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Experimental Verification of the Cast Iron Solidification Model

J. Mendakiewicz
Department of Computational Mechanics and Engineering,
Silesian University of Technology, Konarskiego18A, 44-100 Gliwice, Poland
Corresponding author. E-mail address: jerzy.mendakiewicz@polsl.pl

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Abstract

The article concerns the experimental verification of the numerical model simulating the solidification and cooling processes proceeding in the domain of cast iron casting. The approximate course of the function describing the evolution of latent heat and the value of substitute specific heat resulting from its course were obtained using the thermal and derivative analysis (TDA) method The TDA was also used to measure the cooling curves at the distinguished points of the casting. The results obtained in this way were compared with the calculated cooling curves at the same points. At the stage of numerical computations, the explicit scheme of the finite difference method was applied. The agreement between the measured and calculated cooling curves is fully satisfactory.

Keywords: Application of information technology to the foundry industry, Solidification process, Numerical techniques, Thermal and derivative analysis

1. Introduction

Experimental studies were carried out for the sample casting made from the hypo-eutectic grey cast iron of EN-GJL-200 – EN-GJL-250 (GG-20 – GG-25) class. The temperature measurements have been realized by the system of thermocouples PtRh-Pt installed at selected points of the casting domain (Figures 1 and 2).

The results of thermal and derivative analysis allow ones to find the border solidification temperatures and also to predict the general course of the function describing the evolution of latent heat [1]. For this purpose, the time courses of temperature changes $T(x_i, t)$ and its derivative concerning time $\partial T(x_i, t)/\partial t$ are used.

On this basis, it was possible to determine the general course of the function describing the course of latent heat evolution, and next the function corresponding to the substitute specific heat in the mushy zone domain (austenite and eutectic phases – Figure 3).

2. Mathematical description of cast iron solidification process

The thermal processes in domain of cast iron casting are described by the Fourier-type equation [2-5]

$$x \in \Omega,\ t > 0:\quad C(T)\frac{\partial T(x, t)}{\partial t} = \nabla\left[\lambda(T)\nabla T(x, t)\right] \tag{1}$$

where $C(T)$ is the substitute volumetric specific heat of alloy, $\lambda(T)$ is the thermal conductivity, T, x, t denote the temperature, geometrical co-ordinates and time.

The equation (1) is supplemented by the equation concerning a mould sub-domain

$$x \in \Omega_m,\ t > 0:\quad c_m\frac{\partial T_m(x, t)}{\partial t} = \lambda_m\nabla^2 T_m(x, t) \tag{2}$$

where c_m is the mould volumetric specific heat, λ_m is the mould thermal conductivity.

In the case of typical sand moulds on the contact surface between casting and mould the continuity condition in the form

$$x \in \Gamma_c, t > 0: \begin{cases} -\lambda(T)\, \mathbf{n} \cdot \nabla T(x, t) = -\lambda_m\, \mathbf{n} \cdot \nabla T_m(x, t) \\ T(x, t) = T_m(x, t) \end{cases} \tag{3}$$

can be assumed.

On the external surface of mould also the continuity condition, namely

$$x \in \Gamma_0, t > 0: \quad -\lambda_m\, \mathbf{n} \cdot \nabla T_m(x, t) = \alpha\left[T_m(x, t) - T_a\right] \tag{4}$$

is given (α is the penetration coefficient, T_a is the ambient temperature).
For $t = 0$ the condition in the form

$$x \in \Omega: T(x, 0) = T_0(x), \ x \in \Omega_m: T_m(x, 0) = T_{m0}(x) \tag{5}$$

is also given.

In the case considered, the substitute specific heat for the mushy zone sub-domain is assumed in the form of an interval function (Figure 3) [1]

$$C(T) = \begin{cases} c_L\,, & T \geq T_L \\ a_1 + a_2 T + a_3 T^2 + a_4 T^3 + a_5 T^4, & T_A \leq T < T_L \\ c_{AE}\,, & T_E \leq T < T_A \\ b_1 + b_2 T + b_3 T^2 + b_4 T^3 + b_5 T^4, & T_S \leq T < T_E \\ c_S\,, & T < T_S \end{cases} \tag{6}$$

Here T_L, T_A, T_E, T_S are the border temperatures, c_L, c_S are the constant volumetric specific heats for molten metal and solid states, and

$$c_{AE} = \frac{c_L + c_S}{2} + \frac{Q_{aus2}}{T_A - T_E} \tag{7}$$

where $Q_{aus} = Q_{aus1} + Q_{aus2}$ is the latent heat related to the austenite phase evolution.

The unknown coefficients a_k, b_k, $k = 1, 2, ..., 5$ are determined from two systems of equations, namely

$$\begin{cases} C(T_L) = c_L \\ C(T_A) = c_{AE} \\ \left.\dfrac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_L} = 0 \\ \left.\dfrac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_A} = 0 \\ \displaystyle\int_{T_A}^{T_L} C(T)\,\mathrm{d}T = c_{P1}\left(T_L - T_A\right) + Q_{aus1} \end{cases} \tag{8}$$

and

$$\begin{cases} C(T_E) = c_{AE} \\ C(T_S) = c_S \\ \left.\dfrac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_E} = 0 \\ \left.\dfrac{\mathrm{d}C(T)}{\mathrm{d}T}\right|_{T=T_S} = 0 \\ \displaystyle\int_{T_S}^{T_E} C(T)\,\mathrm{d}T = c_{P2}\left(T_E - T_S\right) + Q_{eu} \end{cases} \tag{9}$$

where Q_{eu} is the latent heat related to the eutectic phase evolution and $c_{p1} = (c_L + c_{AE})/2$, $c_{p2} = (c_S + c_{AE})/2$.

The thermal conductivity of cast iron is

$$\lambda(T) = \begin{cases} \lambda_L\,, & T > T_L \\ \lambda_P\,, & T_S < T \leq T_L \\ \lambda_S\,, & T \leq T_S \end{cases} \tag{10}$$

where λ_L corresponds to the liquid state, λ_S to the solid state and λ_P to the mushy zone sub-domains. This approximation assures the continuity and differentiability of substitute specific heat in the entire considered temperature interval.

It should be noted that the systems of equations (8), (9) can be solved in an analytical way.

3. Finite difference method

To solve the problem the explicit scheme of the FDM in the version proposed in [6] has been applied. The 3D task is considered and the domain was divided using the cubic control volumes. Let us denote a central point of control volume V_{ijk} by $P_{ijk} = P_0$ (a local numbering), while the central points of adjacent control volumes by P_1, P_2 ... P_6. The distances between the central point and adjacent ones are equal to h (regular differential mesh).

In the cited book, mathematical calculations were carried out for the selected FDM variant and it was shown that the difference equations for successive mesh nodes can be written in the form

$$C\,(T_0^f)\,\frac{T_0^{f+1}-T_0^f}{\Delta t}=\sum_{e=1}^{6}\frac{T_e^f-T_0^f}{R_e^f}\Phi_e \tag{11}$$

where f and $f+1$ denote two successive time levels, Δt is a time step, R_e^f is a thermal resistance between the central node P_0 and adjacent node P_e. Φ_e are the shape functions (in the case considered they are equal to $1/h$).

Thermal resistances for the internal node are of the form

$$R_e^f=\frac{0.5h}{\lambda_0^f}+\frac{0.5h}{\lambda_e^f} \tag{12}$$

while it is not important to which sub-domain (casting or mould) the neighboring node P_e belongs.

When the wall of the control volume V_0 in the direction e borders with the environment, then

$$R_e^f=\frac{0.5h}{\lambda_0^f}+\frac{1}{\alpha} \tag{13}$$

where α as previously, is the penetration coefficient. In a such situation in the place of T_e^f, the ambient temperature should be introduced.

Denoting

$$A_e=\frac{\Delta t\,\Phi_e}{C(T_0^f)R_e^f} \tag{14}$$

we can write

$$T_0^{f+1}=\sum_{e=0}^{6}A_eT_e^f\,,\qquad A_0=1-\sum_{e=1}^{6}A_e \tag{15}$$

The stability condition $A_0 > 0$ allows one to find the critical time step for node P_0 and next the computable time step for the whole domain.

It should be pointed out, that the same equations (in the case of the presented above domain discretization) can be obtained using the Control Volume Method algorithm [7].

4. Method of solution and results of computations

The task was solved using the explicit scheme of the finite difference method [6, 8], while the input data were taken from [1].

The domain was divided into 8100 cubic control volumes – Figure 4. The nodes correspond to the central points of these volumes.

The following values of thermophysical parameters appearing in the mathematical model were adopted in the calculations: thermal conductivity of the liquid phase, the transition zone and the solid phase λ_L = 20 W/(m K), λ_P = 30 W/(m K), λ_S = 40 W/(m K), substitute volumetric specific heats of liquid and solid phases c_L = 5.88 MJ/(m^3 K), c_S = 5.4 MJ/(m^3 K), liquidus temperature T_L = 1250ºC, border temperatures T_A = 1200ºC and T_E = 1170ºC, solidus temperature T_S = 1150ºC, thermal conductivity coefficient of the molding sand λ_m = 0.7 W/(m K), volumetric specific heat of the molding sand c_m = 1600·1000 J/(m^3 K), initial mould temperature T_{m0} = 30ºC. The value of the pouring temperature resulted from the experiments. Using the methods of inverse problem solution (see [1]) one obtains Q_{aus1} = 259.2 MJ/m^3, Q_{aus2} = 236.5 MJ/m^3, Q_{eu} = 1260 MJ/m^3. The results of numerical computations are compared with the measurements (the cooling curves recorded by thermocouples placed as shown in Figure 1). Real and calculated cooling curves are shown in Figure 5.

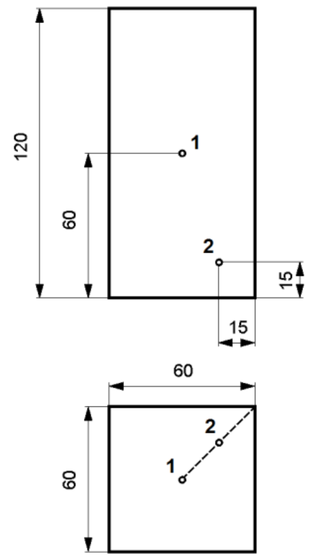

Fig. 1. Test casting – cubicoid

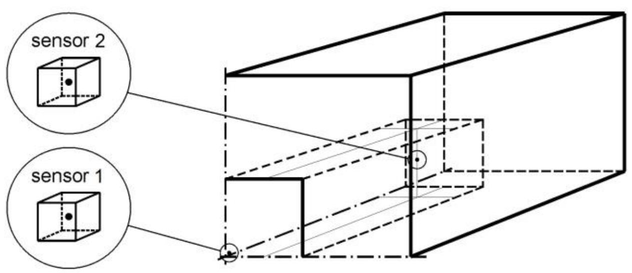

Fig. 2. Domain considered – casting of cuboid shape

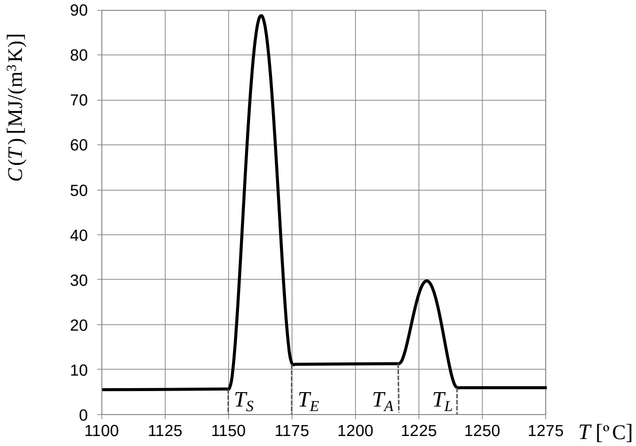

Fig. 3. Substitute thermal capacity of cast iron

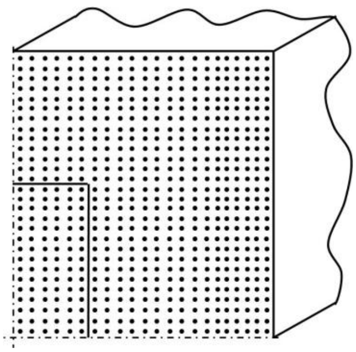

Fig. 4. 3D differential mesh

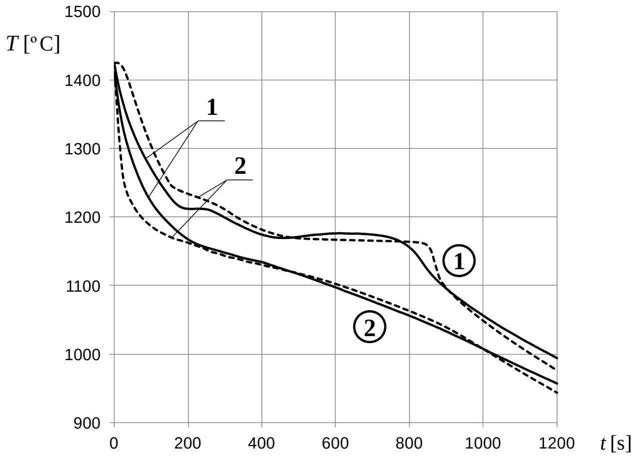

Fig. 5. Real (1) and calculated (2) cooling curves at points 1 and 2

5. Conclusions

The results of the casting cooling numerical model are compared with the measured temperature histories at the selected points from the casting domain.

The application of TDA technique allows one to obtain information concerning not only the real non-steady temperature field but also the courses of cooling rates. So, it gives the possibility of a more precise analysis of numerical simulation quality.

In this paper, the TDA technique has been used to determine the best approximation of the substitute specific heat of the material considered.

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