GEODEZJA I KARTOGRAFIA
t. XLIX, z. 3, s.,159-170 (2000)

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Interrelationships between spatial ortho-cartesian coordinates and nondimensional spatial numerical coefficients

The article is an extension of the suggestion published earlier on determination of the relative position of a point. Introduction of formulas to convert ortho-Cartesian coordinates of points to nondimensional numerical coefficients alongside reverse converting formulas is the main part of the thesis. The said formulas have been completed with demonstrative examples that verify the correctness thereof.

Each position will always associate with the position itself being referred to a certain element according to which it is being determined. The position of a point being referred to an assumed coordinate system and described by means of the measures of appropriate quantities is not always directly useful. The determined positions of points generally make up a starting basis for further proceedings; they may be for example compared. Alongside periodical comparison of points' positions, approached within a single system of reference and belonging to a single geometrical structure of the researched object, done to determine dislocations, it is possible to compare the positions of the points that belong to separate structures. Regardless of a certain specific case of such proceedings, closely connected with the designed position of points being associated with the accomplishment thereof, and leading to deviation determination direct comparison of positions of points, specified within separate systems of reference and belonging to geometrical structures differing from one another, leads merely to determination of obvious, absolute differences of coordinates and their derivatives. As such, they cannot be objects of considerable interest, as opposed to differences or ratios of measures of various homogenous quantities, pertaining to mutually corresponding points of physical objects compared with one another. Within such structures it is usually possible to point out a certain number of reference points that might be directly identified, and which, in turn, due to certain reasons — their position in particular, might be regarded as corresponding to one another. Positions of the other points may be consequently determined according to such ones. Description of the position of a point by means of nondimensional numerical coefficients makes it possible to identify the points that are characterized by an identical relative position as corresponding to one another.

In the tentative description of the relative position of a point, published in the quarterly *Geodezja i Kartografia* (No. 1-2, 1999), appropriate coefficients have been specified with particular regard to the significance of the specific property of such a position, which consists in the fact that it does not depend on actual interrelationships between points of reference. The analytical part, pertaining to the three coefficients that describe positions of points within three-dimensional space, has been restricted down to the necessary general dependences.

The purpose of this is to derive detailed formulas to convert the ortho-Cartesian coordinates of reference points and the determined point to nondimensional numerical coefficients, as well as formulas meant for reverse conversion.

The values of numerical coefficients b, c, v may be calculated with the help of coordinates of point Q (Fig. 1), which is the trace of a straight line passing through point M, parallel to edge $T\ W$ of tetrahedron of reference, upon plane of reference $L\ P\ T$. The coordinates of that point might be described by the already known formulas, which in a slightly modified form will look thus:

$$x_Q = x_M - Vx_Q : V_Q \quad ; \quad y_Q = y_M - Vy_Q : V_Q \quad ; \quad z_Q = z_M - Vz_Q : V_Q \tag{1}$$

where:

$$V_Q = A_5 l_{TW} + B_5\, m_{TW} + C_5 n_{TW} \tag{2}$$

and:

$$\left.\begin{aligned} Vx_Q &= \left(A_5 x_M + B_5 y_M + C_5 z_M + D_5\right) l_{TW} \\ Vy_Q &= \left(A_5 x_M + B_5 y_M + C_5 z_M + D_5\right) m_{TW} \\ Vz_Q &= \left(A_5 x_M + B_5 y_M + C_5 z_M + D_5\right) n_{TW} \end{aligned}\right\} \tag{3}$$

and consenquently:

$$\left.\begin{aligned} l_{TW} &= B_1 C_4 - B_4 C_1 \\ m_{TW} &= B_1 C_4 - B_4 C_1 \\ n_{TW} &= B_1 C_4 - B_4 C_1 \end{aligned}\right\} \tag{4}$$

Coefficients of general equations of three planes $P\ T\ W$, $T\ L\ W$ and $L\ P\ T$, occurring in the foregoing formulas, the planes having been marked with successive numbers 1, 4 and 5, are determined by the following dependence*:

* They may be easily derived starting from determinantal form of the said planes and successive calculation of the values thereof.

$$\left.\begin{aligned}
A_1 &= y_T(z_W - z_P) + y_P(z_T - z_W) + y_W(z_P - z_T)\\
B_1 &= z_T(x_W - x_P) + z_P(x_T - x_W) + z_W(x_P - x_T)\\
C_1 &= x_T(y_W - y_P) + x_P(y_T - y_W) + x_W(y_P - y_T)\\
D_1 &= x_T(y_P z_W - y_W z_P) + y_T(z_P x_W - z_W x_P) + z_T(x_P y_W - x_W y_P)\\
\\
A_4 &= y_L(z_W - z_T) + y_T(z_L - z_W) + y_W(z_T - z_L)\\
B_4 &= z_L(x_W - x_T) + z_T(x_L - x_W) + z_W(x_T - x_L)\\
C_4 &= x_L(y_W - y_T) + x_T(y_L - y_W) + x_W(y_T - y_L)\\
D_4 &= x_L(y_T z_W - y_W z_T) + y_L(z_T x_W - z_W x_T) + z_L(x_T y_W - x_W y_T)\\
\\
A_5 &= y_P(z_T - z_L) + y_L(z_P - z_T) + y_T(z_L - z_P)\\
B_5 &= z_P(x_T - x_L) + z_L(x_P - x_T) + z_T(x_L - x_P)\\
C_5 &= x_P(y_T - y_L) + x_L(y_P - y_T) + x_T(y_L - y_P)\\
D_5 &= x_P(y_L z_T - y_T z_L) + y_P(z_L x_T - z_T x_L) + z_P(x_L y_T - x_T y_L)
\end{aligned}\right\} \tag{5}$$

Geometrical constructions connected with determination of coefficients b, c, υ, i.e. affiliation, parallelism, mutual elements and proportionality, are the so called parallel projection invariants. This property makes it possible to simplify derivation of formulas. It also enables separate approach to components of distances which are parallel projections upon planes formed by appropriate pairs of axes (x, y), (y, z) and (z, x) of the assumed spatial system of rectangular coordinates.

Taking into consideration the earlier deliberations, we will now derive formulas of coefficients b_Q and c_Q, which describe the position of point Q upon plane $L\ P\ T$. Quotients of segment lengths will be replaced by quotients of abscissas increments at selected flat reference system (x, y) corresponding to them. Therefore

quotient

$$\frac{\overline{LQ}}{\overline{PT}} = \pm\sqrt{\frac{(x_Q - x_L)^2 + (y_Q - y_L)^2}{(x_K - x_L)^2 + (y_K - y_L)^2}} = b_Q$$

will be replaced by abscissas quotient

$$\frac{x_Q - x_L}{x_K - x_L} = b_Q \tag{6}$$

quotient

$$\frac{\overline{PK}}{\overline{PT}} = \pm\sqrt{\frac{(x_K - x_P)^2 + (y_K - y_P)^2}{(x_T - x_P)^2 + (y_T - y_P)^2}} = a_Q$$

will be replaced by abscissas quotient

$$\frac{x_K - x_P}{x_T - x_P} = a_Q \tag{7}$$

Egually well one might make use here of ordinate y increment quotient or altitudinal ordinate z quotient.

Abscissa of point K occurring in formulas (6) and (7) will be obtained after a system of equations of straight lines passing through points LQ and PT has been solved. It looks thus:

$$\left.\begin{aligned} y_K - y_L &= \frac{y_Q - y_L}{x_Q - x_L}\left(x_K - x_L\right) \\ y_K - y_P &= \frac{y_T - y_P}{x_T - x_P}\left(x_K - x_P\right) \end{aligned}\right\} \tag{8}$$

After subtracting by sides and appropriate transformations the following will be obtained:

$$\begin{aligned} x_K = {} & \frac{x_L(y_Q - y_L)(x_T - x_P) - x_P(y_T - y_P)(x_Q - x_L)}{(y_Q - y_L)(x_T - x_P) - (y_T - y_P)(x_Q - x_L)} - \\ & - \frac{(y_L - y_P)(x_T - x_P)(x_Q - x_L)}{(y_Q - y_L)(x_T - x_P) - (y_T - y_P)(x_Q - x_L)} \end{aligned} \tag{9}$$

After right side of equation (9) has been substituted for formula (6), and after transformation, the following will be obtained:

$$b_Q = \frac{(y_Q - y_L)(x_T - x_P) - (y_T - y_P)(x_Q - x_L)}{(x_L - x_P)(y_T - y_P) - (y_L - y_P)(x_T - x_P)}$$

Flat coordinates of system (y, z) and (z, x) having been used, followed by exchange of appropriate coordinates in the foregoing formula and its slight transformation, three versions of it will be obtained. As a results three alternative formulas will be at our disposal to calculate coefficient b_Q.
Besides:

$$b_Q = b_M$$

therefore:

$$\left.\begin{array}{ll}
 & b_Q = b_M = \dfrac{(x_L - x_Q)(y_T - y_P) - (y_L - y_Q)(x_T - x_P)}{(y_P - y_L)(x_T - x_L) - (x_P - x_L)(y_T - y_L)} \\
\text{for} & (y_P - y_L)(x_T - x_L) \neq (x_P - x_L)(y_T - y_L) \\
\text{or} & b_Q = b_M = \dfrac{(y_L - y_Q)(z_T - z_P) - (z_L - z_Q)(y_T - y_P)}{(z_P - z_L)(y_T - y_L) - (y_P - y_L)(z_T - z_L)} \\
\text{for} & (z_P - z_L)(y_T - y_L) \neq (y_P - y_L)(z_T - z_L) \\
\text{or} & b_Q = b_M = \dfrac{(z_L - z_Q)(x_T - x_P) - (x_L - x_Q)(z_T - z_P)}{(x_P - x_L)(z_T - z_L) - (z_P - z_L)(x_T - x_L)} \\
\text{for} & (x_P - x_L)(z_T - z_L) \neq (z_P - z_L)(x_T - x_L)
\end{array}\right\} \quad (10)$$

Next in the right side of equation (9) is substituted for formula (7), after transformations the following will be obtained:

$$a_Q = \frac{(y_Q - y_L)(x_L - x_P) - (y_L - y_P)(x_Q - x_L)}{(y_Q - y_L)(x_T - x_P) - (y_T - y_P)(x_Q - x_L)}$$

After further transformations and taking into account the possibility to exchange coordinates x, y for y, z and z, x, the following will be successively obtained:

$$\left.\begin{array}{ll}
 & a_Q = a_M = \dfrac{(y_L - y_Q)(x_P - x_L) - (y_L - y_Q)(y_P - y_L)}{(x_L - x_Q)(y_T - y_P) - (y_L - y_Q)(x_T - x_P)} \\[2ex]
\text{for} & (x_L - x_Q)(y_T - y_P) \neq (y_L - y_Q)(x_T - x_P) \\[2ex]
\text{or} & a_Q = a_M = \dfrac{(z_L - z_Q)(y_P - y_L) - (y_L - y_Q)(z_P - z_L)}{(y_L - y_Q)(z_T - z_P) - (z_L - z_Q)(y_T - y_P)} \\[2ex]
\text{for} & (y_L - y_Q)(z_T - z_P) \neq (z_L - z_Q)(y_T - y_P) \\[2ex]
\text{or} & a_Q = a_M = \dfrac{(x_L - x_Q)(z_P - z_L) - (z_L - z_Q)(x_P - x_L)}{(z_L - z_Q)(x_T - x_P) - (x_L - x_Q)(z_T - z_P)} \\[2ex]
\text{for} & (z_L - z_Q)(x_T - x_P) \neq (x_L - x_Q)(z_T - z_P)
\end{array}\right\} \tag{11}$$

The right sides of equation (10) and (11) having been multiplied by each other, three versions of formula for c_Q=c_M will be obtained, according to which:

$$\left.\begin{array}{ll}
 & c_Q = c_M = \dfrac{(y_L - y_Q)(x_P - x_L) - (x_L - x_Q)(y_P - y_L)}{(y_P - y_L)(x_T - x_L) - (x_P - x_L)(y_T - y_L)} \\[2ex]
\text{for} & (y_P - y_L)(x_T - x_L) \neq (x_P - x_L)(y_T - y_L) \\[2ex]
\text{or} & c_Q = c_M = \dfrac{(z_L - z_Q)(y_P - y_L) - (z_L - z_Q)(x_P - x_L)}{(z_P - z_L)(y_T - y_L) - (y_P - y_L)(z_T - z_L)} \\[2ex]
\text{for} & (z_P - z_L)(y_T - y_L) \neq (y_P - y_L)(z_T - z_L) \\[2ex]
\text{or} & c_Q = c_M = \dfrac{(x_L - x_Q)(z_P - z_L) - (z_L - z_Q)(x_P - x_L)}{(x_P - x_L)(z_T - z_L) - (z_P - z_L)(x_T - x_L)} \\[2ex]
\text{for} & (x_P - x_L)(z_T - z_L) \neq (z_P - z_L)(x_T - x_L)
\end{array}\right\} \tag{12}$$

Formulas for the remaining coefficient, i.e. j_M and υ_M, might be expressed also as functions of two selected coordinates, e.g. x, y.

A triple description of each coefficient provides not merely a possibility of choice, but — if the necessary supernumerary data in the form of a third coordinate are at our disposal — makes it possible to control calculations.

Quotient of lengths of segments QM and TW having been substituted by quotients of appropriate coordinates, three version of the formula to calculate coefficient v_M will be obtained.

$$v_M = \frac{x_M - x_Q}{x_W - x_T} \quad \text{or} \quad v_M = \frac{y_M - y_Q}{y_W - y_T} \quad \text{or} \quad v_M = \frac{z_M - z_Q}{z_W - z_T} \tag{13}$$

$$\text{for} \quad x_W \neq x_T \qquad \text{for} \quad y_W \neq y_T \qquad \text{for} \quad z_W \neq z_T$$

Presentation of the foregoing formulas in a form characterized by the same structure as formulas (10) and (12) is possible, it would not be advisable however, to apply them. The possibility to describe the position of a point by means of nondimensional numerical coefficients should be merely regarded as a proof of a certain coherence of the suggestion in question.

Below there is a numerical example of conversion of Cartesian coordinates, four points of reference L, P, T, W and point to be determined M, to nondimensional numerical coefficients. The necessary data have been listed below in Table 1. The drawing is a geometrical interpretation of the operation and the results thereof.

Table 1

Points	Data				
	L	*P*	*T*	*W*	*M*
x	1	3	5	5	2.5
y	1	5	3	3	2.5
z	0	2	2	6	2.0

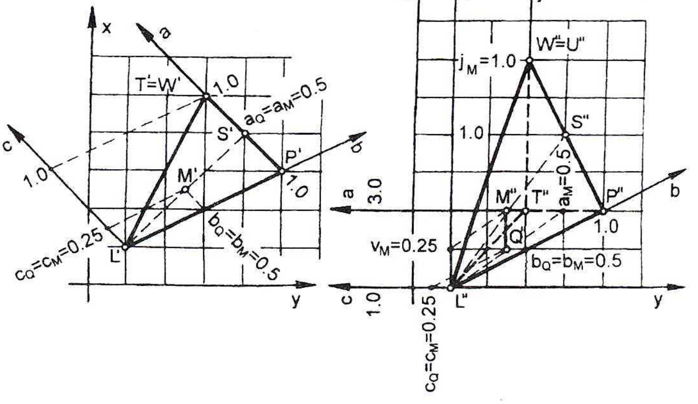

Fig. 1

The following will be successively calculated:

$A_1 = 3(4) + 5(-4) + 3(0) = -8$ $\quad$ $A_4 = 1(4) + 3(-6) + 3(2) = -8$

$B_1 = 2(2) + 2(0) + 6(-2) = -8$ $\quad$ $B_4 = 0(0) + 2(-4) + 6(4) = 16$

$C_1 = 5(-2) + 3(0) + 5(2) = 0$ $\quad$ $C_4 = 1(0) + 5(-2) + 5(2) = 0$

$D_1 = 5(24) + 3(-8) + 2(-16) = 64$ $\quad$ $D_4 = 1(12) + 1(-20) + 0(-3) = -8$

$$A_5 = 5(2) + 1(0) + 3(-2) = 4$$
$$B_5 = 2(4) + 0(-2) + 2(-2) = 4$$
$$C_5 = 3(2) + 1(2) + 5(-4) = -12$$
$$D_5 = 3(2) + 5(-2)+2(-2) = -8$$

In accord with formulas (4):

$$l_{TW} = B_1C_4 - B_4C_1 = 0$$
$$m_{TW} = A_4C_1 - A_1C_4 = 0$$
$$n_{TW} = B_4A_1 - B_1A_4 = 16\,(-8) - (-8)(-8) = -192$$

Applying formulas (2) and (3), the following may be calculated:

$$V_Q = 2304 \quad ; \quad Vx_Q = 0 \quad ; \quad Vy_Q = 0 \quad ; \quad Vz_Q = -12$$

The foregoing values having been substituted for formulas (1) the following will be obtained:

$$x_Q = 2.5 \quad ; \quad y_Q = 2.5 \quad ; \quad z_Q = 1.0$$

As the included data table implies $x_W = x_T$ and $y_W = y_T$. This means that edge $T\,W$ is parallel to axis z of the coordinate system. Due to that ordinates z will be necessary to calculate coefficient v_M. The following will be successively calculated:

$$b_Q = b_M = f(z,y) = \frac{(0-1)(3-5)-(1-2.5)(2-2)}{(5-1)(2-0)-(2-0)(3-1)} = 0.5$$

the following to be checked:

$$b_Q = b_M = f(x,y) = \frac{(1-2.5)(3-5)-(1-2.5)(5-3)}{(5-1)(5-1)-(3-1)(3-1)} = 0.5$$

next:

$$c_Q = c_M = f(z,y) = \frac{(1-2.5)(2-0)-(0-1)(5-1)}{(5-1)(2-0)-(2-0)(3-1)} = 0.25$$

the following to be checked:

$$c_Q = c_M = f(x,y) = \frac{(1-2.5)(3-1)-(1-2.5)(5-1)}{(5-1)(5-1)-(3-1)(3-1)} = 0.25$$

also:

$$\upsilon_M = \frac{z_M - z_Q}{z_W - z_T} = \frac{2-1}{4} = 0.25$$

and finally:

$$j_M = \upsilon_M : c_M = 1.0 \quad ; \quad a_M = c_M : b_M = 0.25:0.5 = 0.5$$

Determination of positions of the points whose relative position has been determined earlier consists in coefficients b, c, υ to be converted to coordinates within a system of reference points. The coordinates of those points, alongside the already mentioned values of nondimensional numerical coefficients, are the input data for such calculations.

The general form of formulas for coordinates x_M and y_M will be derived from the selected version of formulas describing coefficients b_Q and c_Q. It should be borne in mind that although the numerical values of cofficients b_Q and c_Q describing position of points according to points L, P and T are identical for points Q and M, equality of coordinates will not be accordingly thus implied whatsoever.

Thus the following may be written:

$$\left.\begin{aligned} b_Q &= \frac{(x_L - x_Q)(y_T - y_P) - (y_L - y_Q)(x_T - x_P)}{(y_P - y_L)(x_T - x_L) - (x_P - x_L)(y_T - y_L)} \\ c_Q &= \frac{(y_L - y_Q)(x_P - x_L) - (x_L - x_Q)(y_P - y_L)}{(y_P - y_L)(x_T - x_L) - (y_P - y_L)(y_T - y_L)} \end{aligned}\right\} \qquad (14)$$

The foregoing system of equations having been solved, the following will be obtained:

$$\left.\begin{aligned} x_Q &= x_L + b_Q(x_P - x_L) + c_Q(x_T - x_P) \\ y_Q &= y_L + b_Q(y_P - y_L) + c_Q(y_T - y_P) \end{aligned}\right\} \qquad (15)$$

Equations (13) having been transformed, the following will be obtained in turn:

$$\left.\begin{aligned} x_M &= \upsilon_M(x_W - x_T) + x_Q \\ y_M &= \upsilon_M(y_W - y_T) + y_Q \\ z_M &= \upsilon_M(z_W - z_T) + z_Q \end{aligned}\right\} \qquad (16)$$

After appropriate substitutions and transformations the following will be obtained:

$$x_M = x_L + b_Q(x_P - x_L) + c_Q(x_T - x_P) + \upsilon_M(x_W - x_T)$$
$$y_M = y_L + b_Q(y_P - y_L) + c_Q(y_T - y_P) + \upsilon_M(y_W - y_T)$$

and

$$z_M = z_L + b_Q(z_P - z_L) + c_Q(z_T - z_P) + \upsilon_M(z_W - z_T)$$

Since $b_Q = b_Q$ and $c_Q = c_M$, the final form of formulas to - convert nondimensional coefficients b_M, c_M υ_M to Cartesian coordinates x_M, y_M, z_M may be written as follows:

$$\left.\begin{aligned} x_M &= x_L + b_M(x_P - x_L) + c_M(x_T - x_P) + \upsilon_M(x_W - x_T) \\ y_M &= y_L + b_M(y_P - y_L) + c_M(y_T - y_P) + \upsilon_M(y_W - y_T) \\ z_M &= z_L + b_M(z_P - z_L) + c_M(z_T - z_P) + \upsilon_M(z_W - z_T) \end{aligned}\right\} \tag{17}$$

Equations (11) imply that:

$$x_Q = x_M - \upsilon_M(x_W - x_T) \quad ; \quad y_Q = y_M - \upsilon_M(y_W - y_T) \quad ; \quad z_Q = z_M - \upsilon_M(z_W - z_T) \tag{18}$$

Comparing the right sides of formulas (18) and (l) we will obtain the following:

$$\upsilon_M(x_W - x_T)V_Q = V_{X_Q} \quad ; \quad \upsilon_M(y_W - y_T)V_Q = V_{Y_Q} \quad ; \quad \upsilon_M(z_W - z_T)V_Q = V_{Z_Q} \tag{19}$$

Coefficients $V_{X_Q};V_{Y_Q};V_{Z_Q}$ having been substituted for by expressions described by formulas (3), after transformation the following will be obtained:

$$\left.\begin{aligned} & A_5x_M + B_5y_M + C_5z_M + D_5 = \frac{\upsilon_M(x_W - x_T)}{l_{TW}}V_Q \\ \text{for} \quad l_{TW} \neq 0 \qquad & \\ & A_5x_M + B_5y_M + C_5z_M + D_5 = \frac{\upsilon_M(y_W - y_T)}{m_{TW}}V_Q \\ \text{for} \quad m_{TW} \neq 0 \qquad & \\ & A_5x_M + B_5y_M + C_5z_M + D_5 = \frac{\upsilon_M(z_W - z_T)}{n_{TW}}V_Q \end{aligned}\right\} \tag{20}$$

The foregoing formulas determine the relationships between the coordinates of point M, and — as such — may be alternatively applied while controlling the correctness of appropriate calculations. The use of formulas (17) and (20) has been illustrated by an example for which numerical data have been taken from the previous example to specify them in Table 2.

Table 2

Points	Data						
	L	P	T	W	$b_Q = b_M$	$c_Q = c_M$	v_M
x	1	3	5	5			
y	1	5	3	3			
z	0	2	2	6			
					0.5	0.25	0.25

Thus the following may be calculated:

$$x_M = 1.0 + 0.5(3-1) + 0.25(5-3) + (0.25)(5-5) = 2.5$$
$$y_M = 1.0 + 0.5(5-1) + 0.25(3-5) + (0.25)(3-3) = 2.5$$
$$z_M = 0.0 + 0.5(2-0) + 0.25(2-2) + (0.25)(6-2) = 2.0$$

Correctness of calculations will be checked by means of the following dependence:

$$A_5 x_M + B_5 y_M + C_5 z_M + D_5 = \frac{v_M(z_W - z_T)}{n_{TW}} V_Q$$

Substituting data from the table and numerical values calculated earlier, i.e.:

$A_5 = 4$; $B_5 = 4$; $C_5 = -12$; $D_5 = -8$ and $n_{TW} = -192$; $V_Q = 2304$

as well as:

$$v_M = 0.25 \quad ; \quad z_W - z_T = 4$$

we may write the following:

$$4 \times 2.5 + 4 \times 2.5 - 12 \times 2 - 8 = \frac{0.25 \times 4 \times 2304}{-192}$$

thereore: $-12 = -12$

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Received April 27, 2000
Accepted September 21, 2000

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Współzależności współrzędnych ortokartezjańskich przestrzennych i bezwymiarowych przestrzennych współczynników liczbowych

S t r e s z c z e n i e

Artykuł stanowi rozszerzenie opublikowanej wcześniej propozycji, dotyczącej określenia względnego położenia punktu. Zasadniczą część pracy zajmuje wyprowadzenie wzorów do przeliczania ortokartezjańskich współrzędnych punktów na bezwymiarowe współczynniki liczbowe oraz wzorów do przeliczeń w stronę odwrotną. Wzory te uzupełniono poglądowymi przykładami weryfikującymi ich poprawność.

Лех Питонь

Зависимость ортодекартовых пространственных координат и безразмерных пространственных числовых коэффициентов

Р е з ю м е

Статья является расширением раньше опубликованного предложения, касающегося определения относительного положения точки. Основная часть работы это выведение формул для перевода ортодекартовых координат в безразмерные числовые коэффициенты, а также формул для обратных вычислений. Эти формулы дополнены наглядными примерами, проверяющими корректность этих формул.