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Stability analysis and design of state estimated controller for delay fuzzy systems with parameter

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This paper deals with the problem of stabilization by an estimated state feedback for a family of nonlinear time-delay Takagi-Sugeno fuzzy parameterized systems. The delay is supposed to be constant where the parameter-dependent controls laws are used to compensate the nonlinearities which are formulated in terms of linear matrix inequalities (LMIs). Based on the Lyapunov-Krasovskii functionals, global exponential stability of the closed-loop systems is achieved. The controller and observer gains are able to be separately designed even in the presence of modeling uncertainty and state delay. Finally, a numerical example is given to show the applicability of the main result.

Key words: time-delay systems, observer, stabilization

1. Introduction

Takagi-Sugeno (T-S) fuzzy model-based control has received considerable attention in recent years since it allows the parallel distributed compensation concept and linear matrix inequality (LMI) techniques to be systematically applied to complex non-linear systems. T-S fuzzy model-based control can also be used to solve the output feedback control problem. The overall fuzzy model of the system is achieved by smoothly blending the local linear models together through the membership functions. Then, based on this fuzzy model, the control design is worked out by taking full advantage of the strength of modern linear control theory. Such models can approximate exactly a wide class of nonlinear systems. Hence it is important to study their stability or the synthesis of stabilizing controllers in the case of systems [2–5, 7, 20, 21, 25]. Since time delays

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frequently appear in many practical systems, such as chemical processes, and telecommunication systems, the control problem for time-delay systems has received considerable attention [1, 8, 9, 16]. While there is an extensive literature on this topic most of the reported studies focus on linear time-delay systems due to the difficulty of the stability analysis and controller design. The relative existing literature on the technique can be roughly divided into three types: static output feedback control [12], dynamic control [6], and observer-based control [14, 18, 24]. In [15], the authors studied the problem of sampled data control for a class of Takagi-Sugeno fuzzy systems with actuator saturation. Therefore, fuzzy observer-based control is the most popular scheme because the state variables can be reconstructed by a T-S fuzzy observer. In our recent paper [10], under the fact that both the estimator dynamics and the state feedback dynamics are stable we propose a separation principle for Takagi-Sugeno fuzzy control systems with Lipschitz nonlinearities. The considered nonlinearities are Lipschitz or meets an integrability condition which have no influence on the LMI to prove the stability of the associated closed-loop system.

In this paper, we investigate the problem of global stabilization of a class of nonlinear time-delay Takagi-Sugeno fuzzy parameterized systems with constant delay. We use the parallel distributed compensation concept to propose state and output feedback controllers depending on a parameter. Therefore, we use appropriate Lyapunov-Krasovskii functionals to establish global exponential stability of the closed-loop systems. Then, the exponential stability conditions are derived and converted to solving linear matrix inequality (LMI) problems. Based on the developed novel LMI algorithms, the controller and observer gains are able to be separately designed even in the presence of modeling uncertainty and state delay.

The rest of this paper is organized as follows. In Section 2, some preliminary results are summarized and the system description is given. Main results are stated in Section 3. First, parameter-dependent linear state and output feedback controllers are synthesized to ensure global exponential stability of the nonlinear time-delay system. Then, a simulation result that reflects the effectiveness of the proposed approach is given in Section 4.

2. Takagi-Sugeno fuzzy delayed system

Exact mathematical models of most physical systems are difficult to obtain, because of the existence of complexities and uncertainties. However, the dynamics of these systems may include linear or non linear behaviors for small range motion. Lyapunov's linearization method is often implemented to deal with the local dynamics of nonlinear systems and to formulate local linearized approximation. That is, the complex system can be divided by a set of local mathematical models.

Takagi and Sugeno have proposed an effective means of aggregating these models by using the fuzzy inferences to construct.

2.1. Preliminaries and system description

Consider the following time-delay system:

$$\begin{aligned} \dot{x} &= g(x, x(t-\tau)), \\ x(\theta) &= \phi(\theta), \quad \theta \in [-\tau, 0], \end{aligned} \tag{1}$$

where $\tau > 0$ denotes the time delay and $\phi \in C$ is the initial function, where C denotes the Banach space of continuous functions mapping the interval $[-\tau, 0] \rightarrow \mathbb{R}^n$ equipped with the supremum-norm:

$$\|\phi\|_\infty = \max_{\theta \in [-\tau, 0]} \|\phi(\theta)\|.$$

$\|.\|$ being the Euclidean-norm. The map $g\colon \mathbb{R}^n \rightarrow \mathbb{R}^n$ is smooth and satisfies $g(0,0) = 0$. The function segment x_t is defined by $x_t(\theta) = x(t+\theta)$,m $\theta \in [-\tau, 0]$

Definition 1 *[23] The zero solution of system (1) is said to be globally exponentially stable with a decay $\sigma > 0$, if there exists a positive real β such that, for any initial condition $\phi \in C$, the following inequality holds:*

$$\|x(t)\| \leqslant \beta \|\phi\|_\infty e^{-\sigma t}; \quad \textit{for all} \;\; t \geqslant 0.$$

Sufficient conditions for stability of time-delay systems are provided by the theory of Lyapunov-Krasovskii functionals [9], a generalization of the classical Lyapunov theory of ordinary differential equations [13]. The following theorem gives sufficient conditions to ensure that the origin of system (1) is globally exponentially stable [23].

Theorem 1 *If there exist positive numbers λ_1, λ_2, ϱ and a continuous differentiable functional $V\colon C \rightarrow \mathbb{R}_+$ such that:*

$$\lambda_1 \|x(t)\|^2 \leqslant V(x_t) \leqslant \lambda_2 \|x_t\|_\infty, \tag{2}$$

$$\dot{V}(x_t) + \varrho V(x_t) \leqslant 0, \tag{3}$$

then, the zero solution of (1) is globally exponentially stable with the decay rate σ.

2.2. Design of fuzzy control system

The T-S fuzzy model is given by:

Rule i : If $z_1(t)$ is F_{i1} and $z_2(t)$ is F_{i2} ... and $z_p(t)$ is F_{ip}, then

$$\dot{x} = A_i x + B_i u + \widetilde{f_i}(\varepsilon, x, x(t-\tau), u), \quad i = 1, \dots, r \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the control input vector, $A_i(n,n)$ constant matrix, $B_i(n,m)$ matrix control input and the functions $\widetilde{f_i}$ represent the delayed perturbations of each fuzzy subsystem and depends on a small parameter ε for $i = 1, \dots, r$. F_{ij} is the fuzzy set $(j = 1, 2, \dots, p)$, $z(t) =^T (z_1(t), \dots, z_p(t))$ is the premise variable vector associated with the system states and inputs and r is the number of fuzzy rules. Center of gravity defuzzification yields the output of fuzzy system:

$$\dot{x} = \frac{\sum_{i=1}^{r} w_i(z)(A_i x(t) + B_i u(t) + \widetilde{f_i}(\varepsilon, x, x(t-\tau), u))}{\sum_{i=1}^{r} w_i(z)},$$

where $w_i(z) = \prod_{j=1}^{p} F_{ij}(z_j)$ and $F_{ij}(z_j)$ denotes the grade of the number ship function F_{ij}, corresponding to $z_j(t)$.

Let $\mu_i(z)$ be defined as:

$$\mu_i(z) = \frac{w_i(z)}{\sum_{i=1}^{r} w_i(z)}.$$

Then the fuzzy system has the state-space form:

$$\dot{x} = \sum_{i=1}^{r} \mu_i(z) \left(A_i x(t) + B_i u(t) + \widetilde{f_i}(\varepsilon, x, x(t-\tau), u) \right).$$

Clearly,

$$\sum_{i=1}^{r} \mu_i(z) = 1$$

and

$$\mu_i(z) \geqslant 0 \quad \text{for } i = 1, \dots, r.$$

The following assumption is made regarding the T-S fuzzy system: The pairs (A_i, B_i), $i = 1, \dots, r$ are controllable. That is, the nominal fuzzy system is locally controllable.

Based on this assumption, a state feedback control gain K_i can be obtained by pole placement design or Ackerman's formula, such that each local dynamics is stably controlled. The representation of the global control input matrix, denoted by B, is in the form: $B = \sum_{i=1}^{r} \mu_i B_i$. This means that the global control input matrix dominates the control performance. The design of the fuzzy controller can be taken as a linear state feedback control can defined as:

Rule i : If $z_1(t)$ is F_{i1} and $z_2(t)$ is F_{i2} ... and $z_p(t)$ is F_{ip}, then

$$u(t) = -K_i x(t), \quad i = 1, 2, \ldots, r,$$

where K_i is the local state feedback gain. Consequently, the defuzzified result is:

$$u(t) = -\sum_{i=1}^{r} \mu_i(z) K_i x(t).$$

In the sequel, we will consider a fuzzy system with nonlinearities taken as:

$$\widetilde{f_i}(\varepsilon, x, x(t-\tau), u)) = D_i(\varepsilon) f_i(x, x(t-\tau), u(t))$$

and an output $y = \sum_{i=1}^{r} \mu_i(z) C_i x$, $y \in \mathbb{R}^q$ and C_i has an appropriate dimension. $D_i(\varepsilon)$ is a diagonal matrix which depends on ε. Taking $\hat{y}$ defined by

$$\hat{y} = \sum_{i=1}^{r} \mu_i(z) C_i \hat{x}.$$

In this case, an observer can be designed which has the form:

$$\dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(z) \left(A_i \hat{x} + B_i u \right) - L \left(y - \hat{y} \right).$$

We wish to find a gain matrix L such that the error $e(t) = \hat{x}(t) - x(t)$ converge exponentially to zero as t goes to infinity. We assume that the following rules are given concerning the observer of each subsystem.

Rule l : If $z_i(t)$ is F_{i1} and $z_2(t)$ is F_{i2} ... and $z_p(t)$ is F_{ip}, then

$$\dot{\hat{x}} = \left(A_i \hat{x} + B_i u \right) - L_i (y - \hat{y}), \quad i = 1, 2, \ldots, r.$$

It suffices to take $L = \sum_{i=1}^{r} \mu_i(z) L_i$. So,

$$\dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(z) \left(A_i \hat{x} + B_i u \right) - \sum_{i=1}^{r} \mu_i(z) L_i \left(y - \hat{y} \right).$$

The stability of the closed-loop fuzzy system is not guaranteed by its construction where the conception of observers for fuzzy systems are important when we wish to control systems using the an estimated controller which is available via an observer design. Now, in order to prove a result of stabilization by means of an observer one can summarize this fact by the following assumptions on the matrices $(A_i; B_i; C_i)$, the fact that $(A_i; B_i)$ are controllable and $(C_i; A_i)$ are observable for $i = 1, 2, \ldots, r$. The state-feedback gain K_i and the state-observer gain L_i can be obtained by formulating a synthesis tool based on some LMIs.

The aim of this paper is to design an estimated state feedback to stabilize the origin of the following time-delay perturbed fuzzy system depending on a parameter:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r} \mu_i(z) \left(A_i x + B_i u + D_i(\varepsilon) f_i(x, x(t-\tau), u(t)) \right), \\ y(t) &= \sum_{i=1}^{r} \mu_i(z) C_i x(t), \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^n$ is the state, $u \in \mathbb{R}^m$ is the control input, $y \in \mathbb{R}^q$ is the output τ is a positive known scalar that denotes the time delay affecting the state variables and $\varepsilon > 0$ is a parameter. $r \geqslant 2$ is the number of If-then rules, and F_{ij} are the fuzzy sets $(j = 1, \ldots, p)$. $z_1, \ldots, z_p$ are the premise variables which are supposed to be measurable. We set $z = [z_1, \ldots, z_p]$.

It is assumed that $\mu_i(z) \geqslant 0$, for all $i = 1, \ldots, r$ and $\sum_{i=1}^{r} \mu_i(z) = 1$, for all $t \geqslant 0$.

The matrices A_i, B_i, C_i are of appropriate dimension and the perturbed term

$$\begin{aligned} D_i(\varepsilon) f_i(x, x(t-\tau), u(t)) = \Big[& f_{i_1}(\varepsilon, x(t), x(t-\tau), u(t)), \\ & \ldots, \; f_{i_n}(\varepsilon, x(t), x(t-\tau), u(t)) \Big]^T, \end{aligned}$$

where for all $i \in \{1, \ldots, r\}$ and $j \in \{1, \ldots, n\}$ the function $f_{i_j}\colon \mathbb{R}_+^* \times \mathbb{R}^n \times \mathbb{R}^n \times \mathbb{R}^m \longrightarrow \mathbb{R}$ are smooth such and satisfy the following assumption:

$\mathcal{A}_1$: There exist function $\gamma_{i_1}(\varepsilon) > 0$ and $\gamma_{i_2}(\varepsilon) > 0$ such that

$$\| D_i(\varepsilon) f_i(x(t), x(t-\tau), u(t)) \| \leqslant \gamma_{i_1}(\varepsilon) \| x(t) \| + \gamma_{i_2}(\varepsilon) \| x(t-\tau) \|.$$

Remark 1 *Let $D_i(\varepsilon)$ the following diagonal matrix:*

$$D_i(\varepsilon) = \operatorname{diag}\left[\frac{1}{n}, \frac{\varepsilon}{n}, \ldots, \frac{\varepsilon^{n-1}}{n}\right].$$

One can see that if there exist $\gamma_{i_1}(\varepsilon) > 0$ and $\gamma_{i_2}(\varepsilon) > 0$, such that,

$$\sum_{j=1}^{n} \varepsilon^{j-1} f_{i_j}(x, x(t-\tau), u) \leqslant \gamma_{i_1}(\varepsilon) \sum_{j=1}^{n} \varepsilon^{j-1}|x_i| + \gamma_{i_2}(\varepsilon) \sum_{j=1}^{n} \varepsilon^{j-1}|x_i(t-\tau)|$$

then,

$$\|D_i(\varepsilon) f_i(x(t), x(t-\tau), u(t))\| \leqslant \gamma_{i_1}(\varepsilon)\|x(t)\| + \gamma_{i_2}(\varepsilon)\|x(t-\tau)\|.$$

Many published results, concerning the control of the fuzzy system, are based on the parallel distributed compensation (PDC) principle [17, 19, 22]. The fuzzy system is assumed to be locally controllable. The design of the fuzzy controller shares the same antecedent as the fuzzy system and employs a linear state feedback control in the consequent part. The controller is defined as:

$$u = -\sum_{i=1}^{r} \mu_i(z) K_i x\,, \tag{6}$$

where $K_i \in \mathbb{R}^{n\times m}$ is the gain matrix.

In this paper, we will give time-delay independent conditions to ensure global exponential stabilization of the nonlinear time-delay system (5) under Assumption $\mathcal{A}_1$. We will use the parallel distributed compensation to propose a state controller and an output feedback controller. Therefore, we will develop linear matrix inequalities (LMIs) conditions. Indeed, it is worth noting that numerous results on the stabilization of the nonlinear system (5) proposing linear matrix inequality conditions were developed when the nonlinearities satisfies some linear growth condition or it is Lipschitz [10, 11].

Throughout the paper, the time argument is omitted and the delayed state vector $x(t-\tau)$ is noted by x^τ. A^T means the transpose of A. $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and minimal eigenvalue of a matrix A, respectively and I is the matrix identity.

3. Main results

In order to stabilize the fuzzy system by a state estimated feedback law, we first prove a result of stabilization and we consider a fuzzy observer based a fuzzy controller. It is well-known that for linear systems the combination of a stabilizing state feedback and an observer yields a stabilizing estimated feedback controller. This is known as the separation principle. However as the considered fuzzy system is nonlinear, it is not known whether the separation principle holds. Note also that this separation principle is not more available if we consider a TS model with uncertainties.

3.1. Global exponential stabilization

The state feedback controller is given by (6). Thus, the closed-loop system is

$$\begin{aligned}\dot{x} &= \sum_{i=1}^{r}\sum_{j=1}^{r}\mu_i(z)\mu_j(z)\big((A_i - B_iK_j)x + D_i(\varepsilon)f_i(x,x^\tau,u)\big)\\ &= \sum_{i=1}^{r}\mu_i^2G_{ii}x + 2\sum_{i<j}\mu_i\mu_jG_{ij}x + \sum_{i=1}^{r}\mu_iD_i(\varepsilon)f_i(x,x^\tau,u),\end{aligned} \tag{7}$$

where

$$G_{ii} = A_i - B_iK_i$$

and

$$G_{ij} = \frac{1}{2}\left(A_i - B_iK_j + A_j - B_jK_i\right).$$

Theorem 2 *Suppose that $(\mathcal{A}_1)$ hold and there exist symmetric and positive definite matrices P and positive constants ε, σ such that the following inequalities hold,*

$$\begin{aligned} G_{ii}^TP + PG_{ii} &\leqslant -I, \quad i = 1,\ldots,r,\\ G_{ij}^TP + PG_{ij} &\leqslant -I, \quad 1 \leqslant i < j \leqslant r,\end{aligned}$$

and

$$P < a(\sigma,\varepsilon)I,$$

where

$$a(\sigma,\varepsilon) = \frac{1}{2\varepsilon}\min\left(\frac{1}{2\sigma + 2\gamma_1(\varepsilon) + \gamma_2(\varepsilon)}, \frac{e^{-2\sigma\tau}}{2\gamma_2(\varepsilon)}\right),$$

$\gamma_1(\varepsilon) = \sum_{i=1}^{r}\gamma_{i_1}(\varepsilon)$ and $\gamma_2(\varepsilon) = \sum_{i=1}^{r}\gamma_{i_2}(\varepsilon)$, then the fuzzy closed-loop system (7)–(6) is globally exponentially stable.

Proof. Let us choose a Lyapunov-Krasovskii functional candidate as follows:

$$V(x_t) = x^T P x + \frac{1}{2\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s. \tag{8}$$

First, it is easy to see that

$$\begin{aligned} V(x_t) &\leqslant \lambda_{\max}(P)\|x\|^2 + \frac{1}{2\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x_t(s)\|^2 \mathrm{d}s \\ &\leqslant \lambda_{\max}(P)\|x_t\|_\infty^2 + \frac{1}{2\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x_t\|_\infty^2 \mathrm{d}s \\ &\leqslant \left(\lambda_{\max}(P) + \frac{1}{4\sigma\varepsilon}\right)\|x_t\|_\infty^2 \end{aligned}$$

and

$$V(x_t) \geqslant \lambda_{\min}(P)\|x(t)\|^2.$$

Thus condition (2) of Theorem 1 is satisfied with

$$\lambda_1 = \lambda_{\min}(P) \qquad \text{and} \qquad \lambda_2 = \lambda_{\max}(P) + \frac{1}{4\sigma\varepsilon}\,.$$

One can see that by using the change of variable $u = t + s$ one gets

$$\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 = \int_{t-\tau}^{t} \|x(u)\|^2 \mathrm{d}u.$$

Therefore, the time derivative of $V(x_t)$ along the trajectories of system (7) is

$$\begin{aligned} \dot{V}(x_t) = {} & \sum_{i=1}^{r} \mu_i^2 x^T \left(G_{ii}^T P + P G_{ii}\right) x + 2\sum_{i<j}^{r} \mu_i \mu_j x^T \left(G_{ij}^T P + P G_{ij}\right) x \\ & + 2x^T P \sum_{i=1}^{r} \mu_i D_i(\varepsilon) f_i(x, x^\tau, u) + \frac{1}{2\varepsilon}\|x\|^2 \\ & - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s. \end{aligned}$$

On the one hand, we have

$$x^T(G_{ii}^T P + PG_{ii})x \leqslant -\|x\|^2, \quad i = 1, 2, ..., r,$$

and

$$x^T(G_{ij}^T P + PG_{ij})x \leqslant -\|x\|^2, \quad 1 \leqslant i < j \leqslant r.$$

It follows that,

$$\dot{V}(x_t) \leqslant -\|x\|^2 \sum_{i=1}^{r} \sum_{i=1}^{r} \mu_i \mu_j + 2x^T P \sum_{i=1}^{r} \mu_i D_i(\varepsilon) f_i(x, x^\tau, u) + \frac{1}{2\varepsilon}\|x\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s.$$

Since,

$$\sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i \mu_j = 1,$$

then,

$$\begin{aligned}\dot{V}(x_t) &\leqslant -\|x\|^2 + 2x^T P \sum_{i=1}^{r} \mu_i D_i(\varepsilon) f_i(x, x^\tau, u) + \frac{1}{2\varepsilon}\|x\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 \\ &\quad - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s \\ &\leqslant -\frac{1}{2\varepsilon}\|x\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i \|D_i(\varepsilon) f_i(x, x^\tau, u)\| - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 \\ &\quad - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s.\end{aligned}$$

On the other hand by assumption $\mathcal{A}_1$, we obtain

$$\begin{aligned}\dot{V}(x_t) &\leqslant -\frac{1}{2\varepsilon}\|x\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 + 2\|x\|\|P\| \sum_{i=1}^{r} \mu_i \left(\gamma_{i_1}(\varepsilon)\|x\| + \gamma_{i_2}(\varepsilon)\|x^\tau\|\right) \\ &\quad - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s\end{aligned}$$

$$\leqslant -\frac{1}{2\varepsilon}\|x\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 + 2\|x\|\|P\|\sum_{i=1}^{r}\left(\gamma_{i_1}(\varepsilon)\|x\| + \gamma_{i_2}(\varepsilon)\|x^\tau\|\right) - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s$$

$$\leqslant -\frac{1}{2\varepsilon}\|x\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|x^\tau\|^2 + 2\|P\|\gamma_1(\varepsilon)\|x\|^2 + 2\|P\|\gamma_2(\varepsilon)\|x\|\|x^\tau\| - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s.$$

Using the fact that

$$2\|x\|\|x^\tau\| \leqslant \|x\|^2 + \|x^\tau\|^2,$$

we deduce that

$$\dot{V}(x_t) \leqslant -\left[\frac{1}{2\varepsilon} - 2\|P\|\gamma_1(\varepsilon) - \|P\|\gamma_2(\varepsilon)\right]\|x\|^2 - \left[\frac{e^{-2\sigma\tau}}{2\varepsilon} - \|P\|\gamma_2(\varepsilon)\right]\|x^\tau\|^2 - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|x(s+t)\|^2 \mathrm{d}s.$$

Hence, we obtain

$$\dot{V}(x_t) + 2\sigma V(x_t) \leqslant 2\sigma x^T P x - \left[\frac{1}{2\varepsilon} - 2\|P\|\gamma_1(\varepsilon) - \|P\|\gamma_1(\varepsilon)\right]\|x\|^2 - \left[\frac{e^{-2\sigma\tau}}{2\varepsilon} - \|P\|\gamma_2(\varepsilon)\right]\|x^\tau\|^2 \tag{9}$$

$$\leqslant -c(\varepsilon)\|x\|^2 - d(\varepsilon)\|x^\tau\|^2, \tag{10}$$

where

$$c(\varepsilon) = \frac{1}{2\varepsilon} - 2\sigma\|P\| - 2\|P\|\gamma_1(\varepsilon) - \|P\|\gamma_2(\varepsilon)$$

and

$$d(\varepsilon) = \frac{e^{-2\sigma\tau}}{2\varepsilon} - \|P\|\gamma_2(\varepsilon).$$

Since,

$$P < a(\sigma,\varepsilon)I,$$

then $c(\varepsilon) > 0$ and $d(\varepsilon) > 0$. Consequently,

$$\dot{V}(x_t) + 2\sigma V(x_t) \leqslant 0.$$

Thus, condition (3) of Theorem 1 is satisfied. Therefore, the system (7)–(6) is globally exponentially stable.

Corollary 1 *Suppose that* $(\mathcal{A}_1)$ *hold and there exist symmetric and positive definite matrices* P *and a positive constant* ε *such that the following inequalities hold,*

$$G_{ii}^T P + P G_{ii} \leqslant -I, \quad i, j = 1, ..., r,$$

$$G_{ij}^T P + P G_{ij} \leqslant -I, \quad 1 \leqslant i < j \leqslant r,$$

and

$$P < b(\varepsilon) I, \tag{11}$$

where

$$b(\varepsilon) = \frac{1}{2\varepsilon\big(2\gamma_1(\varepsilon) + \gamma_2(\varepsilon)\big)},$$

$\gamma_1(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_1}(\varepsilon)$ *and* $\gamma_2(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_2}(\varepsilon)$, *then the fuzzy closed-loop system* (7)–(6) *is globally exponentially stable.*

Proof. From Theorem 2, we have the closed loop system (7) is globally exponentially stable if

$$\frac{1}{2\varepsilon} - 2\sigma\|P\| - 2\|P\|\gamma_1(\varepsilon) - \|P\|\gamma_2(\varepsilon) > 0$$

and

$$\frac{e^{-2\sigma\tau}}{2\varepsilon} - \|P\|\gamma_2(\varepsilon) > 0.$$

therefore, the constant σ should satisfies

$$\sigma < \frac{\alpha(\varepsilon)}{2\|P\|} \quad \text{and} \quad -2\sigma\tau > \ln\left(2\varepsilon\|P\|\gamma_2(\varepsilon)\right), \tag{12}$$

where $\alpha(\varepsilon) = \frac{1}{2\varepsilon} - 2\|P\|\gamma_1(\varepsilon) - \|P\|\gamma_2(\varepsilon)$. Since we have

$$P < b(\varepsilon) I,$$

then,

$$\alpha(\varepsilon) > 0.$$

Thus,

$$1 - 2\varepsilon\|P\|\gamma_2(\varepsilon) > 4\varepsilon\|P\|\gamma_1(\varepsilon) > 0.$$

It follows that

$$\ln\left(2\varepsilon\|P\|\gamma_2(\varepsilon)\right) < 0.$$

Consequently,

$$\sigma < \frac{1}{2\tau}\ln\left(2\varepsilon\|P\|\gamma_2(\varepsilon)\right).$$

Just take

$$\sigma = \frac{1}{2}\min\left[\frac{\alpha(\varepsilon)}{2\|P\|}, \frac{1}{2\tau}\ln\Big(2\varepsilon\|P\|\gamma_2(\varepsilon)\Big)\right] > 0.$$

In many practical control problems, the physical state variables of systems are partially or fully unavailable for measurement, since the state variables are not accessible by sensing devices and transducers are not available or very expensive. In such cases, observer based control schemes should be designed to estimate the state. Newt, for the perturbed fuzzy system, an observer-based controller is suggested to stabilize the closed-loop dynamic. By combining a Luenberger-like state fuzzy observer and the state fuzzy feedback control law.

3.2. Stabilization by an estimated state feedback

The exponential stabilization of T-S models with a PDC control law is proven. The main property of this control law is that it shares the same premises as the T-S model. If the state is not measurable, conditions of stabilization with a fuzzy observer exist but make the assumption that premises are measurable. We propose the following system:

$$\dot{\hat{x}} = \sum_{i=1}^{r} \mu_i(z)\left(A_i\hat{x} + B_i u - L_i\left(y - \hat{y}\right)\right), \tag{13}$$

Where $\hat{y}$ given by

$$\hat{y} = \sum_{i=1}^{r} \mu_i(z) C_i \hat{x}$$

and the estimated feedback controller is given by

$$u = -\sum_{i=1}^{r} \mu_i K_i \hat{x}. \tag{14}$$

We wish to find L_i, $i = 1, \ldots, r$ such that $e = x - \hat{x}$ converges to zero exponentially as t tends to infinity.

Subtracting (7) from (13), we have the system error

$$\dot{e} = \sum_{i=1}^{r} \sum_{j=1}^{r} \mu_i(z)\mu_j(z) \left(A_i - L_j C_i\right) e + \sum_{i=1}^{r} \mu_i D_i(\varepsilon) f_i(x, x^\tau, u). \tag{15}$$

Thus,

$$\dot{e} = \sum_{i=1}^{r} \mu_i^2 \Upsilon_{ij} e + 2 \sum_{i<j} \mu_i \mu_j(z) \Upsilon_{ij} e + \sum_{i=1}^{r} \mu_i D_i(\varepsilon) f_i(x, x^\tau, u),$$

where

$$\Upsilon_{ii} = A_i - L_i C_i\,,$$

and

$$\Upsilon_{ij} = \frac{1}{2}\left(A_i - L_j C_i + A_j - L_j C_i\right).$$

Then let consider the following theorem.

Theorem 3 *Suppose that* $(\mathcal{A}_1)$ *hold and there exist symmetric and positive definite matrices* $\widetilde{P}$ *and* $\widetilde{Q}$ *and positive constants* ε, σ *such that the following inequalities hold,*

$$\Upsilon_{ii}^T \widetilde{P} + \widetilde{P}\Upsilon_{ii} \leqslant -I, \quad i = 1, \ldots, r,$$

$$\Upsilon_{ij}^T \widetilde{P} + \widetilde{P}\Upsilon_{ij} \leqslant -I, \quad 1 \leqslant i < j \leqslant r,$$

and

$$\widetilde{P} < \widetilde{a}(\sigma, \varepsilon) I,$$

where

$$\widetilde{a}(\sigma, \varepsilon) = \frac{1}{2\varepsilon\left(2\sigma + \gamma_1(\varepsilon) + \gamma_2(\varepsilon)\right)},$$

$\gamma_1(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_1}(\varepsilon)$ *and* $\gamma_2(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_2}(\varepsilon)$, *then the closed loop system* (7)–(14) *is globally exponentially stable.*

Proof. Let us choose a Lyapunov-Krasovskii functional candidate as follows

$$V(e_t) = e^T \widetilde{P} e + \frac{1}{2\varepsilon} \int_{-\tau}^{0} e^{2\sigma s} \|e(s+t)\|^2 \mathrm{d}s.$$

As in the proof of Theorem 1, we have

$$\lambda_{\min}(\widetilde{P})\|e(t)\| \leqslant W(e_t) \leqslant \left(\lambda_{\min}(\widetilde{P}) + \frac{1}{4\sigma\varepsilon}\right)\|e_t\|_\infty .$$

The time derivative of $W(e_t)$ along the trajectories of system (13) is given by

$$\begin{aligned}\dot{W}(e) = &\sum_{i=1}^{r}\mu_i^2 e^T(\Upsilon_{ii}^T\widetilde{P} + \widetilde{P}\Upsilon_{ii})e + 2\sum_{i<j}\mu_i\mu_j e^T(\Upsilon_{ij}^T\widetilde{P} + \widetilde{P}\Upsilon_{ij})e\\ &+ 2e^T\widetilde{P}\sum_{i=1}^{r}\mu_i D_i(\varepsilon)f_i(x,x^\tau,u) + \frac{1}{2\varepsilon}\|e\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|e^\tau\|^2\\ &- \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|e(s+t)\|^2\mathrm{d}s.\end{aligned}$$

On the one hand, we have

$$e^T(\Upsilon_{ii}^T\widetilde{P} + \widetilde{P}\Upsilon_{ii})e \leqslant -\|e\|^2, \quad i = 1,\ldots,r$$

and

$$e^T(\Upsilon_{ij}^T\widetilde{P} + \widetilde{P}\Upsilon_{ij})e \leqslant -\|e\|^2, \quad 1 < i < j < r.$$

Then, one gets

$$\begin{aligned}\dot{W}(e_t) \leqslant &-\|e\|^2\sum_{i=1}^{r}\sum_{i=1}^{r}\mu_i\mu_j + 2\|e\|\|\widetilde{P}\|\sum_{i=1}^{r}\mu_i\|D_i(\varepsilon)f_i(x,x^\tau,u)\|\\ &+ \frac{1}{2\varepsilon}\|e\|^2 - \frac{e^{-2\sigma\tau}}{2\varepsilon}\|e^\tau\|^2 - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|e(s+t)\|^2\mathrm{d}s.\end{aligned}$$

It follows that

$$\begin{aligned}\dot{W}(e_t) \leqslant &-\frac{1}{2\varepsilon}\|e\|^2 + 2\gamma_1(\varepsilon)\|\widetilde{P}\|\|e\|\|x\| + 2\gamma_2(\varepsilon)\|\widetilde{P}\|\|e\|\|x^\tau\|\\ &- \frac{e^{-2\sigma\tau}}{2\varepsilon}\|e^\tau\|^2 - \frac{\sigma}{\varepsilon}\int_{-\tau}^{0} e^{2\sigma s}\|e(s+t)\|^2\mathrm{d}s.\end{aligned}$$

Using the fact that

$$2\|e\|\|x\| \leqslant \|e\|^2 + \|x\|^2 \ \text{ and } \ 2\|e\|\|x^\tau\| \leqslant \|e\|^2 + \|x^\tau\|^2,$$

we deduce that

$$\begin{aligned}\dot{W}(e_t) \leqslant &-\left[\frac{1}{2\varepsilon} - \gamma_1(\varepsilon)\|\widetilde{P}\| - \gamma_2(\varepsilon)\|\widetilde{P}\|\right]\|e\|^2 \\ &- \frac{e^{-2\sigma\tau}}{2\varepsilon}\|e^\tau\|^2 + \gamma_1(\varepsilon)\|\widetilde{P}\|\|x\|^2 \\ &+ \gamma_2(\varepsilon)\|\widetilde{P}\|\|x^\tau\|^2 - \frac{\sigma}{\varepsilon}\int\limits_{-\tau}^{0} e^{2\sigma s}\|e(s+t)\|^2 \mathrm{d}s. \end{aligned} \tag{16}$$

Let

$$U(x_t, e_t) = \eta W(e_t) + V(x_t),$$

where V is given by (8). Using (9) and (16), we get

$$\begin{aligned}\dot{U}(x_t, e_t) + 2\sigma U(x_t, e_t) \leqslant &-\eta\left[\frac{1}{2\varepsilon} - \gamma_1(\varepsilon)\|\widetilde{P}\| - \gamma_2(\varepsilon)\|\widetilde{P}\|\right]\|e\|^2 \\ &- \left[c(\varepsilon) - \eta\gamma_1(\varepsilon)\|\widetilde{P}\|\right]\|x\|^2 \\ &- \left[d(\varepsilon) - \eta\gamma_2(\varepsilon)\|\widetilde{P}\|\right]\|x^\tau\|^2. \end{aligned}$$

Finally, we select η such that

$$\eta < \min\left(\frac{c(\varepsilon)}{\gamma_1(\varepsilon)\|\widetilde{P}\|}, \frac{d(\varepsilon)}{\gamma_2(\varepsilon)\|\widetilde{P}\|}\right).$$

to get

$$\dot{U}(x_t, e_t) + 2\sigma U(x_t, e_t) \leqslant 0.$$

Therefore, the closed-loop system (7)–(14) is globally exponentially stable.

Corollary 2 *Suppose that* $(\mathcal{A}_1)$ *hold and there exist symmetric and positive definite matrices* $\widetilde{P}$ *and* $\widetilde{Q}$ *and positive constants* ε, σ *such that the following inequalities hold,*

$$\Upsilon_{ii}^T\widetilde{P} + \widetilde{P}\Upsilon_{ii} \leqslant -I, \quad i = 1, \ldots, r,$$

$$\Upsilon_{ij}^T\widetilde{P} + \widetilde{P}\Upsilon_{ij} \leqslant -I, \quad 1 \leqslant i < j \leqslant r,$$

and

$$\widetilde{P} < \widetilde{b}(\varepsilon)I, \tag{17}$$

where

$$\widetilde{b}(\varepsilon) = \frac{1}{2\varepsilon\big(\gamma_1(\varepsilon) + \gamma_2(\varepsilon)\big)},$$

$\gamma_1(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_1}(\varepsilon)$ *and* $\gamma_2(\varepsilon) = \sum_{i=1}^{r} \gamma_{i_2}(\varepsilon)$, *then the system error* (7)–(14) *is guaranteed to globally exponentially stable.*

Proof. The fact that the LMI inequality $\widetilde{P} < \widetilde{a}(\sigma, \varepsilon)I$, implies that

$$\sigma < \frac{\widetilde{c}(\varepsilon)}{2\varepsilon}.$$

where $\widetilde{c}(\varepsilon) = \dfrac{1}{2\varepsilon} - \gamma_1(\varepsilon)\|\widetilde{P}\| - \gamma_2(\varepsilon)\|\widetilde{P}\|$. Since, $\widetilde{P} < \widetilde{b}(\varepsilon)I$, then $c(\varepsilon) > 0$. Therefore, in view of (12), we choose

$$\sigma = \frac{1}{2} \min \left[\frac{\widetilde{c}(\varepsilon)}{2\varepsilon}, \frac{\alpha(\varepsilon)}{2\|P\|}, \frac{1}{2\tau} \ln\left(2\varepsilon\|P\|\gamma_2(\varepsilon)\right) \right] > 0.$$

Remark 2 *The conditions* (11) *and* (17) *do not depend on the delay* τ.

Next, we will consider an example to show the applicability of the main result where an observer-based controller for delay fuzzy system is presented. The closed-loop stability is guaranteed based on dual problems concerning the conception of the fuzzy controller and the design the fuzzy observer in presence of a small parameter.

4. Numerical example

Consider the following nonlinear fuzzy planar system,

$$\begin{cases} \dot{x}_1 = x_2 + \varepsilon x_3 \sin(x_2)\sin(x_3) + \varepsilon x_3(t-\tau)\cos(u), \\ \dot{x}_2 = x_3\,, \\ \dot{x}_3 = u\,, \\ \;y = x_1\,. \end{cases} \tag{18}$$

Now one can represent exactly the system by the following two-rule fuzzy model:
Rule 1: If z is F_{11} then

$$\begin{cases} \dot{x}(t) = A_1 x + B_1 u + D_1(\varepsilon) f_1(x, x^\tau, u), \\ y(t) = C_1 x. \end{cases}$$

Rule 2: If z is F_{21} then

$$\begin{cases} \dot{x}(t) = A_2x + B_2u + D_2(\varepsilon)f_2(x, x^\tau, u), \\ y(t) = C_2x, \end{cases}$$

where

$$z = \sin(x_2),$$

$$A_1 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$D_1(\varepsilon)f_1(x, x^\tau, u) = \varepsilon x_3 \sin(x_2)\sin(x_3)\cos(u) + \frac{1}{2\varepsilon}x_3(t-\tau)\sin(u),$$
$$D_2(\varepsilon)f_2(x, x^\tau, u) = -\varepsilon x_3 \sin(x_2)\sin(x_3)\cos(u) + \frac{1}{2\varepsilon}x_3(t-\tau)\sin(u),$$

$$F_{11} = \frac{\sin(x_2)+1}{2} \quad \text{and} \quad F_{21} = \frac{1-\sin(x_2)}{2}.$$

We define the membership functions for rule 1 and 2 as:

$$\mu_1(t) = \frac{1-\sin(x_2(t))}{2} \quad \text{and} \quad \mu_2(t) = 1 - \mu_1(t).$$

It is easy to check that system (18) satisfies Assumption $\mathcal{A}_1$ with $\gamma_{i_1}(\varepsilon) = \gamma_{i_2}(\varepsilon) = \varepsilon$, $i = 1, 2$. Using an LMI optimization algorithm, we obtain:

$$P = \begin{bmatrix} 1.5145 & 0.7341 & 0.0265 \\ 0.7341 & 1.4339 & 0.0548 \\ 0.0265 & 0.0548 & 0.0446 \end{bmatrix},$$

and the following feedback gains

$$K_1 = \begin{bmatrix} 17.4927 & 22.4490 & 8.5714 \end{bmatrix} \quad \text{and} \quad K_2 = \begin{bmatrix} 78.1250 & 62.500 & 12.5000 \end{bmatrix}.$$

Now concerning the observer, let's suppose the following fuzzy observer rules:

Rule 1: If z is F_{11} then

$$\begin{cases} \dot{\hat{x}} = A_1\hat{x} + B_1u - L_1(y - \hat{y}), \\ \hat{y} = C_1\hat{x}; \end{cases}$$

Rule 2: If z is F_{21} then

$$\begin{cases} \dot{\hat{x}} = A_2\hat{x} + B_2u - L_2\left(\hat{y} - y\right), \\ \hat{y} = C_2\hat{x}. \end{cases}$$

Then, we obtain the following observer gain:

$$L_1 = \begin{bmatrix} -34.9854 & -34.9854 & -34.9854 \end{bmatrix}^T$$

and

$$L_2 = \begin{bmatrix} -34.9854 & -34.9854 & -34.9854 \end{bmatrix}^T.$$

and the positive symmetric definite matrices:

$$\widetilde{P} = \begin{bmatrix} 0.8079 & -0.3374 & -0.4560 \\ -0.3374 & 0.7030 & -0.3425 \\ -0.4560 & -0.3425 & 0.7890 \end{bmatrix}.$$

The conditions (11) is satisfied with $\varepsilon < 0.2746$ and (17) is satisfied with $\varepsilon < 0.3645$. One can choose $\varepsilon = 0.2$. For our numerical simulation results given in Figs. 1, 2, and 3, we choose constant delay $\tau = 1$.

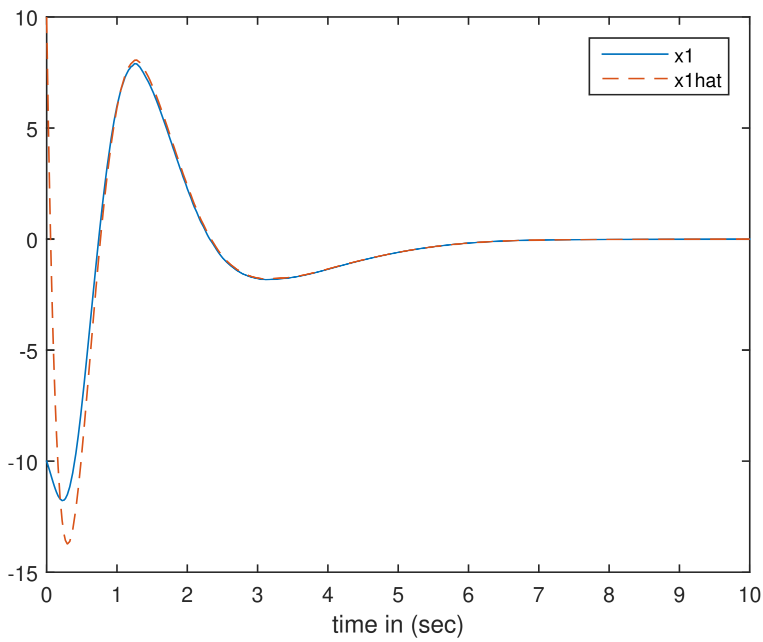

Figure 1: x_1 and its estimated $\widehat{x}_1$

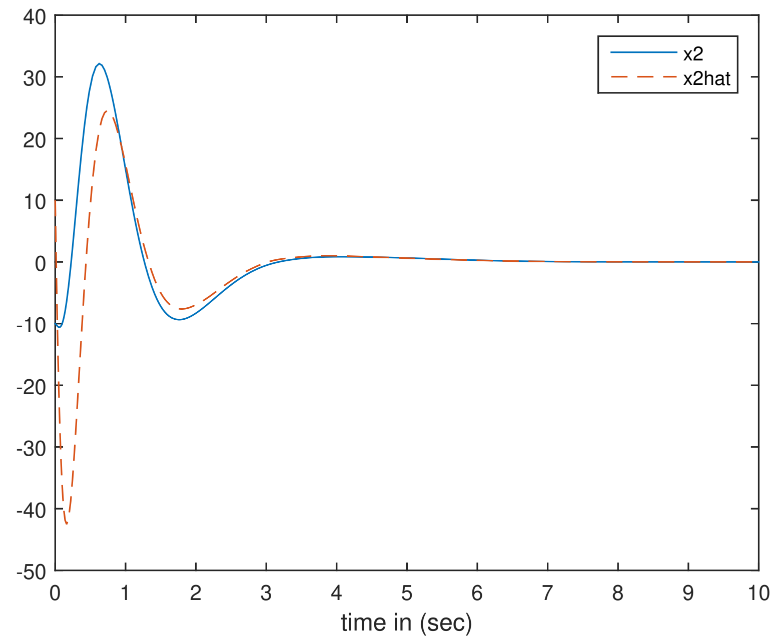

Figure 2: x_2 and its estimated $\widehat{x}_2$

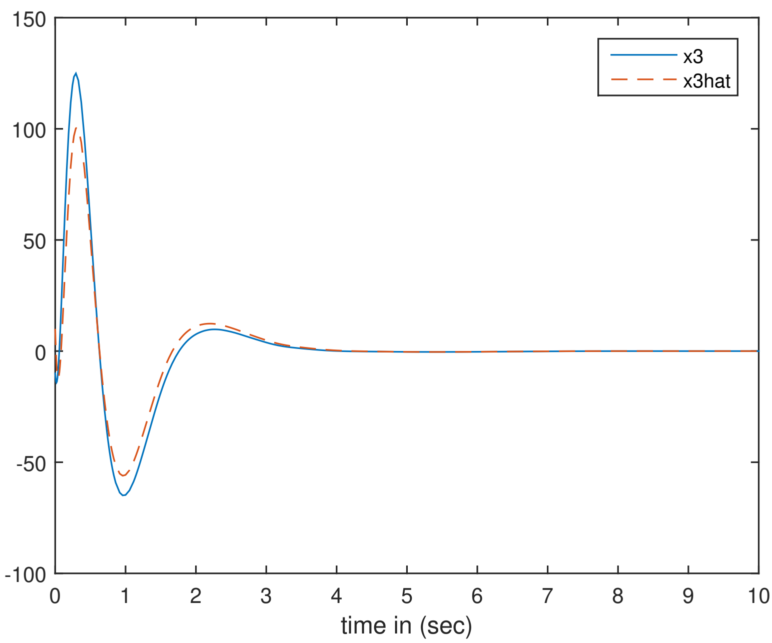

Figure 3: x_3 and its estimated $\widehat{x}_3$

5. Conclusion

In this paper, we have presented state and an estimated feedback controllers for a certain class of nonlinear time-delay fuzzy systems depending on a parameter. We have derived delay-independent conditions to ensure global exponential stability of the resulting closed-loop systems. We have shown that classical LMIs conditions can be used for both the observer and the controller, and they can be designed separately since a separation principle is available. The effectiveness of the proposed theory is illustrated by a computer simulation of a theoretical example.

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