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A concept of solution of correlative observations in photogrammetry

One of the basic processes in photogrammetry consists of identification and measurements of conjugate (homologous) points located within image overlapping. In analytical photogrammetry this process is solved manually by an observer. In digital photogrammetry this process is solved automatically by software and it is called image matching. This process has considerable importance for automation of orientation or aerial triangulation of photographs. The accuracy of image matching process influences the accuracy of determination of image orientation elements and computed point co-ordinates. This article presents the author's idea concerning matching of digital images with regard to correlation between neighbouring pixels. First, the problem of correlation between point co-ordinates will be examined in analytical photogrammetry, what will simplify considerations related to digital photogrammetry.

INTRODUCTION

It was assumed in the publication [2] that one of three successes of last years was "the new matching strategy" connected to correlated observations in digital photogrammetry. The new method is of great interest. This proves that the problem of correlation observation is not important exclusively in geodesy, but also important in digital photogrammetry. In this work the author attempts to approach the problem, starting from correlative observations in analytical photogrammetry. Thus, the basis for formulation of some advice concerning utilisation of such observations in digital photogrammetry will be created.

Considering types of compared data, the image matching process for all proposed solutions can be divided into two groups:

- area-based matching – ABM.
- feature-based matching – FBM.

The ABM method is more accurate than the FBM method and is widely applied in digital photogrammetry. The ABM method, firstly proposed by Ackermann (1984) was improved and developed within next 10 years by Foestner, Grun, Rosenholm and others, with consideration of various geometrical conditions, as: area-based matching using a surface model [13]; muti–point area-based matching [5]; multi-image area-based matching [9] and others.

Photogrammetry is based on mathematical formulation of solutions of geometrical relations, which occur between an image and the terrain surface. Such a mathematical model has to correspond to real relations at the moment of taking photographs. It has been stated that correlation between point co-ordinates (correlation between pixels) occurs as early as the moment of generation and recording of a photographic (or a digital) image. [7], [12].

The theory of correlative observations itself is conventional in its nature, however its efficient and effective practical use in order to solve a particular problem still remains important. The basic task of elaboration of correlative observations in this work is to formulate:

- variance-covariance matrix (correlative matrix) to determine relative orientation elements of a model, which is the smallest unit of aerial triangulation.
- approach to a problem of correlative observations in digital photogrammetry.

The algorithm for solution of correlative observations at single model reconstruction in analytical photogrammetry

The aim of this section is to discuss the algorithm for computation of model relative orientation elements with respect to correlative observations in analytical photogrammetry. The example of calculations is presented at the end of this section.

In order to determine 5 elements of relative orientation measured co-ordinates of certain number of points, located evenly in the model area, are performed. Six Gruber's points are used, according to Fig. 1.

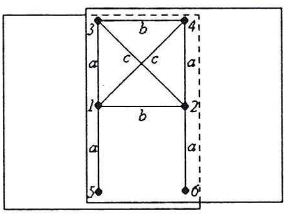

Fig. 1. Six Gruber's points.

One observation equation will be formulated for every point, for the parallax, with an assumption that image point co-ordinates were expressed at the scale, where $z' = z'' = 1$, [14]:

$$(x''y')d\varphi - (1 + y'y'')d\omega - x''d\chi + (x' - x'')by + (x'y'' - x''y')bz + q = \upsilon_q \qquad (1)$$

where:

$d\varphi$, $d\omega$, $d\chi$, by, bz – elements of model relative orientation (ω; φ; χ; – rotation angles);

x, y image point measured co-ordinates, divided by the camera focal length f. Signs (') and (") correspond to the left and the right image, respectively.

The case when correlation between point co-ordinates 1, 2, 3, 4 exists, is considered (similarly for points 1, 2, 5, 6) for solution of the system (1). For this the covariance matrix must be created for these point co-ordinates. Vertical parallaxes (1) of six Gruber's points are defined as the system of equations:

$$\begin{aligned} q_1 &= y_1^{'} - y_1^{''} \\ q_2 &= y_2^{'} - y_2^{''} \\ &\ldots\ldots\ldots\ldots \\ q_6 &= y_6^{'} - y_6^{''} \end{aligned} \tag{2}$$

The equation system (2) is to show that vertical parallaxes are the functions of measured coordinates of homologous points in the left and right image. To solve the system (1) we should form the covariance matrix **C**. General form of the covariance matrix of vertical parallaxes is represented below [8]:

$$\mathbf{C} = \left[\frac{\partial q_i}{\partial y_i}\right] \mathbf{C}_y \left[\frac{\partial q_i}{\partial y_i}\right]^T \tag{3}$$

where:

$$i = 1, 2, 3, \ldots 6$$
$$q_i = \begin{bmatrix} q_1 & q_2 & q_3 & \cdots & q_6 \end{bmatrix}^T$$
$$y_i = \begin{bmatrix} y_1^{'} \; y_1^{''} \cdots y_3^{'} \; y_3^{''} \cdots y_6^{'} \; y_6^{''} \end{bmatrix}^T$$

$\mathbf{C}_y$ – the covariance matrix of ordinates for 6 points (3b); $\partial q_i/\partial y_i$ – Jakobian's matrix (3a):

$$\mathbf{C} = \begin{bmatrix} m_{q_1}^2 & c_{q_1q_2} & . & . & . & c_{q_1q_6} \\ & m_{q_2}^2 & . & . & . & m_{q_2q_6} \\ & & . & . & . & . \\ & & & . & . & . \\ & \text{symetr part} & & & . & . \\ & & & & & m_{q_6}^2 \end{bmatrix}; \left[\frac{\partial q_i}{\partial y_i}\right] = \begin{bmatrix} 1 & -1 & 0 & 0 & . & . & . & 0 & 0 \\ 0 & 0 & 1 & -1 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & 0 & . & . & . & 1 & -1 \end{bmatrix} \tag{3a}$$

$$\mathbf{C}y = \begin{bmatrix} m^2_{y_1} & c_{y_1 y_1} & c_{y_1 y_2} & c_{y_1 y_2} & \cdots & c_{y_1 y_6} \\ & m^2_{y_1} & c_{y_1 y_2} & c_{y_1 y_2} & \cdots & c_{y_1 y_6} \\ & & m^2_{y_2} & c_{y_2 y_2} & \cdots & c_{y_2 y_6} \\ & & & m^2_{y_2} & \cdots & c_{y_2 y_6} \\ & & & & \cdots & \cdot \\ & \text{symmetr.} & \text{part} & & \cdots & \cdot \\ & & & & & \cdot \\ & & & & & m^2_{y_6} \end{bmatrix} \tag{3b}$$

It is assumed that:

- RMS errors of measured image co-ordinates are identical and equal to m_y
- according to [11]: the covariance matrix elements between two points (correlation coefficients) are the function of their distance.Thus:
- $c_{y'i;y''i} = \rho m_y^2$; where ρ - correlation coefficients between ordinates of conjugate point on the left and the right image (point matching) $i = 1, 2,...,6$.
- $c_{y'i;y'k} = \rho_{i,k} m_y^2$; where $\rho_{i,\,k}$ - correlation coefficients between ordinates of points i, k in one image. In our case they are ρ_a; ρ_b; ρ_c.- (Fig. 1).
- $c_{y'i;y''i+m} = 0$; $m = 1, 2, ...,5$ (no point matching).

After introducing these elements to the matrix (3b) and multiplication by the matrix (3a), according to (3), the **C** matrix is obtained:

$$\mathbf{C} = \begin{bmatrix} (1-\rho) & \rho_b & \rho_a & \rho_c & \rho_a & \rho_c \\ & (1-\rho) & \rho_c & \rho_a & \rho_c & \rho_a \\ & & (1-\rho) & \rho_b & 0 & 0 \\ & & & (1-\rho) & 0 & 0 \\ & \text{symmetr.} & \text{part} & & (1-\rho) & \rho_b \\ & & & & & (1-\rho) \end{bmatrix} \tag{4}$$

It should be stressed that the matrix (4) contains two kinds of correlation coefficients. The first coefficient ρ is the correlation coefficient of conjugate point on the left and the right image (homologous points), which is located on the main diagonal of the matrix. Successive coefficients are the correlation coefficients of co-ordinates having the same name (x-x or y-y) between points on one image (left or right), located symmetrically in relation to the main diagonal of the matrix (ρ_a, ρ_b, ρ_c,).

The covariance matrix for 9 or 15 points (or more points located evenly in relation to the image base b) can be created similarly to the formula (4).

From the equation system (1) the formula (5) can be obtained.

$$\mathbf{A}X + L = V \qquad \text{given } \mathbf{C} \tag{5}$$

where: **A**– the matrix of unknown coefficients,

$X = [d\varphi \quad d\omega \quad d\chi \quad dby \quad dbz]^T$ – the vector of unknowns,

$L = [q_1 \quad q_2 \quad .....]^T$ – the vector of vertical parallaxes which are the functions of image observed coordinates,

$V = [v_{q1} \; v_{q2}]^T$ - the vector of corrections for vertical parallaxes,

C – the matrix in the form (4); T – transpose.

The unknowns X will be calculated under the condition $[V^T\mathbf{C}^{-1}V] = \min$.

It is known that the accuracy of the determined model spatial co-ordinates *(X, Y.* and *Z)* depends to large degree on the accuracy of model relative orientation elements. Small changes of relative orientation elements ($d\varphi$, $d\omega$, $d\chi$, dbz, dby) cause considerable changes of computed model spatial co-ordinates. The author [11], in a simplified manner derived the final formula of this relation (in image scale):

$$dX = xdN; \quad dY = ydNs; \quad dZ = -fdN \tag{6}$$

$$dN = \frac{N}{b}\left\{\frac{x-b}{f}dbz + \left(f + \frac{(x-b)^2}{f}\right)d\varphi + \frac{(x-b)y}{f}d\omega - yd\chi\right\}$$

$$dNs = \frac{N}{b}\left\{\frac{2x-b}{2f}dbz + \left[f + \frac{(x-b)(2x-b)}{2f}\right]d\varphi + \left[\frac{2x-b}{2f}y + \frac{fb}{2y}\right]d\omega - \right.$$

$$\left. -\left[y - \frac{b(x-b)}{2y}\right]d\chi + \frac{N}{2}dby\right\}$$

where: x, y – the measured image point co-ordinates related to the right image;

by, bz – the image base components.

Basing on (6), the mean square error for the model spatial co-ordinates m_x^2, m_y^2, m_z^2 equals to:

$$m_x^2 = \left(\frac{N}{b}x\right)^2 A; \quad m_y^2 = \left(\frac{N}{b}y\right)^2 B; \quad m_z^2 = \left(\frac{N}{b}f\right)^2 A \tag{7}$$

$$A = \frac{1}{f^2}\left\{[x-b]^2 m_{bz}^2 + \left[f^2 + (x-b)^2\right]^2 m_\varphi^2 + [(x-b)y]^2 m_\omega^2 + y^2 f^2 m_\chi^2\right\}$$

$$B = \frac{1}{4f^2}\left\{(2x-b)^2 m_{bz}^2 + \left[(x-b)(2x-b) + 2f^2\right]^2 m_\varphi^2 + \frac{1}{y^2}\right.$$

$$\left.\left[y^2(2x-b) + f^2 b\right]^2 m_\omega^2 + \left[2y^2 - b(x-b)\right]^2 \frac{f^2}{y^2} \cdot m_\chi^2\right\} + \frac{N^2}{4} m_{by}^2$$

For image shown in Fig.l points 1, 2, 3, 4 have the following co-ordinates (assuming $a = b$ – for simplicity): 1(0, 0); 2(b, 0); 3(0, b); 4(b, b). Basing on (7) the mean square errors of model spatial point co-ordinates may be calculated; they are presented in Table 1 below:

Table 1. Mean square errors of model point co-ordinates

Point	m_x^2	m_y^2	m_z^2
1	0	0	$H1$
2	$H2$	0	$H3$
3	0	$H4$	$H5$
4	$H6$	$H7$	$H8$

Where:

$$H1 = \left(\frac{N}{b}\right)^2 \left\{b^2 m_{bz}^2 + L^2 m_\varphi^2\right\};\ L = \left(f^2 + b^2\right);\ H2 = N^2 f^2 m_\varphi^2;\ H3 = \left(\frac{N}{b}\right)^2 f^4 m_\varphi^2$$

$$H4 = \left(\frac{N}{2f}\right)^2 \left\{Q^2 m_\varphi^2 + R^2 m_\omega^2 + 9P m_\chi^2 + N^2 f^2 m_{by}^2 + b^2 m_{bz}^2\right\}$$

$$Q = \left(2f^2 + b^2\right);\ R = \left(f^2 - b^2\right);\ P = b^2 f^2$$

$$H5 = \left(\frac{N}{b}\right)^2 \left\{b^2 m_{bz}^2 + L^2 m_\varphi^2 + b^4 m_\omega^2 + P m_\chi^2\right\}$$

$$H6 = (N)^2 \left\{f^2 m_\varphi^2 + b^2 m_\chi^2\right\}$$

$$H7 = \left(\frac{N}{2f}\right)^2 \left\{4 f^4 m_\varphi^2 + L^2 m_\omega^2 + 4P m_\chi^2 + N^2 f^2 m_{by}^2 + b^2 m_{bz}^2\right\}$$

$$H8 = \left(\frac{Nf}{b}\right)^2 \left\{f^2 m_\varphi^2 + b^2 m_\chi^2\right\}$$

In order to explain the created matrix (4) the numerical example is presented.

Numerical example

Data used in this example are the measured co-ordinates and parallaxes of 14 image points on the stereo-pair. To use the formula (1) these data were divided by the camera focal length and presented in Table 2, for 6 Gruber's points.

T a b l e 2 . The coordinates and paralalaxes of image points divided through camera focal length

Co-ordinates of 6 Gruber's points					
Points	x'	y'	x''	y''	q
20 = 1	0.058354	0.077338	-0.573663	0.059217	-0.01812
21 = 2	0.618048	-0.014132	-0.024636	-0.032200	-0.01807
22 = 3	0.123386	0.600774	-0.502333	0.578371	-0.02240
23 = 4	0.653513	0.525816	0.013375	0.499775	-0.02604
24 = 5	-0.040338	-0.497229	-0.698836	-0.523888	-0.02666
25 = 6	0.551908	0.564189	-0.104453	-0.584621	-0.02043

In order to specify the matrix elements (4) practical values $my = mq = \pm 10$ μm are assumed. From definition of the vertical parallax $q = y' - y''$ and basing on (3) $\rho = 0.5$ its minimum values are $\rho_a = \rho_b = 0.3$ and $\rho_c = 0.2$. After introduction and multiplication of these values according to (3) the final matrix **C** is obtained in the form (4).

We execute 3 computational variants:

- variant 1: relative orientation elements are determined from large numbers of vertical parallaxes (14 points) and are considered as "true" values.
- variant 2: relative orientation elements are counted from six Gruber's points, assuming no correlation between vertical parallaxes.
- variant 3: as the variant 2, with assumption of the correlation between vertical parallaxes.

Basing on (5) RMS errors of relative orientation elements for three variants are counted to estimate an internal accuracy. They are presented in Table 3:

T a b l e 3. Root mean square errors of relative orientation elements of a model

RMS errors of model relative orientation elements						
Variants	$\pm m\varphi$	$\pm m\omega$	$\pm m\chi$	$\pm m_{by}$ [mm]	$\pm m_{bz}$ [mm]	$\pm m_0$
1	2'.21	1'.65	0'.87	0.101	0.042	$2.18 * 10^{-4}$
2	4'.80	4'.26	2'.01	0.267	0.104	$4.18 * 10^{-4}$
3	3'.13	2'.32	1'.70	0.180	0.103	$4{,}35 * 10^{-4}$
m_2/m_1	2.17	2.58	2.31	2.61	2.48	Ratio of RMS errors
m_3/m_1	1.41	1.41	1.95	1.78	2.45	
m_3/m_2	0.65	0.54	0.85	0.67	0.99	

The correlation coefficients between relative orientation elements are also calculated; they are presented in Table 4:

T a b l e 4 . The correlation coefficients between relative orientation elements

ρ Variant	$\rho_{\varphi\omega}$	$\rho_{\omega\chi}$	$\rho_{\varphi by}$	$\rho_{\varphi bz}$	$\rho_{\omega\chi}$	$\rho_{\omega by}$	$\rho_{\omega bz}$	$\rho_{\chi by}$	$\rho_{\chi bz}$	ρ_{bybz}
1	-0.10	-0.06	-0.08	**+0.76**	-0.10	**+0.98**	-0.08	-0.17	-0.12	+0.04
2	-0.12	-0.10	-0.12	**+0.75**	+0.08	**+0.98**	+0.01	-0.04	0.00	0.00
3	-0.06	-0.13	-0.08	**+0.51**	+0.08	**+0.95**	-0.01	-0.08	-0.08	+0.01

Basing on Tables 3 and 4 it may be noticed that:

- RMS errors of variant 3 are considerably smaller than for the variant 2 (between 0.54 and 0.99 times).
- the correlation coefficients of relative orientation elements have the highest values for (ω; *by*) and (φ; *bz*) other values are practically equal to 0.

The accuracy analysis of calculated model co-ordinates for three variants to be discussed. For this purpose it is necessary to calculate the ratio of RMS errors between particular variants. Thus it is possible to confirm, to what degree the accuracy of computed model co-ordinates would be improved with the use of the covariance matrix (4).

Basing on Table 1, 3, the RMS errors of three co-ordinates for four points in reconstructed model for three variants and then the ratio of errors (k_x; k_y; k_z) for particular variants, are calculated (Table 5).

T a b l e 5 . The ratio of RMS errors between particular variants

	Ratio of RMS errors	Point	k_x	k_y	k_z
1	variant 2/variant 1	1	0	0	2.19
		2	2.17	0	2.17
		3	0	2.35	2.22
		4	2.18	2.42	2.20
2	variant 3 /variant 1	1	0	0	1,49
		2	1.40	0	1.41
		3	0	1.54	1.50
		4	1.41	1.55	1.46
3	variant 3 /variant2	1	0	0	0.69
		2	0.65	0	0.65
		3	0	0.63	0.68
		4	0.66	0.65	0.69

Basing on Table 5 it may be noticed that for small correlation coefficients (0.2 and 0.3) introduced to the matrix (4) to determine model relative orientation elements (variant 3) the accuracy of model spatial co-ordinates better than for the variant 2 may be obtained. Improved accuracy of the variant 3 in relation to the variant 2 is characterised by coefficient k_x, k_y, k_z, which are 0.65 times smaller.

The single model is the smallest block of aerial triangulation. The accuracy of the block depends on many factors, as the size, quantities and ways of distribution of control points etc., and, first of all, on the single model accuracy.

A concept of a solution of correlative observations in digital photogrammetry

Digital photogrammetry using GPS/TNS techniques to determine image external orientation elements has been highly developed recently. The automatic aerial triangulation has been practically applied for production purposes [10]. The main change between analytical photogrammetry and digital photogrammetry concerns the stage of measurements and interpretation executed by an observer is taken over by computer software. This allows for implementation of fully automated systems in digital photogrammetry. Investigation of matching points of digital stereo-images can be performed solved automatically [1]. The degree of automation depends on different factors, such as image scale, type of terrain, image quality. Practical examples relate to photogrammetric elaboration of mountainous areas, such as Alps in Switzerland [10].

As it was already mentioned in introduction, one of the successes of the last two years is connected with correlative observations. The problem of correlative observations in digital photogrammetry is still interesting at the international scale. It is one of the directions of investigations aiming at technological improvements in digital photogrammetry. In my opinion, this problem refers first of all to the correlation between pixels, which was discussed earlier publications [7], [12]. The obtained accuracy of image matching, with counted probability of point matching with accepted correlation model between pixels, is higher than 25% than for the model assumed without correlation [7], depending on signal-to-noise ratio.

OEEPE-ISPRF experiments [2] relating to automatic aerial triangulation on different digital stations, executed by 21 institutions were published. They confirmed that the block stability depends on the number of measured tie points. Between 100 and 300 points on one stereo-pair are required. The large number of points should be considered. In some systems (HAT, HAT*, Match, FGI), it is required to locate points according to Gruber's schema (Fig. 1). These points can be divided into 15 groups on the image; thus one group has about 9 neighbouring points. Such a number of points in one group result in high correlation among pixels (points). This problem should be taken into account in the process of elaboration.

The aim of monitoring of digital images is to create the 3-D model. The advantage of digital photogrammetry relies on the possibility of direct observations and discussions - by the observer and customers – of the obtained stereoscopic model (3-D). Such possibilities do not exist in conventional photogrammetry; in this case exclusively observer performs measurements. Automation of matching process of monitoring is considered as the basic method, which influences the accuracy of successive processes. The area-based matching (ABM) method is mostly used and implemented basing on the least square rule; sometimes it is called LSM (least squares matching). Observations applied in this method are grey values of pixels. To improve accuracy of the traditional ABM method, the additional constraints, such as differences of scale of the left and right images [4] [6]; the presence of neighbouring pixels co-ordinates [13] and of variable weight models to multi-point LSM [5] are introduced to the observation equation system.

This article presents the author's proposal relating to the ABM method with regard to correlation between the neighbouring pixels and the central pixel of the active window. The correlation between neighbouring pixels in image should be understood with respect to the probability i.e. when a certain event occurs (quantity of photon at the pixel) influences the probability of appearing the second event (quantity of photon at neighbouring pixels) [12].

The publication [12] presents relations between the correlation coefficient of two neighbouring pixels (points) and their intensities of light, which characterise light amplitudes A_1 and A_2

$$\rho_{1,2} = A_1 A_2 \cos\alpha \tag{8}$$

where: 1, 2 – the pixel indexes in one line (or column) ,

α – the spatial angle between two light rays reflected from the ground with the vertex in the camera projection point (Fig. 3). The values of amplitudes A in the image are characterised by magnitudes of grey level g:

$$\rho_{1,2} = g_1 g_2 \cos\alpha \tag{9}$$

Grey values of all pixels will be normalised on one image in the following way:

$$\hat{g}_{ij} = \frac{g_{ij}}{g_{(pq)\max}} \tag{10}$$

where: $i = 1, 2,\dots p,\dots n; j = 1, 2,\dots, q,\dots m$ – the pixel index; p, q – the pixel index of maximum grey value. The values $\hat{g}_{ij}$, change in the interval (0 –1).

Rosenholm [11] proposed normalisation of grey value for the matching process as follows:

$$\Delta g = c_1 + g^l c_2 + x c_3 + y c_4 + d c_5 + d g^l c_6 \tag{11}$$

where: Δg – the grey value difference between points in the left and right windows (images);

g – the grey value of point (pixel) in the left window;

d – the radius from the centre point of the left image to the image point;

x, y – the image point co-ordinates of the left image; c_1 c_2... c_6 – coefficients to be determined. From (9) the following formula may be derived:

$$\rho_{1,2} = \hat{g}_1 \cdot \hat{g}_2 \cos\alpha \tag{12}$$

Looking at Fig. 3 the angle value from scalar product may be determined:

$$\cos\alpha = \frac{\mathbf{s}_c \cdot \mathbf{s}_{i,j}}{|\mathbf{s}_c| \cdot |\mathbf{s}_{i,j}|} \tag{13}$$

where: $s_c=[x_c\ y_c\ -f\,]^T$; $s_{i,j}=[x_{i,j}\ y_{i,j}\ -f]^T$ – vectors of the central pixel (point c) co-ordinates, and neighbouring pixels (points' (i, j)).

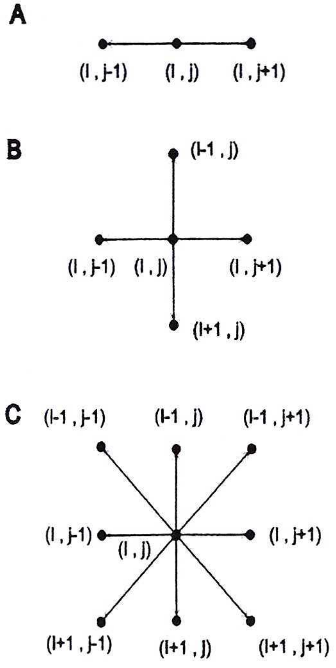

Fig. 2. The pixel centres

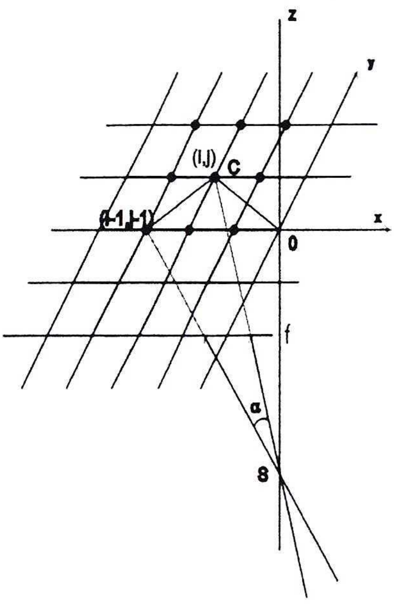

Fig. 3. The spatial angle α

In the ASM method measured observations are grey values. Now we will try to form the covariance matrix assuming the accepted model of correlation between neighbouring pixels. The error equations system for a simple, one-dimensional case may be written as follow [5]; [11]:

$$\begin{aligned} \upsilon(x_i) &= -g'^{l}(x_i)x_0 + \left[g^l(x_i) - g^r(x_i)\right] \\ \upsilon(x_i) &= \left[n^l(x_i) - n^r(x_i)\right] \end{aligned} \tag{14}$$

where: i – pixel index; l; r – the left and right image number; x – one-dimensional axis x;
$n^l(x_i)$; $n^r(x_i)$ – noises of the left and right image;
$g^l(x_i)$, $g^r(x_i)$ – measured grey values of left and right image;
$g'^{l}(x_i)$ – the gradient of the left image;
x_0 – the unknown shift of the transformation.

The equation system (14) should be solved under the condition $\upsilon^T\mathbf{C}^{-1}\upsilon = \min$. For this target we must form the matrix **C**.

We will introduce simple signatures:

$$P_i = \left[g^l(x_i) - g^r(x_i)\right] = x_i^{'} - x_i^{''} \tag{15}$$

Basing on (3) we can build the matrix **C** using the created matrix C*x* and $[\partial p_i / [\partial x_i]$ – Jakobian's matrix. For the needs of illustration, we assume 3 neighbouring pixels with numbers 1, 2, 3 on definite lines (Fig. 2A) and errors of measurements are equal to m_x. The matrix **C** for the system (14), used in order to solve three observation equations will have the form:

$$\mathbf{C} = \begin{bmatrix} (1-\rho) & \frac{1}{2}\cdot(1-\rho-\rho_{1,2}) & \rho_{1,2} \\ \frac{1}{2}\cdot(1-\rho-\rho_{1,2}) & 1 & \frac{1}{2}\cdot(1-\rho-\rho_{1,2}) \\ \rho_{1,2} & \frac{1}{2}\cdot(1-\rho-\rho_{1,2}) & (1-\rho) \end{bmatrix} \cdot 2 \cdot m_x^2 \tag{16}$$

where: ρ – the correlation coefficient of the central pixel in the left and right windows, $\rho_{1,2}$ – the correlation coefficients between the central pixel with the left or right one in the image.

The system (14) will be solved by the iteration method, assuming the initial value of the gradient as equal $g^l(x_i)$. The gradient $g'^l(x_i)$ may be also calculated with higher accuracy by means of Robert's method:

$$g'^l = \sqrt{2}\left[g^l(x_{i+1}) - g^l(x_i)\right] \tag{17}$$

The matrix (16) can be created in a similar manner for more neighbouring pixels, as it is shown in Fig. 2B and 2C. There are two kinds of correlation coefficients in the matrix (16). One of them refers to the correlation between pixels in one image, which coefficients have been determined according to (12). The second ρ refers to the correlation between two central points of the window in the left and right images (window matching). This correlation coefficient may be determined for the general case (different scale and different orientation of two images), in the following way [6]:

$$\rho = \frac{\sum_{i=1}^{n}\sum_{j=1}^{m}\left((g^l)T(i,j)) - g_{sT}^l\right)\cdot\left(g^r(i,j) - g_s^r\right)}{\sqrt{\sum_{i=1}^{n}\sum_{j=1}^{m}\left((g^l)T(i,j)) - g_{sT}^l\right)^2 \sum_{i=1}^{n}\sum_{j=1}^{m}\left(g^r(i,j) - g_s^r\right)^2}} \tag{18}$$

where: m; n – sizes of a window; $g^l(i, j)$, $g^r(i, j)$ – grey level values in the left and right window; g_s^r – the mean grey level value of the right window; g^l_{sT} – the mean grey level value in the left transformed window; $T(i, j) = \mathbf{Q}_P[i, j, 1]^T$ – new co-ordinates of (i, j) after affine transformation.

Basing on (12) and (18) all elements of the matrix (16) will be counted; i.e. the matrix of type (16) is fully constructed. The ABM process will be explicitly solved under the condition $v^T\mathbf{C}^{-1}v = \min$.

If some geometrical constraints related to image matching are added to the proposed idea, a global concept of image matching will be obtained, as it was mentioned at the beginning.

CONCLUSIONS

It should be admitted, that the new tendency in development of digital photogrammetry is grows basing on analytical photogrammetry. The new digital input data result in new ideas and new methods of solutions. The general rule to elaborate correlative observations is continually obligatory wherever we deal with measurements. The problem of correlation between point co-ordinates in analytical photogrammetry is somehow similar to the problem of correlation between pixels in digital photogrammetry. Two ways may be applied in order to consider the problem of correlation in analytical or digital photogrammetry:

- to construct a system, which possesses instruments correcting this correlation.
- to consider this correlation in the process of data processing by means of software packages,

Solving the problem of correlation in photogrammetric observations, both in the case of analytical and digital photogrammetry, allows to increase the accuracy and probability of delineated terrain co-ordinates.

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Koncepcja rozwiązania problemu skorelowanych obserwacji fotogrametrycznych

S t r e s z c z e n i e

Jednym z podstawowych procesów w fotogrametrii jest identyfikacja wraz z wykonaniem pomiarów punktów homologicznych znajdujących się w pasie podwójnego pokrycia zdjęć. W fotogrametrii analitycznej proces ten jest rozwiązywany przez obserwatora, natomiast w fotogrametrii cyfrowej, automatycznie poprzez oprogramowanie. Proces ten jest nazywany spasowaniem obrazów (image matching). Proces spasowania ma znaczenie przy automatyzacji wykonania orientacji w aerotriangulacji bloku zdjęć. Dokładność spasowania obrazów wpływa na dokładność wyników wyznaczania elementów orientacji i wartości współrzędnych punktów.

Artykuł przedstawia koncepcję spasowania obrazów cyfrowych z uwzględnieniem korelacji między sąsiadującymi pikselami.

Луонг Чин Ке

Концепция решения проблемы коррелированных фотограмметрических наблюдений

Р е з ю м е

Одним из основных процессов в фотограмметрии является идентификация и выполнение измерений трансформированных центральных точек, находящихся в зоне двойного перекрытия снимков. В аналитической фотограмметрии этот процесс решается наблюдателем, а в цифровой фотограмметрии автоматически, через программу. Этот процесс называется совмещением изображений (image matching). Процесс совмещения имеет значение в случае автоматизации выполнения ориентирования в фототриангуляции блока снимков. Точность совмещения изображений влияет на точность результатов определения элементов ориентации и величины координат точек.

В статье представлена концепция совмещения цифровых изображений с учётом корреляции между соседними пикселями.