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The effect of topography and quality of a digital terrain model on the accuracy of terrain corrections for centimetre quasigeoid modelling

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Abstract: Modelling quasigeoid with centimetre accuracy requires taking into account irregularities of topography in the vicinity of a gravity station, i.e. the terrain correction to surveyed gravity. Accuracy of determination of the terrain correction affects quality of quasigeoid model determined. It depends on the resolution and accuracy of terrain data that usually is provided in the form of a digital terrain model DTM.

Investigations were conducted with the use of the Digital Terrain Elevation Data – DTED2 model developed for Poland according to the NATO-STANAG 3809 standard, as well as global models SRTM3 and SRTM30 (The Shuttle Radar Topography Mission). Also height data from the gravity database was considered.

The prism method of determination of terrain corrections was applied in majority of numerical tests. Practical method for determining the optimum radius of the integration cap considering roughness of topography as well as required accuracy of terrain corrections was developed. The effect of vertical and horizontal uncertainty of a DTM as well as its resolution on the quality of the terrain corrections was investigated. The terrain corrections obtained using a prism method were also compared with the respective ones calculated using the FFT approach. The usefulness of the available topography data for precise terrain correction computation in Poland was discussed.

The results of the investigations were used to determining the strategy of computation of the terrain corrections to point gravity data in the gravity database for Poland. The "2005" terrain correction set calculated for 1 078 046 gravity stations contributes to the increase of precision of gravimetric quasigeoid models developed for Poland.

Keywords: Terrain correction, digital terrain model, DTED, SRTM, prism method, FFT, quasigeoid

1. Introduction

Modelling of quasigeoid that determines the shape of the Earth is considered as one of major tasks of geodesy. Recent surveying techniques based on the use of precise global navigation systems require quasigeoid models of high precision. The surveyor can efficiently replace a laborious and time consuming spirit levelling with GNSS levelling, providing the quasigeoid model at a centimetre accuracy level is available.

Although quasigeoid models based on GNSS/levelling data – called "best fitted models" – are most suitable for surveying practice, the GNSS/levelling points are too sparse to ensure required accuracy of quasigeoid heights calculated between them. Densely distributed gravity data may efficiently support such a model (Osada et al., 2005). On the other hand the gravimetric quasigeoid model is a powerful tool to verify and validate GNSS/levelling quasigeoid models. To calculate the gravimetric quasigeoid model, that is nothing but a solution of a boundary value problem, with the use of either the Stokes' approach (geoid model) or the rigorous Molodensky's approach, the gravitational effect of terrain – the terrain correction – must be taken into consideration, particularly in mountainous regions (e.g. Forsberg, 1984). The importance of implementing the terrain corrections to the process of quasigeoid modelling was reported by many researchers in the last decade. The consideration of roughness of topography around gravity stations resulted in change of quasigeoid heights, for example in East Australia and Tasmania at the level of a few decimetres (Zhang and Featherstone, 1997), while in Chinese part of Himalayas – up to over 2 m (Zhang et al., 1998).

To model geoid using the Stokes' approach, gravity anomalies are required on the geoid. Gravity surveyed at the Earth surface must then be reduced to the geoid. The reduction must not change the total mass of the Earth. Moreover, after the reduction no masses must remain outside the geoid.

One of the components of the Faye gravity reduction is the terrain correction c that represents the deviation of the actual topography from the Bouguer plate of the gravity station P (Heiskanen and Moritz, 1967). Computation of terrain corrections, that is one of the most laborious tasks in precise geoid/quasigeoid modelling, requires the knowledge of the density of the upper crust of the Earth, the height of a gravity station P above the geoid as well as the height characteristics of the terrain within a certain radius from P. Particularly important is a very good knowledge of topography around a gravity station P (Sideris, 1984).

The conventional methods of the terrain correction computation use either gridded or raw height data (Ferland, 1984). The area around the gravity station P is subdivided into zones. The terrain correction is expressed by a sum of the contributions of the zones. Those methods are rather time consuming, especially when high accuracy of the terrain correction determination is required and high resolution height data is available.

The extensive research on methodology of terrain corrections computation using conventional methods was conducted for the test area in western Canada (Blais et al, 1983; Blais and Ferland, 1984). The results of simulation analysis that carefully investigated contributions of near zone, intermediate zone and distant zone, show that when neglecting errors in the height information, an accuracy for the terrain correction of 1.0–1.5 mGal can be achieved in such rough topography, that ensures accuracy of 2 mGal in Bouguer anomalies. It was also shown that the critical part of the topographic data is what corresponds to the near zone around the gravity station. To reach the accuracy of 1.0–1.5 mGal in the terrain correction, topographic map data corresponding to 1:50 000 are acceptable for distant zones and most of the intermediate zones (Blais and Ferland, 1984).

Faster approaches of the terrain correction computation involve the Fast Fourier Transform (FFT) technique. Majority of DTMs can directly be applied to the FFT algorithms for the terrain correction computation since they process height data provided on a regular grid.

In the gravity database for Poland, established in 1974–1992 by the Polish Geological Institute for geological and geophysical prospective, the terrain corrections were computed only at the stations where the slopes exceeded 6° within the radius of 100 m from the station. In the mountainous regions, i.e. Carpatians, Sudeten, and Holy Cross Mountains, the terrain corrections were calculated within the radius of 22.5 km. In the inner zone – up to 100 m – the approach of the circular diagram of Lukavtchenko based on measured terrain slope angles was used. In the intermediate zone ranging from 100 m to 1500 m, the Kane's approach based on mean heights for 200 m × 200 m sectors, determined from the maps at the scale of 1:25 000 was applied. The terrain correction due to the outer zone ranging from 1.5 km to 22.5 km was calculated using the Bott's method based on mean heights for 1 km × 1 km sectors (Królikowski, 2004).

Data from the existing gravity database for Poland was qualitatively and quantitatively analysed in the framework of the research project led by the Institute of Geodesy and Cartography, Warsaw, in 2002–2005. It was verified and updated to fulfil the requirements of centimetre quasigeoid modelling in Poland. One of the tasks of the project was to compute new terrain corrections at all 1 078 046 stations of the gravity database using the newly developed high-resolution DTM for Poland. High-resolution DTMs are required for precise determination of terrain corrections in mountainous regions. The use of a high-resolution height model ensures the reduction of systematic error in the terrain correction (Forsberg, 2005).

The classical rectangular prism method was used for computing the terrain corrections in Poland. Summed up positive and negative mass contributions, representing areas where the topography is either above or below a Bouguer plate passing through the computation point P provides the terrain correction to the vertical component of gravity at P. For each station P the heights of the prisms correspond to the difference between the elevation of P and the elevation of the elements of the DTM used. The sides of the bases of the prisms are determined by grid points of the DTM. A constant density $\rho = 2.67$ g/cm^3 was used for all prisms derived from the DTM. The terrain correction at P was calculated in planar coordinates using the following formula (Torge, 2001; Forsberg, 2005)

$$c_P = G\rho \int_{-\infty}^{\infty}\int \int_{z=H_P}^{z=H(x,y)} \frac{z - H_P}{\left[(x-x_P)^2 + (y-y_P)^2 + (z-H_P)^2\right]^{3/2}} dxdydz \qquad (1)$$

where

x_P, y_P – planar coordinates of the gravity station P,

H_P – elevation of the gravity station P,

x, y – planar coordinates of the central point of the current prism (the element of topography),

z – elevation of the central point Q of the current prism,
dx, dy – north and east grid spacings of the DTM,
dz – difference between the elevation of P and the elevation of current element of the DTM (height of the prism),
G – gravitational constant,
ρ – density of the upper crust of the Earth (density of topography).

Planar coordinates x, y in the horizontal plane of P are obtained from geodetic coordinates considered in spherical approximation as spherical coordinates with respect to the sphere of the radius R = 6371 km.

Since the planar approximation of the Earth' surface is considered insufficient when a centimetre accuracy of a geoid/quasigeoid model is needed (Rózsa, 1998), the heights of the prisms that correspond to the elements of the DTM were corrected for the curvature of the Earth in the process of the terrain correction computation. The respective correction dh to the height z is expressed as follows

$$dh = \frac{r^2}{2R} \tag{2}$$

where r is the distance of the central point of the current prism from P. The example given by Ferland (1984) illustrates the effects of planar and spherical approximations. The gravity anomaly for the circular disc of 111.2 km in diameter, 30 km thick and a density of 1 g/cm^3 was computed assuming the mass to be just below the surface. The difference between assuming a flat top disc and the effect of sphericity was approximately 10% while the difference between the spherical and ellipsoidal case was always less than 0.4%. The conclusion drawn by Ferland was that ellipticity is negligible for terrain corrections.

Terrain corrections can also be computed with a FFT technique using the formula that is the combination of convolutions (∗) (Forsberg, 2005)

$$c_P = \frac{1}{2}G\rho\left[z^2 * f - 2H_P(z * f) + H_P^2 f_0\right] \tag{3}$$

where f is the kernel

$$f = \left(x^2 + y^2\right)^{-\frac{3}{2}}$$

and

$$f_0 = \int_{-\infty}^{\infty}\int \frac{1}{r_0^3}dxdy, \quad r_0 = \sqrt{(x_P - x)^2 + (y_P - y)^2}$$

$$f * z = F^{-1}\left[F \cdot G\right]$$

with F – Fourier transform of f, G – Fourier transform of z, and F^{-1} – inverse Fourier transform.

When f_0 evaluated with FFT over a finite domain, it turns into a constant obtained from the 0 (DC) value of the Fourier transform of the kernel.

The terrain correction c is applied to surveyed gravity when calculating Faye gravity anomalies Δg^F or complete Bouguer gravity anomalies Δg^B, i.e.

$$\Delta g_P^F = g_P^{obs} - \gamma_Q + F_P + c_P \tag{4}$$

$$\Delta g_P^B = g_P^{obs} - \gamma_Q - B_P + F_P + c_P \tag{5}$$

where γ_Q is the normal gravity at the point Q on the telluroid that corresponds to P at the Earth surface, F_P is the free-air reduction (practically $F_P = 0.3086\ H_P$), and B_P is the so-called incomplete Bouguer reduction (practically $B_P = -0.1119\ H_P$).

The TC program of the GRAVSOFT package used (Forsberg, 2005) computes terrain corrections with a prism method. Since the TC program provides the terrain correction (1) that could rather be applied to the gravity disturbance $\delta g_P = g_P^{obs} - \gamma_P$ than to the gravity anomaly $\Delta g_P = g_P^{obs} - \gamma_Q$, the indirect effect should be considered. Thus, before applying to (4) and (5), the terrain correction c_P calculated with (1) must first be reduced by a simple free-air reduction

$$\delta g = -0.3086\delta\zeta \tag{6}$$

where $\delta\zeta$ is the increment of the height anomaly due to irregularities of topography.

2. Height data used in the analysis

Height data of sufficient resolution and accuracy is required to compute terrain corrections to gravity data. Heights used for computing the terrain corrections were traditionally obtained from topographic maps using sophisticated laborious procedures. Availability of high resolution and accurate DTMs together with progressing in computational technology, make possible to calculate more accurate terrain corrections and to extend computation to flat areas to fulfil the requirements of centimetre geoid/quasigeoid modelling.

Three DTMs: DTED2, SRTM3 and SRTM30 were used in the present work for computing terrain corrections.

The DTED (Digital Terrain Elevation Data) digital terrain model has been elaborated in Poland according to the NATO-STANAG 3809 standard (NGA, 1996). The horizontal datum of the model is WGS84 while the elevations are referred to MSL. The model covers completely the territory of Poland (between 49°–55°N and 14°–24°E) and is available at two resolution levels: level 1 of 15′ × 15′ resolution, and level 2 (DTED2) of 1″ × 1″ (49°–50°N) and 1″ × 2″ (50°–55°N) resolution. The DTED2 model was derived by digitization of 1:50 000 topographic maps in '1942' geodetic datum, with 10 m contour line interval, with support from a military geodetic control modernized in late 90. Estimated vertical accuracy of the model varies from 2 m to 7 m, while its horizontal accuracy equals to 15 m (Krynski et al., 2005).

The SRTM (Shuttle Radar Topography Mission) data, acquired during the 11-day mission in February 2000, covers land areas over nearly 80% of Earth's land surfaces between 54° south latitude and 60° north latitude (Bamler, 1999). As a product of radar interferometry survey the SRTM provides elevations that are not always referred to actual ground level. Both natural and human-made land coverage affect the model (Showstack, 2003).

The SRTM3 model of $3'' \times 3''$ resolution, released to public in 2004, is a preliminary one as not yet fully consistent with map accuracy standards (JPL, 2004). The horizontal datum of the model is WGS84 while the elevations are referred to EGM96 geoid. The accuracy of the model is uniform over the whole area covered. The absolute vertical accuracy is specified as 16 m and the absolute horizontal accuracy is 20 m (Bamler, 1999; JPL, 2004).

The SRTM30 model of $30'' \times 30''$ resolution is a generalization of the SRTM3 model. Figure 1 shows the coverage of Poland and neighbouring regions with the three DTMs considered.

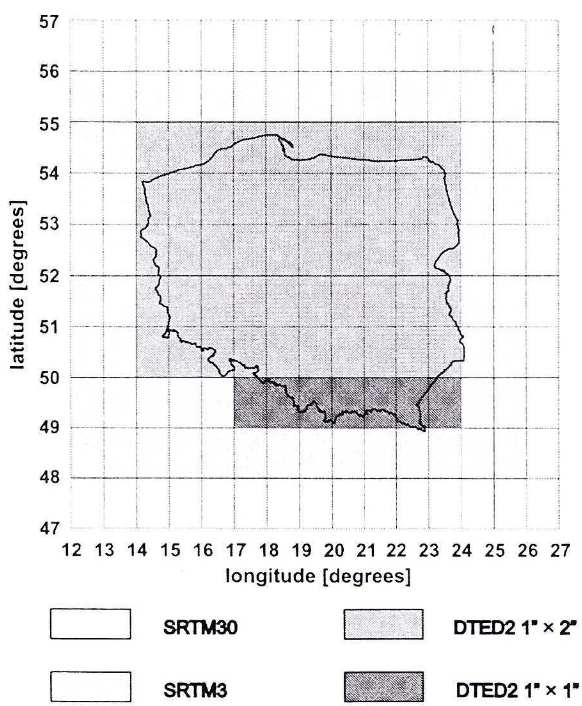

Fig. 1. Coverage of Poland and neighbouring regions with the three DTMs: DTED2, SRTM3 and SRTM30

3. Numerical experiments

A number of numerical experiments on computation of the terrain corrections have been performed. They considered the determination of the optimum maximum radius of computation of terrain corrections, the effect of vertical and horizontal errors of the terrain model and its resolution on calculated terrain corrections as well as comparison of terrain corrections obtained using the prism and the FFT methods.

3.1. Determination of the optimum maximum radius of computation of the terrain corrections

Contribution of the deviation of actual topography from the Bouguer plate of P to the terrain correction decreases with growing distance d_i from P. In addition, it converges much faster in flat than in hilly areas. On the other hand, the computation time using the prism method increases proportionally to d^2. To calculate the terrain correction with sufficient accuracy and in a reasonable time, the determination of the optimum radius d of integration of prisms is needed.

A number of experimental terrain models $2d_{max} \times 2d_{max}$ (Fig. 2) were generated to investigate the relation between the deviations of topography and the required radius d. In the models, only the elements distant by d_i in x, y coordinates, from the station P are deviated in height by Δh from the Bouguer plate of P. For each Δh and for varying d_i the convergence of the resulting terrain correction was investigated. The radius d was considered the optimum maximum radius when the contribution of modelled topography from the band determined by radii d and d_{max} becomes smaller than the assumed accuracy ε of the terrain correction (Fig. 3) (Grzyb, 2004).

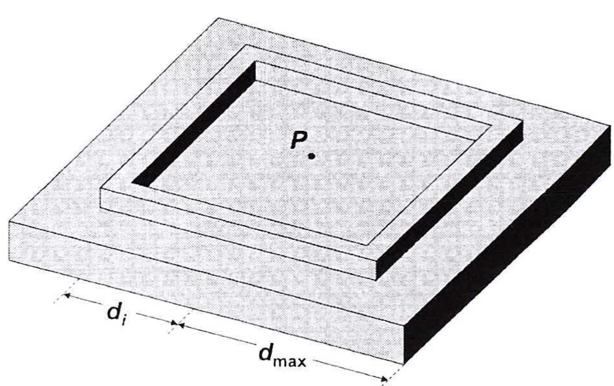

Fig. 2. The concept of the experimental terrain model used for determination of the optimum maximum radius d of integration of prisms when computing the terrain correction

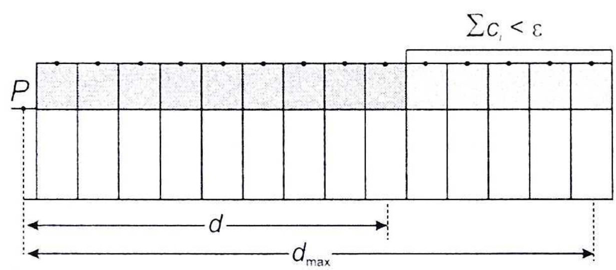

Fig. 3. The concept of determination of the optimum maximum radius d of integration of prisms when computing the terrain correction

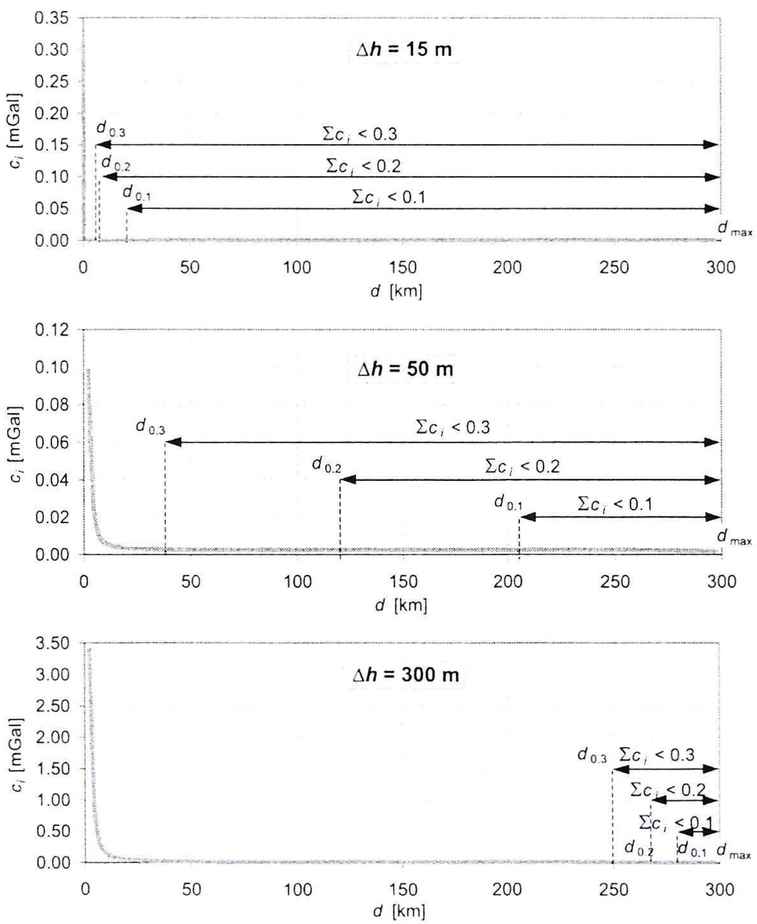

Fig. 4. Determination of the optimum maximum radius d of integration of prisms when computing the terrain correction for Δh = 15 m, Δh = 50 m, Δh = 300 m and for ε = 0.1 mGal, ε = 0.2 mGal, ε = 0.3 mGal

Table 1. The optimum maximum d [km] for different distortions Δh of terrain elevation

Accuracy ε of the terrain correction [mGal]	Distortion Δh of terrain elevation relative to a gravity station [m]		
	15 (flat)	50 (hily)	300 (mountainous)
0.1	14.9	205	280
0.2	0.22	118	265
0.3	0.06	33	249

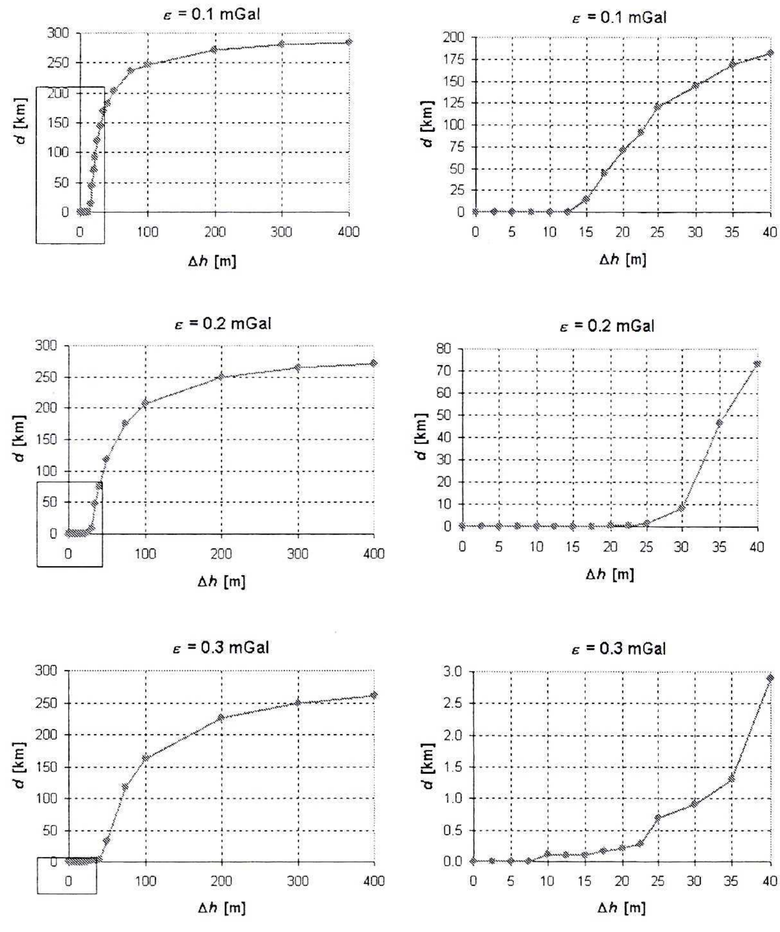

Fig. 5. The optimum maximum radius d of integration of prisms when computing the terrain correction for different types of topography and for ε = 0.1 mGal, ε = 0.2 mGal, ε = 0.3 mGal

Taking into consideration that the accuracy of terrain corrections is about 10% of their estimated values (Blais et al., 1983) and assuming roughly that the uncertainty of 1 mGal in gravity anomaly results in 1 cm error in calculated geoid height (Duchnowski and Baran, 2004) the numerical experiments were performed for $\varepsilon \in$ [0.1, 0.2, 0.3 mGal] what would correspond to the contribution of the error of determination of the terrain correction to uncertainty of gravity anomaly at the level of 10%, 20% and 30%, respectively. Since 80.0%, 90.8%, 96.6% terrain corrections from the gravity database for Poland do not exceed 1 mGal, 2 mGal and 3 mGal, respectively the choice of ε equal to 0.1, 0.2, 0.3 mGal in numerical tests seem fully appropriate. Strategy of the determination of the optimum maximum d for Δh = 15 m, Δh = 50 m, Δh = 300 m is shown in Figure 4, and the test results are given in Table 1.

The results in Table 1 that concern Δh = 300 m might not be quite representative due to the limited d_{max} = 300 km used in the experiments. They clearly show, however, that the radius d of integration of prisms dramatically decreases when lowering accuracy demand towards the computed terrain corrections.

The results of further numerical experiments for $\Delta h \in$ [2.5, 5, 7.5, 10, 12.5, 15, 17.5, 20, 22.5, 25, 30, 35, 40, 50, 75, 100, 200, 300, 400 m] that cover the range of irregularities of topography in Poland are given in Figure 5. They practically allow for the determination of the optimum maximum radius d of integration of prisms for arbitrary distortion Δh of terrain elevation relative to a gravity station and for the accuracy of the terrain correction required.

When calculating terrain corrections, Δh can practically be determined as equal to standard deviation of heights in the region around gravity station P that is a measure of dispersion of heights.

3.2. The effect of vertical and horizontal errors of DTM on the terrain corrections

Both vertical and horizontal uncertainties of a DTM affect computed terrain corrections. The numerical tests of evaluating that effect were performed at three test areas 1 (52.2°N–52.5°N; 16.1°E–16.6°E), 2 (50.2°N–50.6°N; 19.5°E–19.9°E) and 3 (49.6°N–49.8°N; 19.2°E–19.8°E) (Fig. 6). The topography of the test areas is shown in Figure 7.

The statistics of heights from DTED2 and from the gravity database for the test areas including 200 km band around are given in Table 2 and Table 3, respectively.

Table 2. Statistics of heights from the DTED2 and SRTM3 contributing to the terrain corrections computed within the test areas (heights from the test area enlarged with a band of 5 km and 200 km for the DTED2 and SRTM, respectively) [m]

Test area	Digital terrain model	Number of points	Min	Max	Mean	Std dev.
1	DTED2	1652281	51	142	87	12
	SRTM3	17880171	1	1594	136	126
2	DTED2	1735767	238	502	357	53
	SRTM3	17331561	0	2576	307	231
3	DTED2	2800350	250	1725	589	195
	SRTM3	33635217	0	2576	325	229

T a b l e 3. Statistics of heights from the gravity database for the test areas [m]

Test area	Number of points	Min	Max	Mean	Std dev.
1	2787	67	132	88	**11**
2	7252	283	489	388	**47**
3	3619	328	1366	596	**171**

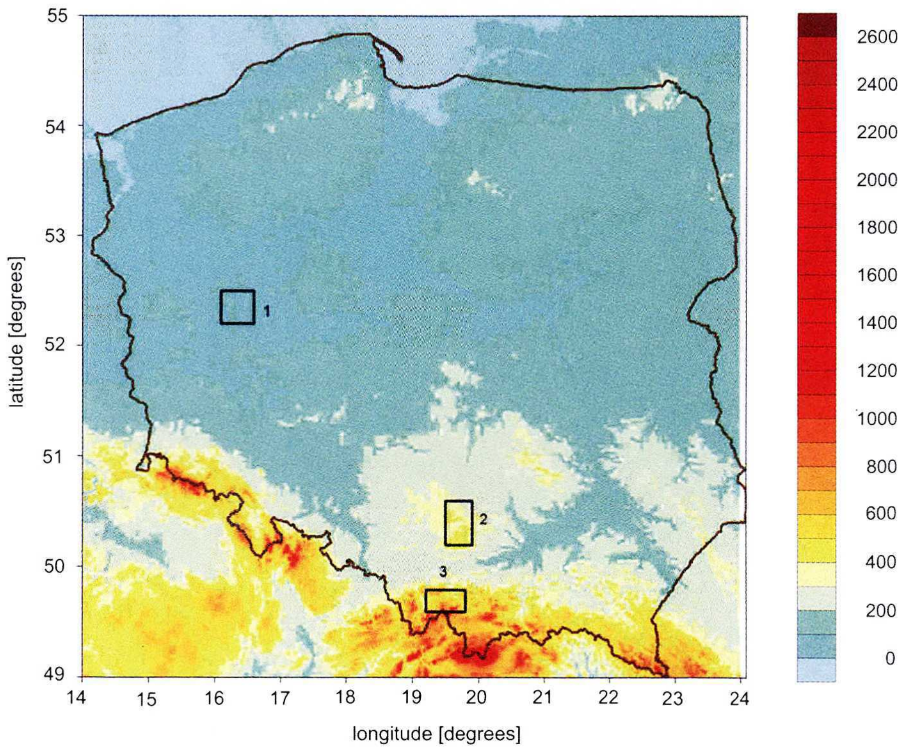

Fig. 6. Location of the test areas and topography [m]

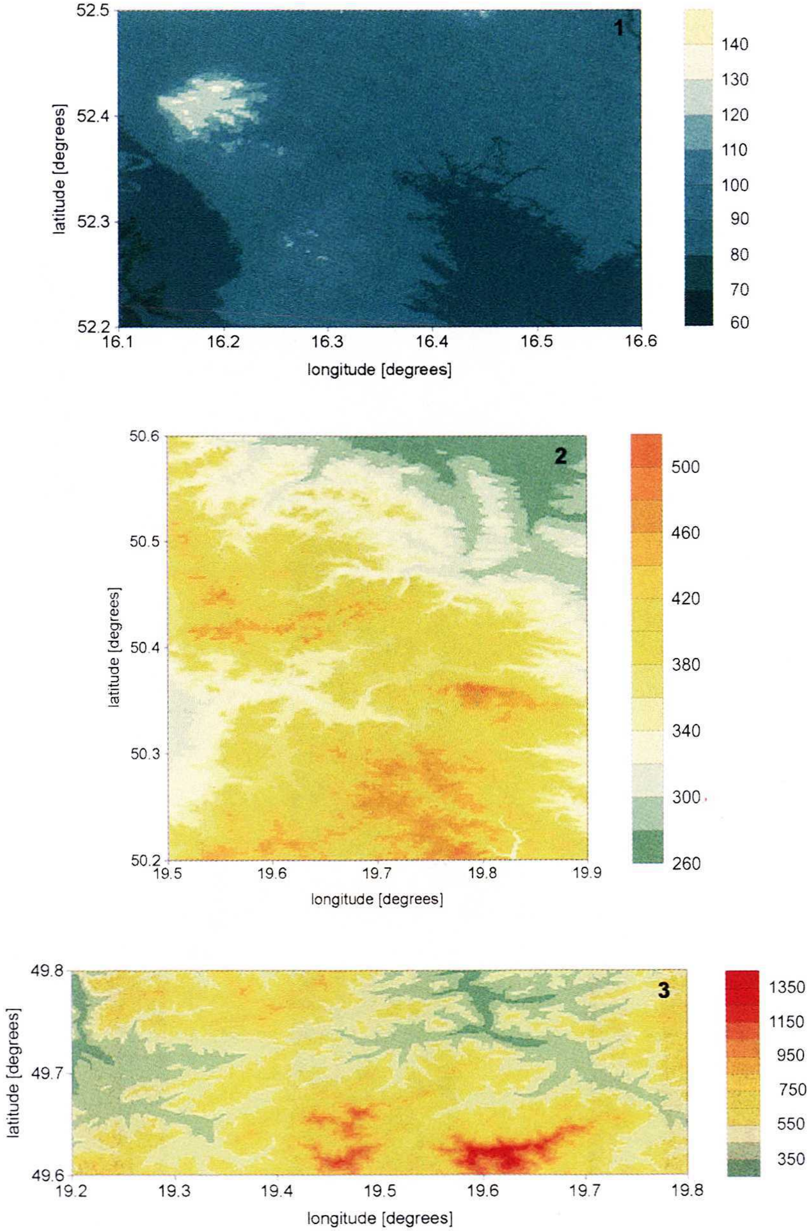

Fig. 7. The topography of the test areas 1, 2 and 3 [m]

The reference terrain corrections were calculated with the use of the DTED2 data (Fig. 8); their statistics is given in Table 4.

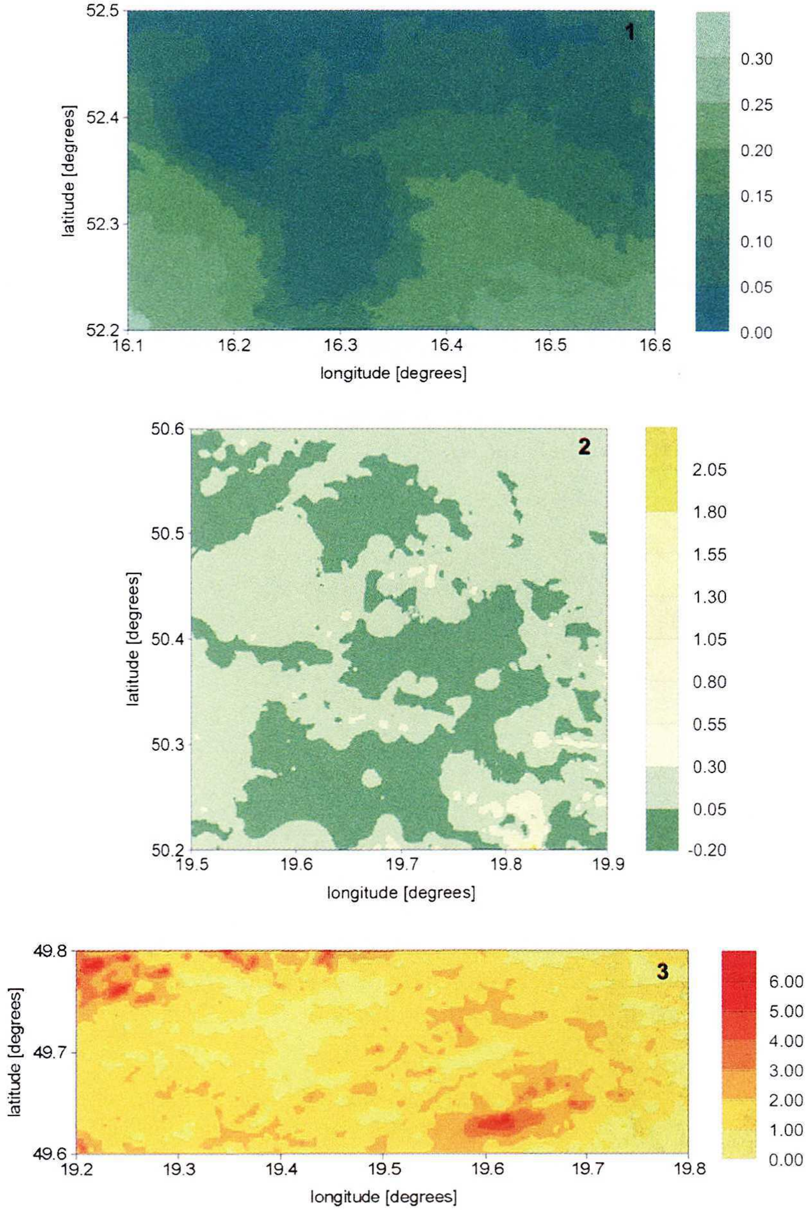

Fig. 8. The topography of the test areas 1, 2 and 3 [m]

Table 4. Statistics of the terrain corrections in the test areas [mGal]

Test area	Min	Max	Mean	Std dev.
1	−0.105	0.331	0.112	**0.066**
2	−0.408	2.146	−0.002	**0.226**
3	−0.082	11.055	1.569	**0.842**

To evaluate the effect of vertical and horizontal errors of a DTM on the computed terrain corrections the heights of the DTED2 have been distorted with random errors of standard deviations of 2.01 m, 4.02 m and 7.29 m for the test areas 1, 2, and 3, respectively. Variances of those distortions correspond to the sums of variances reflecting vertical uncertainty of the model in the selected areas – provided by the developer of the DTED2, and variances reflecting vertical uncertainty due to the horizontal error of the model – also provided by its developer. Those last ones were determined using average slopes in each test area. The uncertainties of the height data of the DTED2 in the test areas 1, 2, and 3 are given in Table 5.

Table 5. Uncertainties of the height data of the DTED2 in the test areas 1, 2 and 3

Test area	Horizontal error [m]	Average slope [°]	Vertical error due to horizontal error [m]	Vertical error [m]	Total vertical error [m]
1	16	0.61	**0.17**	**2**	**2.01**
2	16	1.33	**0.37**	**4**	**4.02**
3	16	7.23	**2.03**	**7**	**7.29**

The height data of the DTED2 distorted with random errors were used for computing a new set of the terrain corrections in the test areas. Those corrections were then compared with the respective reference ones. The statistics of the obtained differences is given in Table 6. The results indicate that in mountainous regions the accuracy of the DTED2 is not sufficient to determine terrain corrections with accuracy better than 0.1 mGal. The existing height data in Poland certainly need further wider investigation in terms of accuracy.

Table 6. Statistics of differences between terrain corrections computed from "error-free" DTED2 height data and from heights affected with vertical errors [mGal]

Test area	Min	Max	Mean	Std dev.
1	−0.135	0.009	−0.022	**0.017**
2	−0.637	0.255	−0.085	**0.066**
3	−2.084	0.370	−0.311	**0.246**

3.3. The effect of the resolution of height data on the terrain corrections

To ensure high accuracy of the terrain corrections required for centimetre quasigeoid modelling, a model that adequately describes topography is needed. Besides accuracy of a DTM, its resolution substantially affects the quality of computed terrain corrections.

The effect of resolution of a DTM on the computed terrain corrections was empirically investigated at three test areas (Fig. 6). As the reference terrain corrections were used those based on the original DTED2 height data, i.e. on a grid of $1'' \times 2''$ (31 m × 40 m) in the test areas 1 and 2, or $1'' \times 1''$ (31 m × 20 m) in test area 3. Next, the terrain corrections were calculated using the height data averaged to grid points sparser by factors 2, 4 and 8, respectively. The statistics of the differences between the obtained terrain corrections and the respective reference ones is given in Table 7.

Table 7. Differences between corresponding terrain corrections calculated using height data of different resolution [mGal]

Test area	Grid spacing				Min	Max	Mean	Std dev.
	ϕ [″]	λ [″]	y [m]	x [m]				
1	2	4	62	79	−0.013	0.020	0.001	0.002
	4	8	124	159	−0.011	0.090	0.001	0.004
	8	16	247	318	−0.011	0.113	0.002	0.005
2	2	4	62	79	−0.176	0.332	0.004	0.022
	4	8	124	159	−0.147	0.605	0.016	0.044
	8	16	247	318	−0.138	1.236	0.040	0.095
3	2	2	62	40	−0.309	0.281	0.006	0.049
	4	4	124	79	−0.447	0.590	0.033	0.081
	8	8	247	159	−0.342	1.677	0.128	0.161

The results shown in Table 7 indicate that in the flat regions (~70% of Poland) there is no need to use the DTM as dense as the DTED2 for computing the terrain corrections with accuracy better than 0.1 mGal. In those regions a DTM of resolution 100 m × 100 m seems quite sufficient. The use of the DTED2 model in hilly regions ensures the determination of terrain corrections with accuracy of 0.1 mGal. To reach, however, such accuracy in the determination of terrain corrections in mountainous regions, a DTM of higher resolution is needed.

The influence of an accuracy and resolution of elevation data on terrain corrections was invetigatd by Blais and Ferland (1984). They showed that the accuracy of 1.0–1.5 mGal for gravimetric terrain corrections computed using height data of 1 km resolution for gravity stations in the Norman Range in Canada is generally easily achievable. The results presented in this paper are compatible with those of Blais and Ferland despite of much higher resolution of the terrain model used.

3.4. Comparison of terrain corrections obtained using the prism method with those computed using the FFT method

The conventional approach of computing the terrain correction with the use of the prism method is considered as a rigorous one. The rigorousness of the fast and more efficient FFT method is not as clear. The investigations performed on the test area in Canadian Rocky Mountains (2100 m range of elevations) with the use of 1 km × 1 km grid height data showed that the accuracy of the FFT method is about the same as that of the prism method (Sideris, 1984). The maximum RMS value of the differences between terrain corrections was about 0.5 mGal.

Resolution of DTMs available has much improved during last two decades. On the other hand modelling quasigeoid with a centimetre accuracy requires better quality of the terrain correction. Comparison of terrain corrections obtained with using two different approaches enables to verify the results and further provides data for estimation the quality of a quasigeoid model.

The terrain corrections were computed with both the prism and the FFT methods in a dozen or so virtual gravity stations located at the grid points of the DTED2 model, using height data provided by that model within 50 km radius in three test areas of flat (52°N–54°N; 21°E–23°E), hilly (50°N–52°N; 20°E–22°E) and mountainous (49°N–50°N; 19°E–21°E) topography of different elevation range. The statistics of height data from the DTED2 in the considered the test areas and of elevations of virtual gravity stations are given in Table 8 and Table 9, respectively.

Table 8. Statistics of heights from the DTED2 in the test areas [m]

Test area	Number of points	Min	Max	Mean	Std dev.
flat	25920000	74	224	130	**24**
hilly	25920000	89	610	200	**56**
mountainous	25920000	190	2632	645	**305**

Table 9. Statistics of heights of virtual gravity stations in the test areas [m]

Test area	Number of points	Min	Max	Mean	Std dev.
flat	14	119	159	128	**11**
hilly	13	248	327	288	**23**
mountainous	13	586	1026	750	**150**

The terrain corrections computed with the use of a prism method are shown in Table 10. Differences between the corresponding terrain corrections computed with both methods without the indirect effect are given in Table 11.

Table 10. Statistics of terrain corrections computed with the use of the prism method in the test areas [mGal]

Test area	Number of points	Min	Max	Mean	Std dev.
flat	14	0.004	0.136	0.022	**0.035**
hilly	13	0.074	0.636	0.188	**0.147**
mountainous	13	0.423	5.340	1.524	**1.613**

Table 11. Statistics of the differences between the terrain corrections computed using the prism method and the FFT method in the test areas [mGal]

Test area	Number of points	Min	Max	Mean	Std dev.
flat	14	−0.001	0.024	0.004	**0.006**
hilly	13	0.000	0.053	0.019	**0.014**
mountainous	13	−0.076	0.098	0.007	**0.056**

Differences between the terrain corrections obtained using the prism approach and the FFT approach (Table 11), besides the effect of methodical difference, they additionally reflect the inconsistency in the procedure applied. In the prism method used the curvature of the Earth is taken into consideration while the FFT algorithm applied is based on a planar approximation what results in different modelling of Bouguer plate. Moreover the FFT solution is interpolated to the point of the gravity station. The largest differences occur in mountainous region characterized with the largest roughness of topography. The consistency of the results obtained corresponds to the accuracy of gravity data in the gravity database for Poland. The TCFOUR program of the GRAVSOFT package was used to compute terrain corrections with the FFT approach.

4. The terrain corrections for gravity stations from the gravity database for Poland

Since height data is available on regular grids, the FFT approach is commonly chosen as an efficient and fast computational tool for terrain correction computations, in particular when a complete grid of terrain effects is required over a large area. Recent computational technology enables, however, for applying in such cases a prism approach, although computing time is much longer. Considering the prism approach more suitable for numerical analysis and accuracy estimation, that method has been chosen for generating the terrain corrections for gravity stations from the gravity database for Poland.

In the gravity database for Poland established in 1974–1992 by the Polish Geological Institute, the terrain corrections (called in the paper a "1992" terrain correction set) were computed only at some stations. The prism approach was applied within the radius of 22.5 km to height data from the gravity database that corresponded to the levelled heights of nearly 300 000 gravity stations. Heights of the prisms were determined as a simple average over the prism area.

In the recent computation of the terrain corrections (called in the paper a "2005" terrain correction set) also the prism method was used but the height data was taken from the DTED2 model. The elevations of gravity stations used in calculations were interpolated from that model. Distribution of gravity stations in Poland, together with differences between their levelled heights and the corresponding ones obtained from the DTED2 model, is shown in Figure 9. Standard deviation σ of the differences equals to 6.9 m. Stations where the differences are within the range of 3σ (97.7% of all stations) are marked with grey dots. The remaining stations are marked with blue and red dots, depending on the sign of the difference (Krynski et al., 2005).

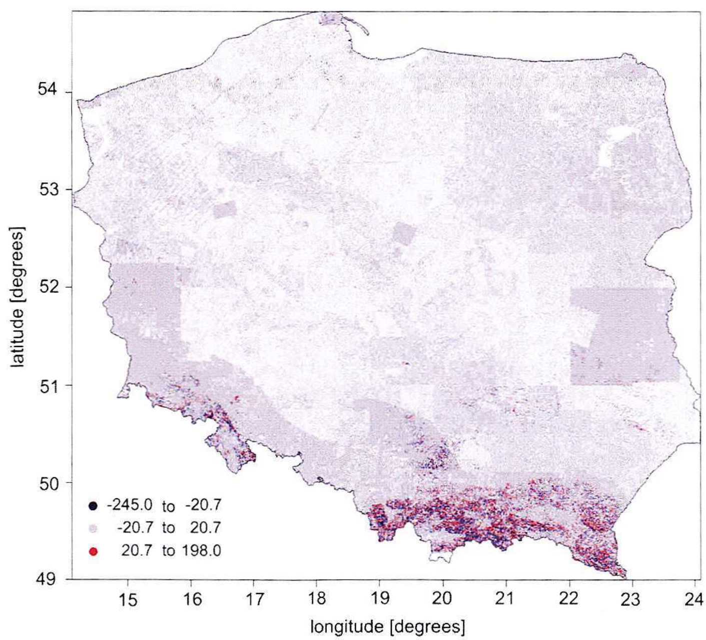

Fig. 9. Distribution of gravity stations in Poland with differences between their levelled heights and the corresponding ones obtained from the DTED2 model

Differences in heights of gravity stations indicate a substantial inconsistency between those two datasets, that exceeds the estimated accuracy of heights in those datasets. The discrepancy between the corresponding heights might substantially affect the calculated terrain corrections, in particular when it takes place in the close vicinity of the station at which the terrain correction is computed. For example, if the difference in corresponding heights reaches 35 m then ignoring it, i.e. not moving the station to the surface of a DTM, results in changing the terrain correction by as much as 4.1 mGal.

The terrain corrections at 1 078 046 gravity stations from the gravity database in Poland have been computed using the DTED2 data within the radius of 5 km and the SRTM3 data within the range of radii from 5 km to 200 km – in Central Poland, and respectively using the SRTM3 and SRTM30 data at the edges of the country (the "2005" terrain correction set). The average (in red) terrain corrections and standard deviations (in black) of the terrain corrections in 1° × 1° blocks are given in Figure 10.

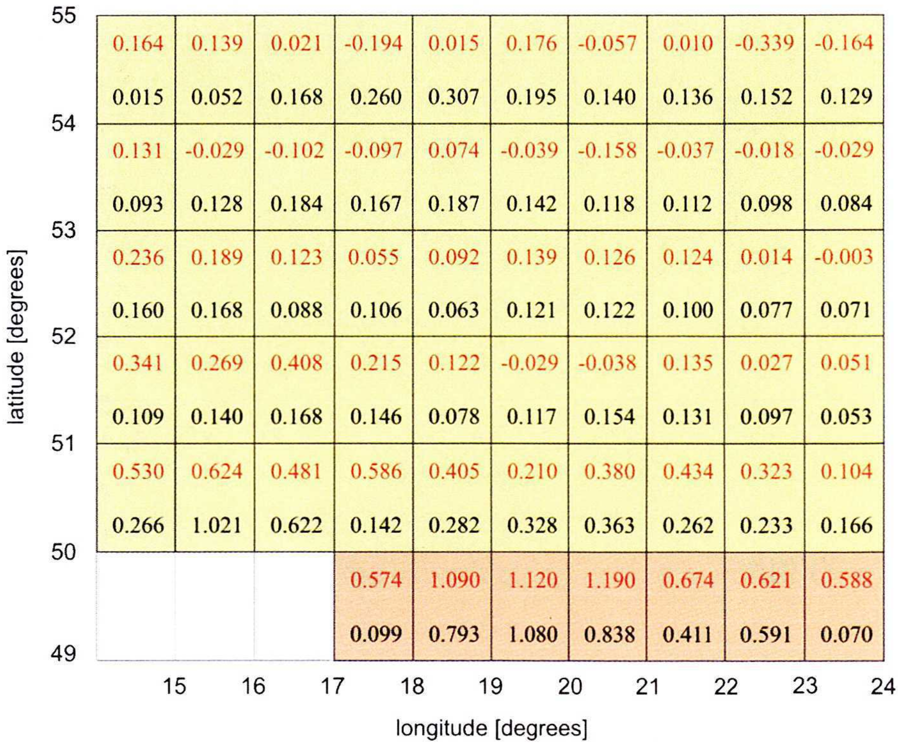

Fig. 10. The average (in red) terrain corrections and standard deviations (in black) of "2005" terrain corrections in 1° × 1° blocks [mGal]

The map of "2005" terrain corrections at the gravity stations from the gravity database for Poland is presented in Figure 11. Maximum terrain correction in Poland – excluding the region of Tatra Mountains where no terrain corrections were computed because of a lack of gravity stations there – reaches 22.3 mGal. Only 10% of all calculated terrain corrections exceed 0.5 mGal, and 3% are larger than 1 mGal.

The "1992" terrain corrections existing in the gravity database were compared with recently computed "2005" terrain corrections. It must be pointed out here that both sets of terrain corrections were computed in different ways assuming different parameters. The statistics of differences between corresponding terrain corrections is given in Table 12.

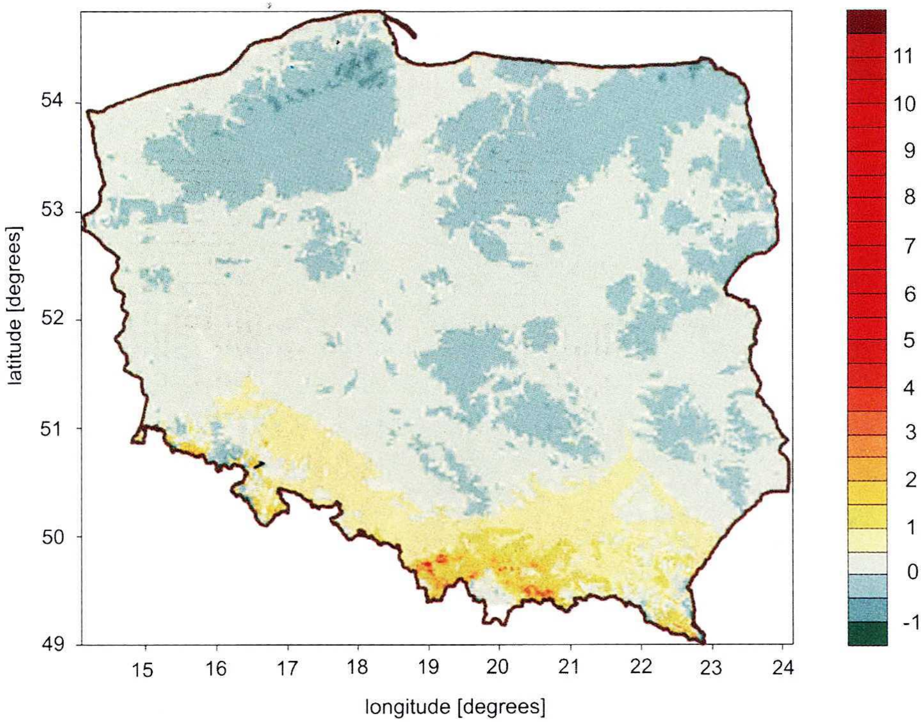

Fig. 11. The map of calculated "2005" terrain corrections at all gravity stations from the gravity database for Poland [mGal]

Table 12. Statistics of differences between "1992" terrain corrections descended from the gravity database and the newly determined "2005" terrain corrections [mGal]

Number of points	Min	Max	Mean	Std dev.
288 507	−8.135	10.260	−0.050	0.616

Distribution of gravity stations at which the terrain corrections were mutually compared (it shows simultaneously those gravity stations for which the "1992" terrain corrections were computed) with distinguishing differences between the "1992" and the "2005" terrain corrections, within the intervals defined by a single standard deviation (σ = 0.616 mGal) (Table 12) and a triple standard deviation is given in Figure 12. The histogram of the computed differences in terrain corrections is shown in Figure 13.

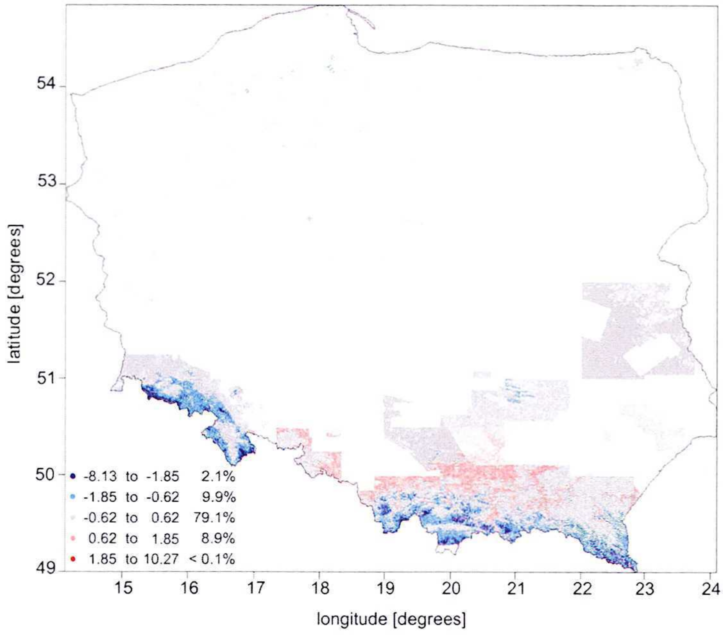

Fig. 12. Differences between the "1992" and "2005" terrain corrections [mGal]

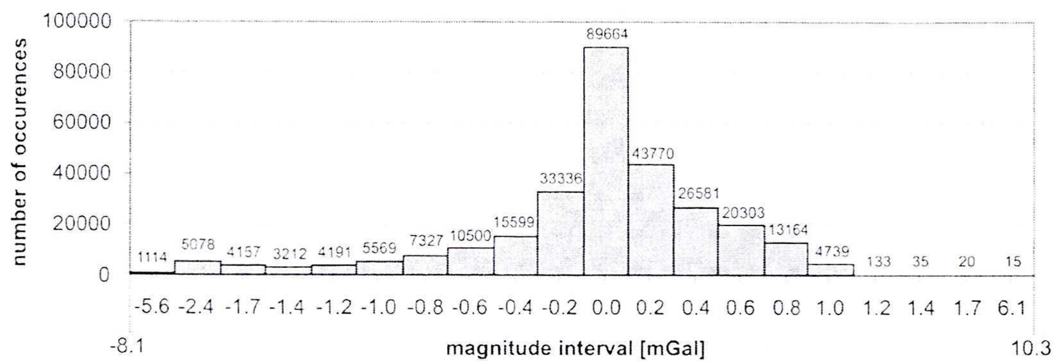

Fig. 13. Histogram of differences between the "1992" and "2005" terrain corrections

5. Conclusions

Both approaches, the prism method and the FFT method of the terrain correction computation, applied to high resolution height data provide similar results, sufficiently accurate for centimetre quasigeoid modelling. Due to laboriousness caused by complexity of the algorithm, computer time and problems with processing large datasets, an important step to be done prior to computations is to specify the required accuracy ε of the computed terrain corrections, in order to choose the most appropriate height data in terms of resolution and accuracy and to determine the optimum size of the area covered by height data used for computing the terrain corrections.

The size of the area covered by height data used for computing the terrain corrections, e.g. the radius d of integration of prisms, depends on required accuracy ε. It dramatically decreases when lowering accuracy demand towards the computed terrain corrections. Numerical experiments with computing the terrain corrections using the prism method based on high resolution DTMs enabled to precisely determine the optimum maximum radius d of integration of prisms for a required accuracy ε of the solution. For example, for mean deviation of heights Δh = 50 m in the area of integration, the optimum maximum radius d of integration equals to 33 km, 118 km and 205 km, for ε = 0.3 mGal, ε = 0.2 mGal, ε = 0.1 mGal, respectively.

Height data available in Poland seem sufficient in terms of resolution for computing the terrain corrections in most areas (over 80%) with an accuracy required for centimetre quasigeoid modelling. There is no need to use the DTM as dense as the DTED2 model for computing the terrain corrections with accuracy better than 0.1 mGal. In that case a DTM of resolution 100 m × 100 m seems quite sufficient. The use of the DTED2 model in hilly regions ensures the determination of terrain corrections with accuracy of 0.1 mGal. To reach, however such accuracy in the determination of terrain corrections in mountainous regions a DTM of higher resolution is needed.

The existing height data in Poland need further investigation in terms of accuracy and reliability. Also the strategy of integrating the available height data for optimum terrain correction computation should be developed.

Numerical experiments show that in flat regions of Poland (about 70%) a flat Bouguer plate is sufficient as reference for computing the terrain corrections for precise quasigeoid modelling. In the mountainous regions a spherical Bouguer plate should be used.

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References

Bamler R., (1999): *The SRTM Mission: A World-Wide 30 m Resolution DEM from SAR Interferometry in 11 Days*, In D. Fritsch, D. Spiller (eds.), Photorgammetric Week 99, Wichmann Verlag, Heidelberg, pp. 145-154.

Blais J.A.R., Lodwick G.D., Ferland R., (1983): *Gravimetric terrain corrections in western Canada*, Canadian Journal of Earth Sciences, Vol. 20, No 2, pp. 259-265.

Blais J.A.R., Ferland R., (1984): *Optimization in gravimetric terrain corrections*, Vol. 21, No 5, pp. 505-515.

Denker H., (2004): *Evaluation of SRTM3 and GTOPO30 Terrain Data in Germany*, Proceedings of the IAG Symposium "Gravity. Geoid and Space Missions – GGSM2004", IAG Symposia, Vol. 129, (eds.) C. Jekeli, L. Bastos, J. Fernandes, 30 August – 3 September 2004, Porto, Portugal, pp. 218-223.

Duchnowski R., Baran L.W., (2004): *Analysis of geodetic, astronomic, geological and satellite data for their use for integrated geoid modelling* (in Polish), University of Warmia and Mazury, Report for the Institute of Geodesy and Cartography, Warsaw.

Ferland R., (1984): *Terrain Corrections for Gravity Measurements*, UCSE Reports No 20009, Division of Surveying Engineering, The University of Calgary, Calgary.

Forsberg R., (1984): *A study of terrain reductions, density anomalies and geophysical inversion methods in gravity field modelling*, OSU Report No 355, Department of Geodetic Science and Surveying, The Ohio State University, Columbus, Ohio.

Forsberg R., (2005): *Terrain Effects in Geoid Computations*, International School for the Determination and Use of the Geoid, Budapest, pp. V1-V38.

Grzyb M., (2004): *The use of Fourier transform for determination of gravimetric terrain correction with elevation data from digital terrain model* (in Polish), MSc thesis, Warsaw University of Technology.

Heiskanen W.A., Moritz H., (1967): *Physical Geodesy*, W.H. Freeman & Co., San Francisco.

JPL, (2004): *SRTM – The Mission to Map the World*, Jet Propulasion Laboratory, California Institute of Technology, http://www2.jpl.nasa.gov/srtm/index.html.

Królikowski C., (2004): *Qualitative and quantitative analysis of the existing gravity data (terrestrial and seaborne – from Poland and neighbouring regions)* (in Polish), Polish Geological Institute, Report for the Institute of Geodesy and Cartography, Warsaw.

Krynski J., Mank M., Grzyb M., (2005): *Evaluation of digital terrain models in Poland in view of a cm geoid modelling*, Geodesy and Cartography, Vol. 54, No 4, pp. 155-175.

NGA, (1996): *Performance specification Digital Terrain Elevation Data (DTED)*, National Geospatial-Intelligence Agency, Document MIL-PRF-89020A.

Osada E., Krynski J., Owczarek M., (2005): *A robust method of quasigeoid modelling in Poland based on GPS/levelling data with support of gravity data*, Geodesy and Cartography, Vol. 54, No 3, pp. 99-117.

Rózsa, S., (1998): *Determination of terrain correction in Hungary and the surrounding area*, Reports of the Finnish Geodetic Institute, 98:4, Masala.

Showstack R., (2003): *Digital Elevation Maps Produce Sharper Image of Earth's Topography*, EOS Transactions, American Geophysical Union, Vol. 84, No 37, pp. 363.

Sideris M.G., (1984): *Computation of Gravimetric Terrain Corrections Using Fast Fourier Transform Techniques*, UCSE Reports, No 20007, The University of Calgary, Division of Surveying Engineering, 114 pp.

Torge W., (2001): *Geodesy* (3rd Edition), Walter de Gruyter, Berlin.

Zhang K.F., Featherstone W.E., (1997): *A preliminary evaluation of the terrain effects on gravimetric geoid determination in Australia*, Gravity, Geoid and Marine Geodesy, International Association of Geodesy Symposium, Vol. 117, Berlin, Springer-Verlag, pp. 565-572.

Zhang, K.F., Featherstone W.E., Ding X.L., (1998): *An accuracy estimation of gravimetric terrain corrections*, Annales Geophysicae, Vol. 16 (Supplement I), European Geophysical Society, p. C384.

Ocena wpływu topografii terenu i jakości DTM na dokładność wyznaczania poprawek terenowych w aspekcie modelowania centymetrowej quasigeoidy

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Streszczenie

Przy wyznaczaniu centymetrowej quasigeoidy niezbędne jest uwzględnienie nieregularności topografii występujących wokół stacji grawimetrycznej, czyli wprowadzenie do pomierzonego przyspieszenia siły ciężkości poprawek terenowych. Dokładność obliczania poprawek terenowych ma wpływ na dokładność wyznaczanego modelu quasigeoidy. Zależy ona od dokładności i rozdzielczości danych wysokościowych oraz użytych do wyznaczania poprawek terenowych parametrów.

W badaniach przeprowadzonych w ramach niniejszej pracy wykorzystano opracowany przez Zarząd Geografii Wojskowej, według standardu NATO-STANAG 3809, numeryczny model terenu DTED2 (Digital Terrain Elevation Data) dla obszaru Polski oraz modele SRTM3 (The Shuttle Radar Topography Mission) i SRTM30 dla obszaru Polski i obszarów przyległych.

Porównano wyniki testowe obliczenia poprawki terenowej uzyskane przy użyciu metody prostopadłościanów i metody wykorzystującej transformaty Fouriera. Poprawki terenowe obliczano metodą prostopadłościanów polegającą na sumowaniu wpływów nadwyżek lub niedoborów mas pochodzących od graniastosłupów o podstawach prostokątnych na składową pionową przyspieszenia siły ciężkości. Opracowano praktyczną metodę wyznaczania wymiary obszaru, z jakiego topografia powinna być uwzględniana przy obliczaniu poprawki terenowej. Analizowano również wpływ błędów wysokości, a także błędów położenia punktów modelu na dokładność uzyskiwanych poprawek terenowych. Przedyskutowano użyteczność dostępnych danych dotyczących topografii terenu do obliczania precyzyjnych poprawek terenowych w Polsce.

Uzyskane wyniki badań wykorzystano do określenia strategii obliczenia poprawek terenowych dla ponad miliona punktów grawimetrycznych zawartych w bazie danych grawimetrycznej dla Polski. Dzięki obliczonemu dla 1 078 046 punktów grawimetrycznych zbiorowi poprawek terenowych możliwe będzie zwiększenie precyzji obliczanych dla obszaru Polski modeli quasigeoidy grawimetrycznej.