GOSPODARKA SUROWCAMI MINERALNYMI – MINERAL RESOURCES MANAGEMENT

2023 Volume 39 Issue 1 Pages 149–171

DOI: 10.24425/gsm.2023.144631

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The selling price of raw material surpluses in an *n*-person market game model – exemplified by copper raw materials

Introduction

Mineral resources are a unique category of the naturally available components of the Earth. This particular category is overwhelmingly economically non-renewable, and is generally used up at a much faster rate than it is replenished. Minerals and raw materials obtained from them (useful products of mineral processing) are components of many production processes, and the economic development of countries is sometimes heavily dependent on minerals. The production and consumption of mineral resources can be a determinant of economic development and welfare (Sachs and Worner 1995, 1997; OECD 2008; OECD 2008a; Lei et al. 2013; Maier et al. 2014; Havranek et al. 2016; Anggraeni et al. 2017; Krausmann et al. 2018; Sanchez et al. 2020; Scheel et al. 2020; Xi et al. 2020; CIE 2021).

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Minerals are also unique goods traded on commodity and financial markets. On the commodity markets, transactions of raw materials refer to the physical exchange and trading of a commodity, while in the financial markets, various forms of money capital are bought and sold, at different terms and based on various financial instruments. It is noteworthy, that the largest and most important commodity exchanges are currently giant financial markets, apart from the physical exchange of mineral raw-materials. With regard to the physical (real) exchange of mineral raw materials, convenient places of sale may be a few commodity exchanges (e.g. London Metal Exchange – LME, Intercontinental Exchange – ICE, etc.), wherein the trading participants are provided with the same conditions for concluding exchange transactions with identical and simultaneous access to market information, e.g. stock quotes, turnover or stock-exchange reserves (LME 2018).

One of the more interesting aspects of commodity trading is the comprehensive supply-demand relationship between economic factors and consumer habits, which may lead to a high volatility of prices (Davis 2010; Henckens et al. 2016; Tokimatsu et al. 2017; Buchholz and Brandenburg 2018; Tilton 2018). In the entire assessment of the market quotations, it is important to realise that the mineral raw materials are products of a multi-stage and usually long chain of production cycles. The extraction of primary mineral resources is a complex process, comprising geological exploration, mining, processing, transport and final use. All of these activities are conducted in specific legal, social, fiscal and technological environments, and must be executed in line with the principles of economics and (in advanced societies) with respect for the natural environment (Maier et al. 2014; Sonderegger et al. 2020; Berger et al. 2020). While raw materials are key components of manufactured goods, their extraction and consumption depend mainly on factors such as climate, seasonality, industrial demand and resources. The static availability of the mineral resource base is determined not only by the current production/consumption and the volume of resources in the deposits. Competent exploration, technological progress, and advances in appropriate knowledge (Wright and Czelusta 2004; Tilton et al. 2018; Ponomarenko et al. 2021) and moreover recycling, anthropogenic deposits and the production of synthetic mineral resources may be crucial for maintaining the sustainability of the resource base (Meinert et al. 2016; IRP 2019).

The economic and social development of states is a dynamic and continuous process. The UNCTAD methodology (2019) defines three states of development of domestic economies: resource-independent countries, resource-dependent countries and highly resource-dependent countries. The resources generally mean mineral resources (metals, fossil fuels, rock minerals, etc.), but food products and agricultural produce should also be included in this group. UNCTAD experts recognize that the economies of countries in which the share of mineral and food products (often limited to only one raw material) exceeds the level of 60% in total exports become economically dependent on other countries. There are, of course, exceptions to this rule, although raw material monoculture is a burden rather than an advantage when it comes to economic development. UNCTAD analyses (UNCTAD 2019) show that in the years 2013–2017, in as many as 102 countries (out of 189 covered by the study), the share of raw materials in exports exceeded the level of 60%. The most difficult situation in this respect was South African countries, where the scale of dependence generally exceeded a level of 90%. Angola, the record-breaker in this context, had only one export product (crude oil) and a similar situation existed in Iraq and Chad. It is worth mentioning here that crude oil is the basic export product of many developing countries, and among the top ten countries most dependent on exports of raw materials, as many as seven owe their position to oil.

More economically developed countries (e.g. Norway or Russia) do not so often base their income solely on the export of hydrocarbons. However, none of the indicated states of economy are in a permanent state, rather they are subject to fluctuations. Such fluctuations are determined by, among other factors, prices of raw materials, the discovery of important and economically strategic mineral deposits, the world economy status, the development of alternative and modern industries (e.g. industry 4.0) and others. In some countries, mineral resources constitute an enormous (often the only) source of income and wealth. This abundance of resources, however, does not always lead to sustained economic growth and development and can have the opposite effect, which is called the "resource curse" or the so-called "Dutch disease" (Davis 1995; Mikesell 1997; Robinson et al. 2006; Same 2008; Mironov and Petronevich 2015; Bahar and Santos 2018; Damette and Seghir 2018; Marques and Pires 2019). Indeed, it is often the case that countries dependent on a select set of minerals have weaker state and local government institutions, spend less on education and are more corrupt (Di John 2011).

1. The export and import of mineral raw materials

The mining sector generally provides an insignificantly small number of jobs in mining. The efforts of national governments to generate more jobs are associated with the construction of processing and refining facilities. Such facilities help in the obtaining of more refined raw material, often of a significantly higher market price than only initially enriched minerals. Therefore, the export limitations of low-processed raw materials enforce the readiness and tendency of investors to create jobs on the domestic market. Raw-material export restrictions are also an instrument used to pursue other goals, among others, generating income for governments, controlling the export of illegally mined minerals, improving the condition and protection of the environment, balancing exchange rates or a set of diverse goals resulting from the preferences of influencers (van der Ploeg 2010).

Regardless of the reasons for export restrictions, the development of many countries in the world depends on the export of raw materials of various degrees of refinement/processing. Developed countries are assumed to be in a state of non-commodity dependency, while developing countries are more dependent on raw materials (Junne and Komen 1989; UNCTAD 2008; von der Goltz and Barnwal 2014; DEVE 2016; COE-RESOURCES 2016). The economically strongest Asian countries (Japan, South Korea, and the People's Republic of China) almost do not export minerals, and if so, they implement restrictions in the form of export quotas or customs duties. The aforementioned countries, especially China, are the world's largest importers of raw materials, making many countries dependent on exports, and this includes not only developing African, South American and Asian countries.

For decades, Europe has been dominated by the economic model based on processing of raw materials, and the lowest amounts of mineral resources among European Union countries are exported by the Czech Republic, Germany, Slovakia, Hungary and Ireland (EUROSTAT 2022).

The importance of Poland in terms of raw material exports, as compared with other European countries, is becoming smaller (Figure 1). Over the last 25 years, Polish exports increased tenfold, and the share of raw materials in it decreased from 29 to 20 percent.

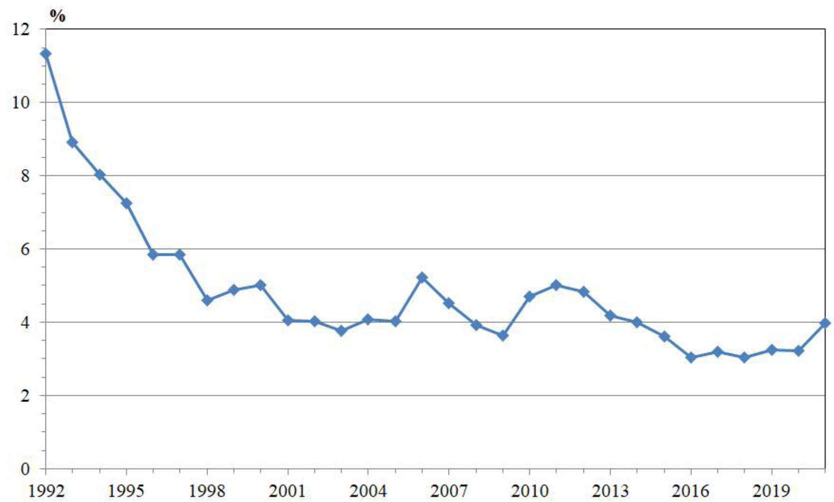

Fig. 1. The share of exports of metallic raw materials in Poland in relation to all exported goods (WORLDBANK 2022)

Rys. 1. Udział eksportu surowców metalicznych w Polsce w stosunku do ogółu eksportowanych towarów

Despite the relative wealth and abundance of the resource base (Szuflicki et al. eds. 2022), raw material production in Poland is characterized by a permanent surplus of imports over exports. In this regard, the qualitative balance of turnover is negative in all groups of materials, and only a few groups of raw materials are exported (Table 1). Of note, with regard to the listed mineral exports, rhenium should also be added to the raw materials listed in the table; however, real figures for rhenium are difficult to obtain as it is reported together with niobium. From among those indicated in Table 1, metallic raw materials (refined copper) and coking coal are of the greatest importance in terms of monetary value. In the total value of exports of minerals in 2020 (Figure 2), coke and semi-coke, petroleum products and hard coal had the largest share in the group of energy resources (PGI-NRI 2022). Moreover, raw materials and copper metallurgy products, precious metals, raw materials and iron and silver metallurgy products as well as aluminum raw materials dominated in the group of metallic raw materials. In addition, nitrogen and multi-component fertilizers, salt and sodium compounds, silicon and phosphorus dominated in the group of chemical raw materials, while insulation materials, refractory materials, gypsum, cement, as well as block and broken stones dominated in the group of rock raw materials.

Table 1. The percentage of domestic production of mineral raw materials exported in 2020 (acc. to Galos and Lewicka eds. 2021)

Tabela 1. Odsetek produkcji krajowej surowców mineralnych będący przedmiotem eksportu w roku 2020

Raw material	Export/domestic production (%)
Unalloyed tin	82
Metallic zinc	43
cadmium	96
Refined copper	52
Lead – concentrates	58
Refined lead	22
Selenium	26
Nickel sulphate	100
Silver	98
Coke coal	22
Gold	37
Gypsum and anhydrite	5
Synthetic graphite	95
Fire clay	78
Building and road stones	5
Kaolin	12
Industrial quartzite	18
Glass sand	11
Sulphur	16
Soda ash	57
Rock salt	8
Lime	6

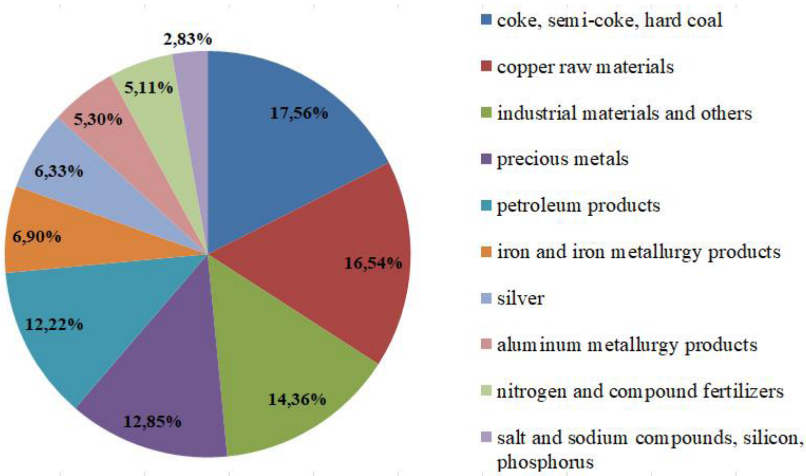

Fig. 2. Volume of Polish mineral resources exports in 2020 (PGI-NRI 2022)

Rys. 2. Wielkość eksportu polskich surowców mineralnych w 2020 r.

An important turning point in the balance of the Polish mineral resources turnover was the accession to the EU structures. Since 2004, significant changes, both in terms of quantity and value, have been recorded in the export and import of mineral resources. In 2005, the general level of trade did not change so much, while in 2006, the value of both exports and imports of mineral resources significantly increased, even though the volume of trade remained at a similar level as in the previous year. In the years 2007–2008, the volume and value of imports, however, increased significantly, while exports in terms of volume decreased. Unfortunately, the global economic crisis of 2009 resulted in a decrease in the level of trade in terms of the volume and value of mineral raw materials, both in export and import, but a reverse trend has been observed since 2010, with significantly higher quantitative and qualitative indexes (PGI-NRI 2022). The 2011–2020 decade had proved to be a period of economic fluctuations without any significant disturbances, but it is worth noting significant fluctuations in the field of imports and exports of energy resources (Table 2).

Table 2 does not contain natural gas (data covered by statistical confidentiality), mineral fertilizers and bitumen, but it is noteworthy, especially in the context of fuel and energy management, that the indicated lack of data on gas imports clearly reduces the total quantity and value of the imported mineral resources. Publicly available yearbooks of the Central Statistical Office only report the volume of gas imports (without specifying the directions of trade). Import of natural gas, from the level of 9–10 billion m^3 annually in the years 2009–2010, increased to approx. 14.95 billion m^3 in 2018, only to reach almost 16.5–16.7 billion m^3 in the years 2019–2020. Generally, natural gas exports are small – in 2019, they amounted to approx. 0.67 billion m^3. However, a year later, the number doubled and amounted to 1.32 billion m^3 (GUS 2021).

Table 2. Value of trade of mineral raw materials in Poland between 2011 and 2020 (mln PLN, acc. to Galos and Lewicka eds. 2021)

Tabela 2. Wartość obrotów surowcami mineralnymi w Polsce w dekadzie 2011–2020 (mln PLN)

Raw materials group/year	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Energy										
Export	471	583	1,077	1 031	384	286	488	488	455	228
Import	52,860	63,964	56,905	53,574	36,904	27,996	34,218	49,378	45,971	30,761
Balance	–52,389	–63,381	–55,828	–52,542	–36,519	–27,709	–33,730	–48,889	–45,516	–30,534
Metallic and metallurgical										
Export	17,596	19,573	16,700	16,455	16,348	14,553	15,670	15,454	16,839	16,870
Import	10,763	10,926	11,375	11,988	13,421	13,452	17,507	18,035	17,197	16,646
Balance	6,833	8,647	5 325	4,468	2,927	1,101	–1,837	–2,581	–358	225
Non-metallic										
Export	2,395	2,616	2,686	2,669	2,438	2,243	3,025	2,742	2,906	2,674
Import	5,695	5,685	5,146	5,221	4,923	4,963	5,398	6,280	6,923	6,837
Balance	–3,300	–3,069	–2,460	–2,552	–2,486	–2,720	–2,373	–3,539	–4,017	–4,163
Total										
Export	20,462	26,932	25,310	23,986	22,615	20,072	22,750	21,794	23,283	22,353
Import	69,317	85,459	77,756	74,816	58,256	49,234	62,459	80,556	78,079	58,483
Balance	–48,855	–58,527	–52,446	–50,829	–35,640	–29,162	–39,709	–58,762	–54,795	–36,130

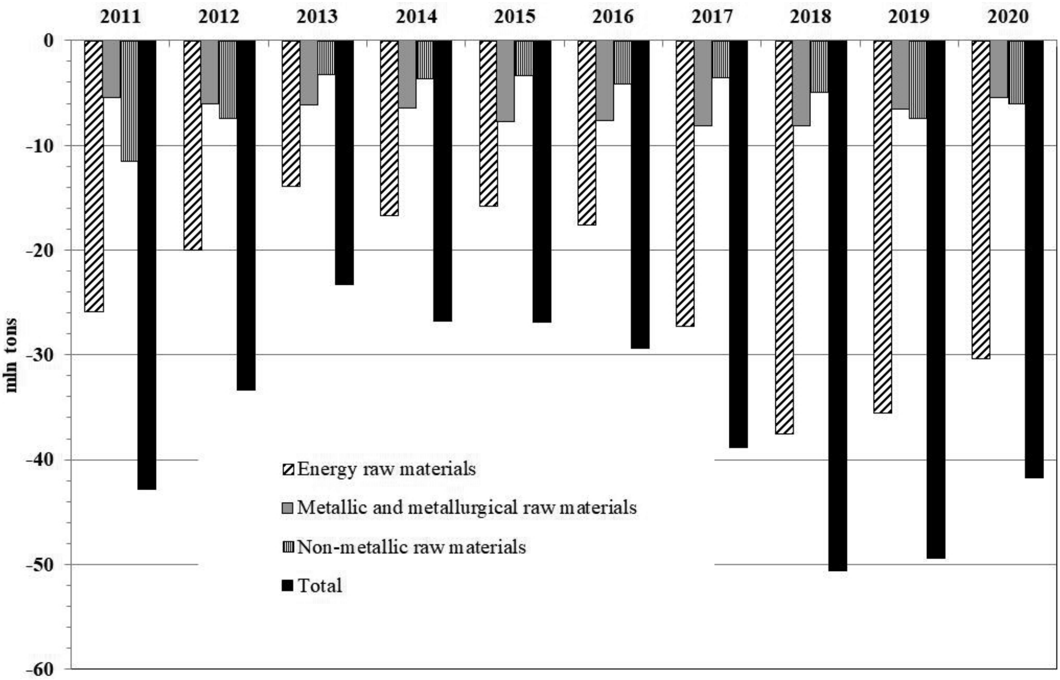

Fig. 3. The quantitative balance of trade in mineral resources in Poland (according to PIG-PIB 2021)

Rys. 3. Bilans ilościowy obrotów surowcami mineralnymi w Polsce

The balance of the turnover of raw materials remained negative in terms of quality and quantity between 2011 and 2020 (Figure 3, Table 2), and if the volume of natural gas imports to Poland was taken into account, the balance would be even worse.

In 2020, as in the previous years, energy resources had by far the largest share in the imported mineral resources value, while other resources (chemical and construction) had a marginal share. Surpluses were observed only in metallic and metallurgical raw materials, despite the unfavorable import/export volume ratio. Regarding the aforementioned, a positive balance of exports over imports is usually a result of favorable unit prices (sales of more expensive metals and the import of cheaper metals). Thus, it is worth noting that the volume of imported raw materials exceeds the export quotas in all groups. The negative unfavorable tendency of trade balances in the last decade (this trend has continued for decades) is subject to cyclical fluctuations, sometimes recording clear declines in value (2015–2016) and periods of generally long cycles with significant excesses of the value of imports over exports.

The dominant directions of Polish exports in terms of sales value have remained unchanged for years. The key partner is Germany. In 2020, about 25% of all mineral resources by value were exported to Germany, followed by the Czech Republic (~11%), the People's Republic of China (~6%) and the United Kingdom (~5%). In terms of quantity, the directions of exports are similar to the qualitative ones and include Germany, the Czech Republic and Austria, where in 2020, approximately 21, 20 and 6% of the total export volumes were sold, respectively (PGI-NRI 2022). As for import, the first three places in terms of the volume of mineral resources were taken by Russia, Ukraine and Germany. However, this classification is out of date due to the military conflict in Ukraine (Lewicka et al. 2022). It is worth adding that the value of the raw materials imported from Russia accounted for almost 40% of the total value of imported mineral products.

As mentioned before, the extraction and processing of copper and silver resources in Poland is currently the most important in terms of value, next to the exploitation of hard coal and lignite. The production of mined copper in Poland has exceeded the level of 400,000 thousand tonnes annually in the last decade, while the production of refined copper has generally exceeded 550,000 tonnes, of which more than half were exported (Table 3). According to the rules introduced by the WTO (World Trade Organization), Poland does not apply quota practices and restrictions in the form of customs duties, and surpluses of electrolytic copper and coke are sold on external markets.

Table 3. Production and export of refined copper in Poland (Szuflicki et al. 2022; Galos and Lewicka 2021), in thous. tons

Tabela 3. Produkcja i eksport miedzi rafinowanej w Polsce, w tys. ton

	2011	2012	2013	2014	2015	2016	2017	2018	2019	2020
Production	571.0	565.8	565.2	576.9	574.3	535.6	522.0	501.8	565.6	560.4
Export	327.7	333.3	343.5	308.8	298.2	253.8	251.7	209.7	287.3	291.8
Export/production ratio (%)	57.4	58.9	60.8	53.5	51.9	47.4	48.2	41.8	50.8	52.1

2. Outline of game theory

Game theory (GT) is a branch of mathematics that analyses situations of conflict or cooperation between multiple entities. Decisions are made in interactive conditions, where in addition to own usefulness, the player must take into account the usefulness and possible strategies of other players, and the basis for GT considerations is the presumption of all game participants behaving rationally. The fundamental terminology of game theory includes the following four basic concepts:

- Player (agent) – a reasoning, autonomous participant of the game (e.g. a single person, group of people, non-thinking participants, e.g. natural or economic conditions);
- Movement (strategy) – possible procedure or complete action plan that takes into account all possible situations;

- Payoff (winnings, usefulness) – the value of the game result (benefits) for players, both the measurable (e.g. financial profit) and the non-measurable, such as development or prestige;
- Information – knowledge or lack of knowledge of all possible moves in the course of the game including both one's own and one's opponents.

The first type of game considered by mathematicians was two-player zero-sum games. Such games are used to model a conflict situation in which cooperation between game participants is not possible. Both players, having opposing interests, compete with each other, striving to maximize their own payoffs, and their sum is zero. In non-zero-sum games, the interests of the players are neither exactly opposite nor fully compatible. Competition in the game is preserved, but cooperation between players is possible. It is worth noting that possible cooperation between players requires information exchange, which is not always possible.

Modelling behavior between more than two game participants is the domain of n-person games. N-person games are systems typical of real economic, political, social and other games. The same rules apply to these games as to two-player games, and the so-called "characteristic function" is used to define and present the games. A game in the form of a characteristic function is defined by a set of $N = \{1, 2, ..., n)$ players and "v" function that assigns a number $v(S)$ to each subset S (coalition) $S \subseteq N$. Other players that do not join the coalition are treated as the so-called "anti-coalition" $(N – S)$. $V(S)$ is the value of the win (expected value, price or game value) that the players belonging to S will gain if they form a coalition, regardless of the moves of the other participants of the game (Rapoport 1970; Luce and Raiffa 1989; Thomas 2003). For the correct description of the game, it is required to calculate the function value for each possible coalition, i.e. for each possible subset of players. The resulting function ν is called the game's "characteristic". From a formal point of view, it should be mentioned that the value of the coalition that no-one joined (empty) is 0, and that if two (or more) players or two (or more) disjointed coalitions decide to merge and form a larger alliance, they can always secure a payoff equal to at least the sum of the individual payoffs, although the coordination of moves generally leads to an increase in payoffs.

The use of game theory tools in *n*-person class games in the commodity markets has been the subject of a few research projects, mostly in relation to oil and gas trading and concession-tendering complications. Shenoy's (Shenoy 1980) pioneering analyses were aimed at identifying the imputation that defined the strategy of oil-importing countries; Hendricks et al. (Hendricks et al. 1994) proved the influence of information asymmetry on the strategic behavior of entities applying for exploration and production concessions in tender procedures with an implicit price; Porter (Porter 1995) specified the economic conditions of the decision-making environment and their impact on the strategic behavior of entities participating in tenders in the aspect of incomplete information; Grais and Zheng (Grais and Zheng 1996) proposed solutions to the Stackelberg game model for three gas-trading entities (supplying country, transit country and receiving country); Boyce (Boyce 1997) investigated the optimal strategies in business relations of three mining enterprises with different positions and market potential; Thomas (Thomas 2003) analyzed the interactions between crude oil consumers in the "Oil market game" model; Zweifel et al. (Zweifel et al. 2009) discussed cooperative and non-cooperative arrangements for the transport of Russian gas to Western Europe; Massol and Tchung-Ming (Massol and Tchung-Ming 2010) pointed to the need for logistical cooperation (especially in terms of transport) of LNG exporters; Krzak (Krzak 2012) considered the concepts of cooperation between local aggregate producers, and in a later monographic work (Krzak 2013), he formulated further proposals for the use of n-persons games with regard to the division of concessions and tenders for the exploration and production of hydrocarbons among many entities; Esmaeili et al. (Esmaeili et al. 2015) analyzed the issue of sharing access to collective oil and gas resources by looking for the concept of conflict solutions in the management of shared resources, while the Cournot-Nash game model in relation to the rare metals market was analyzed in the work of Ding et al. (Ding et al. 2021).

3. Export of copper raw materials – a game theory-based model of the market game

According to the data provided by the SWAiD analytical platform (Analysis and Decision Support System) of the Central Statistical Office, in the first quarter of 2022, copper was exported from Poland to several countries, including seven EU members. The highest export volumes were recorded for Germany, Italy and France (Table 4). The values presented in the table refer to refined copper in the form of cathodes and their parts, falling within CN code 740311. The values come from the INTRASTAT statistical system (a statistical system covering trade between EU Member States introduced in 1 January 1993 in the area of the European Single Market), kept by the Analytical Centre of the Tax Administration Chamber in Warsaw represent actual data from customs entries and declarations. Filling in the INTRASTAT declaration is required after exceeding a certain threshold value of goods, determined annually by the President of the Central Statistical Office and published in the form of a regulation of the Prime Minister. The INTRASTAT reports provide detailed statistics on copper exports, broken down by kind and type of raw material and product (e.g. copper matte, cement copper, unrefined copper, refined copper and unwrought copper alloys, copper waste and scrap, alloys, powders and flakes, rods and sections, wire, sheets and strips, sheets, tubes and pipes, connectors, etc.).

The three largest buyers of copper (Germany, Italy and France) were included in further considerations and modelling. The average price of refined copper was adopted based on the quotations of the London Metal Exchange for the period of January–March 2022, which is USD 9,900/t. To convert the value of sales into Polish Zloty, the tables of average exchange rates published by the National Bank of Poland were used, which in the said period was PLN 4.2 for one USD. The unit value of refined copper on the domestic market was equated with the average stock exchange price of USD 9,900/t (P). It was assumed that in the domestic market, potential buyers would pay no more than the current exchange price, and a higher sale price would discourage them from buying and they would start looking for a cheaper supplier. Foreign clients are willing to pay a little more and the tendency is unevenly defined. It was also assumed that the price does not include other transaction costs (transport, insurance, possible freight cost, risk, etc.) and is only the price of the raw material. Moreover, it was assumed that Italy declares the highest price for buying surpluses, while Germany declares the lowest.

Table 4. Directions and value of electrolytic copper exports to EU countries in the first quarter of 2022 (GUS-SWAID 2022)

Tabela 4. Kierunki i wartość eksportu miedzi elektrolitycznej do krajów UE w I kwartale 2022

Direction	Tonnage (t)	Value of PLN	Unit value (PLN/t)	Unit value (USD/t)
Germany	42,532.2	1,770,142,892	41,619	9,909
Italy	24,883.2	1,055,037,389	42,400	10,095
France	8,034.4	340,071,789	42,327	10,077
Czech	765.2	31,717,693	41,449	9,870
Austria	290.8	11,709,433	40,265	9,587
Belgium	216.7	8,639,017	39,869	9,493
Slovakia	46.0	1,985,340	43,160	10,276

The trading system described above can be modelled using a four-player game in which Germany (N) is willing to pay USD 9,909/t, France (F) 10,077 USD/t, and Italy (W) 10,095 USD/t. In addition, the following relationships between the offered prices of copper exists: the price on the domestic market is lower than the price paid by Germany, which is lower than the price paid by France, and finally the price paid by Italy is higher than the price paid by France (or may also be equal to it). Therefore, it is legitimate to express the relation with the following formula: $\nu(P) < \nu(N) < \nu(F) \leq \nu(W)$. The characteristic function of this game is as follows:

- $\nu(\varnothing) = 0$, by definition;
- $\nu(P) =$ USD 9,900/t, the coalition of Germany, France and Italy is not able to force Poland to sell the raw material, thus the remaining copper surplus in Poland is worth USD 9,900/t to that coalition;
- $\nu(N) = \nu(F) = \nu(W) = \nu(N, F) = \nu(N, W) = \nu(F, W) = \nu(N, F, W) = 0$, the potential buyers, individually or in any coalition, cannot force the seller to sell them the copper;

- $v(P, N)$ = 9,909 USD/t Poland and Germany may form a coalition where, for the price of 9,909 USD/t, the excess copper supply will be sold to Germany, thus Poland will improve its payoffs, while the other countries, i.e. France and Italy, would have to pay at least USD 9,909/t to be able to receive the raw material;
- $v(P, F) = v(P, N, F)$ = 10,077 USD/t, t, since Poland and France may conclude a transaction for the sale of the raw material at the price of USD 10,077/t, which is related to the fact that Germany would also have to purchase copper at this price;
- $v(P, W) = v(P, N, W) = v(P, F, W) = v(P, N, F, W)$ = 10,095 USD/t, by analogy, Poland and Italy may conclude an agreement on the supply of the raw material, thus Germany and France would be obliged to purchase copper at the same price, regardless of whether they are a three-person or four-person coalition.

By adopting the strategic position of Poland being expressed as striving to maximize payments, the most likely arrangement is a coalition of Poland and Italy, when $v(F) < v(W)$, or a coalition of Poland, France and Italy when $v(F) = v(W)$. Individual agreements between individual countries may, however, force different price fluctuations; however, further considerations are based on the assumption that it is Poland, as the owner of copper surpluses that decides on the coalition and seeks a solution that satisfies all parties willing to pay slightly different rates for the raw material. It was also preliminarily assumed that there is no alternative way of supplying the cheaper raw material from other sources (from other producers).

The above-mentioned values of the game in the form of a characteristic function are summarized in Table 5. Two questions arise here: which of the coalitions will be formed and how the payoffs will be divided. The second question is of key importance here, as the payoff of a single player is a more important element for him than the mere fact of forming a coalition and being a part of it. If x_i means a payoff of the n-th player, than the set of payoffs of all players is the n vector of real numbers $(x_1, x_2, ..., x_n)$ called "imputation (division) of the game". This vector should meet two basic conditions:

- individual rationality

$$\forall_{i \in N} x_i \geq v(\{i\}) \tag{1}$$

- collective rationality (Pareto optimality).

$$\sum_{i \in N} x_i = v(G) \tag{2}$$

Characteristic function $v(i)$ represents the i player payoff, i.e. the price paid by individual countries, which they guarantee without cooperation with other participants of the game, whereas the value $v(G)$ is the payoff of the so-called "grand coalition" when all participants of the trade decide to form it. The aforementioned imputation thus meets the criteria of individual and collective rationality, and the final result of the n-person game is to find a single

Table 5. Characteristic function of the market game of trading in copper raw materials surplus (coalition payoff value in USD/t)

Tabela 5. Funkcja charakterystyczna gry rynkowej obrotu nadwyżkami surowców miedzi (wartości wypłat koalicji w USD/t)

Coalition S	Ø	{P}	{N}	{F}	{W}	{P, N}	{P, F}	{P, W}
ν(S)	0	9,900	0	0	0	9,909	10,077	10,095

Coalition S	{N, F}	{N, W}	{F, W}	{P, N, F}	{P, N, W}	{P, F, W}	{N, F, W}	{P, N, F, W}
ν(S)	0	0	0	10,077	10,095	10,095	0	10,095

imputation or set of imputations that can provide a rational solution to the game defined in this way. Coalitions in this game do not provide any higher payoffs than the countries can achieve by selecting their strategies according to the characteristic function. Therefore, there is no incentive to form the coalitions, although it is important from the point of view of a strategic copper supplier to achieve the highest payoff, thus the search for a solution to the game seems justified.

The first to be indicated is the imputation belonging to the game core (Scarf 1967). If $x = (x_P, x_N, x_F, x_W)$ is the core of the game, then from the core conditions:

- $x_P + x_N + x_F + x_W = 10{,}095$,
- $x_P \geq 9{,}900$,
- $x_N \geq 0$,
- $x_F \geq 0$,
- $x_W \geq 0$,
- $x_P + x_N \geq 9{,}909$,
- $x_P + x_F \geq 10{,}077$,
- $x_P + x_W \geq 10{,}095$,
- $x_N + x_F \geq 0$
- $x_N + x_W \geq 0$,
- $x_F + x_W \geq 0$,
- $x_P + x_N + x_F \geq 10{,}077$,
- $x_P + x_N + x_W \geq 10{,}095$,
- $x_P + x_F + x_W \geq 10{,}095$,
- $x_N + x_F + x_W \geq 0$.

The fact that $x_P + x_F + x_W \geq 10{,}095$ and $x_P + x_N + x_F + x_W = 10{,}095$ results in $x_N = 0$, further assuming that $x_P + x_F \geq 10{,}077$ i $x_P + x_W \geq 10{,}095$ and $x_P + x_F + x_W = 10{,}095$ (assumed that $x_N = 0$) implies that $x_F = 0$. In the next step, it is known that $x_P + x_F \geq 10{,}077$, and the value of the total payment is $x_P + x_W = 10{,}095$, thus the core of the game is a single imputation $C(\nu) = \{(x, 0, 0, 10{,}095 - x)$, where $10{,}077 \leq x \leq 10{,}095$, which means the same as $\nu(F) \leq \nu(P) \leq \nu(W)$. When interpreting this result, it should be expected that Poland will sell the surplus back to Italy at the price of "*x*". Obviously, such a coalition will be most beneficial to the owner of the resource. However, the above imputation does not exhaust the statement made about the importance of forming a coalition to maximize the payment for the exported raw material. For this purpose, the so-called "Shapley value" was calculated, which determines the way of dividing the surplus (profit) in a cooperative game (Shapley 1952). Despite the usefulness requirements (it must be the same for each player), the Shapley value has the significant advantage of assigning only one solution to the game that takes into account the rationality and strength of individual players resulting from the characteristic function (Rapoport and Kahan 1984; Hausken and Mohr 2001). The value calculation is based on the use of the so-called "Shapley theorem" (Shapley 1952):

$$x_i(v) = \frac{1}{n!} \sum_{i \in S} (s-1)!C(n-s)!C\left[v(S) - v\left(S - \{i\}\right)\right] \tag{3}$$

s – number of members in the coalition,
n – number of all players,
$v(S)$ – the value of the payoff of a coalition composed of s members.

The set of values for all players (the so-called "Shapley game vector") is a certain proposal for the distribution of payoffs in a coalition game and due to the way it is constructed, it is interpreted as the average expected distribution in the game after it is played repeatedly. It is worth noting that a significant drawback of the Shapley's concept is that this imputation does not always belong to the core of the game when it is not empty. Apart from the mathematical calculations, the final value of Shapley's game vector is $x_S = \{x_P = 10{,}027.75$; $x_N = 0.75$; $x_F = 28.75$; $x_W = 37.75\}$. The highest value of the grand coalition is obviously introduced by the owner of the raw material, Poland, and the resulting vector of payments indicates, which does not come as a surprise, that Poland is most willing to cooperate with Italy. Cooperation with Italy is the most advantageous, and the proposed value of payoff for Poland (USD 10,027.75/t) provides information about the level of the expected payoff from the assumed maximum payoff of USD 10,095/t. Considering iteratively individual coalition arrangements, it is an advantage of the alliance of *P* and *N* that there is a higher price of the raw material and the difference between the values of *P* and *N* is the so-called "marginal contribution", USD 9/t in this case. Here, the value of the marginal contribution by France is already much higher.

For the current settlement amounts, Poland's total revenue from the sale of surplus copper to Germany, France and Italy amounts to USD 753,610,123. The value obtained in the Shapley imputation $v(P) = 10{,}027.75$ USD/t can be considered as the minimum price at which Poland will be willing to sell the surplus of the raw material, thus generating almost 3 million higher profits (total marginal contribution). Therefore, the question arises of how and whether to renegotiate the prices of supplies based on the obtained results. The willingness of countries (France and Italy) to pay more indicates the legitimacy of renegotiating prices and adopting them at the level resulting from the Shapley value. The new price, unlike the one offered by Germany, would undoubtedly be more attractive to these countries. It is convenient to adopt the same settlement price for countries, although it may not fully reflect the contractual arrangements. Perhaps it would be reasonable to consider how to divide the total marginal contribution among the countries purchasing the raw material based on the current values of settlements on the one hand, and taking into account the indications resulting from Shapley's value on the other. If France and Italy would still be willing to continue to pay the current rates, the price for Germany would be USD 9,979/t.

A slightly different interpretation and method of division is provided by the Gately point (Gately 1974). If a split payoff in the form of a Shapley imputation $x = (x_P, x_N, x_F, x_W)$ was proposed in the game and Poland is necessary for the formation of a grand coalition (it has a surplus of raw material), the situation with its failure to join the coalition will result in Germany, France and Italy losing jointly $x_N + x_F + x_W - v(N, F, W)$, i.e. the difference between what results from imputation and what they could gain in a situation of cooperation (the assumption that Poland is a reliable exporter and ensures the delivery of goods remains valid all the time). It is obvious that Poland will also suffer a loss by not joining the coalition ($x_P - v(P)$) and will be forced to sell the goods on the domestic market at USD 9,900/t or look for another market to sell. The ratio of these two values – the loss of the coalition of Germany, France and Italy and Poland's loss – is defined according to Gately's (1974) proposal as a tendency to break the grand coalition. In the analyzed example, Poland is the only player willing to break this coalition. Generally, for any n-person game, the inclination of the player i to break the grand coalition with imputation x is given by the expression (Gately 1974):

$$d_i(x) = \frac{\sum_{j \neq i} x_j - v(N - i)}{x_i - v(i)} \tag{4}$$

With a high rate of $d(x)$, a player may break the coalition and lose; however, the other players lose more. The player's maximum propensity to break a grand coalition will be minimized when the propensity coefficients are equal to each other. Again, omitting the mathematical approximations, the Gately's imputation amounts to $x_G = \{x_P = 10{,}067.40; x_N = 0.31; x_F = 11.80; x_W = 15.49\}$.

Similarly, assuming the Gately imputation as a starting point, Poland may demand a price of USD 10,067.40/t for the raw material and increase its total profit by almost USD 6 million. Then, if France and Italy remain ready to pay at the current level, the unit rate of copper raw material paid by Germany should amount to USD 10,049/t, which would be an amount higher by USD 140/t than the settlement price in the contracts concluded so far.

Conclusions

An attempt was made in this study to use the n-person model of the game to create a strategy of selling surplus raw materials. The analysis covered the refined copper surplus produced by the Polish copper industry and the largest importers of the internal market of the European Union. It was proposed to establish prices between recipients and producers based on the export quotas in Q1 2022. The price dispersion between the rates paid by the buyers was an attempt to evaluate the hypothetical coalitions between the trading participants, on the basis of which, the characteristic functions of the four-player game were determined, for which the solution indicated a possible mathematical approximation of the price diversification paid by the customers. Of the many approaches to solving n-person games, the game core, Shapley imputation and Gately point were selected. The latter is probably one of the most important and the most frequently used methods of fair distribution in game theory, but its importance decreases in games in which psychological factors or individual characteristics of players (e.g. the ability to bargain) play a significant role – and these are common to market entities in mass trade. Moreover, Gately point is not always a good solution for games in which grand coalition payoffs are lower than any payoffs for smaller coalitions, as it does not accurately reflect the power of the coalition; this was not the situation, however, in the analyzed case.

The position of Poland is of highest importance in all deliberations, and its decisions determine further decisions, generating possible, and multiple arbitration solutions. Gately's solution would generate higher revenues than Shapley's value and would probably be preferable by the raw material supplier, while Shapley's solution is more beneficial to customers. The reason for the hypothetical efforts of France and Italy to reduce the price paid (base rates above the game theory-based solutions) may be the level of the marginal contribution or the coalition strength indicator of the Gately's result. The adoption of any of the solutions would involve an increase in the rate paid by Germany each time. In Tables 6 and 7, several possible solutions are given.

The indicated scenarios can be a theoretical background for real negotiations. Undoubtedly, the privileged position of the supplier, assuming the lack of a competitive market, may, albeit in a narrow scope, indicate a limited price dictate. Buyers (France and Italy in particular) may, based on modelling, demand lower rates with respect to the baseline, which would always imply the increase of price for Germany, which if adopted as a benchmark, would be significantly lower and could be a kind of compensation for the largest purchase volume in terms of quantity. Much closer fee rates between buyers would define the adoption of Gately's point as the starting solution.

Apart from the inconveniences indicated above, both solutions indicated the legitimacy of renegotiating contracts, from the point of view of the producer and equally that of some recipients, resulting from the assumptions of the game based on the base sales rates. The obtained results enable various contractual arrangements in line with the preferences of trading participants as well as those resulting from the importance of the partners in the

Table 6. Selected raw material sales prices in relation to Shapley's value as the primary solution

Tabela 6. Wybrane ceny sprzedaży surowca w odniesieniu do wartości Shapleya jako rozwiązania prymarnego

	Base price (USD/t)	Base sales value (USD)	Sales value for the Shapley value (USD)	Underpayment (–)/ overpayment (+) (USD)	Sale prices (USD/t)			
					status quo (F) and (W)	status quo (F), Shapley value (W)	status quo (W), Shapley value (F)	marginal contribution reduction (F) and (W)
Germany	9,909.00	421,451,570	426,502,269	–5,050,699	9,979.10	10,018.45	9,988.41	10,007.00
France	10,077.00	80,962,649	80,566,955	395,694	10,077.00	10,077.00	10,027.75	10,046.25
Italy	10,095.00	251,195,904	249,522,509	1,673,395	10,095.00	10,027.75	10,095.00	10,057.25
Total		753,610,123	756,591,732					

Table 7. Selected raw material sales prices in relation to Gately point as the primary solution

Tabela 7. Wybrane ceny sprzedaży surowca w odniesieniu do wartości Gatelego jako rozwiązania prymarnego

	Base price (USD/t)	Base sales value (USD)	Sales value for the Shapley value (USD)	Underpayment (–)/ overpayment (+) (USD)	Sale prices (USD/t)				
					status quo (F) and (W)	status quo (F), Gately point (W)	status quo (W), Gately point (F)	coalition strength reduction (F) and (W)	coalition strength reduction (W) and Gately point (F)
Germany	9,909.00	421,451,570	428,188,670	–6,737,100	10,049.44	10,065.59	10,051.25	10,061.11	10,060.32
France	10,077.00	80,962,649	80,885,519	77,130	10,077.00	10,077.00	10,067.40	10,063.20	10,067.40
Italy	10,095.00	251,195,904	250,509,128	686,776	10,095.00	10,067.40	10,095.00	10,079.51	10,079.51
Total		753,610,123	759,583,317						

coalition. The proposed price variants obviously do not cover the entire pool of available resolutions, so perhaps it is justified to continue searching for an objectivized solution based on the rules of fair distribution typical of game theory by Brams and Taylor (Brams and Taylor 1996) or a more in-depth interpretation of the value of marginal contributions or coalition strength indicators. It is also important that the commodity markets are extremely dynamic markets, with generally unpredictable fluctuations and volatility, and the opportunity to link a fair imputation with, for example, the stock price, appears to be a tempting challenge.

This research and paper preparation was funded by AGH University of Science and Technology, Faculty of Geology, Geophysics and Environment Protection; subsidy number: 16.16.140.315.

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THE SELLING PRICE OF RAW MATERIAL SURPLUSES IN AN *n*-PERSON MARKET GAME MODEL – EXEMPLIFIED BY COPPER RAW MATERIALS

Keywords

raw-material trade, *n*-person game, Shapley value, Gately point

Abstract

Mineral-resource mining is a pillar of many state economies and, in many cases, it determines the welfare of the society. The mining of mineral resources provides the market with the raw materials that are traded and drives the economic and social development of countries, although it can also be a source of tensions and crises (e.g. the “curse of wealth”, “Dutch disease”). The trade of raw materials is conducted by exchanges, bilateral deals and other forms of transactions, and is regulated by trade regulations and contract agreements, and in most cases, constitutes a source of income for exporters. In this paper, the use of game-theory modelling for creating the selling price of mineral products on the basis of Polish export quotas for refined copper raw materials is proposed. Using a characteristic function created on the basis of reported export values, possible cooperation arrangements are defined and solutions are calculated for an *n*-person game of hypothetical coalitions of the major (in terms of volume) recipients of refined Polish copper, i.e. Germany, Italy and France. Alternative markets and possible supplies of cheaper raw material are excluded from the analyses, while the price spread between the rates paid by the buyers is taken into consideration. Among the many possibilities, the game core, the Shapley imputation and the Gately point are arbitrarily adopted as permissible solutions to the defined system. The obtained results are used for a speculative analysis relating to the possibility of renegotiating prices between the producer and recipients of the raw material. Marginal contributions resulting from Shapley's solution are taken into account as is the power of individual trading-participant coalitions. The paper demonstrates that the recognition and adoption of solutions based on the *n*-personnel game model as impartial would require the redefinition of contracts and the rates paid for the raw material.

CENA ZBYTU NADWYŻEK SUROWCOWYCH W MODELU *n*-OSOBOWEJ GRY RYNKOWEJ NA PRZYKŁADZIE SUROWCÓW MIEDZI

Słowa kluczowe

obrót surowcowy, gra *n*-osobowa, wartość Shapleya, punkt Gatelego

Streszczenie

Górnictwo kopalin bywa filarem gospodarek państw i decyduje niekiedy o poziomie dobrobytu społeczeństw. Produkcja górnicza, przeróbka i przetwórstwo kopalin dostarcza na rynek pożądane surowce, będące przedmiotem obrotu handlowego, warunkując rozwój gospodarczo-społeczny krajów, jakkolwiek bywa też źródłem napięć i kryzysów (np. klątwa bogactw, choroba holenderska). Obrót surowcowy, realizowany za pośrednictwem sformalizowanych rynków (giełd), wymian bilateralnych bądź innych form transakcji jest na ogół źródłem przychodów dla eksporterów i co równie ważne ujęty jest w karby regulacji handlowych i umów kontraktowych. W artykule, na bazie kwot eksportowych rafinowanych surowców miedzi z Polski, zaproponowano wykorzystanie modelowania teoriogrowego w zagadnieniu kreowania ceny sprzedaży produktów mineralnych. Za pomocą funkcji charakterystycznej, skonstruowanej na bazie raportowanych wartości eksportu, zdefiniowano możliwe układy kooperacyjne a następnie skalkulowano rozwiązania *n*-osobowej gry dla hipotetycznych koalicji, największych tonażowo odbiorców miedzi rafinowanej z Polski: Niemiec, Włoch i Francji. Przyjęto w analizie brak istnienia alternatywnych rynków i możliwych dostaw tańszego surowca stamtąd oraz uwzględniono rozrzut cenowy pomiędzy stawkami płaconymi przez nabywców. Spośród wielu możliwości, rdzeń gry, imputacja Shapleya oraz punkt Gatelytego zostały arbitralnie przyjęte jako dopuszczalne rozwiązania zdefiniowanego układu. Uzyskane wyniki wykorzystano do spekulatywnej analizy odnoszącej się do sposobności renegocjacji cen pomiędzy producentem a odbiorcami surowca. Uwzględniono w rozważaniach wkłady marginalne wynikające z rozwiązania Shapleya, jak również wzięto pod uwagę siłę koalicyjną poszczególnych uczestników obrotu. Wskazano, że uznanie i przyjęcie jako bezstronne rozwiązań opartych na modelu gry *n*-osobowej wymagałoby redefiniowania umów oraz stawek kwotowych płaconych za surowiec.