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PAN POLSKA AKADEMIA NAUK

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AN ALTERNATIVE APPROACH FOR DISSEMINATION OF MASS UNIT FOLLOWING THE NEW DEFINITION OF THE KILOGRAM

Adriana Vâlcu

Romanian Measurement Society – RMS, Unirii Bv. no. 61, 030828, Bucharest, Romania (formerly National Institute of Metrology, Romania), (✉ *adivaro@yahoo.com,* +40 723 822 621)

Abstract

The paper proposes an alternative approach to the dissemination of the mass unit in the context of the new definition of the kilogram. Considering the fact that redefinition allows mass to be directly realized at any value, the paper presents a model of the dissemination of mass which can be used for different series in grams, where the measurements are performed in the downward direction, but using 1 g as the reference standard (whose mass value is assumed to be determined after the redefinition using the capacitive or electrostatic techniques). The subdivision method presented (suitable for E_1 weights) has as its starting point the approach used by Mihailov-Romanowski for the calibration of series in kilograms which uses an orthogonal system of equations. Thus, according to this method, a solution for obtaining the orthogonality of a system can be the use as defining standard of the ratio between the mass having the highest nominal value in the set and the standard (unit). The results obtained for a set of weights from 10 to 1 g using the subdivision method, in accordance with the Mihailov-Romanowski principle, are validated with those obtained with the multiplication method, where the measurements start from 1 to 10 g, as in the case of the kilogram series. The mass values obtained with both methods are equal, while the estimated uncertainties are slightly different, yet insignificant. The results obtained previously for the same sequence of weights using the traditional dissemination method, where the 1 kg standard is used as reference, are also presented in the paper. The results show that only three weights out of six have a mass value insignificantly different by 1×10^{-4} mg compared to those obtained with the methods presented in this article, but, in terms of uncertainty, there are some differences.

The way of disseminating the mass unit presented in this article can be extended to other different sequences of nominal values such as: (5 . . . 1) g, (20 . . . 1) g, (50 . . . 1) g or (500 . . . 100) g if the reference standard is 100 g.

Keywords: new definition of the kilogram, efficiency of the weighing design, subdivision method, multiplication method, mass dissemination.

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1. Introduction

Until November 2018, the kilogram was the single *SI* base unit based on a material artefact. Starting from May 2019, the world said "au revoir" to "*Le Grand K*" which had defined the kilogram for more than one hundred years.

Even if continuing to define the international unit of mass using an object made in the 19th century was considered "scandalous" [1], we cannot fail to appreciate the fact that both the definition and the standard "succeeded together": they were able to withstand the extraordinary evolution which took place in science and technology during the 20th century and the beginning of the 21st century.

The new definition of the kilogram is of particular importance because, the last definition referring to an artefact (the main advantage of the previous definition was its simplicity thus being easily understandable to everyone) was eliminated.

The new definitions of the four SI base units (the kilogram, the ampere, the kelvin and the mole) came into force as of May 2019 and, starting from this date, all SI units have been defined in terms of constants. Thus, the fixed numerical values of the Planck constant (h), the elementary charge (e), the Boltzmann constant (k), and the Avogadro constant (NA) are basis for these new definitions [2].

Currently, there are two main ways which have been taken into consideration to realize the *SI* unit of mass [3]: one of them uses the Kibble balance (the kilogram being defined through the fixed value of Plank constant) and the second one is the *X-ray crystal density* (XRCD) method (where the kilogram is defined through the fixed numerical value of the Avogadro number).

The redefinition has allowed mass to be directly realized at any value [4, 5] and at any location using a suitably scaled instrument. This may be an advantage for masses smaller than 1 kg [6]. Thus, with Kibble balances, the mass of weights having different nominal values can be determined ($e.g.$, 50 g to 1 kg) while using other techniques (capacitive, electrostatic) small mass standards ($e.g.$, 10 mg to 1 g) can be determined [7].

The subdivision method presented in this article (suitable especially for E_1 weights) has as its starting point the approach used by Mihailov and Romanowski [8] for the calibration of series in kilograms which employs an orthogonal system of equations. Thus, according to this method, a solution for obtaining the orthogonality of a system can be the use of the ratio between the mass having the highest nominal value in the set and the standard (unit) as the defining standard.

In this way, the defining standard is attributed to two different masses: (10) which is the highest mass in the set and (1), the lowest one. In this paper this principle is extrapolated to a group of weights having nominal values from 10 g to 1 g. The steps used in this method are presented in Section 2.2. With the same group of weights and the same equations involved in the calculations, a parallel is made between the results obtained by both methods: subdivision, where measurements start from 10 g to 1 g, and multiplication (presented in Section 2.3), where measurements start from 1 g to 10 g (as in the case of kilogram series).

One can conclude that the same system of equations prepared for the subdivision method may as well be used for the multiplication method, but the advantage owed to orthogonality is lost as covariances are no longer all equal to zero.

The paper also presents the previous results obtained for the same sequence of weights using the traditional dissemination method (briefly described in Section 2.4) where the 1 kg standard is used as reference.

This paper also offers an opportunity to choose a new appropriate procedure for the realization of the mass scale because, even now, the 1 kg mass standard is still used as starting point in the dissemination of the mass unit. The way of disseminating the mass unit presented in this article can

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be extrapolated to other sequences of nominal values, such as: (5 . . . 1) g, (20 . . . 1) g, (50 . . . 1) g, *etc.* If a laboratory decides to have a weight of 100 g as the reference mass, the method can be used as well, for the sequence of (500 . . . 100) g.

2. The mass and uncertainty determination

2.1. The optimization of the design matrix by evaluating the efficiency of the weighing design

The possible mass comparisons for the interval 10 to 1 are presented in Table 1. Starting from this Table, an efficient design matrix can be chosen so that the variances of the unknowns are as small as possible.

Table 1. Possible mass comparisons between (10 . . . 1) g.

Det. No.	**Weights**					
	10	5	2	2*	1	1*
1	–1	1	1	1	1	0
2	–1	1	1	1	0	1
3	0	1	–1	–1	–1	0
4	0	1	–1	–1	0	–1
5	0	0	1	–1	1	–1
6	0	0	1	–1	–1	1
7	0	0	1	–1	0	0
8	0	0	1	0	–1	–1
9	0	0	0	1	–1	–1
10	0	0	0	0	1	–1

Efficiency is especially useful when comparing the designs involving the same masses and balances, even if the number of mass comparisons is different. It is desirable for the efficiency of a design to be large, as this will indicate that the variances are small [9]. For an efficient scheme, it is advantageous that each mass is used approximately the same number of times and as often as possible. In the scheme, the standard weight is shown in grey, whereas the "test" weight(s) is shown in black.

To establish the design matrix X of the comparisons, several versions were performed and the efficiency of the design for each of them was calculated. As shown in Table 2, using 12 equations (combinations of weights in accordance with Table 1), for the design $(1, 1, 1, 1, 2, 2, 0, 2, 2, 0)$ an efficiency of 1.20 was obtained, while for the design $(2, 1, 0, 1, 2, 0, 2, 1, 2, 1)$ the efficiency obtained was 0.85.

Table 2. Efficiency for different weighing design and number of times the weights are used in the design matrix.

Model of matrix design	**Number of times the weights are used**					**Efficiency**
	5	**2**	**2***	**1**	**1***	
12 equations: (1, 1, 1, 1, 2, 2, 0, 2, 2, 0)	4	10	10	10	10	1.20
12 equations: (2, 1, 0, 1, 2, 0, 2, 1, 2, 1)	4	9	10	8	8	0.85
13 equations: (2, 1, 0, 1, 0, 2, 2, 2, 1, 2)	4	10	9	8	8	0.83

Using 13 equations, for the design $(2, 1, 0, 1, 0, 2, 2, 2, 1, 2)$ an efficiency of about 0.83 was obtained.

In Table 2 the efficiency for different weighing designs is presented and the number of times the weights are used in the design matrix.

Finally, the design $(1, 1, 1, 1, 2, 2, 0, 2, 2, 0)$ was chosen for matrix X, having 12 equations of condition. The content of the brackets indicates that the first comparison of Table 1 appears once, the fifth appears twice, *etc.* From the Table 1 one can observe that this model is chosen as the value for the efficiency is greater, namely 1.20.

Also, it can be seen that for this matrix design, each mass is used the same number of times or as often as possible. In Table 3 the design matrix derived accordingly is presented.

Table 3. The chosen weighing design.

$X =$

Weights				
5	**2**	**2***	**1**	**1***
1	1	1	1	0
1	1	1	0	1
1	–1	–1	–1	0
1	–1	–1	0	–1
0	1	–1	1	–1
0	1	–1	1	–1
0	1	–1	–1	1
0	1	–1	–1	1
0	1	0	–1	–1
0	1	0	–1	–1
0	0	1	—1	–1
0	0	1	—1	–1

The efficiency for this design was determined in the following manner: once all weighings are completed, the first step is to form design matrix, X, which contains information on the equations used (the weighing design). The vector containing the standard deviation of each comparison (in µg) is represented by s and the vector of measured values y_i is represented by Y. The elements of β are the values of the unknown departures.

For calculating the efficiency, only standard deviation of each comparison is used.

$$Y = \begin{pmatrix} y_1 + 10r \\ y_2 + 10r \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \end{pmatrix}, \qquad s = \begin{pmatrix} s_1 = 2.00 \\ s_2 = 0.57 \\ s_3 = 1.50 \\ s_4 = 1.20 \\ s_5 = 0.38 \\ s_6 = 0.38 \\ s_7 = 0.12 \\ s_8 = 0.12 \\ s_9 = 0.25 \\ s_{10} = 0.25 \\ s_{11} = 0.13 \\ s_{12} = 0.13 \end{pmatrix}, \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \end{pmatrix}. \tag{1}$$

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Standard deviations of comparisons, represented by s, are introduced as diagonal elements in the variance-covariance matrix, denoted by G:

$$G = \text{diag}\left(s_1^2,\ s_2^2\ \ldots\ s_n^2\right). \tag{2}$$

Having the design matrix X, the matrix K can be defined as:

$$K = G^{-1/2} \cdot X\,. \tag{3}$$

Calculating K^{T}, which is the transpose of K, one can determine the inverse $(K^{\mathrm{T}} \cdot K)^{-1}$:

$$\left(K^{\mathrm{T}} \cdot K\right)^{-1} = \begin{pmatrix} 0.296 & -0.057 & -0.059 & -0.028 & -0.030 \\ -0.057 & 0.053 & 0.045 & 0.026 & 0.019 \\ -0.059 & 0.045 & 0.049 & 0.022 & 0.025 \\ -0.028 & 0.026 & 0.022 & 0.017 & 0.009 \\ -0.030 & 0.019 & 0.025 & 0.009 & 0.017 \end{pmatrix}. \tag{4}$$

If v_i are the diagonal elements of $(K^{\mathrm{T}} \cdot K)^{-1}$ corresponding to the i^{-th} mass, s_m is the largest of the s_i, then the efficiency of the design, represented by the matrix X is defined as [9]:

$$E = \sum v_i^{-1} \cdot h_i^2 \cdot \frac{s_m^2}{(n-1)}\,, \tag{5}$$

where n is the number of comparisons, and h is a k dimensional vector, the i-th element of which is the ratio between the nominal mass of the i-th unknown weight and the reference.

Table 4 presents the calculation of the efficiency for design $(1, 1, 1, 1, 2, 2, 0, 2, 2, 0)$ containing 12 equations of condition.

Table 4. Calculation (in µg) of the efficiency for design $(1, 1, 1, 1, 2, 2, 0, 2, 2, 0)$.

Nominal value	**5**	**2**	**2***	**1**	**1***	
$1/v_i$	3.38	18.81	20.26	60.51	57.80	
h	0.5	0.2	0.2	0.1	0.1	
$h_i^2 \cdot 1/v_i$	0.85	0.75	0.81	0.60	0.58	***E*=1.20**
s_m^2	4.0					
$(h_i^2 \cdot 1/v_i) \cdot s_m^2/(n-1)$	0.28	0.25	0.27	0.20	0.19	
Variance of the weights $V(b) = hh^t u_r^2 + v_i$	1.30	0.21	0.21	0.06	0.06	

In the formula of the variance, the first term on the right side represents the contribution to $V(b)$ of each weight due to the variance of the reference and the second term is one resulting from variances of the mass comparisons, from matrix $(K^{\mathrm{T}} \cdot K)^{-1}$.

2.2. The calibration method for (10. . . 1) g using the Mihailov-Romanowski principle

A selection rule for a better weighing design (in addition to that presented in Section 2.1) should be that the measurements matrix (design matrix) is orthogonal.

Orthogonality is considered to be one of the most important properties for the design of an experiment [10] and, by introducing orthogonality in the mass calibration techniques, the type A uncertainty of the results can be minimized [11].

Even if, in the calibration of weights, the type A uncertainty is a component smaller than others (which are part of the standard uncertainty), it is preferable for its value to be as small as possible, to ensure that the design matrix model is the most efficient.

In the mass measurements, when searching for a better design, the objective is to obtain a minimum value of either the variance, if the weights are used independently from each other, or the covariance, if the weights are used in combination. This can be accomplished with an increased number of weighings by adding and repeating elementary measurements [12].

In Table 3 the design matrix derived for set 10, 5, 2, 2*, 1, 1* is presented. Some equations from Table 1 are omitted while others are used more than once to obtain an orthogonal design. Thus, starting with measurement y_5, all the following measurements are repeated two times.

In this calibration, it is assumed that the mass of weight (10) defines a "temporary unit of mass", $(10) = M = 10r$ where r is equal by definition to one tenth of the highest mass, (10). Thus, all other masses are now expressed as functions of r, using the classical procedure of subdivision.

In the second step, it will be necessary to know r in terms of the reference standard, represented by weight (1):

$$10(1) = 10r + N1\,, \tag{6}$$

$N1$ represents the measurements where the reference standard (1) is used, in accordance to Table 3.

$$N1 = y_1 - y_3 + y_5 + y_6 - y_7 - y_8 - y_9 - y_{10} - y_{11} - y_{12}\,. \tag{7}$$

Having $N1$ calculated, the mass $(10) = 10r$ can be easily determined from (6), because the value of (1) g reference standard is known from the previous calibration.

The value of (10) g (introduced in the vector of Y) is used to start the calculation of the other weights. Using the least squares method, the unknown values are calculated:

$$\langle\beta\rangle = \left(X^{\mathrm{T}} \cdot X\right)^{-1} \cdot X^{\mathrm{T}} \cdot Y. \tag{8}$$

The signification of each term from (8) is presented in Section 2.1.

$(X^{\mathrm{T}} \cdot X)^{-1}$ is termed the inverse of $(X^{\mathrm{T}} \cdot X)$ and has the following form:

$$\left(X^{\mathrm{T}} \cdot X\right)^{-1} = \begin{pmatrix} 0.25 & 0 & 0 & 0 & 0 \\ 0 & 0.10 & 0 & 0 & 0 \\ 0 & 0 & 0.10 & 0 & 0 \\ 0 & 0 & 0 & 0.10 & 0 \\ 0 & 0 & 0 & 0 & 0.10 \end{pmatrix}. \tag{9}$$

Determination of variances

From (6) it follows that:

$$\operatorname{var} r = \operatorname{var}(1) + \left(\frac{1}{10}\right)^2 \cdot \operatorname{var} N1, \tag{10}$$

where: $\operatorname{var} N1 = 10\ \mathrm{s}^2$; s is the standard deviation of the observations calculated according to (14) and $var\,(1)$ is the variance of the reference standard.

Having the variance of r, we can calculate the remaining variances as [12, 13]:

$$\operatorname{var}(\beta_j) = c_{jj} \cdot s^2 + h_j^2 \cdot \operatorname{var} r, \tag{11}$$

where: c_{jj} are the diagonal elements of the matrix $(X^{\mathrm{T}} \cdot X)^{-1}$ and the factor $h_j = m_j/m_r$, has the same signification as in (5).

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Note that (11) contains two terms. One includes elements of the diagonal matrix which is the benefit of having chosen an orthogonal design. The other term represents the price we have paid for using the largest weight as a temporary standard.

2.3. The calibration of (10. . . 1) g using the multiplication method

To ensure the validity of results, according to Section 7.7.1, Paragraph *f* of [14], a calibration using a different method (by multiplication) was performed.

Usually, the multiplication method is used for the calibration of kilogram series and consists in comparing the weights starting from the lowest nominal value to the highest one.

In this paper, the method used for kilogram series is extrapolated for (1 . . . 10) g using 1 g as the reference standard. In addition, this method is used to validate the results obtained in Section 2.2, by comparing the mass values and uncertainties resulted for both methods.

The system of equations contains the same measurements combinations of weights as shown in Section 2.2. Table 5 contains the weighing design for the multiplication method.

Table 5. The weighing design for (1 . . . 10) g using the multiplication method.

$X =$

Weights					
1	**1***	**2***	**2**	**5**	**10**
–1	–1	1	0	0	0
–1	–1	1	0	0	0
–1	–1	0	1	0	0
–1	–1	0	1	0	0
1	–1	–1	1	0	0
1	–1	–1	1	0	0
–1	1	–1	1	0	0
–1	1	–1	1	0	0
–1	0	–1	–1	1	0
0	–1	–1	–1	1	0
1	0	1	1	1	–1
0	1	1	1	1	–1

As shown in Section 2.1, the vector of measured values y_i is represented by Y and the elements of β are the values of the unknown departures resulting from the least squares calculation.

$$Y = \begin{pmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \\ y_5 \\ y_6 \\ y_7 \\ y_8 \\ y_9 \\ y_{10} \\ y_{11} \\ y_{12} \end{pmatrix}, \qquad \beta = \begin{pmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ \beta_4 \\ \beta_5 \\ \beta_6 \end{pmatrix}. \tag{12}$$

The least squares solutions are calculated as shown in (8). The inverse of $(X^{\mathrm{T}} \cdot X)$ has the following form:

$$\left(X^{\mathrm{T}} \cdot X\right)^{-1} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0.2 & 0.2 & 0.2 & 0.5 & 1.0 & 1 \\ 0 & 0.2 & 0.5 & 0.4 & 1.0 & 2.0 & 2 \\ 0 & 0.2 & 0.4 & 0.5 & 1.0 & 2.0 & 2 \\ 0 & 0.5 & 1.0 & 1.0 & 2.7 & 5.0 & 5 \\ 0 & 10.0 & 2.0 & 2.0 & 5.0 & 10.0 & 10 \\ 1 & 1 & 2 & 2 & 5 & 10 & 0 \end{pmatrix}. \tag{13}$$

Determination of variances

The standard deviation s of the observations is calculated by:

$$s = \sqrt{\frac{1}{\nu} \sum_{i=1}^{n} \mathrm{res}_i^2}\,, \tag{14}$$

where:

– the residual *res* are the elements of vector $\langle e \rangle$:

$$\langle e \rangle = \mathrm{res}_i = Y - \langle Y \rangle\,, \tag{15}$$

$\langle Y \rangle$ is the adjusted mass difference of the weighing equations with:

$$\langle Y \rangle = X \cdot \langle \beta \rangle\,, \tag{16}$$

and

– $\nu = n - k$ represents the degrees of freedom (with n being the number of performed observations and k the number of unknown).

From matrix (13) it can be seen that, for this method, the advantage due to the orthogonality is lost because the covariances are no longer all equal to zero.

The matrix representing variance – covariance for $\langle \beta \rangle$ is given by:

$$V_\beta = s^2 \left(X^{\mathrm{T}} \cdot X\right)^{-1}. \tag{17}$$

For a particular unknown weight, the complete variance is given by:

$$\mathrm{var}(\beta_j) = c_{jj} \cdot s^2 + h_j^2 \cdot \mathrm{var}_{\mathrm{ref}}\,, \tag{18}$$

where: c_{ij} are the the diagonal elements of V_β, $\mathrm{var}_{\mathrm{ref}}$ is the variance associated to the reference standard and $h_j = m_j/m_r$, has the same signification as in (11). The off-diagonal elements of matrix V_β give the covariance between the weights.

2.4. The calibration of weights using the traditional dissemination of mass, starting from 1 kg

Even now the 1 kg mass standard is still used as the starting point in dissemination of the mass unit. Thus, in designing the scheme all the masses from 1 kg to 1 g are broken down into decades. The first decade, containing 12 equations, includes the 1 kg standard. For subsequent decades,

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the role of the standard is taken by the 1 from the previous decade; thus, the weights having nominal values of 100 g and 10 g masses become intermediate standards, whose uncertainty is propagated directly to masses in the decade they head and hence to those in subsequent decades (each of them containing 12 equations) [15].

Therefore, starting from 1 kg, to calibrate the sequence of (10 . . . 1) g, the whole set of weights from (500 . . . 1) g should be calibrated. The calibration method as well the calculation of mass and uncertainty of the weights are described in detail in many works, such as [16, 17].

A comparison between the results obtained using this traditional dissemination of mass and those obtained according to Sections 2.2 and 2.3 are presented in Section 3.

3. Numerical example: results

The observed mass differences Y in mg (including buoyancy corrections) and the vectors $\langle\beta\rangle$ with the unknown masses, in mg, calculated according to (8), for both methods (presented in Sections 2.2 and 2.3) are given below.

Subdivision–Orthogonal method

$$Y = \begin{pmatrix} y_1 = 0.0214 \\ y_2 = 0.0214 \\ y_3 = -0.0152 \\ y_4 = -0.0135 \\ y_5 = -0.0006 \\ y_6 = -0.0006 \\ y_7 = -0.0052 \\ y_8 = -0.0052 \\ y_9 = -0.0159 \\ y_{10} = -0.0159 \\ y_{11} = -0.0132 \\ y_{12} = -0.0132 \end{pmatrix} \quad \langle\beta\rangle = \begin{pmatrix} 5\text{ g} = 0.0035 \\ 2\text{ g} = 0.0028 \\ 2\text{ g}* = 0.0057 \\ 1\text{ g}_{\text{ref}} = 0.0104 \\ 1\text{ g} = 0.0084 \end{pmatrix},$$

Multiplication method

$$Y = \begin{pmatrix} y_1 = -0.0159 \\ y_2 = -0.0159 \\ y_3 = -0.0132 \\ y_4 = -0.0132 \\ y_5 = -0.0006 \\ y_6 = -0.0006 \\ y_7 = -0.0052 \\ y_8 = -0.0052 \\ y_9 = -0.0152 \\ y_{10} = -0.0135 \\ y_{11} = 0.0044 \\ y_{12} = 0.0044 \end{pmatrix} \quad \langle\beta\rangle = \begin{pmatrix} 10\text{ g} = 0.0170 \\ 5\text{ g} = 0.0035 \\ 2\text{ g} = 0.0028 \\ 2\text{ g}* = 0.0057 \\ 1\text{ g}_{\text{ref}} = 0.0104 \\ 1\text{ g} = 0.0084 \end{pmatrix}.$$

For the subdivision-orthogonal method, the mass of 10 g weight, calculated according to (6), has the value of 10 g + 0.0170 mg.

A centralization of the results containing mass values and standard uncertainty obtained for the methods described in Sections 2.2 and 2.3 is presented in Table 6. The results obtained using the traditional method of dissemination, *i.e.* according to Section 2.4, are also included in this table.

In order to compare the performance of the different methods, the uncertainty reported in Table 6 was evaluated with the next contributions: reference weight, weighing process and resolution of the mass comparator (this component was added to (11) and (18)). For the determinations containing the weights of 10 g and 5 g, a mass comparator with a resolution of 1 µg was used, while for the other measurements, a mass comparator with a resolution of 0.1 µg was used. Uncertainty contribution associated to buoyancy correction was negligible.

Analysing the results from the Table 6, one can see that the mass values obtained using the subdivision-orthogonal method are equal to those obtained from the multiplication method, having only some insignificant differences in uncertainty estimation. Also, the results obtained with the traditional method show that only three weights out of six have an mass value insignificantly different by 1×10^{-4} mg when compared to those obtained with the methods described in

Table 6. Mass and standard uncertainty obtained for each method.

Weight	**Subdivision–Orthogonal method**		**Multiplication method**		**Traditional method starting from 1 kg**	
Nominal value	β (mg)	u (k = 1) (mg)	β (mg)	u (k=1) (mg)	β (mg)	u (k = 1) (mg)
10 g	0.0170	0.0027	0.0170	0.0030	0.0170	0.0010
5 g	0.0035	0.0014	0.0035	0.0015	0.0034	0.0005
2 g	0.0028	0.0006	0.0028	0.0006	0.0028	0.0003
2* g	0.0057	0.0006	0.0057	0.0006	0.0056	0.0003
1g $_{ref}$	**0.0104**	**0.0002**	**0.0104**	**0.0002**	0.0104	0.0002
1* g	0.0084	0.0003	0.0084	0.0004	0.0083	0.0002

Sections 2.2 and 2.3, but, in terms of uncertainty, there are some differences, especially at 10 g and 5 g: 0.0017 mg and 0.0009 mg respectively.

Figure 1 shows the results obtained for each calibration weight (mass and standard uncertainty).

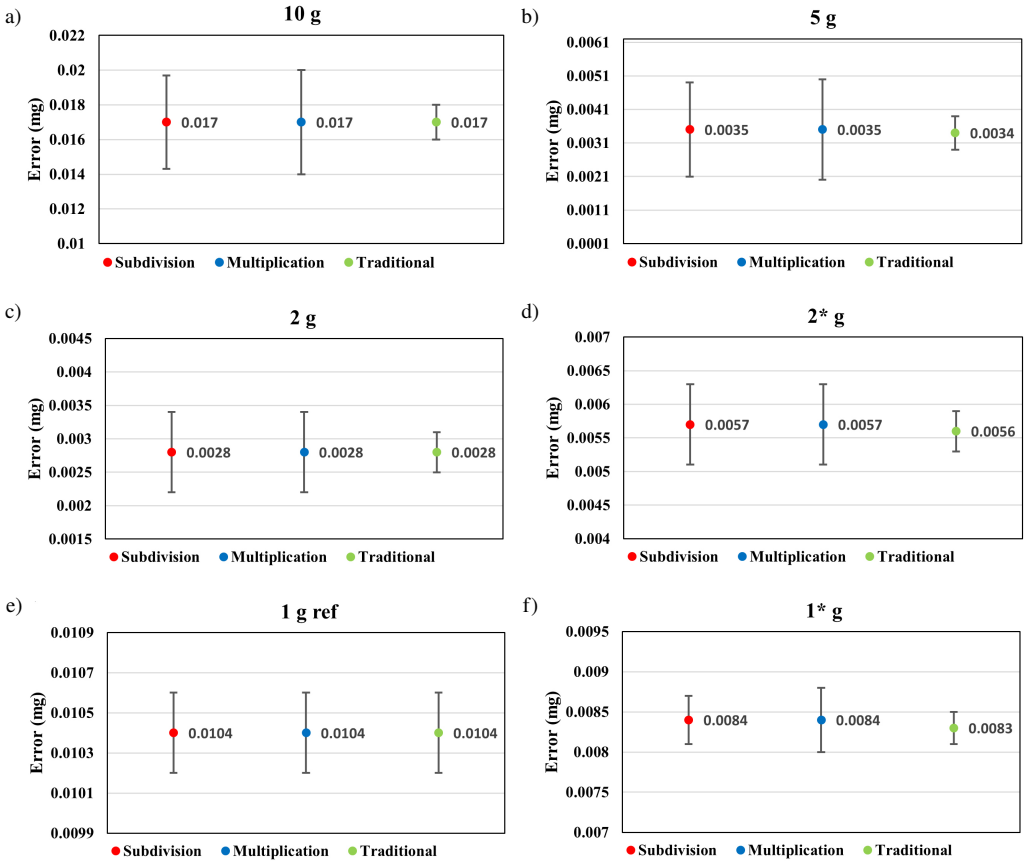

Fig. 1. Mass and standard uncertainty for: a) 10 g, b) 5 g, c) 2 g, d) 2* g, e) 1 g ref and f) 1* g.

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In Figs. 2 and 3, the correlation graph between the results (mass and uncertainty) from the subdivision, multiplication and traditional methods is presented. Both the graphs and the value of the correlation coefficient, r (with $r = 0.995 \ldots 1$) indicate that there is a perfect (or strong) positive linear relationship between the results.

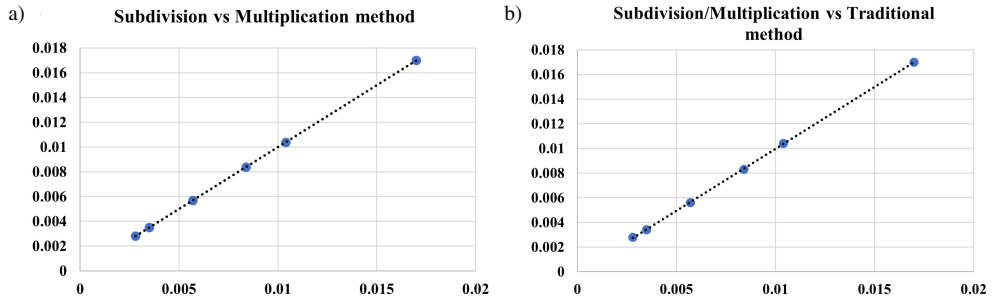

Fig. 2. The correlation graph between the mass results from: (a) subdivision vs multiplication and (b) subdivision/multiplication vs traditional method, ($r = 1$).

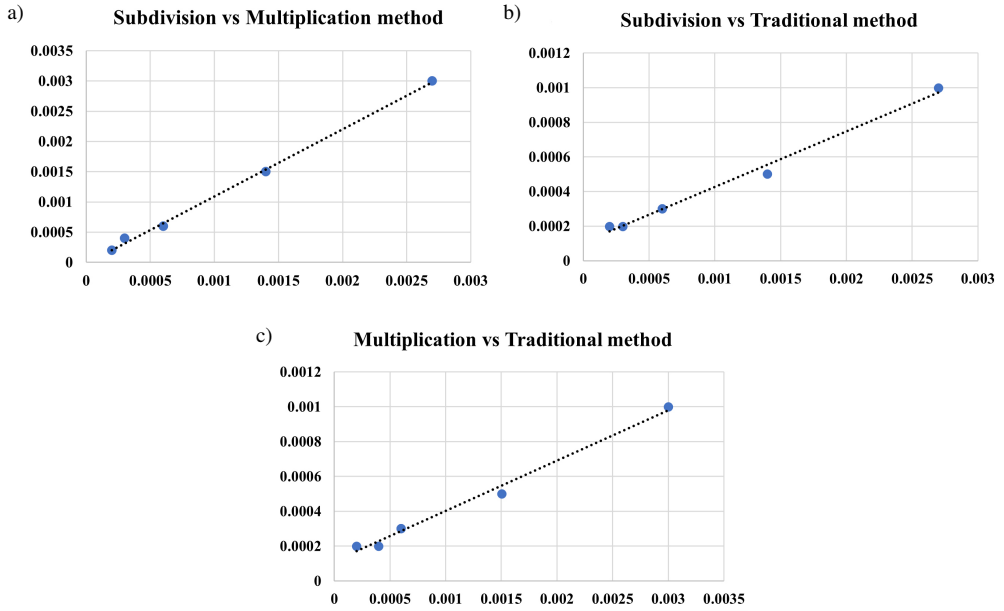

Fig. 3. The correlation graphs between the uncertainty results from: (a) subdivision vs multiplication, (b) subdivision vs traditional and (c) multiplication vs traditional, ($r = 0.995 \ldots 0,999$).

4. Conclusions

The paper proposes an alternative approach to the dissemination of the mass unit following the new definition of the kilogram.

Considering that the redefinition allows the mass to be directly realized at any value and at any location using an appropriately scaled instrument, a model of the mass dissemination for the series

in grams (especially suitable for class E_1 weights) is presented in the paper using an adaptive subdivision method, having 1 g as the reference standard (whose mass value is assumed to be determined after the redefinition). To ensure the validity of results, according to [14], a calibration using a different method (by multiplication) was performed.

A comparison between the results obtained for the three methods: subdivision, multiplication and traditional dissemination of mass are presented in Table 6.

By analysing the results, one can see that the mass values obtained using the subdivision-orthogonal method, are equal to those obtained from the multiplication method, having only some insignificant differences in the estimation of uncertainty.

Comparing these mass values and uncertainties with those achieved in traditional dissemination from the "old kilogram", only three weights out of six have an insignificant different mass value by 1×10^{-4} mg, but, in terms of uncertainty, there are some differences, especially at 10 g and 5 g: 1.7×10^{-3} mg and 9×10^{-4} mg respectively.

The results are also represented in the correlation graphs. Both the graphs and the value of the correlation coefficient, r (with $r = 0.995 \ldots 1$) indicate that there is a perfect or strong positive linear relationship between the results.

From (11) and (18), one can see that an improvement in the standard deviation of the weighing process could lead to a decrease in the value of the uncertainty for the weights. This could be achieved by increasing the number of weighings, especially for measurements where the weights of 10 g and 5 g are involved. If possible, another way to reduce the standard uncertainty of the weights could be performing at least the sequence of measurements from 5 g to 2 g on the same mass comparator, having the resolution of 0.1 µg.

Even if some differences were obtained in terms of uncertainty in the calibration certificate the results should be reported according to Chapter 5.2. of [18], according to which, the expanded uncertainty, U, for k = 2, of the conventional mass, shall be less than or equal to one-third of the maximum permissible error. Thus, from this point of view, one can consider that this condition was met for all the methods.

The main advantage of this adaptive subdivision method is the fact that the number of measurements is substantially reduced (when compared to the traditional method) because only 12 equations corresponding to the sequence of (10 . . . 1) g are used, whereas the previous 24 equations are no longer performed.

The way of disseminating the mass unit presented in this article can be extended to other different sequences of nominal values such as: (5 . . . 1) g, (20 . . . 1) g, (50 . . . 1) g, or (500 . . . 100) g if the reference standard is 100 g.

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Adriana Vâlcu received the Ph.D. degree from Politehnica University of Bucharest, Romania in 2010. Formerly she worked at the National Institute of Metrology and currently she is a scientific researcher at the Romanian Measurement Society and a technical assessor at the RENAR (Romanian Accreditation Institution). She has authored 1 book, 2 book chapters, over 50 published scientific papers (in various publications and conferences) and more than 25 unpublished works. Her research activity focuses on the development and implementation of new calibration methods in the mass field (for both weights and weighing instruments).