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SIMULATIVE INVESTIGATION OF ROBOT'S ACCURACY AND RUNNING SPEED FOR AT PLANNING OF A TRAJECTORY

The paper presents the algorithms for kinematic analysis, trajectory planning, dynamics of kinematic chain and driving units elaborated for manipulators and robots with kinematic chains of serial structure with revolute pairs with perpendicular or parallel axes. Elastic deflections of driving units as well as action of external forces on end-effector have been taken into account. The simulating software was created using the modular structure of modeling process. The application of software for testing the robots accuracy and running speed acc. to ISO 9283 is also presented.

1. Introduction

A dynamic model, experimentally verified on the basis of test results for a real object, is used for analyzing the impact of model parameters on manipulator's properties and for parameters modification to obtain desired properties [8]. The robotics uses kinematic and dynamic models of manipulator [1], [5], e.g. for planning the trajectory [3], [4], [9] and programming control units [2], [6].

In 1990, the standard ISO 9283 [10] was implemented, which determines international standards of tests and kinematic and geometric calculations of capacities of manipulators and industrial robots. These capacities include running speed, representation accuracy of a given movement, and positioning repeatability. One of the testing procedures is the accuracy test of an end-effector's point's path on the cube diagonal sections circumferences, done with planes crossing its vertices and on the sections circumferences, done with planes perpendicular to opposite cube's walls and halving them.

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The test is based on determination of errors made in representation of move of when movement directions vary perpendicularly at small radii and at velocity of a set-up value. The running speed index is the maximum velocity value, at which the absolute error of path representation does not exceed the assumed value.

In this work, used the author own algorithms of kinematics, trajectory planning, dynamics of kinematic chain and driving units as well as the programme for robot movement simulation [7], meeting the basic requirements and applicable for practical use in simulation tests.

In the kinematic analysis, the matrix method with author,s original simplifications was used, in the dynamic analysis, the 2nd type Lagrange's equations method was applied.

The mathematical model of electromechanical unit with a DC motor was created using the Kirchhoff's voltage equations. The model takes into account the driving shaft's torsional flexibility, suppression of its torsional vibrations, feedback control regarded to position, velocity and acceleration. Neglected was the mechanical and magnetic hysteresis, the armature's reaction, role of eddy currents and magnetic saturation. A single driving unit was considered as a system with two degrees of freedom. For describing relationship between active and reactive loads specified with movement equations for the machine's kinematic chain, the kinetostatic equivalence equations were used. Obtained were systems of second order differential equations relativy status variables to the external ones.

The point movement trajectory of the end-effector are described in form of a sequence of lines and arcs connected one to another, in any position and in a plane at any angle against the planning system in Cartesian coordinates. The proposed algorithm describes movement trajectories for a point of end-effector at specific orientation being used in typical processes or in tests of manipulators and robots.

Positions, velocities and accelerations described in the link coordinates were obtained from a solution of reversed task and used as ''patterns of movement" for drive units control systems.

The dynamic equations for the kinematic chain together with dynamic equations of driving units, with regard to the control system, were solved against angular accelerations and presented in a form of status equation, which was solved by numerical methods. The so obtained kinematic runs are the result of planning the trajectory under consideration of kinematic and dynamic possibilities, drive capacities and force interactions among elements.

2. Formulation of the trajectory modeling task

The trajectory, specified in the Cartesian coordinate base system, consists of lines and circular arcs (Fig. 1a). The input data are entered as sets of four numbers determining coordinates of a point and the radius of circle $P_i(x_{r,i}, y_{r,i}, z_{r,i})$, r_i, whereas it is assumed that the point belongs to the circle. The radius r_i is equal to zero at the first and the last point of trajectory as well as where the robot has to be stopped between the first and the last point of trajectory. In that case the set coordinates are considered as coordinates of a contact point to a zero radius circle and is marked as $(x_{rs,i}, y_{rs,i}, z_{rs,i})$. Three adjacent points of trajectory are joined with lines and arcs laying on the same plane "p", defined with axes x_p, y_p of coordinate systems $\{x_p, y_p, z_p\}$ with the origin in point $(x_{rs,p}, y_{rs,p}, z_{rs,p})$ an the axis x_p crossing the points: $(x_{rs,p}, y_{rs,p}, z_{rs,p})$ and $(x_{r,p+1}, y_{r,p+1}, z_{r,p+1})$. The number of all planes "p" is lower by two than the number of the introduced points i_t, so that all given relationships are specified with regard to "p" and calculated from $p = 1$ to $p = i_t - 2$.

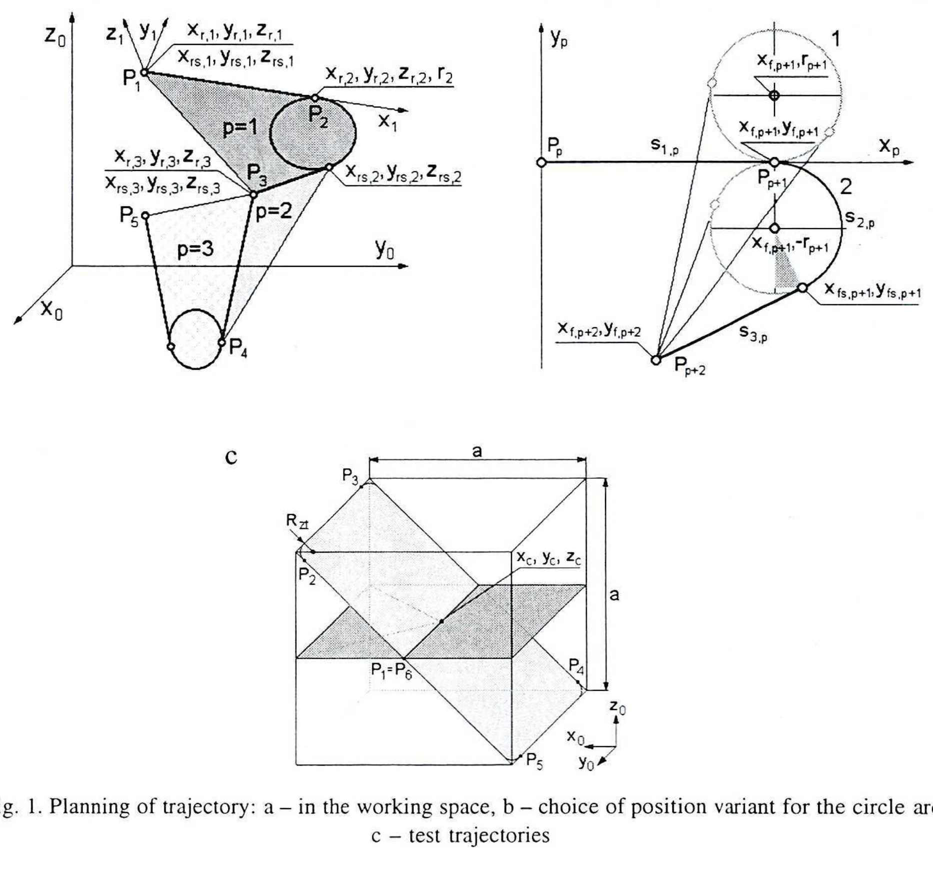

Fig. 1. Planning of trajectory: a – in the working space, b – choice of position variant for the circle arc, c – test trajectories

The process of planning line-arc trajectories can be divided into the following steps:

1. Specification of coordinates for trajectory points in the base reference system $\{x_0, y_0, z_0\}$.
2. Projection of three subsequent points of trajectory on a plane set by axes x_p, y_p of the reference system $\{x_p, y_p, z_p\}$, whose origin lays at the first point P_p and the axis x_p crosses the contact point P_{p+1}.
3. Determination of contact point coordinates $(x_{fs,+1}, y_{fs,+1})$ for trajectory segments $s_{2,p}$, $s_{3,p}$, i.e. the arc-line joint in the reference system $\{x_p, y_p\}$ (Fig. 1b).
4. Transformation of contact point coordinates calculated in the flat coordinate system $\{x_p, y_p\}$, to the base reference system $\{x_0, y_0, z_0\}$.

To determine coordinate values of points P_p, P_{p+1}, P_{p+2} in the reference system $\{x_p, y_p, z_p\}$, a transformation of coordinate system $\{x_0, y_0, z_0\}$ must be made.

After a rotation and translation of the system $\{x_0, y_0, z_0\}$, one obtains the coordinate values for trajectory points $(x_{f,p}, y_{f,p}, z_{f,p})$, $(x_{f,p+1}, y_{f,p+1}, z_{f,p+1})$, $(x_{f,p+2}, y_{f,p+2}, z_{f,p+2})$ in the reference system $\{x_p, y_p, z_p\}$:
– rotation (axes of immobile system get parallel to new system's axes),

$$\begin{bmatrix} x_{ro,p} & x_{ro,p+1} & x_{ro,p+2} \\ y_{ro,p} & y_{ro,p+1} & y_{ro,p+2} \\ z_{ro,p} & z_{ro,p+1} & z_{ro,p+2} \end{bmatrix} = \begin{bmatrix} l_{x,p} & m_{x,p} & n_{x,p} \\ l_{y,p} & m_{y,p} & n_{y,p} \\ l_{z,p} & m_{z,p} & n_{z,p} \end{bmatrix} \begin{bmatrix} x_{rs,p} & x_{r,p+1} & x_{r,p+2} \\ y_{rs,p} & y_{r,p+1} & y_{r,p+2} \\ z_{rs,p} & z_{r,p+1} & z_{r,p+2} \end{bmatrix}, \tag{1}$$

whereas: $\{x_{ro,p}, y_{ro,p}, z_{ro,p}\}$ – coordinates of point P_p, in the rotated system, $l_{k,p}$, $m_{k,p}$, $n_{k,p}$ – directional cosines for axes x_p, y_p, z_p $(k = x, y, z)$,
– translation (the first point P_p is relocated to the origin of coordinate system),

$$\begin{bmatrix} x_{f,p} & \{x_{f,p+1}\} & \{x_{f,p+2}\} \\ y_{f,p} & y_{f,p+1} & \{y_{f,p+2}\} \\ z_{f,p} & z_{f,p+1} & z_{f,p+2} \end{bmatrix} = \begin{bmatrix} x_{ro,p} & x_{ro,p+1} & x_{ro,p+2} \\ y_{ro,p} & y_{ro,p+1} & y_{ro,p+2} \\ z_{ro,p} & z_{ro,p+1} & z_{ro,p+2} \end{bmatrix} - \begin{bmatrix} x_{ro,p} & x_{ro,p} & x_{ro,p} \\ y_{ro,p} & y_{ro,p} & y_{ro,p} \\ z_{ro,p} & z_{ro,p} & z_{ro,p} \end{bmatrix} \tag{2}$$

The further planning of trajectory sections takes place in the plane coordinate system $\{x_p, y_p\}$. There are two possibilities of evaluating the equation for circle arc (proper – 2 and improper – 1) against position of the point P_{p+2}, (Fig. 1b). Depending on the chosen variant, the coordinates of contact points $(x_{fs,p+1}, y_{fs,p+1})$ and of dislocation $s_{1,p}$, $s_{2,p}$, $s_{3,p}$ on individual sections in subsequent planes p are calculated, and then the calculated coordinates are converted into the base reference system.

$$\begin{bmatrix} x_{rs,p+1} \\ y_{rs,p+1} \\ z_{rs,p+1} \end{bmatrix} = \begin{bmatrix} l_{x,p} & l_{y,p} & l_{z,p} \\ m_{x,p} & m_{y,p} & m_{z,p} \\ n_{x,p} & n_{y,p} & n_{z,p} \end{bmatrix} \begin{bmatrix} x_{fs,p+1} + x_{ro,p} \\ y_{fs,p+1} + y_{ro,p} \\ z_{ro,p} \end{bmatrix}. \tag{3}$$

The time parameterization can be done in two ways: – by assuming a specific pattern of trajectory movement – by specifying a multinominal pattern of movement using the method of current updating at the kinematic values, at which the movement occurs at the edge of maximum values of velocity and acceleration in mobile joints (e.g. specified by the manufacturer), and in this case the trajectory is run within an approximately optimum time. For the calculated values of dislocation $s_{tr}(t)$, velocity $v_{tr}(t)$ and acceleration $a_{tr}(t)$, the vector coordinates for position, velocity and acceleration at the trajectory in the plane system are determined and then converted into the reference system of the working space:

$$\begin{bmatrix} x_r \\ y_r \\ z_r \end{bmatrix} = \begin{bmatrix} l_{x,p} & l_{y,p} & l_{z,p} \\ m_{x,p} & m_{y,p} & m_{z,p} \\ n_{x,p} & n_{y,p} & n_{z,p} \end{bmatrix} \begin{bmatrix} x_f + x_{ro,p} \\ y_f + y_{ro,p} \\ z_{ro,p} \end{bmatrix},$$

$$\begin{bmatrix} \dot{x}_r \\ \dot{y}_r \\ \dot{z}_r \end{bmatrix} = \begin{bmatrix} l_{x,p} & l_{y,p} \\ m_{x,p} & m_{y,p} \\ n_{x,p} & n_{y,p} \end{bmatrix} \begin{bmatrix} \dot{x}_f \\ \dot{y}_f \end{bmatrix}, \quad \begin{bmatrix} \ddot{x}_r \\ \ddot{y}_r \\ \ddot{z}_r \end{bmatrix} = \begin{bmatrix} l_{x,p} & l_{y,p} \\ m_{x,p} & m_{y,p} \\ n_{x,p} & n_{y,p} \end{bmatrix} \begin{bmatrix} \ddot{x}_f \\ \ddot{y}_f \end{bmatrix} \tag{4}$$

If an algorithm for describing test trajectories is used (Fig. 1c), determined are: the length of cube a, coordinates (x_c, y_c, z_c) of the cube's centre position in the base reference system and two equal radii $r_i = R_{zt}$. The coordinates of further points of trajectory are calculated from the relationship:

$$\begin{array}{ll} P_1(x_c,\ y_c + a_y,\ z_c - z_a), 0, & P_2(x_c + x_a,\ y_c + a_y,\ z_c + z_a), R_{zt}, \\ P_3(x_c + a_x,\ y_c - y_a,\ z_c - z_a), R_{zt}, & P_4(x_c + x_a,\ y_c - y_a,\ z_c - z_a), R_{zt}, \\ \mathrm{P}_5(x_c - x_a,\ y_c + y_a,\ z_c - z_a), R_{zt} & P_6(x_c,\ y_c + a_y,\ z_c + z_a), 0, \end{array} \tag{5}$$

where

$x_a = y_a = z_a = 0.5(a - R_{zt}\sqrt{2})$, $a_x = a_y = a_z = 0.5a$ – for diagonal sections,
$x_a = y_a = 0.5a - R_{zt}$, $z_a = 0$, $a_x = a_y = 0.5a$, $a_z = 0$ – for non-diagonal sections,

(x_c, y_c, z_c) – coordinates of cube centre position in the base reference system, whereas the plus and minus signs before a_z and z_a mean accordingly the position of point P_i in the upper and lower cube vertex.

Errors of creation the trajectory's coordinates are determined from the relationships

$$\delta x_n = x_r - x_n,\ \delta y_n = y_r - y_n,\ \delta z_n = z_r - z_n \tag{6}$$

and Δ_n error of absolute distance between the point of end-effector and the set path

$$\Delta_n = \sqrt{\delta x_n^2 + \delta y_n^2 + \delta z_n^2}, \tag{7}$$

where

(x_n, y_n, z_n) – position coordinates of the end-effector's point in the base reference system obtained from the movement simulation (n – number of end-effector).

3. Kinematic and dynamic model

The matrix method, together with author's own algorithms simplifying the calculation procedure for kinematic chains with revolute pairs, was used for determining vectors of position and velocity of mass centres and vectors of angular velocity.

The vector $\boldsymbol{r}_{s,i}$ of the mass centre position for element i in the immobile coordinate system has been determined from the relationship

$$\boldsymbol{r}_{s,i} = \boldsymbol{T}_{1,i}\boldsymbol{r}_{s,i,j}, \quad \text{whereas} \quad \boldsymbol{r}_{s,i} = [x_{s,i} \quad y_{s,i} \quad z_{s,i} \quad 1]^T,\ \boldsymbol{T}_{1,i} = \prod_1^i \boldsymbol{A}_i, \tag{8}$$

where $\boldsymbol{r}_{s,i,i} = [x_{s,i,i}\ y_{s,i,i}\ z_{s,i,i}\ \ 1]^T$ – of the mass centre position for element i determined in its reference system, $\boldsymbol{T}_{1,i}$ – product of subsequent transformation matrices $\boldsymbol{A}_i$ describing transformation of system i-1 into the system i.

The vector of the mass centre velocity $\boldsymbol{v}_{s,i}$ for element i is the derivative of the vector of position

$$\boldsymbol{v}_{s,i} = \boldsymbol{D}_i\boldsymbol{r}_{s,i,i}, \quad \text{whereas} \quad \boldsymbol{v}_{s,i} = [v_{s,x,i}\ v_{s,y,i},\ v_{s,z,i}\ \ 1]^T, \tag{9}$$

whereas

$$\boldsymbol{D}_i = \boldsymbol{Q}_i\boldsymbol{A}_i \quad \text{for} \quad i = 1, \quad \boldsymbol{D}_i = (\boldsymbol{D}_{i-1} + \boldsymbol{T}_{1,i-1}\boldsymbol{Q}_i)\boldsymbol{A}_i \quad \text{for} \quad i = 2...n, \tag{10}$$

where the first derivative's differential matrix

$$\boldsymbol{Q}_i = \dot{\Theta}_i \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{11}$$

n – number of all kinematic chain's elements.

The vector of the angular velocity $\boldsymbol{\omega}_{i,i}$ for element i in the element-bound coordinate system is determined by the relationship

$$\boldsymbol{\omega}_{i,i}^T = (\boldsymbol{\omega}_{i-1,i-1} + \boldsymbol{\omega}_{i,i-1})^T\boldsymbol{B}_i, \quad \text{where} \quad \boldsymbol{\omega}_{0,0} = [0\ \ 0\ \ 0]^T, \boldsymbol{\omega}_{i,i-1} = [0\ \ 0\ \ \dot{\Theta}_i]^T, \tag{12}$$

$\boldsymbol{B}_i$ – the tertiary of matrix $\boldsymbol{A}_i$ obtained by deleting the last column and the last line.

The general form of movement equations for any kinematic chain with rotating pairs has been obtained by using the 2nd type Lagrange's equations, having assumed the rotation angles Θ_i between elements i-1 and i as generalized coordinates.

After determining the kinetic and potential energy, performing operations directed by the Lagrange's equation and grouping terms by loads caused by inertia, centrifugal, Coriolis and gravitational forces, and after performainy Jacobian transformation of moments and forces acting on the working tool, one determines the values of moment vectors $\boldsymbol{M}_i$ by the relationship

$$\boldsymbol{M}_i = \sum_{j=1}^{n}\boldsymbol{D}_{1,i,j}\ddot{\Theta}_j + \sum_{j=1}^{n}\boldsymbol{D}_{2,i,j}\dot{\Theta}_j^2 + \sum_{j=2}^{n}\sum_{k=j}^{n}\boldsymbol{D}_{j+1,k,i}\dot{\Theta}_{j-1}\dot{\Theta}_k + g\boldsymbol{E}_i + \boldsymbol{J}_{6,n}^T\boldsymbol{F}_{n,n}, \tag{13}$$

where: $\boldsymbol{D}_{1,i,j}$ – inertia forces matrix, $\boldsymbol{D}_{2,i,j}$ – centrifugal forces matrix, $\boldsymbol{D}_{j+1,k,j}$ – Coriolis forces matrix, $\boldsymbol{E}_i$ – gravitation forces matrix, $\boldsymbol{F}_{n,n}$ – vector of moments and forces acting on the end-effector in the element's system of reference, g – gravity acceleration, $\boldsymbol{J}_{6,n}$ – Jacobi's matrix sized $6 \times n$, j – column number in the inertia matrix, n – number of elements, Θ_i – rotation angle between elements i-1 and i.

Moments of forces loading kinematic pairs $\boldsymbol{M}_i(t)$ are balanced by moments of elasticity forces $\boldsymbol{M}_{kl,i}(t)$ of components of individual drive units considered as systems with two degrees of freedom. Two masses, one constant and another with moment of inertia depending on configuration of the kinematic chain elements, are joined with massless elastic and energy dissipating elements to consider elastic and suppressing properties of all components of passive part of the drive unit.

The values of elasticity force moments $\boldsymbol{M}_{kl,i}(t)$ for driving units are determined by the relationship

$$\boldsymbol{M}_{kl,i}(t) = k_{\varphi i}[\Theta_{1,i}(t) - \Theta_{2,i}(t)] + l_{\varphi i}[\dot{\Theta}_{1,i}(t) - \dot{\Theta}_{2,i}(t)], \tag{14}$$

$k_{\varphi,i}$ – coefficient of torsional rigidity, $l_{\varphi,i}$ – coefficient of torsional suppression, $\Theta_{1/2,i}$ – angular dislocation in the rotation pair, whereas indices "1" and "2" mean accordingly values determined for the driving unit (with neglected elastic deformations) and driven element (with considered elastic deformations), i – element number.

The form of kinetostatic equivalence equations for masses with moment of inertia $J_{1,i}$ reduced to the gearbox shaft output point is, according to the d'Alembert's rule

$$J_{1,i}\ddot{\Theta}_{1,i}(t) = \boldsymbol{M}_{n,i}(t) - \boldsymbol{M}_{kl,i}(t), \tag{15}$$

where the engines' driving moments $\boldsymbol{M}_{n,i}(t)$ are determined from the relationship

$$\boldsymbol{M}_{n,i}(t) = \boldsymbol{M}_{s,i}(t)\, i_{p,i}, \tag{16}$$

$i_{p,i}$ – gear reduction ratio, $J_{1,i}$ – moment of inertia of rotor and gear's rotating components reduced to the gear's output shaft's axis of rotation, $\boldsymbol{M}_{s,i}(t)$ – electrical moment of motor.

The electric moment of motor is determined as follows

$$\boldsymbol{M}_{s,i}(t) = c_i\, i_{a,i}(t), \tag{17}$$

where: c_i – motor constant, $i_{a,i}(t)$ – armature current.

The values of armature current and voltage as well as armature's angular velocity are corelated by the Kirchhoff's equation, whose form for the direct current motor armature circuit is

$$u_{a,i}(t) = c_i i_{p,i} \frac{d\Theta_{1,i}(t)}{dt} + R_{a,i} i_{a,i}(t) + L_{a,i} \frac{di_{a,i}(t)}{dt}, \tag{18}$$

where: $L_{a,i}$ – armature circuit inductance, $R_{a,i}$ – armature circuit resistance, $u_{a,i}(t)$ – armature powering voltage, $\Theta_{1,i}(t)$ – angular dislocation of the gear's output shaft, where the index $i = 1 \div n$, i – number of individual drive, n – number of all drive units.

Based on (13), (14) and (15), and taking into account that moments of load forces $\boldsymbol{M}_i(t)$ are balanced by moments of elasticity forces $\boldsymbol{M}_{kl,i}(t)$, we may write as follows

$$\boldsymbol{M}_{kl,i}(t) = \boldsymbol{M}_i(t) = \sum_{j=1}^{n} \boldsymbol{D}_{1,i,j} \ddot{\Theta}_{2,j}(t) + \boldsymbol{M}_{ocg,i}(t), \tag{19}$$

where $\boldsymbol{M}_{ocg,i}(t)$ – sum of moments of centrifugal, Coriolis, gravitation and working tool load balancing forces.

The values for positions, velocities and accelerations in the joint coordinates, obtained from the solution of the kinematics reversed task for a given trajectory of movement is considered as a "patterns of movement" for driving units' control systems. The values for voltage of drive motors' power supply $u_{a,i,n}$ in the current sampling time step are calculated from the relationship

$$u_{a,i,n} = u_{a,i,n-1} - [K_{\varphi,i} \quad K_{\omega,i} \quad K_{\varepsilon,i}][\Theta_{\Delta,i} \quad \dot{\Theta}_{\Delta,i} \quad \ddot{\Theta}_{\Delta,i}]^T, \tag{20}$$

where: $K_{\varphi,i}$, $K_{\omega,i}$, $K_{\varepsilon,i}$ – error increase coefficients for position, velocity and acceleration, $\Theta_{\Delta,i}$, $\dot{\Theta}_{\Delta,i}$, $\ddot{\Theta}_{\Delta,i}$ – differences between set and real values of position, velocity and acceleration in the joint coordinates, $u_{a,i,n-1}$ – value of the drive motor power supply voltage in the rotating pair i at the time step "n-1". The movement equations (19), being solved against $\ddot{\Theta}_{2,j}$ together with drive units dynamics equations with considered control system, is shown in form of status equations to be integrated numerically.

4. Structure of simulation programme

A typical simulation programme contains some mathematical relationships, permanently included in the programme in a form of procedures. Such a way of programming has been applied for algorithms with closed complication of description: trajectory planning, working element loading and dynamics of driving units. The equations of the machine carrying chain kinematics and dynamics, whose degree of sophistication depends on the number of elements, are generated symbolically – in a form of text files containing formulas. The task of derivation programs for symbolic is the automatic transformation of symbolic relationships into kinematics and dynamics calculation procedures.

As a result of program execution movement equation in the open form as well as numerical results and diagrams of kinematic and dynamic parameters are obtained. The programme is with movement animation procedures provided.

The calculations made in the trajectory planning programme – *trajectory*, whose block diagram is shown in Fig. 2, are carried out on the basis of the algorithm presented below:

1. entering input data in the form of sets of four numbers specifying point coordinates and circle radii,
2. search for extreme values of point coordinates and circle radii, for crude checking if those points belong to the working zone,
3. calculation of directional coefficients for coordinate system axes, where the axes x_i and y_i determine a plane parallel to the plane containing three subsequent points of planned trajectory, and then determination of a plane system of coordinates $\{x_p, y_p\}$,

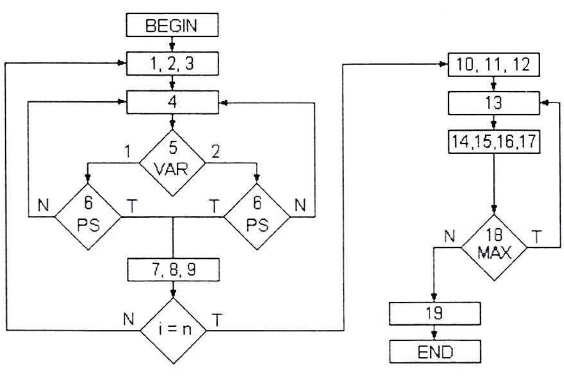

Fig. 2. Block diagram of the "trajectory" programme

4. calculation of contact points in the plane coordinate system $\{x_p, y_p\}$,
5. VAR – choice of variant for circle arc position against point P_{p+2}, from among two possibilities proposed by the programme,
6. PS – choice of proper solution determining the position of contact point between the circle arc and the line,
7. transformation of contact point coordinates from the plane coordinate system $\{x_p, y_p\}$ into the immobile coordinate system $\{x_0, y_0, z_0\}$,

8. calculation of the traveled way $s_{1,p}$, $s_{2,p}$, $s_{3,p}$ after three subsequent sections of trajectory and calculation of the course of trajectory point coordinates in the plane coordinate system,
9. transformation of subsequent trajectory points coordinates from the plane coordinate system into the immobile coordinate system,
10. calculation of the traveled way on the trajectory,
11. calculation of start, steady movement and braking times for individual stages of trajectory according to the assumed velocity pattern,
12. calculation, from the reverse task, of start in and end in angular positions in revolute pairs,
13. calculation of dislocations, velocities and accelerations in the trajectory movement assumed according to the kinematic run and the maximum values,
14. calculation of vector coordinates for dislocation, velocity and acceleration on a trajectory in a plane coordinate system,
15. transformation of the calculated vector coordinates for dislocation, velocity and acceleration in a plane coordinate system into the base reference system,
16. calculation, from the reverse task, of time courses of angular dislocation, velocity and acceleration in kinematic pairs,
17. calculation of the Jacobian matrix determinant value,
18. MAX – checking whether the calculated values of dislocation belong to permissible ranges (geometric constructional limits) and whether linear velocities and speeds in kinematic pairs do not exceed maximum values (kinematic limits); if the permissible values are exceeded, the maximum values assumed for this kinematic pattern for this trajectory are reduced proportionally to the excess value,
19. graphic presentation of time courses of angular dislocation, velocity and acceleration in kinematic pairs for the whole path of movement.

The planned movement path of the end-effector, even if completely included in the working space, may appear impossible to execute because of singularities. In the vicinity of singular positions, the velocities increase rapidly, and the Jacobian determinant value approaches zero. As the results of the consecutive speed limitations in the *MAX* stage of programme, it may appear that the values of kinematic parameters are close to zero, and in this case the trajectory must be replaced by another one.

The structure of programme for symbolic determination of kinematic and dynamic equation is presented in Fig. 3, where:

01 – generating the transformation matrix,

02 – drawing the kinematic chain,

03 – symbolic multiplication of matrices,
04 – squaring the velocity coordinates,
05, 06 – simplification and reduction of expressions,
07 – creation of calculation subroutines,
08 – generation of numeric data for procedure of checking the correctness of created equations by means of work and energy balance,
09 – vectors of revolute pair position,
10 – vectors of element mass centre position,
11 – velocity squares for element mass centres,
12 – angular velocities,
13 – compresion of results obtained by numerical matrix operations with those obtained from symbolic relationships,
14 – counting the number of components to evaluate operational effects of subroutines for reduction and simplification as well as the assessment of the equations' sophistication degree,
15 – differentiation by generalized dislocation, generalized velocities and time,
16 – ordering, reduction and simplification of expressions,
17 – grouping expressions,
18 – creating subroutine 19 for calculation of loading forces moments in kinematic pairs.

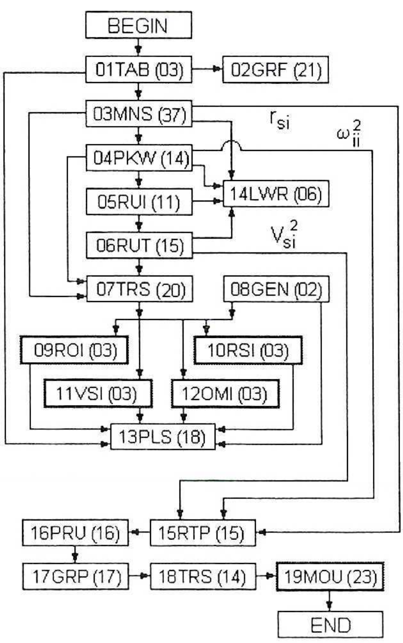

Fig. 3. Block diagram of the programme for symbolic calculations

The programme supplemets the generated kinematic and dynamic calculation procedure by subroutines for planning the trajectory, and dynamic of driving units, and a user interface with an unfolding *menu* containing sets of tables for selecting movement patterns, planning trajectories, entering numerical data, programme performed functions and tables with numeric results.

5. Numeric example

The simulation test were carried out on an example of industrial robot IRb/p–60. Robots of the series IRb/p–6 and IRb/p–60 belong to the most known in Poland, as they were used in the automotive industry, mainly for spot pressure welding, melt welding and grinding. They were manufactured in Poland in 1972–1990.

The trajectory used for testing the dynamic accuracy of robots and running on the circumference of a cube's section by a plane perpendicular to opposite walls and dividing them into halves, was composed of five lines and four circle arcs to create a closed contour in a form of a square with rounded vertices. The values of centre position coordinates for a cube with an edge of $a = 0.6$ m in the base reference system were assumed as follows: $x_c = 1.7$ m, $y_c = 0$, $z_c = 1$ m, so that the values of coordinates of gripper's trajectory points are: $P_1(1.70, 0.30, 1.00),0$, $P_2(2.00\text{-}R_{zt}, 0.30, 1.00)$, R_{zt}, $P_3(2.00, \text{-}0.30+R_{zt}, 1.00),R_{zt}$, $P_4(1.40+R_{zt}, \text{-}0.30, 1.00),R_{zt}$, $P_5(1.40, 0.30\text{-}R_{zt}, 1.00),R_{zt}$, $P_6(1.70, 0.30, 1.00),0$, whereas $R_{zt} = 0.03$ m. The maximum velocity is $v_{\mathrm{tr,max}} = 0.50$ m/s, the maximum acceleration is $a_{\mathrm{tr,max}} = 6.00$ m/s^2. The joint tilting drive is disconnected. During the movement, the joint is aligned horizontally because of the circumferential and parallel driving power transmission unit is applied. Because of a relatively small value of reduced rigidity coefficient for the driving power transmission unit, about 2500 Nm/rad, the joint makes oscillation movements due to inertial interactions, and this exerts an impact on representation errors for the given movement. To consider the role of the joint's free vibrations in the movement representation accuracy, four configuration coordinates were taken into account in this numeric example.

The courses of movement trajectories: the planned one and the one obtained from simulation, together with the robot silhouette in projections on the coordinate system planes are shown in Fig. 4. The time courses of displacements and velocities in revolute pairs are shown in Fig. 5. Fig. 6 present the courses of differences between assumed and real (obtained from simulation) coordinates of the gripper's point $\delta_{k,5}$ ($k = x, y, z$) as well as absolute value of gripper's distance Δ_5 from the given path. The maximum values of absolute distances between a point and a given path for ten assumed values of trajectory movement velocity are shown in Fig. 7.

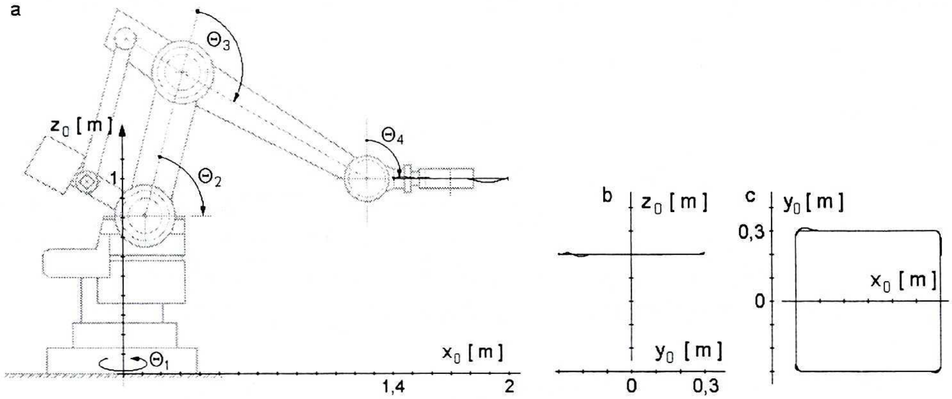

Fig. 4. Run of trajectory: a, b, c – projected on planes of the base system of reference

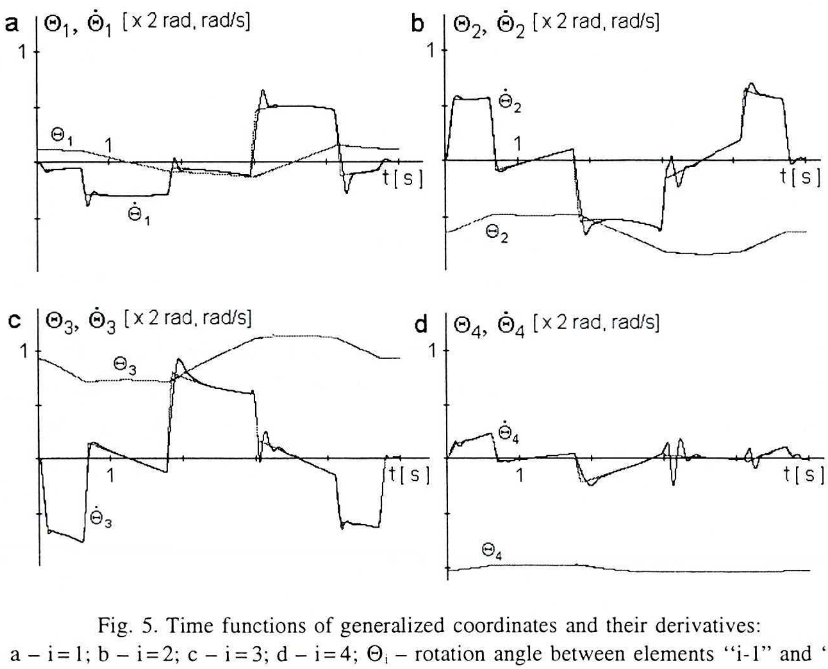

Fig. 5. Time functions of generalized coordinates and their derivatives:
a – $i=1$; b – $i=2$; c – $i=3$; d – $i=4$; Θ_i – rotation angle between elements "i-1" and "i"

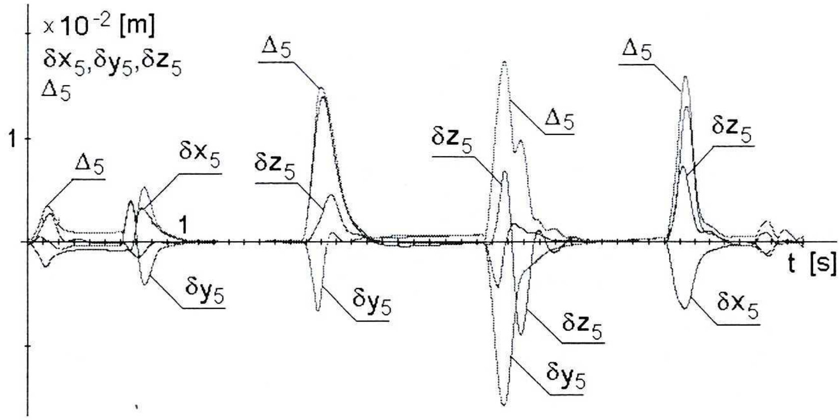

Fig. 6. Time functions of errors of reaching the assumed coordinates of trajectory

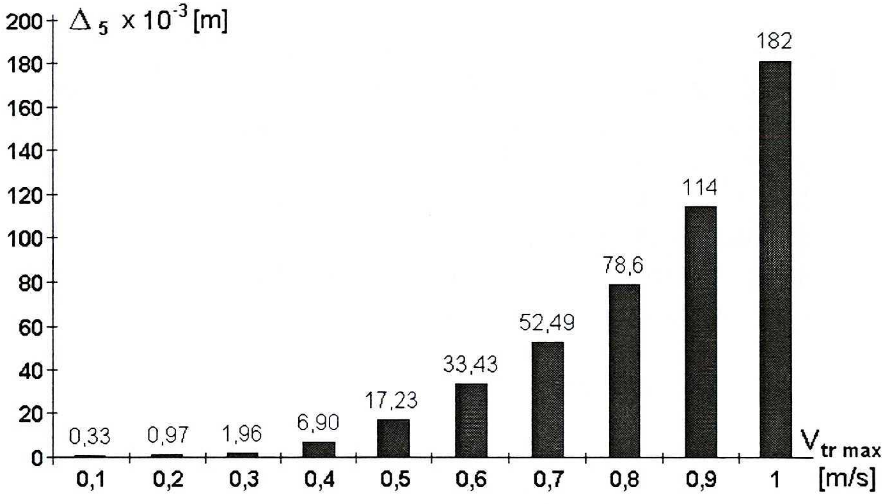

Fig. 7. Maximum values of absolute deviations of point's position from the assumed path for set-up values of movement velocity on the trajectory

6. Conclusions

1. The planning of movement path must start from the analysis of working space in Cartesian coordinates to determine coordinates of cube vertices in the working space.
2. The proposed modeling algorithm for a movement path consisting of any number of lines and circle arcs with specific dimensions, makes it possible model the trajectories used for testing the representation accuracy of a given movement.
3. The simulation programme facilitates performing the simulation of accuracy of trajectory implementation and can be use for

- simulation testing of movement accuracy of manipulation machinery,
- choice of movement velocity and trajectory course,
- determining the working space areas where the movement performance error is minimal,
- determining the most convenient position of the base coordinate system with respect to the trajectory,
- identification and choice of dynamics model parameters.

4. On the basis of the results obtained from simulation, the following conclusions can be drawn:
   - for the gripper's point velocity equal to 0,5 m/s (Fig. 4–6) the absolute error of movement representation at the test trajectory exceeds 0,017 m,
   - for the accuracy of movement representation stated by the manufacturer and amounting to about 0,4 mm, the robot's running speeds is about 0,1 m/s,
   - the absolute error of the given movement representation is a function depending on the squared velocity of movement on the trajectory,
   - the inaccuracy of movement path representation could result from the following reasons:
     - low rigidity of drive transmitting elements (movement transmission in parallel and circumferential way),
     - vibration processes caused by a strong dynamics of relatively heavy robot elements, including the counterweight, which can be replaced by an inertia free, spring type relieving system, similar to the second element's drive unit.

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[10] ISO 9283.

Badanie symulacyjne dokładności i szybkobieżności robota przy planowaniu trajektorii

Streszczenie

Dla manipulatorów i robotów z łańcuchami kinematycznymi o strukturze szeregowej, z parami obrotowymi o osiach wzajemnie prostopadłych lub równoległych, opracowano algorytmy obliczeniowe kinematyki, planowania trajektorii, dynamiki łańcucha kinematycznego i układów napędowych. Uwzględniono podatność i tłumienie elementów układów napędowych oraz działanie sił zewnętrznych na człon roboczy. Wykorzystując modułową strukturę procesu modelowania opracowano oprogramowanie symulacyjne. W pracy przedstawiono zastosowanie oprogramowania do badania dokładności i szybkobieżności robota zgodnie z wymogami normy ISO9283.