsymbol{f}_{[8:16]}\left(\boldsymbol{C}_F^I\right) = \boldsymbol{C}_F^I\left(\boldsymbol{C}_F^I\right)^T - \boldsymbol{I}. \end{cases} \tag{16}$$

The MATLAB function *fsolve* was used to obtain the solution (15). The function uses the Levenberg-Marquardt method [8] to calculate the approximate solution of equation (16) by minimizing the objective function in the form of (15). To start the calculations an initial value $\boldsymbol{C}_F^I$ is needed, which may be the last known attitude.

4.2. Solution to the Wahba problem

The rotation matrix $\boldsymbol{C}_F^I$ may be calculated as a solution to the Wahba problem to find the matrix which when given several measurements minimizes the functional $J\left(\boldsymbol{C}_F^I\right)$:

$$J\left(\boldsymbol{C}_F^I\right) = \frac{1}{2}\sum_{i=1}^{N} a_i \left\|\boldsymbol{C}_F^I \boldsymbol{g}_{iF} - \boldsymbol{g}_{iI}\right\|, \tag{17}$$

where:
N – number of measurements,
a_i – positive weight for i-th measured vector,
$\boldsymbol{g}_{iF}, \boldsymbol{g}_{iI}$ – i-th direction versor in BFF and ECI respectively,
$\|\ldots\|$ – the Euclidean norm.

For two-direction versors obtained from the sun sensor $\boldsymbol{g}_{SF}$ and the earth sensor $\boldsymbol{g}_{EF}$ equation (17) has the form:

$$J\left(\boldsymbol{C}_F^I\right) = \frac{1}{2}a_S\left\|\boldsymbol{C}_F^I \boldsymbol{g}_{SF} - \boldsymbol{g}_{SI}\right\| + \frac{1}{2}a_E\left\|\boldsymbol{C}_F^I \boldsymbol{g}_{EF} - \boldsymbol{g}_{EI}\right\|, \tag{18}$$

where a_S and a_E are positive weights for the measured directions the Sun and to the Earth, respectively.

The Wahba problem has an analytical solution for two vectors [7] based on equation (18) transformed to the form:

$$J\left(\boldsymbol{C}_F^I\right) = a_S + a_E - \operatorname{tr}\left(\boldsymbol{C}_F^I \boldsymbol{B}^T\right), \tag{19}$$

$$\boldsymbol{B} = a_S \boldsymbol{g}_{SI}\boldsymbol{g}_{SF}^T + a_E \boldsymbol{g}_{FI}\boldsymbol{g}_{EF}^T. \tag{20}$$

The direct Wahba problem solution is reformulated to maximizing trace of $\boldsymbol{C}_F^I \boldsymbol{B}^T$, which is achieved using singular value decomposition of matrix $\boldsymbol{B}$.

The numerical solution to the Wahba problem was also considered here, as in prospective applications of the attitude algorithm also other sensors (for instance, magnetic field) may be implemented.

The minimum of the $J\left(\boldsymbol{C}_F^I\right)$ defined by equation (18) subjected to constraints of (14) is calculated using the MATLAB function *fmincon*, which uses the Interior-Point Algorithm [9] to solve the problem.

4.3. TRIAD method

The TRIAD (TRIaxial Attitude Determination) is an analytical method to calculate satellite attitude using two-direction measurements. The attitude matrix $\boldsymbol{C}_F^I$ is calculated in two steps: as the rotation $\boldsymbol{C}_F^T$ from BFF to the new TRIAD coordinate system $O_T t_1 t_2 t_3$ and then rotation $\boldsymbol{C}_T^I$ from $O_T t_1 t_2 t_3$ to ECI:

$$\boldsymbol{C}_F^I = \boldsymbol{C}_T^I \boldsymbol{C}_F^T. \tag{21}$$

The rotation matrices $\boldsymbol{C}_F^T$ and $\boldsymbol{C}_T^I$ are defined as:

$$\boldsymbol{C}_T^I = \begin{bmatrix} \boldsymbol{t}_{1I} & \boldsymbol{t}_{2I} & \boldsymbol{t}_{3I} \end{bmatrix}, \tag{22}$$

$$\boldsymbol{C}_F^T = \begin{bmatrix} \boldsymbol{t}_{1F} & \boldsymbol{t}_{2F} & \boldsymbol{t}_{3F} \end{bmatrix}^T, \tag{23}$$

where the columns of matrices are unit vectors $\boldsymbol{t}_{1I}$, $\boldsymbol{t}_{2I}$ and $\boldsymbol{t}_{3I}$ are calculated in ECI and $\boldsymbol{t}_{1F}$, $\boldsymbol{t}_{2F}$ and $\boldsymbol{t}_{3F}$ are measured in BFF.

The TRIAD coordinate system $O_T t_1 t_2 t_3$ is defined by three orthonormal unit vectors $\boldsymbol{t}_1$, $\boldsymbol{t}_2$ and $\boldsymbol{t}_3$. The vector $\boldsymbol{t}_1$ is one of the measured unit direction vectors. As it is used for defining axes directly, it is recommended to select the vector for which the most accurate measurements are available. The vector $\boldsymbol{t}_2$ is perpendicular to the two measured unit directions versors and vector $\boldsymbol{t}_3$ completes the orthonormal triad. $\boldsymbol{t}_3$ is calculated as the vector product of $\boldsymbol{t}_1$ and $\boldsymbol{t}_2$.

For earth and sun sensors, the direction vectors are measured in the BFF coordinate system and respective unit vectors in the ECI coordinate system are known for given satellite position and time. The direction versors in BFF are $\boldsymbol{g}_{1F}$ and $\boldsymbol{g}_{2F}$, and corresponding versors in ECI are $\boldsymbol{g}_{1I}$ and $\boldsymbol{g}_{2I}$. The axes of the $O_T t_1 t_2 t_3$ coordinate system in BFF and ECI are equal to:

$$\begin{cases} \boldsymbol{t}_{1F} = \boldsymbol{g}_{1F} \\ \boldsymbol{t}_{2F} = \dfrac{\boldsymbol{g}_{1F} \times \boldsymbol{g}_{2F}}{\|\boldsymbol{g}_{1F} \times \boldsymbol{g}_{2F}\|}, \\ \boldsymbol{t}_{3F} = \boldsymbol{t}_{1F} \times \boldsymbol{t}_{2F} \end{cases} \quad \begin{cases} \boldsymbol{t}_{1I} = \boldsymbol{g}_{1I} \\ \boldsymbol{t}_{2I} = \dfrac{\boldsymbol{g}_{1I} \times \boldsymbol{g}_{2I}}{\|\boldsymbol{g}_{1I} \times \boldsymbol{g}_{2I}\|}. \\ \boldsymbol{t}_{3I} = \boldsymbol{t}_{1I} \times \boldsymbol{t}_{2I} \end{cases} \tag{24}$$

The TRIAD method is limited to two-direction measurements, and it does not profit from all measured data.

5. DATA FUSION OF STAR TRACKER AND EARTH-SUN SENSORS

Star tracker measurements are fused with quaternions from other sensors. First an attitude star tracker measurements quaternion $\boldsymbol{q}_{STF}$ is calculated from equation (11) by replacing the transformation matrix $\boldsymbol{C}_F^I(\boldsymbol{q})$ with $\boldsymbol{C}_F^I\left(\boldsymbol{q}_{STF}\right)$.

The integrated value of satellite attitude quaternion $\hat{q}$ is obtained by combining quaternion $\boldsymbol{q}_{SE}$ calculated from the sun and earth sensor measurements with the star tracker measurement $\boldsymbol{q}_{STF}$. Two methods of data fusion were explored in this study: weighted average and Kalman filter. Both methods facilitate prospective extension to more attitude quaternions to be fussed.

5.1. Weighted average method

To calculate the weighted quaternion, the spherical linear interpolation [10] method was used. The combined quaternion is calculated as:

$$\hat{\boldsymbol{q}} = \frac{\boldsymbol{q}_{SE}\sin(w_{SE}\Omega) + \boldsymbol{q}_{STF}\sin(w_{STF}\Omega)}{\sin\Omega}, \tag{25}$$

where $w_{SE} + w_{STF} = 1$.

The weights w_{SE}, w_{STF} may be selected arbitrarily, but the accuracy of measurements may be some hint for the selection. The angle Ω is calculated from the dot product of two quaternions as:

$$\Omega = \operatorname{acos}(\boldsymbol{q}_{SE} \cdot \boldsymbol{q}_{STF}). \tag{26}$$

For $\boldsymbol{q}_{SE} = \pm\boldsymbol{q}_{STF}$ equation (25) is singular, which was considered in the final attitude algorithm structure.

5.2. Kalman filter application

The Kalman filter was used for recursive quaternion estimation using subsequent measurements. Two model processes were investigated, one estimating the attitude quaternion directly, and the other one estimating the attitude quaternion error. It was done for comparison of the efficiency of two different process models, as in attitude systems the errors are estimated more often.

In the first approach (Fig. 5), the estimated state vector was a satellite attitude quaternion $\hat{\boldsymbol{q}}$ describing transformation from ECI to BFF; the state equation was:

$$\boldsymbol{q}_{k+1} = \boldsymbol{q}_k, \tag{27}$$

where k and $k+1$ are indices of subsequent measurements (in subsequent time steps).

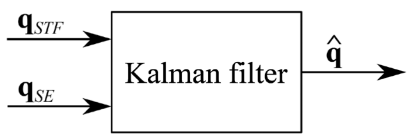

Fig. 5. Linear Kalman filter estimating attitude quaternion

The measurement vector was composed of both measurements: $\boldsymbol{q}_{SEk}$ and $\boldsymbol{q}_{STFk}$:

$$z_k = \begin{bmatrix} \boldsymbol{q}_{SEk} & \boldsymbol{q}_{STFk} \end{bmatrix}^T. \tag{28}$$

The covariance matrix $\boldsymbol{Q}$ of the state noise was:

$$\boldsymbol{Q} = \sigma \boldsymbol{I}_{4\times4}, \tag{29}$$

where σ is the assumed variance of state disturbances.

The covariance matrix R of observation noise was diagonal in the form:

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{R}_{SE} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{R}_{STF} \end{bmatrix}, \tag{30}$$

where $\boldsymbol{R}_{SE}$ and $\boldsymbol{R}_{STF}$ are covariance matrices of an attitude quaternion error from sun and earth sensors and star tracker, respectively.

In the second Kalman filter implementation (Fig. 6) the state variable was the quaternion error $\boldsymbol{\delta}_q$ given by:

$$\boldsymbol{\delta}_q = \boldsymbol{q}_{SE} - \boldsymbol{q}_{STF}. \tag{31}$$

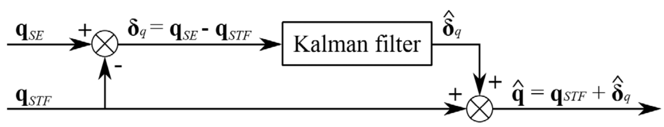

Fig. 6. Linear Kalman filter estimating quaternion difference

The state equations had the form:

$$\boldsymbol{\delta}_{q(k+1)} = \boldsymbol{\delta}_{q(k)}, \tag{32}$$

and observation vector z_k was assumed as:

$$z_k = \boldsymbol{\delta}_{q(k)}. \tag{33}$$

The state $\boldsymbol{Q}$ and observation $\boldsymbol{R}$ noise covariance matrices were described as:

$$\boldsymbol{Q} = \omega \boldsymbol{I}_{4\times4}, \qquad \boldsymbol{R} = \rho \boldsymbol{I}_{4\times4}. \tag{34}$$

The values ω and ρ were variations of the state and observation disturbances.

The final attitude quaternion was calculated as:

$$\hat{\boldsymbol{q}} = \boldsymbol{q}_{STF} + \hat{\boldsymbol{\delta}}_q. \tag{35}$$

The covariance matrices of states and observer noises are usually selected as specific for cases of applied sensors and environment stochastic properties.

6. SIMULATION STUDY

The validation of the algorithm was done using simulated data. The error of attitude quaternion was defined as rotation from estimated $\hat{\boldsymbol{q}}$ to the true $\boldsymbol{q}$ attitude quaternion using the θ_{ER} angle calculated as:

$$\theta_{ER} = 2\operatorname{acos}|q_{e0}|, \tag{36}$$

where q_{e0} is the first component (scalar component) of an attitude error quaternion corresponding to the rotation from $\hat{q}$ to the true $\boldsymbol{q}$.

The simulations were performed for a satellite 600 km above the Earth, in equatorial plane at the ECI position $r_{FI} = \begin{bmatrix} 4.9292 & 4.9292 & 0 \end{bmatrix}^T \begin{bmatrix} 10^6\,\text{m} \end{bmatrix}$. The satellite attitude quaternion in ECI was $\boldsymbol{q} = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$ and the direction to the Sun in ECI was $\boldsymbol{g}_{SI} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$.

The simulated sensor measurands data were calculated using "inverse relations" to the formulae of sensor models with added random noise. The random noise added to the sun sensor error was within 1° and to the earth sensor error – within 2°. The noise was generated once and used for all simulations to facilitate a comparison of results of various algorithms for the same set of input data.

The results of fusing data from sun and earth sensors for 100 simulation points (steps) are illustrated in Fig. 7 and detailed numbers are given in Table 2.

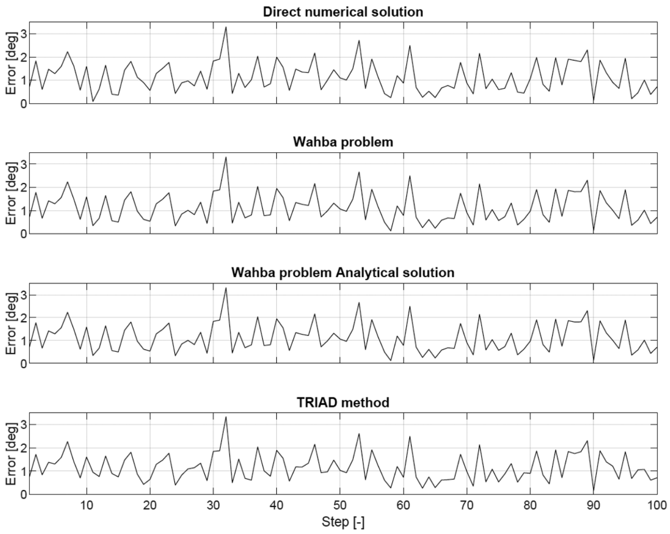

Fig. 7. Errors for sun and earth sensor integration

Table 2
Simulation results of sun and earth sensor integration

	Direct *numerical*	Wahba *numerical*	Wahba *analytical*	TRIAD
Average error [deg]	1.1378	1.1272	1.1272	1.1672
Maximum error [deg]	3.3001	3.3089	3.3089	3.3425
Average iterations [–]	7	17	non-iterative	non-iterative
Maximum iterations [–]	7	42	non-iterative	non-iterative
Total calculation time [s]	1.3906	7.9241	0.0389	0.0247

The number of simulation points reflects the number of static experiments performed so it seems sufficient to compare the efficiency of the methods.

The attitude was estimated with similar error values for all algorithms integrating data from direction sensors. The lowest average error was obtained for both numerical and analytical solution of Wahba problem, and the lowest maximum error was obtained by direct numerical solution. The TRIAD method took the shortest computation time. The numerical solution of the Wahba problem required the largest number of iterations and the longest computation time to converge comparing to the direct numerical solution. But its analytical counterpart provided a similar result in a much shorter time.

Simulations of the weighted average method were performed for biased quaternions $\boldsymbol{q}_{SE}$ and $\boldsymbol{q}_{STF}$. The quaternion $\boldsymbol{q}_{SE}$ describes an attitude expressed in Euler angles (yaw, pitch, and roll) as $45° + 0.9\delta$, 45°, 45° with weight $w_{SE} = 0.1$ and quaternion $\boldsymbol{q}_{STF}$ describes an attitude reflecting Euler angles $45° - 0.1\delta$, 45°, 45° with weight $w_{STF} = 0.9$, where δ is a simulated bias angle. The quaternion errors for selected bias angles are given in Table 3. The error of the weighted average method increases with the growing bias value but is substantially less than the assumed bias in input data.

Table 3
The error of the weighted average method

Bias	Error
$\delta = 2°$	$1.3° \cdot 10^{-7}$
$\delta = 5°$	$2.0° \cdot 10^{-6}$
$\delta = 15°$	$5.4° \cdot 10^{-5}$

In two implementations of the Kalman filter denoted here as (KF1) and (KF2), simulations were performed for parameters given in Table 4. The quaternion $\boldsymbol{q}_{SE}$ of sun and earth sensor measurement was calculated using the TRIAD method with the Sun direction as a base vector, and the star tracker quaternion $\boldsymbol{q}_{STF}$ was calculated from noisy star tracker measurements. The assumed maximum sensor errors were sun sensor – 1°, earth sensor – 2°, star tracker pointing – 0.002°, star tracker rolling – 0.02°. The values of observer covariance matrices were based on real sensor data given in specifications.

Table 4
Parameters of Kalman filters

	KF1	KF2
process covariance	$\boldsymbol{Q} = 10^{-9}\boldsymbol{I}_{4\times4}$	$\boldsymbol{Q} = 10^{-7}\boldsymbol{I}_{4\times4}$
observation covariance star tracker	$\boldsymbol{R}_{STF} = 10^{-6}\boldsymbol{I}_{4\times4}$	
observation covariance earth/sun sensors	$\boldsymbol{R}_{SE} = 10^{-2}\boldsymbol{I}_{4\times4}$	
observation covariance quaternion difference		$\boldsymbol{R} = 10^{-2}\boldsymbol{I}_{4\times4}$
initial state vector:	$\hat{\boldsymbol{q}}_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}^T$	
initial states covariance	$\boldsymbol{P} = 10^{-6}\boldsymbol{I}_{4\times4}$	$\boldsymbol{P} = 10^{-6}\boldsymbol{I}_{4\times4}$

The results of the simulation are presented in Fig. 8 and the final error values in Table 5. In the cases considered, the average

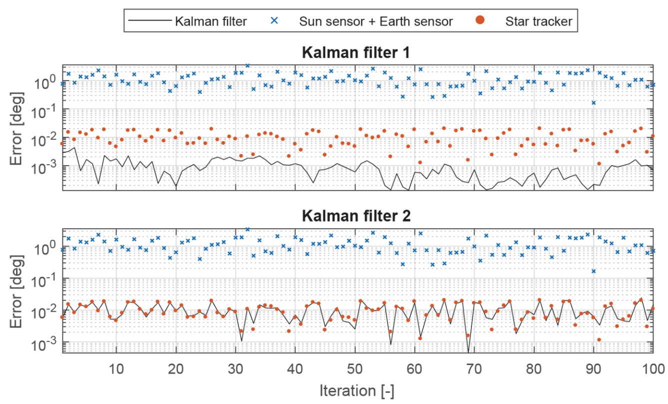

Fig. 8. Kalman filtering results

error from the KF1 method is about 11 times smaller than the error of KF2. It illustrates mainly the difference in the results for different KF models, using more precise sensor data the KF2 would improve the results.

Table 5

Summary of estimation errors for Kalman filter method

	q_{SE}	q_{STF}	KF1	KF2
Average error [deg]	1.1672	0.0099	0.0009	0.0101
Maximum error [deg]	3.3425	0.0199	0.0043	0.0239
Standard deviation [deg]	0.6029	0.0054	0.0007	0.0056

7. CONCLUSIONS

The objective of this study was to design an algorithm for satellite attitude determination, which integrates measurements from three visual sensors: sun sensor, earth sensor, and star tracker and to investigate various methods for estimating satellite attitude. The architecture of an attitude calculation software was designed to facilitate embedding various methods for combining sensor data. The measurements of directions from the sun and earth sensors were combined to calculate satellite attitude using one of the four methods: direct numerical solution, numerical and analytical solutions of Wahba problem and the TRIAD method. The final satellite attitude was computed by fusing the data from sun and earth sensor direction measurements and data from a star tracker. The efficiency of the methods was investigated by simulations. The methods of attitude estimation from the sun and earth sensors provided an attitude with similar errors close to the accuracies of individual measurements. The TRIAD method required the lowest and the numerical solution of the Wahba problem – the highest computational time. Two applications of the Kalman filter were compared for integration sensor data corrupted by random noise. The Kalman filter, based on state vector which directly reflected satellite attitude quaternion, estimated satellite attitude with the average error about 11 times less than the error of the input from the star tracker. The Kalman filter, where the state vector was the measurement error, estimated attitude with the average and maximum error higher than the error of input from the star tracker.

The results presented show guidelines for practical implementation of the investigated methods.

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