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## Dynamic perspective projection – methods of connecting the reference plane and the projection centre with the moving vessel

In this paper, the definitions of the space and projection systems, used in the mathematical model, have been shown. The projection system, situation of the plane and the projection centre have been established. The concept of the model of perspective projection process has been described: the stages of the geometrical data process (3D objects included in Electronic Navigation Chart- ENC) realized during projection process.

Separately were considered the issues of changing of the reference plane and the projection centre with respect to the geocentric reference system and movement of the ship as the geometrical object containing the reference system. They constitute a basis for preparing the conception of connecting the moving ship with the observer reference system using three established, nonlinear points (with known position with respect to the geocentric reference system) placed on ship's board and with the horizontal topocentric reference system.

### INTRODUCTION

Let  $E^3$  will be 3D Euclidean space over a body of real numbers  $R$ ,  $E^3$  will be associated Euclidean one,  $O$  will be the point  $E^3$  and  $(e_1, e_2, e_3)$  will be the orthonormal base of the  $E^3$ . Ortocartesian reference system (global reper) of the space  $E^3$  will be the system:

$$\mathfrak{S} = \{O, (e_1, e_2, e_3)\} \quad (1)$$

Point  $O$  describes the beginning (or the base point), and  $(e_1, e_2, e_3)$  the base of the system  $\mathfrak{S}$ . Let  $\mathfrak{S}$  will be the established reference system of the space  $E^3$ . For any point  $P$ :

$$P = xe_1 + ye_2 + ze_3 \quad (2)$$

The set  $p = (x, y, z)$  will be called ortocartesian coordinates of the point  $P$  with respect to the reference system  $\mathfrak{S}$ .

Let  $\mathfrak{S} = \{O, (e_1, e_2, e_3)\}$  and  $\mathfrak{S}' = \{O', (e'_1, e'_2, e'_3)\}$  will be ortocartesian reference systems of the space  $E^3$ .

The reference system  $\mathfrak{S}$  (called the geocentrical) will be connected with the Earth in this way, that its versors  $e_1, e_2, e_3$  will determine the axes  $X, Y, Z$  of this system. The axis  $Z$  will agree with the axis of rotation of the Earth. Axes  $X$  and  $Y$  will be placed in the equator's plane: the axis  $X$  will be placed in the prime meridian and the axis  $Y$  in the meridian  $90^\circ E$ .

The reference system  $\mathfrak{S}'$  (called the observer's) will be connected with the position of the vessel and its beginning and the base will be determined with respect to the reference system  $\mathfrak{S}$ .

The observer's reference system  $\mathfrak{S}' = \{O', (e'_1, e'_2, e'_3)\}$  will be described in the space of locations:

$$E^3_{\mathfrak{S}} = \{P \in E^3, P = O + xe_1 + ye_2 + ze_3, (x, y, z) \in \mathbb{R}^3\} \quad (3)$$

by the pair  $(n, k)$  noncolinear (in particular ortonormal) vectors and the point  $O'$ , which will be its beginning.

Versors of the system  $\mathfrak{S}'$  will be obtained by means of following equations:

$$e'_1 = \frac{k \times n}{k \times n} \quad (4)$$

$$e'_2 = e'_3 \times e'_1 \quad (5)$$

$$e'_3 = \frac{n}{n} \quad (6)$$

#### *Conception model of the perspective projection's process*

The perspective projection belongs to the class of the geometrical planar projections. The projection is made on the plane, however, linear projection beams are used.

Visual effect of the single-point perspective projection is similar to the effect occurring in the human optic system. In the perspective projection the real image of an observed 3D object originates as a result of projection of each point by projection beams radiated from the projection centre and crossing the projection plane.

Assuming that a chart should be prepared for observer's position (for ship's position), it is suggested that the projection plane  $\Pi$ , reference system  $\mathfrak{S}''$  and projection centre  $S$  position are given with respect to  $\mathfrak{S}'$  in following way:

$$\Pi = \{P \in E^3, P = O' + x'e'_1 + y'e'_2, (x', y') \in \mathbb{R}^2\} \quad (7)$$

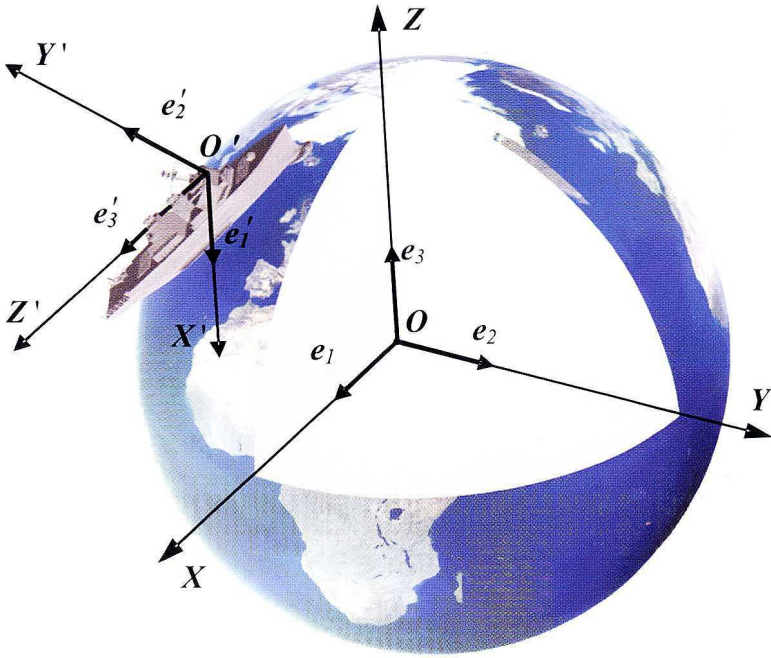


Fig. 1. Location of the reference systems  $\mathcal{S}'$  i  $\mathcal{S}$  in the space  $\mathbf{E}^3$ .

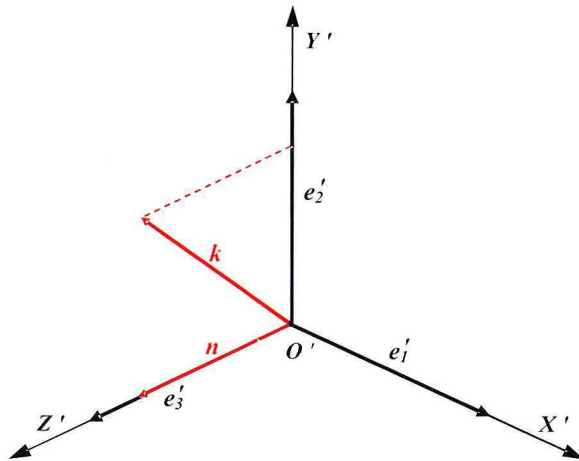


Fig. 2. The reference systems  $\mathfrak{S}'$

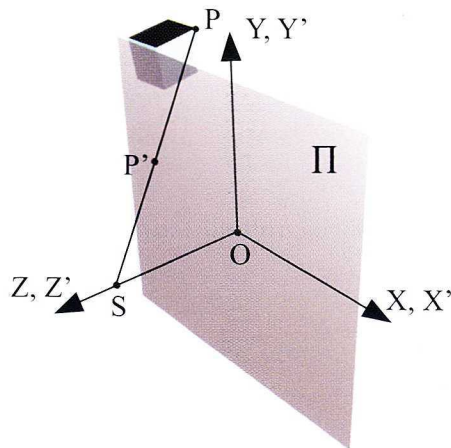


Fig. 3. Projection of the point  $P$  on the projection plane  $\Pi$

$$\mathfrak{S}^{\Pi} = \{O', (e'_1, e'_2)\} \quad (8)$$

$$S = (0, 0, d) \quad (9)$$

It follows that the projection is made in the observer reference system  $\mathfrak{S}'$ . However, before projection, the base  $(e'_1, e'_2, e'_3)$  transformation and the centre  $O$  of the geocentric reference system  $\mathfrak{S}$  in relation to the base  $(e'_1, e'_2, e'_3)$  and the centre  $O'$  of the observer reference system  $\mathfrak{S}'$ , should be made.

The transformation, which consists of translation of the point  $O$  to the point  $O'$  and rotation of the base  $(e_1, e_2, e_3)$  with respect to the base  $(e'_1, e'_2, e'_3)$ , is done using the following transformation matrix:

$$\mathbf{M}_{TR} = \begin{bmatrix} x'_1 & y'_1 & z'_1 & -(x'_1x_0 + y'_1y_0 + z'_1z_0) \\ x'_2 & y'_2 & z'_2 & -(x'_2x_0 + y'_2y_0 + z'_2z_0) \\ x'_3 & y'_3 & z'_3 & -(x'_3x_0 + y'_3y_0 + z'_3z_0) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (10)$$

where:

$$e'_1 = (x'_1, y'_1, z'_1),$$

$$e'_2 = (x'_2, y'_2, z'_2),$$

$$e'_3 = (x'_3, y'_3, z'_3),$$

$$O' = (x_0, y_0, z_0).$$

In suggested model, it is assumed that the geometrical data contained in ENC are given in the homogeneous ortocartesian coordinates. So, there is possible to compound the afinic transformations: rotation, translation and scaling. In this case, it allows to compound the translation matrix and the rotation one and obtain the compound transformation matrix  $\mathbf{M}_{TR}$ .

Coordinates of the point  $P' = (x'_p, y'_p)$  on the plane  $\Pi$  (called the flat one) in the reference system  $\mathfrak{S}^{\Pi}$  can be obtained by means of the equations:

$$x'_p = \frac{x_p}{\frac{|z_p|}{d} + 1}, \quad (11)$$

$$y'_p = \frac{y_p}{\frac{|z_p|}{d} + 1}, \quad (12)$$

where:  $P = (x_p, y_p, z_p)$  – coordinates of the projected point,  $S = (0, 0, d)$  – coordinates of the projection centre.

*Position changing of the projection plane and the projection centre  
with respect to the geocentric reference system*

The projection plane and the projection centre are rigidly connected with the observer reference system  $\mathfrak{S}'$ . To change the location  $\Pi$  and  $S$  with respect to  $\mathfrak{S}$ , it is necessary to change the location of vectors  $\overrightarrow{O'E'_1}=e'_1$ ,  $\overrightarrow{O'E'_2}=e'_2$ ,  $\overrightarrow{O'E'_3}=e'_3$ , (location of the centre  $O'$  and ends ( $E'_1, E'_2, E'_3$ ) of versors ( $e'_1, e'_2, e'_3$ )) of the observer reference system in the location space  $E^3_{\mathfrak{S}}$ . This transformation is the compound of the translation with respect to  $\mathfrak{S}$  and the rotation  $\mathfrak{S}'$  with the centre  $O'$  with respect to the system  $\{O', (e_1, e_2, e_3)\}$ .

This transformation can be written as:

$$\xi = \overrightarrow{OO'} \quad (13)$$

$$e'_i = M_R e_i \quad (i = 1, 2, 3) \quad (14)$$

where  $\xi$  is the translation vector of the point  $O$  to the point  $O'$  and  $M_R$

$$M_R = \begin{bmatrix} x'_1 & y'_1 & z'_1 \\ x'_2 & y'_2 & z'_2 \\ x'_3 & y'_3 & z'_3 \end{bmatrix} \quad (15)$$

is the rotation matrix of the base  $(e'_1, e'_2, e'_3)$  with respect to the base  $(e_1, e_2, e_3)$ .

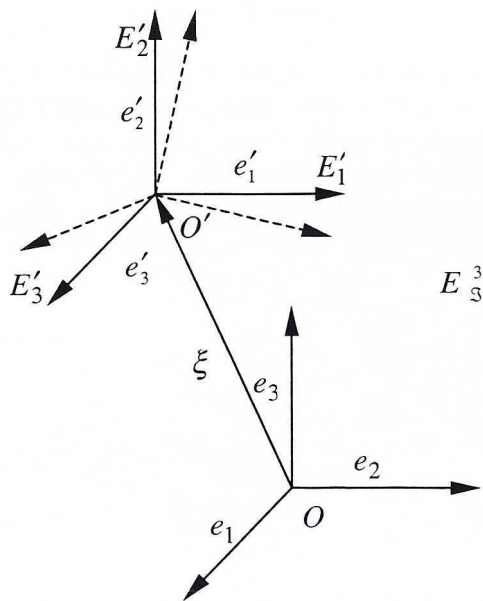


Fig. 4. Geometrical interpretation of the location changing  $\Pi$  and  $S$  with respect to  $\mathfrak{S}$

Rotation of the base  $(e'_1, e'_2, e'_3)$  can be described by means of several groups of parameters:

- space orientation angles and Euler's angles,
- direction cosines,
- quaternions and Cayley-Klein's parameters.

Because of speed of the numerical calculation, the most frequently used method is quaternions one.

*Movement of the observer geometrical reference system*

When location of vectors  $\overrightarrow{O'E'_1}=e'_1, \overrightarrow{O'E'_2}=e'_2, \overrightarrow{O'E'_3}=e'_3,$  with respect to the geocentrical reference system  $\mathfrak{S}$  (location of the centre  $O'$  and ends  $(E'_1, E'_2, E'_3)$  of versors  $(e'_1, e'_2, e'_3)$ ) of the observer reference system  $\mathfrak{S}'$  in the location space  $E^3_{\mathfrak{S}}$  are functions of the time variable  $t (t \in \tau)$ , then the system  $\mathfrak{S}'$  is in movement  $\mathfrak{R}^{\mathfrak{S}}$  with respect to the reference system  $\mathfrak{S}$  in the time range  $\tau$ .

There is assumed, that the system  $\mathfrak{S}'$  is in rigid (Euclidean) movement with respect to the system  $\mathfrak{S}$  in the time range  $\tau$ .

$$\forall P_1, P_2 \in E^3_{\mathfrak{S}}, \forall t', t'' \in \tau \quad |\overrightarrow{P_1(t')P_2(t')}| = |\overrightarrow{P_1(t'')P_2(t'')}| \tag{16}$$

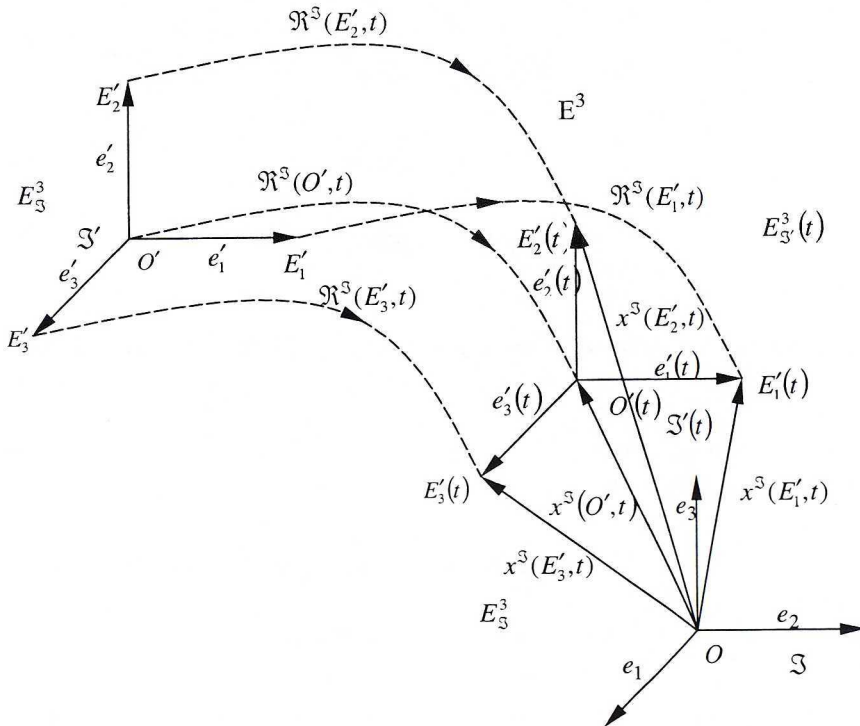


Fig. 5. Geometrical interpretation of the movement  $\mathfrak{R}^{\mathfrak{S}}$  of the observer reference system  $\mathfrak{S}'$  with respect to the geocentrical reference system  $\mathfrak{S}$

Location  $\mathfrak{S}'$  with respect to  $\mathfrak{S}$  can be expressed:

$$e'_1 = \overrightarrow{O'E'_1} = \overrightarrow{O'(t)E'_1(t)} \quad (17)$$

$$e'_2 = \overrightarrow{O'E'_2} = \overrightarrow{O'(t)E'_2(t)} \quad (18)$$

$$e'_3 = \overrightarrow{O'E'_3} = \overrightarrow{O'(t)E'_3(t)} \quad (19)$$

$\mathfrak{R}^{\mathfrak{S}}$  unambiguously describes movement of the location space  $E^{\mathfrak{S}}$  with respect to the reference system  $\mathfrak{S}$ :

$$E^{\mathfrak{S}}(t) = E^{\mathfrak{S}}, \quad (20)$$

for  $t \in \tau$ .

This movement is unambiguously described at each moment  $t$  ( $t \in \tau$ ) by location  $O'(t)$ ,  $E'_1(t)$ ,  $E'_2(t)$ ,  $E'_3(t)$ . This location is determined by means of vector movement functions:

$$x^{\mathfrak{S}}(O', t) = \overrightarrow{OO'(t)} \quad (21)$$

$$x^{\mathfrak{S}}(E'_1, t) = \overrightarrow{OE'_1(t)} \quad (22)$$

$$x^{\mathfrak{S}}(E'_2, t) = \overrightarrow{OE'_2(t)} \quad (23)$$

$$x^{\mathfrak{S}}(E'_3, t) = \overrightarrow{OE'_3(t)} \quad (24)$$

*Connecting the observer reference system with the ship's hull reference system  
by three nonlinear points*

The hull reference system  $\mathfrak{S}^K = \{O^K, (e_1^K, e_2^K, e_3^K)\}$  is rigidly connected with the ship's hull, which axes  $X^K$ ,  $Y^K$ ,  $Z^K$  determine crossing lines of three structural planes of the ship:

- the plane  $Y^K, O^K, Z^K$ , called the symmetry plane (cutting the hull into two symmetrical parts),
- the plane  $X^K, O^K, Y^K$ , called midship section plane,
- the plane  $X^K, O^K, Z^K$ , called the basic plane.

The chart image should be the most similar to the image perceived by the officer of the watch on the bridge. It is suggested to initial the set of the base and the centre of the observer system with relation to the ship in following way:



$$\begin{aligned} e'_1 &= e_1^K \\ e'_2 &= e_2^K \\ e'_3 &= e_3^K \\ O' &= O^M \end{aligned}$$

where  $O'$  describes established point  $O^M$  of the bridge.

Then the projection plane  $\Pi$  is perpendicular to the symmetry plane and the basic one. The chart image, which results from the projection of the geometrical objects contained in ENC on the plane  $\Pi$ , shows the manoeuvre area in front of the ship's bow, marked as  $C$ .

Let be known the location of three nonlinear points and the established point  $O^M$  of the bridge  $((P_1, P_2, P_3, O^M) \in C)$ , with relation to  $\mathfrak{S}^K$ . Points  $P_1, P_2, P_3$  describe the following vectors:

$$n = \overrightarrow{P_1P_2} \tag{25}$$

$$k = \overrightarrow{P_1P_3} \tag{26}$$

which determine the base  $(e_1^{K'}, e_2^{K'}, e_3^{K'})$  in the location space  $E_{\mathfrak{S}^K}^3$ . The rotation matrix of the base  $(e_1^K, e_2^K, e_3^K)$  to the base  $(e_1^{K'}, e_2^{K'}, e_3^{K'})$  can be expressed:

$$\mathbf{M}_R = \begin{bmatrix} x_1^{K'} & y_1^{K'} & z_1^{K'} & 0 \\ x_2^{K'} & y_2^{K'} & z_2^{K'} & 0 \\ x_3^{K'} & y_3^{K'} & z_3^{K'} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{27}$$

where

$$\begin{aligned} e_1^K &= (x_1^{K'}, y_1^{K'}, z_1^{K'}), \\ e_2^K &= (x_2^{K'}, y_2^{K'}, z_2^{K'}), \\ e_3^K &= (x_3^{K'}, y_3^{K'}, z_3^{K'}). \end{aligned}$$

The matrix  $\mathbf{M}_R^T$ :

$$\mathbf{M}_R = \begin{bmatrix} x_1^{K'} & y_1^{K'} & z_1^{K'} & 0 \\ x_2^{K'} & y_2^{K'} & z_2^{K'} & 0 \\ x_3^{K'} & y_3^{K'} & z_3^{K'} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^T = \begin{bmatrix} x_1^{K'} & x_2^{K'} & x_3^{K'} & 0 \\ y_1^{K'} & y_2^{K'} & y_3^{K'} & 0 \\ z_1^{K'} & z_2^{K'} & z_3^{K'} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{28}$$

is the matrix of reverse rotation, i. e. rotation of the base  $(e_1^{K'}, e_2^{K'}, e_3^{K'})$  to the base  $(e_1^K, e_2^K, e_3^K)$ .  $\xi_M = \overrightarrow{P_1O^M}$  is the vector of translation of the point  $P_1$  to the point  $O^M$ .

Let the ship  $C$  is in the rigid movement  $\mathfrak{N}^3$  with respect to the geocentric reference system  $\mathfrak{S} = \{O, (e_1, e_2, e_3)\}$ , in the time range  $\tau$  and location (position) of three nonlinear points  $(P_1, P_2, P_3) \in C$  with respect to  $\mathfrak{S}$  (in the location space  $E_3^3$  are known).

Let points  $P_1, P_2, P_3$  describe at each moment  $t$  ( $t \in \tau$ ) vectors:

$$n(t) = \overrightarrow{P_1(t)P_2(t)} \quad (29)$$

$$k(t) = \overrightarrow{P_1(t)P_3(t)} \quad (30)$$

determining the base, that, after transformation by the translated rotation matrix  $\mathbf{M}_R^T$ , is the base  $(e'_1, e'_2, e'_3)$  of the observer reference system  $\mathfrak{S}'$  (called the initial set base). Additionally the point  $P_1$  after translation by the vector  $\xi_M$  describes the centre of the observer's reference system (point  $O'$ )  $\mathfrak{S}'$ .

Therefore, in the model, it is suggested to use the constant translation of the plane and the projection centre with respect to the ship permitting displaying the manoeuvring area in each observation sector.

This operation consists of initial change of the setting the base and location of the observer reference system's centre  $\mathfrak{S}'$ .

#### *Connecting the observer reference system with the horizontal topocentric reference system*

Horizontal topocentric reference system  $\mathfrak{S}^H = \{O^H, (e_1^H, e_2^H, e_3^H)\}$  is obtained by transformation  $\mathfrak{S} = \{O, (e_1, e_2, e_3)\}$ . Transformation  $\mathfrak{S}$  is a compound of the rotation of the base  $(e_1, e_2, e_3)$  and translation of the point  $O$  to the point  $O^H$ , which is placed on the Earth.

Rotation of the base of the system  $\mathfrak{S}$  can be described by following compound matrix of transformations:

$$\mathbf{M}_R^{(\lambda, \psi)} = \begin{bmatrix} -\sin \psi \cos \lambda & -\sin \psi \sin \lambda & \cos \psi & 0 \\ -\sin \lambda & \cos \lambda & 0 & 0 \\ \cos \psi \cos \lambda & \cos \psi \sin \lambda & \sin \psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (31)$$

where:  $\psi$  – geocentric latitude of the point  $P$ ,  $\lambda$  – longitude of the point  $P$ .

The above is a compound of the rotation on angles:  $\lambda$  versors  $e_1$  and  $e_2$  (with constant  $e_3$ ),  $\psi$  versors  $e_2$  and  $e_3$  (with constant  $e_1$ ) the base of the system  $\mathfrak{S}$  and the transformation, which lies in conversion of positions versors determining the axes  $OX$  and  $OZ$ .

The base  $(e_1^H, e_2^H, e_3^H)$  can also be determined by parameterization of 3D space  $V$  using spherical coordinates  $r, \lambda$  and  $\psi$ .

The parameterized function (the chart) of the space  $V$  with respect to the reference system  $\mathfrak{S}$  describes following equation:

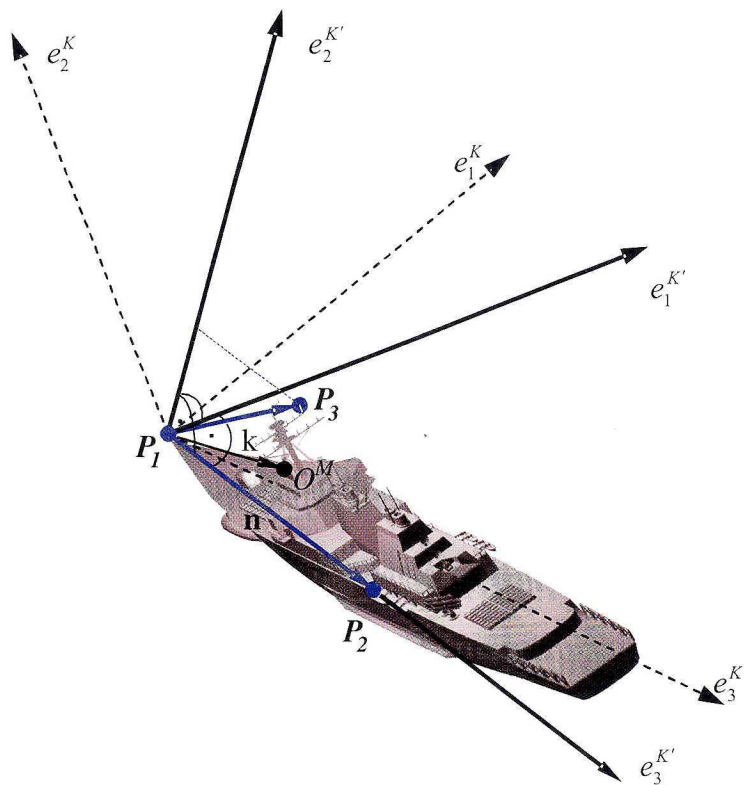


Fig. 6. Location of the observer reference system  $\mathcal{S}'$  connected with the ship by three nonlinear points  $P_1$ ,  $P_2$ ,  $P_3$ .

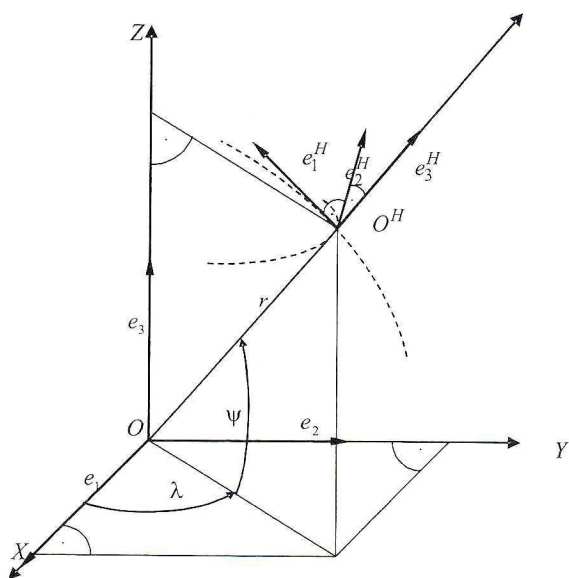


Fig. 7. Location of the observer reference system  $\mathcal{S}$  and  $\mathcal{S}''$

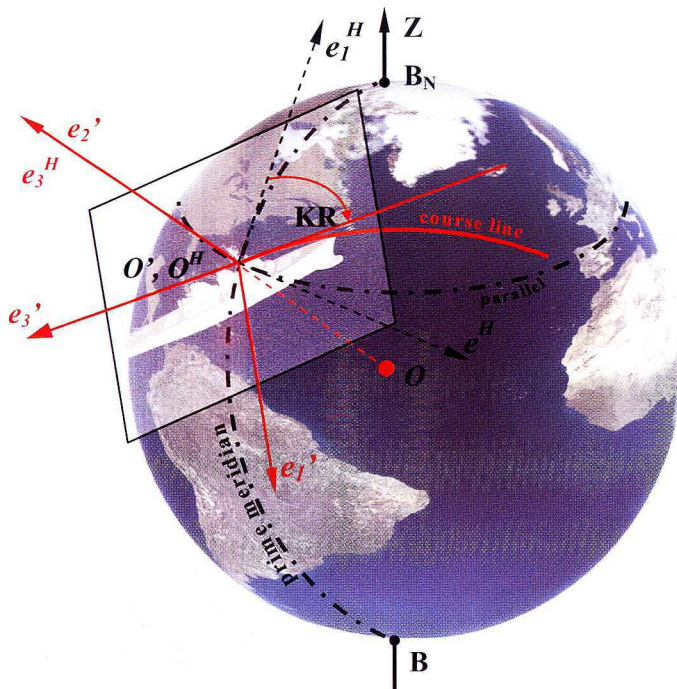


Fig. 8. Graphic interpretation of transformation of the base of the horizontal topocentric reference system  $\mathfrak{S}''$  toward the initial set base of the observer's reference system  $\mathfrak{S}'$

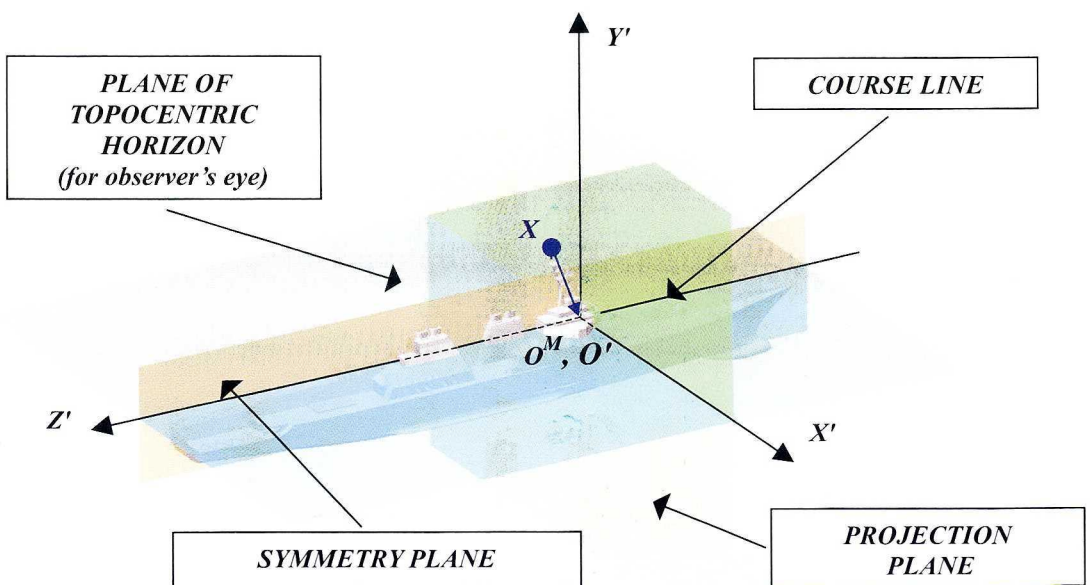


Fig. 9. Location of the projection plane  $\Pi$  with respect to the vessel C

$$x = r \cos \psi \cos \lambda e_1 + r \cos \psi \sin \lambda e_2 + r \sin \psi e_3 \quad (32)$$

Calculating derivatives of the function with respect to  $r$ ,  $\lambda$ ,  $\psi$ , we obtain vectors of the local base in 3D space  $V$ :

$$g_r = \cos \psi \cos \lambda e_1 + \cos \psi \sin \lambda e_2 + \sin \psi e_3 \quad (33)$$

$$g_\lambda = -r \cos \psi \sin \lambda e_1 + r \cos \psi \cos \lambda e_2 \quad (34)$$

$$g_\psi = -r \sin \psi \cos \lambda e_1 - r \sin \psi \sin \lambda e_2 + r \cos \psi e_3 \quad (35)$$

which, after normalizing:

$$e_1^H = \frac{g_\psi}{|g_\psi|} = -\sin \psi \cos \lambda e_1 - \sin \psi \sin \lambda e_2 + \cos \psi e_3 \quad (36)$$

$$e_2^H = \frac{g_\lambda}{|g_\lambda|} = -\sin \lambda e_1 + \cos \lambda e_2 \quad (37)$$

$$e_3^H = \frac{g_r}{|g_r|} = \cos \psi \cos \lambda e_1 + \cos \psi \sin \lambda e_2 + \sin \psi e_3 \quad (38)$$

are the base for each of points on the Earth surface with known geocentric latitude  $\psi$  and longitude  $\lambda$ .

Let be known location of the points  $P$ ,  $O^M \in C$ , with respect to  $\mathfrak{S}^K$ . These points determine the translation vector  $\xi_M = \overrightarrow{PO^M}$  of the point  $P$  to the point  $O^M$ .

Let be known: the real course  $KR$  and the position (geocentric latitude  $\psi$  and longitude  $\lambda$  of the point  $P \in C$ ) of the vessel  $C$  being in movement  $\mathfrak{R}^3$ , in time range  $\tau$ , with respect to the reference system  $\mathfrak{S}$ .

Let for each moment  $t$  ( $t \in \tau$ ) the point  $P$  determines:

$$O^M(t) = P(t) + \xi, \quad (39)$$

$$\mathfrak{S}^H(t) = \{O^H(t) = O^M(t), (e_1^H(t), e_2^H(t), e_3^H(t))\} \quad (40)$$

$$O'(t) = O^M(t) \quad (41)$$

and versors  $e_2^H, e_3^H$  of the system  $\mathfrak{S}^H$  and the angle  $K_0 = KR + 90^\circ$  vectors:

$$n(t) = \mathbf{M}_R^{(K_0)}(t) \cdot e_2^H(t) \quad (42)$$

$$k(t) = e_3^H(t) \quad (43)$$

determining the initial set base  $(e'_1, e'_2, e'_3)$  of the observer's reference system  $\mathfrak{S}'$ , where

$$\mathbf{M}_R^{(K_0)} = \begin{bmatrix} \cos K_0 & \sin K_0 & 0 & 0 \\ -\sin K_0 & \cos K_0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (44)$$

is the rotation matrix for the angle  $K_0$  of versors  $e_1^H, e_2^H$  (with constant  $e_3^H$ ) of the base of the horizontal topocentric reference system  $\mathfrak{S}^H$ .

Transformation of the base of the reference system  $\mathfrak{S}$  to the bases  $\mathfrak{S}^H, \mathfrak{S}'$ , can be also written using following compound rotation matrix:

$$\mathbf{M}_R^{(\lambda, \psi, K_0)} = \begin{bmatrix} -\cos K_0 \sin \psi \cos \lambda - \sin K_0 \sin \lambda & \sin K_0 \cos \lambda - \cos K_0 \sin \psi \sin \lambda & \cos K_0 \cos \psi & 0 \\ \cos \psi \cos \lambda & \cos \psi \sin \lambda & \sin \psi & 0 \\ \sin K_0 \sin \psi \cos \lambda - \cos K_0 \sin \lambda & \sin K_0 \sin \psi \sin \lambda + \cos K_0 \cos \lambda & -\sin K_0 \cos \psi & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (45)$$

The base of the observer's reference system  $\mathfrak{S}'$  obtained as a result of transformation of the base of the horizontal topocentric reference system  $\mathfrak{S}^H$ , has the versor  $e_1'$  collinear with the course line of the vessel. It results that for each moment  $t (t \in \tau)$  the projection plane  $II$  is perpendicular to the plane of the topocentric horizon (determined by the versors  $e_1^H$  and  $e_2^H$ ) and the real course line of the moving vessel  $\mathfrak{R}^3$ , in time range  $\tau$ .

This conception of connecting the observer's reference system  $\mathfrak{S}'$  with the vessel also supposes the possibility of input the constant offset of the location of the reference plane and projection centre with respect to the initial set for presentation the maneuver area in the specific observation sector.

## CONCLUSIONS

1. The perspective projection enables obtaining 3D real image, which is the quickest and the most comprehensible by the human taking majority of information by the eyesight.
2. 3D image of the navigation area created during the movement of the vessel, enables better space orientation of the officer of the watch increasing efficiency of analyzing, working out, synthesizing and concluding possibilities.
3. Reconstructed 3D image of the area and sea bottom being created on the basis of the geometrical data included in the Electronic Navigation Chart and compared with images from the camcorder, sonar or radar enables using the comparative methods of navigation.

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**Dynamiczne odwzorowanie perspektywiczne – metody związania płaszczyzny  
i środka rzutowania z jednostką pływającą, będącą w ruchu**

Streszczenie

Celem niniejszego artykułu było przedstawienie wyników badań prowadzonych nad nową formułą odwzorowawczą do dynamicznego, trójwymiarowego, realistycznego zobrazowania informacji geometrycznej o środowisku geograficznym. Formułą tą nazywaną dynamicznym odwzorowaniem perspektywicznym, określono odwzorowanie perspektywiczne na płaszczyznę z jednym stałym punktem rzutowania w pozycji obserwatora, przy czym zarówno płaszczyzna rzutowania jak i punkt rzutowania dowiązane są do jednostki pływającej będącej w ruchu. W artykule przedstawiono opis matematyczny dynamicznego odwzorowania perspektywicznego, opartego na dwóch koncepcjach związania jednostki pływającej (będącej w ruchu) z układem rzutowania (płaszczyzną i punktem rzutowania), tj.:

- poprzez trzy ustalone, niewspółliniowe punkty (o znanym położeniu względem geocentrycznego układu odniesienia) znajdujące się na jednostce pływającej,
- poprzez ustalony punkt (o znanym położeniu względem geocentrycznego układu odniesienia) i kurs rzeczywisty jednostki pływającej.

Pierwsza koncepcja, w której wiązami są trzy ustalone, niewspółliniowe punkty znajdujące się na jednostce pływającej, pozwoli na dynamiczne zobrazowanie rejonu żeglugi, zmieniające się dynamicznie nie tylko ze względu na ruch jednostki pływającej, ale także ze względu na jej przechyły poprzeczne i podłużne oraz nurzanie. Dlatego też, zobrazowanie to nie będzie zawsze użyteczne dla nawigacji, szczególnie przy złych warunkach hydrometeorologicznych, ale winno dać dobrą podstawę do pełniejszego wykorzystania porównawczych metod pozycjonowania, chociażby przez możliwość porównywania z obrazami z kamery wizyjnej, czy sonaru.

Друга концепція, повинна дозволити на динамічне зображення реjonу жеглуґи, в часі руху і в часі зміни курсу јодностки плавучеј. Зображення то бјдзє odporne на пречеһы попречне і подлуґне oraz nurzanie јодностки плавучеј, dlatego та концепція wydaje się бјć najkorzystniejsza до презентова́ннн на mostku nawigacyjnym Elektronicznej Mapy Nawigacyjnej.

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### **Динамическое перспективное отображение как метод связи плоскости и центра проекции с перемещающейся плавучей единицей**

#### **Резюме**

Целью статьи является представление результатов исследований, проведённых по новой проекционной формуле для динамического, трёхмерного, реального графического представления геометрической информации о географической среде. Эта формула, называемая динамическим перспективным отображением, определяет перспективное отображение на плоскость с одним постоянным пунктом проекции в позиции наблюдателя, при чём плоскость проекции и пункт проекции привязаны к плавучей единице. В статье представлено математическое описание динамической перспективной проекции, основано на концепциях привязки плавучей единицы к системе проекции (к плоскости и точке проекции), а именно:

- через три определённые, несолинейные пункты (с известным положением относительно геоцентрической системы координат), расположенные на плавучей единице,
- через определённый пункт (с известным положением относительно геоцентрической системы координат) и действительный курс плавучей единицы.

Первая концепция, в которой связями являются три определённые, несолинейные пункты, находящиеся на плавучей единице, даёт возможность динамического изображения района судоходства, изменяющегося динамически не только из-за движения плавучей единицы, но тоже из-за её продольных и поперечных кренов и погружений. Поэтому, этот вид изображения не всегда будет пригодным для навигации, прежде всего в плохих гидрометеорологических условиях, но оно создавать хорошую основу для более полного использования сравнительных методов определения положения, хотя бы через возможность сравнения с изображениями из видеокамеры или сонара.

Вторая концепция должна создать возможным создание динамического изображения района судоходства во время движения и во время изменения курса плавучей единицы. Это изображение является устойчивым к поперечным и продольным креном, а также к погружениям плавучей единицы и поэтому кажется, что эта концепция будет самой пригодной для представления на штурманском мостике Электронной Навигационной Карты.