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Mathematics and Art

At first glance mathematics and art might appear very distant, perhaps even directly opposed to one another, but the fact is that they have quite a lot in common. How are they interlinked, and what do these links tell us?

ANNA SZWAJA

Prof. Robert A. Wolak works at the Institute of Mathematics at the Jagiellonian University and is President of the Friends of the Academy of Fine Arts in Kraków. Between 2014 and 2016 he was a member of the board of the Polish Mathematical Society. He studies topological and geometric aspects of foliation theory.
robert.wolak@uj.edu.pl

Robert A. Wolak

Institute of Mathematics
Jagiellonian University in Kraków

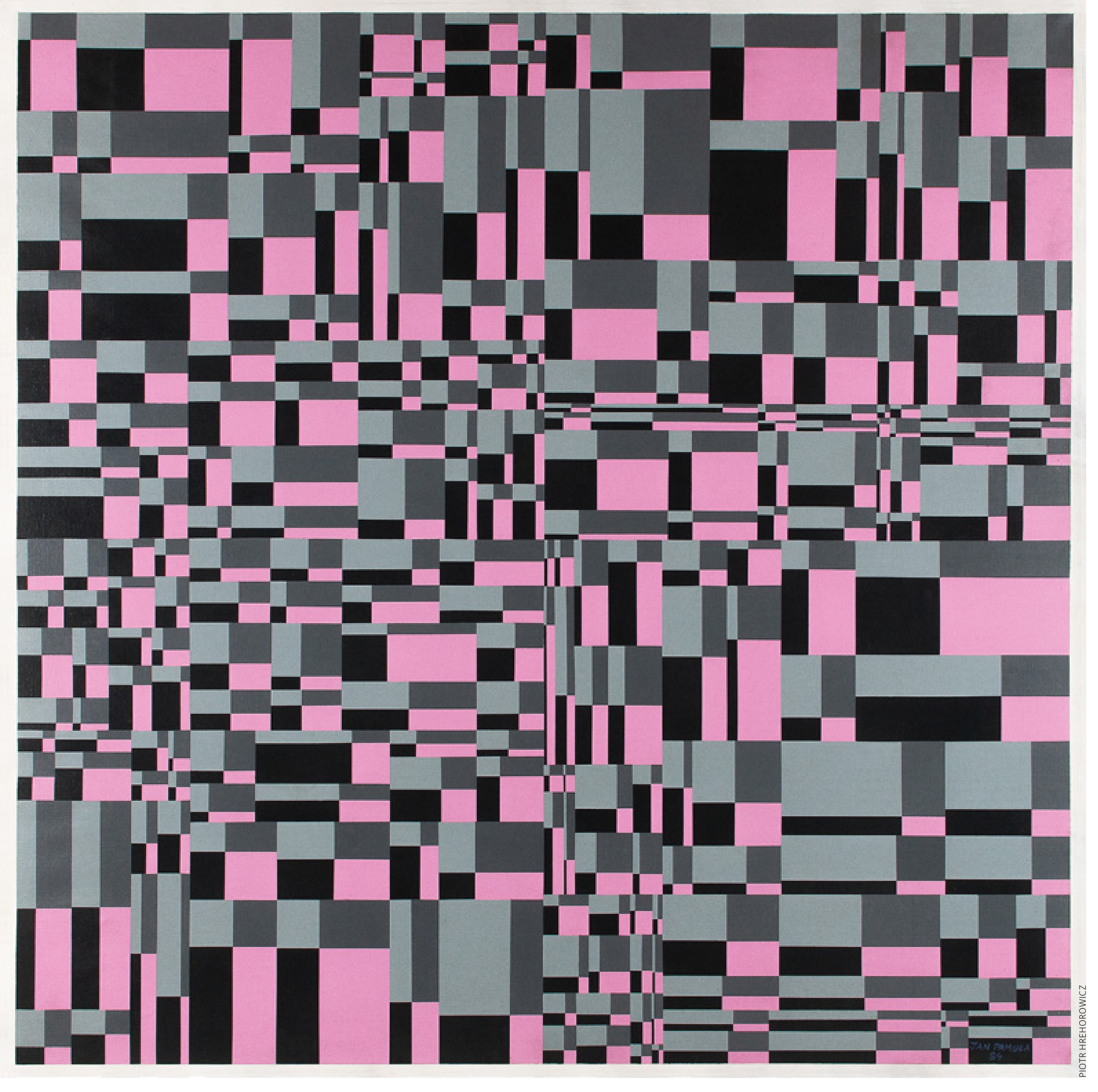
To understand the relationship between mathematics and art, we must first understand what each of them are individually. What we learn at school gives us little idea of the history of mathematics, and even less of what kind of science it is today.

Mathematics through the ages

When we look back at ancient Egypt and Babylon, mathematics served as a tool for describing the world and created tools and procedures for answering questions faced by humankind. Mathematicians operated with numbers and specific geometric figures of given dimensions. The Ancient Greeks developed a different, more abstract approach to the same science, based on philosophy and studying ideal rather than real objects. An important role was played by Pythagoras and the mystical and mathematical school he founded in Croton in 529 B.C.E. In his dialogue *Timaeus*, Plato wrote that God created the world following an "idea" ("form") and "numbers." It is said that above the entrance to the Platonic Academy there was inscribed the phrase "Let None But Geometers Enter Here." Geometry and mathematics were regarded as the foundation of all science, tools for describing the world and paving the way to real knowledge, and understanding the Creator and the true world – one we only experience in the shadows. This abstract definition of science is perhaps best encapsulated by Euclid's *Elements*. Two- and three-dimensional geometry as depicted in *Elements* was seen as an unsurpassed paragon of scientific theorizing, and not just in mathematics. Myriad scholars and thinkers have attempted to use this perfectly logical method, assuming certain primal concepts and axioms, to develop theories in many fields, including theology.

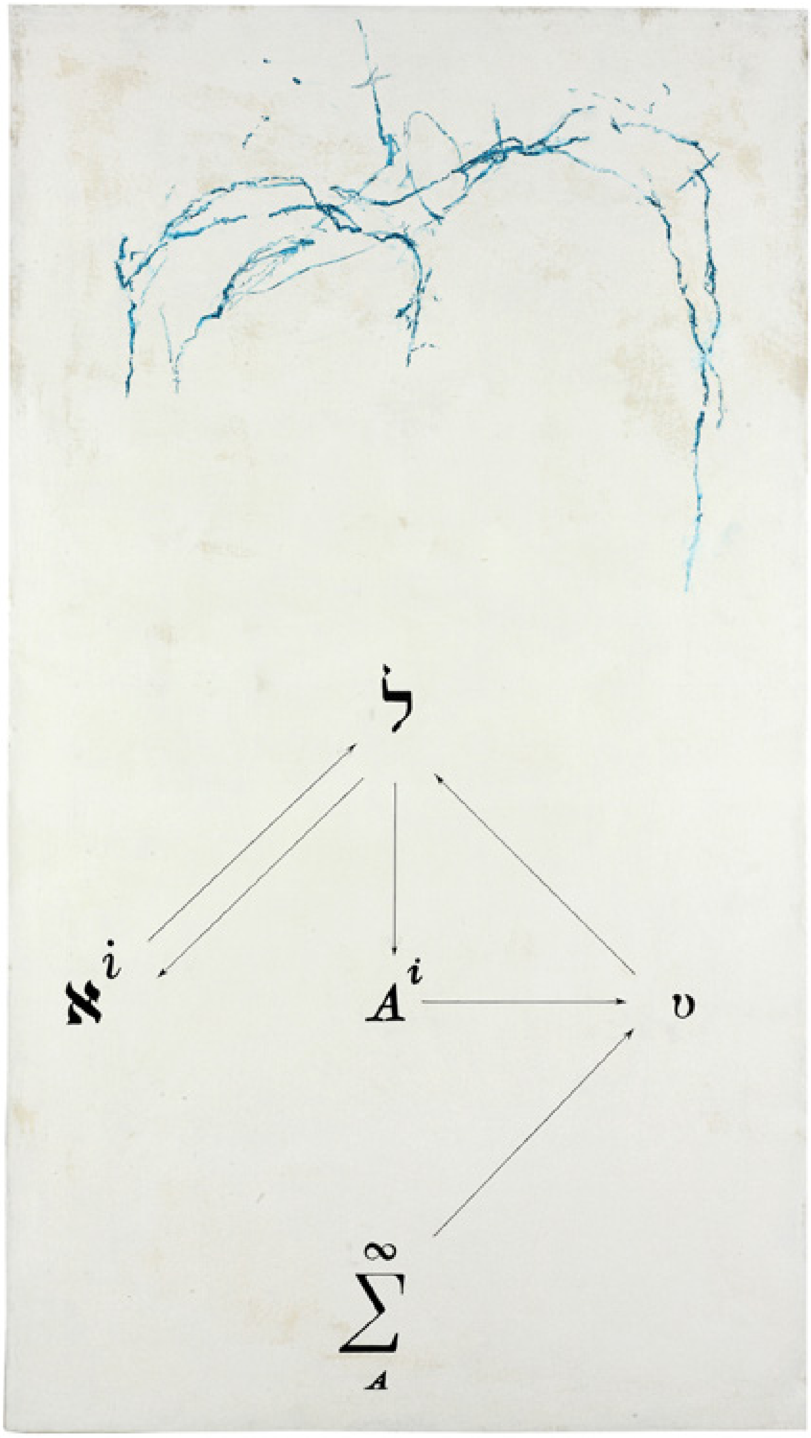
Defining art

Science has its own objectively designed tools, but what, then, is art? There are many theories. What's the difference between "real" art and craftsmanship, or the production of decorative objects that bring us pleasure? Is there a difference, in fact? Are they both art? Is it art to "talk" about oneself? There are many such questions, and even more different answers. Rooted in Hellenistic traditions, the current trend suggests that artists should seek true ideas. This means that scientists and artists aim to understand the world using different tools. Scientists are limited



PIOTR HREHOROWICZ

Jan Pamuła, "Computer series I," 1984, acrylic on canvas, 120×120 cm, collection of Renata and Grzegorz Król



MACIEJ ŻYWOLEWSKI

Marian Warzecha, Untitled, 1973, mixed technique, 115×65 cm. Courtesy of the Nautilus Gallery

to using "objective" tools, while artists are free from such objectivism.

Greek artists from the Classical period drew upon mathematical theories in depicting the world of ideas and the divine *universum*; they believed that male and female figures must be harmonious, with every part of the body in perfect proportion to the others. The sculptor Polykleitos created the canonical male body shape. Proportions also play an important role in architecture, in particular the golden ratio or "divine proportion," which can be found in the buildings of Ancient Rome, Islamic architecture and works by 20th-century architects.

Tessellation and Islamic art

Orthodox Islam prohibits the human form from being depicted in art. This has propelled the development of sublime decorative art with a strong mystical subtext. Artists working over the centuries covered hundreds of square meters of walls and floors of palaces, public buildings and mosques with dazzling, abstract mosaics. They take on myriad forms and colors, and their order and symmetry are striking.

As they looked upon such mosaics, mathematicians asked themselves whether they represented the pinnacle of achievement. Could there be other relatively regular ways of filling a plane surface? To begin with, they assumed that the entire infinite plane must be filled with polygon tiles of a single type. Additionally, the sides of the tiles had to fit exactly against one another. This introduced the concept of tessellation. The vertices of the polygons created a regular network of points filling the entire surface, and their arrangement determined the type of tessellation. Over time, the rules became more relaxed, for example allowing different types of tiles on the same surface, but each time a "regular" network of points was created. Mathematicians asked themselves: how many types of such patterns are there? They noted that every type has a specific group of symmetries, rigid movements and plane isometries which transpose points of the grid onto others of the same grid. Such groups were named plane crystallographic groups (or "wallpaper groups"), and in 1891 Yevraf Fyodorov (and, independently, George Pólya in 1924) proved that there are exactly seventeen different groups of isometries. During the second half of the 20th century, mathematicians visiting Alhambra in Spain spotted more and more of these plane crystallographic groups on the palace walls. Eventually, it was shown that all 17 groups can be found as groups of symmetries of selected decorative walls at Alhambra – assuming, of course, that the patterns have no boundaries and cover the entire surface.

Gothic architecture

Gothic architecture is another important example of mathematics influencing art. In 10th- and 11th-century Europe, understanding of Greek geometry, in particular Euclid's *Elements*, was highly limited; it is even likely that Pythagoras's theorem was not widely known. Things finally began to change following the Reconquista of the Iberian Peninsula, in particular the reclamation of Toledo – a major cultural and scientific center – from Arab rule in 1085. In the early

12th century, Archbishop Raymond of Toledo founded a translation school with the aim of translating all kinds of scholarly texts by Greek, Arab and Jewish authors from Arabic into Latin. Many treatises by Aristotle and Euclid's *Elements* were translated there for the first time.

Newly-founded universities taught mathematics as part of a quadrivium – the liberal arts covering the four subjects of arithmetic, geometry, music and astronomy – and it was regarded as the foundation of all knowledge. The increasingly elaborate tracery of Gothic churches goes to show that cathedral construction would have been impossible without a thorough understanding of Euclid's *Elements*.

Renaissance architecture

A few centuries later, Filippo Brunelleschi stood on the steps of the cathedral in Florence, pondering how best to depict in his painting the square and baptistery that he saw before him. It is likely he was familiar with Euclid's *Optics* and Vitelon's *Perspectiva*. He understood that light propagates along straight lines, and so in order to present a realistic image of an object, he needed to imagine straight lines running between his eyes and points on the object. The points where these lines intersect with the painting's surface create the image, and the viewer will recognize the object so depicted. For the illusion to be complete, the viewer's eyes need to be in exactly the same position as those of the artist creating it. When a tiled floor is depicted following these principles, we see a surprising effect. The edges of square tiles form two families of parallel lines which are perpendicular to one another. Let's place the plane of the painting perpendicularly to the plane of the floor, such that the lower edge is parallel with one of the lines from one of the families. In the painting they appear as lines parallel with the bottom edge, while lines from the other family are not parallel and intersect at a single point where the line leading from the artist's eye and running perpendicular to the plane intersects that plane. As such, this point of the painting does not correspond to any of the points of the depicted scene; it will be somewhere beyond the plane, leading into infinity. This concerned artists, theologists and philosophers. At the time, Florence was being visited by Nicholas of Cusa – a cardinal, mathematician, philosopher and one of the first scholars to ponder the concept of infinity. From the theological perspective, he was fascinated by the question of how infinite, boundless God could have become incarnate in a finite, limited human. It is no accident that the most rigorous depictions of linear perspective with a clear vanishing point are found in depictions of the Annunciation. Linear perspective soon became a fundamental element of painting, and scholars published textbooks explaining how to draw the perspective grid and how to use it to depict people and objects to create a convincing illusion. The perspective technique known as anamorphosis gained in popularity soon after – an ingenious trick requiring the viewer to occupy a specific vantage point to see a recognizable image. It's worth noting here that the first treatise on anamorphosis is the booklet *Perspectivae stereo pars specialis* by Jan Ziarnko (born ca. 1575 in Lviv and died ca. 1630 in Paris), which was published in Paris in 1619 and was heavily based on Euclid's *Elements*, including theorems and proofs. All such treatises proposed ways to construct line grids, which would be perceived by the eye as two families of parallel lines perpendicular to each other. Their authors tried to provide geometric reasoning behind such constructions, but one of the simplest and most convenient of them remained out of reach. Abraham Bosse's book *Manière universelle de M. des Argues pour pratiquer la perspective par petit-pied comme le géométral…* (Paris, 1648) finally proved this construction technique, based on Desargues's theorem – one of the fundamental statements of projective geometry. This treatise also includes the first proof of that theorem. It turns out that linear perspective, the vanishing point and the peculiar perspective of anamorphosis are explained through projective geometry, where a plane or 3D space is supplemented with infinite points that correspond to classes of parallel lines. In such spaces, perspective is projected based on bundles of straight lines, similarly to linear perspective where the projection is based on the bundle of straight lines originating from the artist's eye. It is hardly a surprise that Descartes, Gerard Desargues, Blaise Pascal, Abraham Bosse and most likely also Jan Ziarnko knew one another very well and regularly met at Marin Mersenne's Académie Parisienne.

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Cubism

Science and art entered a fascinating period from around the mid-19th century onwards. Works by Nikolai Lobachevsky, János Bolyai and Bernhard Riemann undermined the widespread belief in the Euclidian nature of our world. Scientists began to understand that the universe may actually be curved and time can be seen as the fourth dimension. It feels difficult nowadays to comprehend the shock caused by the publications of examples of non-Euclidean geometries, but it led to the collapse of the entirety of physics and astronomy modelled on Euclidian spaces. In the popular impression, Hermann Minkowski's spacetime would make it possible to travel in time as well as space. The world we inhabit was understood to have more dimensions than three or even four, and geometers started imagining solids in the fourth and fifth dimension. Esprit Jouffret penned the popular *Traité élémentaire de géométrie à quatre dimensions* (Elementary Treatise on the Geometry of Four Dimensions, Paris 1903). Henri Poincaré played an important role in disseminating new mathematical concepts and scientific discoveries, dedicating a lot of his time and energy to hosting popular lectures which attracted vast audiences in Paris. The avant-garde poet Guillaume Apollinaire wrote in *Les Peintres Cubistes*, defending his friends Pablo Picasso and Georges Braque: "Geometry is to the plastic arts what grammar is to the art of the writer." Cubist painting is vastly different from art from earlier periods. One of its early proponents was Paul Cezanne, but it has roots in Cycladic art and in masks and sculptures from Sub-Saharan Africa. All this is true and many aspects of cubism can be explained this way, but it is harder to elucidate one of its most significant aspects: depicting an object from different perspectives/viewpoints on a single plane. This seems to be directly borrowed from what geometers of the period were developing.

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Contemporary Polish art

Contemporary art features myriad examples of references to mathematical traditions. Elements of mathematical language and its symbols appear in numerous visual artworks. Frequently the artist is simply fascinated by the artistic values of symbols, but there are also many others who seek deep inspiration in mathematics and mathematical processes. Algorithms can be used to objectify the creative process. For example, Ryszard Wieniawski's monochromatic paintings feature a regular grid of squares colored in black or white, decided by flipping a coin.

Jan Pamuła has been using computer algorithms almost from the very beginning of his career. During his scholarship at the École nationale supérieure des beaux-arts in Paris, he worked with IT experts to develop a program creating complex grids of rectangles filling a rectangular or, more frequently, square image. He then transferred the grid onto canvas and painted the rectangles by hand to create beautiful abstract art – colorful maps of a certain reality. To begin with, he limited himself to four colors, and the paintings served as an artistic commentary on the "four color map theorem" which had been proven with computer assistance around that time.

Marian Warzecha presents yet another approach. He has always been interested in language, in particular syntax. His works and collages have always featured quotes, letters, numbers, and geometric figures. To start with, their arrangement is "random," but in subsequent artworks all elements follow a certain order. Over time, fragments of old manuscripts come to be replaced with mathematical formulas which eventually dominate. In some paintings, they are accompanied by a schematic male figure or a colorful paintbrush mark. His artworks are presented in sets known as meta-sets, accompanied by documentation explaining the goal of the given exhibition. The artworks are austere and ascetic, and – just like the documentation – they are difficult to understand even for viewers familiar with set theory and category theory. The documentation, prepared together with professional mathematicians, is based on academic texts and points out certain paradoxes of the theories. Such ascetic depictions encourage reflection on the problems of fundamental mathematical theorems.

Intuition and logic

To finish, I would like to recall Henri Poincaré's notes on the creative process. He believed that logic and intuition play an important role in mathematical discoveries. He wrote, "Logic, which alone can give certainty, is the instrument of demonstration; intuition is the instrument of invention." Intuition was not something he resorted to when he could not find logical proof; he believed that formal arguments can reveal errors in intuition, while logical arguments are the only means of confirming observations. He also believed that pure logic could create nothing new.

Intuition seems to be the tool of art and logic the tool of science. The truly great works which expand our understanding of the world are created when logic supports intuition or vice versa. Mathematics and art can survive without one another, but true masterpieces arise when one inspires the other. ■