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Target functions for surface matching

Surface matching is a fundamental task that must be solved whenever we want to merge data sets of same physical surface obtained with different sources. In general, the points in two sets are in different reference systems, not identical with different accuracy, distribution and density.

To perform surface matching the different target functions are proposed.

Proposed target function in section 1.4 is based on the condition of equality of triangle areas (TIN). This target function can be used for surface patches of pattern set S1 generated also in squares (DEM).

Conception of target function provided in section 1.5 is based on the combination of two conditions. The first of them is a condition of fitting two normal vectors of surface patches generated by square model (DEM) in set S1 (as a pattern) and of triangle created with three points in set S2 (as a candidate). The second is one of four target functions upper represented.

INTRODUCTION

In last years powerful data sets have been obtained from aerial or satellite imagery, airborne laser scanning (LIDAR), Synthetic Aperture Radar (SAR), Interferometer Synthetic Aperture Radar (IFSAR), Inertial Platforms (INS), Global Positioning System (GPS), digital mapping, Geographical Information System (GIS) of a same terrain surface. The skill of fusion of these data for concrete task is one of the important directions in terrain investigation.

For accurately generating DEM from stereo imagery the point sets of same surface derived from IFSAR or LIDAR were proposed to use [1, 6], epecially for urban area [3, 8, 10]. Since the past three years LIDAR has enjoyed explosive growth to use about 25% per year [7]. The airborne laser scanning altimetry can obtain as many as 5000 points (in 3D) per second with vertical accuracy from ±0.15 m to ±3.0 m on hard surface to ±0.3 m to ±0.5 m on soft (vegetation) surface and hick terrain and with horizontal accuracy equal to ± 0.75 m on all extremely hilly terrain [2, 11].

The surface matching is useful to solve the problem of comparing two surfaces obtained from two data sets of the same terrain. It means that for integrating different data sets of same terrain surface obtained from different flying platforms the problem of surface matching should be given to investigate. However, the two sets of same physical surface are in different density, different reference systems and the points are not identical in two sets. Between two data sets there is a consistency of mathematical model, in all of matching methods as well as area, feature or relation-based matching, the true mathematical model of the photogrammetric application is not considered and estimated, and rather, matches are based on an assumed similarity measure. In surface matching the true mathematical model existing between two data sets should be constructed by transformation parameters. This model is the target (or goal) function.

Three target functions presented in section 1.1, 1.2, 1.3 for mathematical model forming between two data sets of same surface were proposed by [4, 9]. The first of them is relied on the coplanarity condition. The second is on the condition of minimizing ΔZ differences between two surfaces. The third is based on the condition of minimizing distances along surface normal.

In this paper, the new target functions for surface matching will be presented in sections 1.4 and 1.5. The idea of target function in section 1.4 is based on equality of triangle (or squares) areas. The target function presented in section 1.5 is based on combination of two conditions. The first of them is related with fitting two normal vectors of square generated in set Sl and of triangle created with three points chosen in set S2. The second is taken from one of four target functions presented in upper sections.

1. *Target functions for surface matching*

Lets S1 = $\{p_1, p_2,... p_n\}$ be a surface (pattern) described by n discrete points that are randomly distributed. Lets S2 = $\{q_1, q_2,... q_n\}$ be a second surface (candidate) described by m discrete points q. In general $n \neq m$, suppose that two sets are describing the same physical surface. However, the points of two sets are in different reference systems and in different distribution, no points in the two sets are identical. Suppose further that between two sets there is an existence of mathematical model, simply for example, 3D similarity transformation.

The point q_i of the second surface S2 (candidate) is transformed to Sl by 3D similarity transformation as follows:

$$q'_i = s\mathbf{R}\boldsymbol{q}_i - \mathbf{R}_0 \qquad (1)$$

where: s – the unknown scale factor; $\mathbf{R}$ – the 3D rotation matrix of unknowns ω, φ, χ; $\mathbf{R}_0$ – the translation of unknowns X_0, Y_0, Z_0.

Assuming small rotation angles $d\omega, d\varphi, d\chi$ and small scale factors ds the equation (l) will be expressed in the scalar form:

$$\begin{aligned} X_{q'} &= X_q ds - Y_q d\chi + Z_q d\varphi - X_0 \\ Y_{q'} &= X_q d\chi + Y_q ds - Z_q d\omega - Y_0 \\ Z_{q'} &= -X_q d\varphi + Y_q d\omega + Z_q ds - Z_0 \end{aligned} \tag{2}$$

The target functions for surface matching are in turn represented.

1.1. Target function based on coplanarity condition

In beginning of this section all of the assumptions for providing the target function based on coplanarity condition were introduced. Suppose now that surface patches of S1 are generated in TIN model (Fig. 1a). Single surface patch in Sl is defined by three points P_a, P_b, P_c [4]. Let's q'_i be a transformed point of q_i, belonged into S2. We impose the condition that q'_i has to be lain on the surface patch $P_a\, P_b\, P_c$. It means four points P_a, P_b, P_c, q'_i lie on the same plane (coplanarity condition), then we have $D = 0$ (eq.3). Equation (3) is the target function based on coplanarity condition. Using target function (3) it would be possible to solve the transformation parameters appeared in (l).

$$D = \begin{vmatrix} Xq'_i & Yq'_i & Zq'_i & 1 \\ Xp_a & Yp_a & Zp_a & 1 \\ Xp_b & Yp_b & Zp_b & 1 \\ Xp_c & Yp_c & Zp_c & 1 \end{vmatrix} = 0 \tag{3}$$

1.2. Target function based on ΔZ-difference

Suppose that we have generated surface patches of set Sl in TIN model as a pattern [9]. This surface patch is expressed by (Fig. 1b):

$$Z = AX + BY + C \tag{4}$$

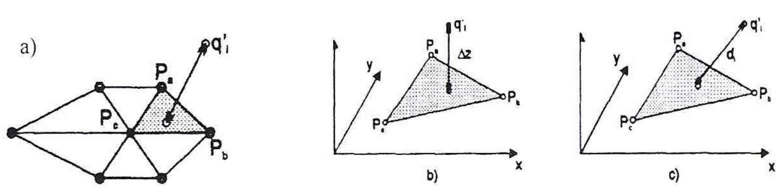


Fig. 1. a) generated surface patches; b) target junction of ΔZ-difference; c) target function of distance along surface normal

where: A, B, C – the coefficients calculated from three points P_a, P_b, P_c of TIN $P_a P_b P_c$. The point q_i in set S2 is transformed to the set Sl by equation (l). Suppose further that transformed point q'_i is on plane $P_a\, P_b\, P_c$. We create the difference in Z-axis as follows:

$$\Delta Z = Z_{q'} - Z_p \tag{5}$$

The value of Z_p is obtained by replacing X and Y in (4) with $X_{q'}$ and $Y_{q'}$ of (eq. 2); $Z_{q'}$ is the third equation of system (2). Considering ΔZ as an observation, we determine the unknowns $d\omega$, $d\varphi$, $d\chi$, ds, X_0, Y_0, Z_0.

1.3. Target function based on distance along surface normal

The idea of mathematical model is illustrated in Figure (1c). The shortest distance d from transformed point q'_i to the surface patch P_a P_b P_c presented in normal equation [9] is:

$$d = q'_i \cdot \boldsymbol{h} - \boldsymbol{l} \tag{6}$$

where: $\boldsymbol{l}$ – the vector of the system origin to surface patch, $\boldsymbol{h}$ – vector of directional cosines, $\boldsymbol{h} = [\cos\alpha, \cos\beta, \cos\gamma]^T$

Replacing $\boldsymbol{q}'_i$ in (6) with the right hand side of (1) we have new form of observation equation:

$$d = (s\boldsymbol{R}q_i - \boldsymbol{R}_0)\boldsymbol{h} - \boldsymbol{l} \tag{7}$$

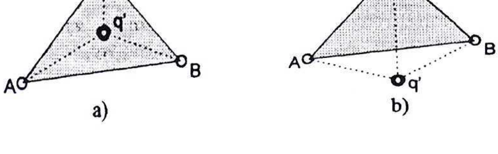
Linearising this equation the unknowns will be determined. Results obtained from experiment [9] explained that the idea based on minimizing distance along surface normal is better than idea based on the minimizing ΔZ-difference, especially for terrain with large slope angle (large than 50°).

1.4. Target function based on area equality

If we want to impose condition that the transformed point q'_i from q, lies on the pattern surface patch SP generated with three points A, B, C; then, we have the following condition of equal areas (Fig. 2a):

$$S_{q'\mathrm{ABC}} = S_{q'\mathrm{AB}} + S_{q'\mathrm{BC}} + S_{q'\mathrm{CA}} = S_{\mathrm{ABC}} \tag{8}$$

If the point q'_i is not belonging to triangle ABC, then sum of areas in left equation (8) will be bigger than S_{ABC} (Fig. 2b).



a) b)

	g1	g2	...	gj
SP1	1	0	0	0
SP2	0	0	0	1
...	0	1	0	0
SPi	0	0	1	0

c)

Fig. 2. The point q' lies on inside (a) and outside (b) of triangle ABC, c) the result of matching process

The general formula for calculation of triangle area is:

$$S = \frac{1}{2}\sum_{i=1}^{3} X_i (Y_{i+i} - Y_{i-i}) \tag{9}$$

On the basing (9) we have

$$S_{Q'\mathrm{ABC}} = K.X_{q'} + L.Y_{q'} + M \tag{10}$$

Where $X_{q'}$, $Y_{q'}$ – the transformed co-ordinates of point q' taken from (2). Basing on (8) and (10) we have the condition equation of equal areas as follows:

$$K.X_{q'} + L.Y_{q'} + N = 0 \tag{11}$$

where

$$K = \frac{1}{2}\left[(Y_B - Y_A) + (Y_C - Y_B) + (Y_A - Y_C)\right]$$

$$L = \frac{1}{2}\left[(X_A - X_B) + (X_B - X_C) + (X_C - X_A)\right]$$

$$N = M - S_{ABC}$$

Basing on the formula (2) the equation (1 l) is now in the last form

$$(KX_q + LY_q)ds + (LX_q - KY_q)d\chi + KZ_q d\varphi - LZ_q d\omega - KX_0 - LY_0 + N = 0 \tag{12}$$

For calculating six unknown parameters we need less six points q_i. After obtaining six calculated parameters the seventh unknown parameter Z_0 can be determined on the basing of formula (2) as follows:

$$Z_0 = (-X_q d\varphi + Y_q d\omega) + Z_q ds) - Z_{q'} \tag{13}$$

where

$$Z_{q'} = AX_{q'} + BY_{q'} + C$$

and A, B, C – the coefficients taken from plane equation of triangle ABC.

The process of correspondence between the group of points q_i in the set S2 and the surface patches generated as the TIN triangles in the set Sl must be simultaneously established with determining the seven parameters of the similarity transformation. The process of matching is same as in [4]. Six points q in set S2 were first selected. We can match them with all generated surface patches of set Sl. For every such combination the system of six equations in type of (12) is established and six unknown parameters will be calculated. Once again all possible combination of new six points q will be selected and process of calculation should be repeated. Basing on the set of calculated parameters obtained from combinations the correct solution will be selected. The results of matching process are written in table presented on Fig. 2c. The SP_i ($i = 1, 2, 3,..., n$) are the generated surface patches of TIN in set Sl. The g_j ($j = 1, 2, 3,..., k$) are the groups of six points q selected in set S2. Number l means that selected group of points q are belong to corresponding patch in Sl. On the contrary, number O means that doesn't.

1.5. Combined target function

The combination between fitting condition of normal vectors and one of four presented upper target functions is proposed. Combined target function should be used for points selected in set S2 to fit the surface patches generated in square forms from set Sl (DEM) (Fig. 3a).

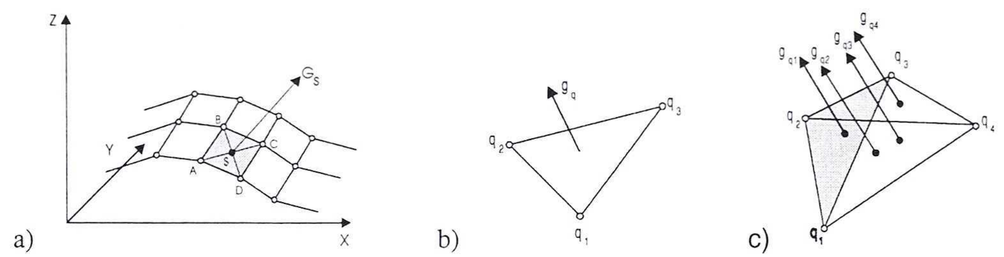


Fig. 3a. a) generated DEM in set S1 with the gradient vector of square; b) the gradient vector of triangle in set S2; c) four normal vectors of triangles built on four chosen points in set S2

The height of the middle point S interpolated with bilinear equation is based on following equation:

$$Z_s = a_0 + a_1Xs + a_2\mathrm{Y}s + a_3XsYs \tag{14}$$

The gradient vector (normal) in middle point S is:

$$G_s = [A_1\ A_2 - 1]^T \tag{15}$$

where:

$$A_1 = (a_1 + a_3Y_s)$$

$$A_2 = (a_2 + a_3X_s)$$

The values of a_0, a_1, a_2, a_3 were calculated from (14) basing on the four points A, B, C, D. In the set S2 three points are selected and create a triangle (Fig. 3b) with general form (eq. 4):

$$B_1X_q + B_2Y_q + B_3 - Z = 0 \tag{16}$$

where: B_1, B_2, B_3 – the coefficients determined on the base of coplanarity condition of three points q. The three values B_1, B_2, -1 are the co-ordinates of normal vector $\boldsymbol{g}_q$. It means $\boldsymbol{g}_q$ = [B_1 B_2 -1] suppose the three points q are belonged to square ABCD. In this case two vectors $\boldsymbol{G}_s$, $\boldsymbol{g}_q$ have following mathematical relationship by similarity transformation:

$$\boldsymbol{G}_s = s\mathbf{R}\boldsymbol{g}_q + \mathbf{R}_0 \tag{17}$$

where: s, $\mathbf{R}$, $\mathbf{R}_0$ – the marks are as same as (1), $\boldsymbol{G}_s = [A_1\ A_2\ \text{-}1]^T$, $\boldsymbol{g}_q = [B_1\ B_2\ \text{-}1]^T$.

In the equation (2) there are seven unknowns ds, $d\chi$, $d\varphi$, $d\omega$, X_0, Y_0, Z_0. We also consider that the equation (17) guarantees the two vectors $\boldsymbol{G}_s$, $\boldsymbol{g}_q$ which are fitted themselves, but three transformed points q' may not lie in the surface ABCD. We impose the condition that three transformed points q' have to be lain in the patch ABCD. For this purpose we have system of two types of equation:

$$\begin{aligned} \boldsymbol{G}_s &= s\mathbf{R}_q + \mathbf{R}_0 \text{ (a)} \\ \Delta Z &= Z_{q'} - Z_p \quad \text{(b)} \end{aligned} \tag{18}$$

where: ΔZ – the formula taken from (5).

When we have three chosen points in S2 (Fig.3b) the system (18) has six equations, but seven unknowns. We need fourth point q in set S2 (Fig.3c). From four points in S2 we can create four independent triangles with their corresponding normal vectors which have to be imposed to normal vector G_s (Fig.3a). It is clear that four points in S2 have to be chosen in order that the radius from their center to them is not bigger than that of the circle passing through four points A, B, C, D in set S1. In this case the number of equation in (18) will be equal to 16. In the case, when we know the scale factor s (for example, known corresponding distances in two set Sl, S2), fourth point q in the set S2 is not needed. In this case, three points q in the set S2 are required only.

We consider that to determine seven transformation parameters, every target functions in sections 1.1, 1.2, 1.3 needs lest 7 points in set S2, but combined target function for generated DEM in set Sl we require only 4 points q in set S2. It is clear that probability of hitting simultaneously of small number of points q in set S2 to surface patches in Sl is bigger than that of hitting simultaneously of big number of points q to same surface patches.

2. *Geometrical stability of normal equation system constructed from target functions*

It is known that seven unknowns ds, $d\omega$, $d\varphi$, $d\chi$, X_0, Y_0, Z_0 are determined from normal system created on the base of target function. Therefore, geometrical stability of normal equation system puts into port on the accuracy of determined unknowns. Examining so stability of normal equation system formed on the ground of minimizing target function we can check the efficiency of proposed target functions. For this purpose the investigation relies on determination of certain number characterizing given normal equation system. In practice Togg' number T is used which expresses with following formula:

$$T = \frac{|\lambda_i|_{\max}}{|\lambda_i|_{\min}} \tag{19}$$

where

$$\lambda_{i\ \min} = D\left(\frac{n - 1}{sp}\right)^{n-1}$$

$$\lambda_{i\ max} = sp - (n - 1)\sqrt{\frac{D}{sp}}$$

At this: n – the rank of matrix of normal equation system,
D – the value of determinant of normal equation system,
sp – the trace of matrix of normal equation system,
$\lambda_{i\min}$, $\lambda_{i\max}$ – minimum and maximum eigenvalues of the matrix of normal equation system.

Tagg's number has such feature, that with greater Tagg's number will be smaller accuracy of determined unknowns of normal equation system i.e. stability of normal equation system will be changeable into poor. Basing on this way we can confirm about the efficiencies of proposed target functions in surface matching.

CONCLUSION

Merging of two point data sets of the same terrain surface obtained by different sensors can be solved with surface matching. The general assumption of two data sets of point is that the points in two sets are randomly distributed (no identical) with different density, accuracy and in the different reference system.

For surface matching point and surface patches modeled in the form of triangles or squares considered as features are used. The process of surface matching is implemented to solve simultaneously two problems of correspondence and transformation. For this purpose the target functions have been proposed.

In section 1.4 i 1.5 the contents of target functions proposed by author are presented.

The target function based on equal areas (Eq. 8) is performed to determine transformation parameters (Eq. 12; Eq. 13). This target function can be not only used for surface patches of Sl, generated in TIN model, but also for surface patches, generated in squares (DEM). The required smallest number of point in set S2 is six.

The target function in section 1.5 is based on the combination of two particular target functions. The first of them is based on the fitting of two normal vectors of squares generated in set Sl and of the angle created with three selected points q in set S2. To calculate the transformation parameters four points q in set S2 are required.

For choosing rational target function further numerical experiments have to be investigated.

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Funkcje celu stosowane do spasowania powierzchni terenu

Streszczenie

W ostatnich trzech latach problem integracji danych otrzymanych z różnych źródeł jest jednym z głównych kierunków badań. Dane dotyczące tego samego terenu takie jak LIDAR, SAR, IFSAR i inne są wykorzystywane do generowania DEM, ekstrakcji budynków na terenach zaludnionych, badania deformacji terenu w wyniku powodzi, wulkanów itd. Do rozwiązania tego zadania wykorzystano funkcje celu jako podstawowe do spasowania powierzchni (surface matching technique) pozwalające na jednoczesne prowadzenie dwu rozwiązań: utworzenia relacji pomiędzy cechami dwóch powierzchni (correspondence task) i określenia ich parametrów (transformation task).

Zaproponowane w tej pracy dwie funkcje celu są reprezentowane w podrozdziałach 1.4 i 1.5. Pierwsza jest oparta o warunek przynależności kolejnych transformowanych punktów z drugiego zbioru do generowanych trójkątów (TIN models) pierwszego zbioru. Natomiast, druga funkcja celu została oparta o warunek minimalizacji różnic odpowiednich wektorów normalnych elementów powierzchni trójkątów generowanych z drugiego zbioru i kwadratów generowanych w pierwszym zbiorze.

Люонг Чинг Ке

Целевые функции применяемые для приспособления поверхности местности

Резюме

В последние три года проблема интеграции данных получаемых с разных источников является одним из главных направлений исследований. Данные, касающиеся той-же самой местности, такие как LIDAR, SAR, IFSAR и другие, используются для определения цифровой модели местности, экстракции зданий в населённых пунктах, исследования деформаций местности в результате наводнений, деятельности вулканов и др. Для решения этой задачи были использованы целевые функции как основные для приспособления поверхности (surface matching technique), дающие возможность одновремнного ведения двух задач: создания соотношений между признаками двух поверхностей (correspondence task) и определния их параметров (transformation task).

Предложенные в этой работе две целевые функции представлены в подразделах 1.4 и 1.5. Первая функция основана на условии принадлежности очередных трансформированных пунктов с другого множества к генерированным треугольникам (TIN models) первого множества. Другая целевая функция основана на условии минимизации разниц соответственных векторов нормальных элементов поверхности треугольников генерированных с другого множества и квадратов генерированных в первом множестве.