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Research paper

The application of the immanent tensegrity properties to control the behavior of double-layered grids

Justyna Tomasik[1], Paulina Obara[2]

Abstract: The paper focuses on the static behavior of double-layered tensegrity grids. Due to the specific characteristics, like the self-stress states and infinitesimal mechanisms, tensegrities can be used as deployable structures. For such structures, the possibility of the control of the behavior is very important. The main purpose of the work is to prove that the control of tensegrity structures with mechanisms is possible. The stiffness of such structures is found to depend not only on the geometry and material properties, but also on the initial prestress level and external load. In the case, when mechanisms do not exist, structures are insensitive to the initial prestress. It is possible to control the occurrence of mechanisms by changing the support conditions of the structure. Grids built with modified Simplex modules are considered. Two-stage analysis is performed. Firstly, the presence of the characteristic tensegrity features is examined and then, on that basis, the structures are classified into one of two classes. Next, the influence of the level of initial prestress on the behavior of structures under static load is analyzed. To evaluate this behavior, a geometrically non-linear model is used.

Keywords: double-layered tensegrity grids, infinitesimal mechanism, self-stress state, qualitative analysis, quantitative analysis

[1]MSc., Eng., Kielce University of Technology, al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland, e-mail: jtomasik@tu.kielce.pl, ORCID: 0000-0002-2055-6714
[2]DSc., PhD., Eng., Kielce University of Technology, al. Tysiąclecia Państwa Polskiego 7, 25-314 Kielce, Poland, e-mail: paula@tu.kielce.pl, ORCID: 0000-0001-8994-4419

1. Introduction

Tensegrities are the structures composed of compressed elements (struts) and tensed elements (cables). However, these systems distinguish from conventional systems rod-like structures due to the existence of some specific mechanical and mathematical properties. Some tensegrities are characterized by the presence of the self-stress states and the mechanisms. The self-stress state can be defined as a system of self-equilibrated normal forces that satisfy homogeneous equations of equilibrium. The absence of those forces makes tensegrity structures unstable, i.e. geometrically variable. To ensure the stabilization, initial prestress must be introduced to the structure. Additionally, the modification of the prestress level enables the control of the static parameters of the structure.

The above mentioned features are auspicious in the context of the possible use of the tensegrity systems in adaptive and deployable structures, for example footbridges. In [1–3], the authors proposed a footbridge built with pentagonal ring tensegrity modules and presented schemes of folding/unfolding of such structure. Due to the use of continuous cables, the number of actuators is diminished. In [4,5], the lightweight tensegrity structure built with V-expander module is proposed as a solution which enables the access to the shore for people with disabilities. Numerical and experimental studies were performed. In [6], a structure built with an expanded Octahedron module is presented. In order to ensure the stability, additional cables had to be added and that example is not characterized by the presence of mechanisms. In turn, in [7] a footbridge built with modified Simplex modules is considered in terms of the smart structures and different aspects of the "smartness" were analyzed.

In the paper, deployable footbridges built with modified Simplex modules are considered. The aim of the work is to prove that the controlling the behaviour of these structures is possible. For this purpose, the parametric analysis, including the influence of the level of initial prestress and the change of support conditions, is carried out. A nonlinear static analysis assuming the hypothesis of large displacements is used. The analysis contains in two steps. Firstly, the immanent tensegrity properties, that is, self-stress states and infinitesimal mechanisms are identified (qualitative analysis). Depending on the presence of those features, the behavior of double-layered tensegrity grids under external load differs significantly and this is considered in the second step of the analysis (quantitative analysis). At this stage, the impact of the level of initial prestress on the displacements and effort of structures is investigated.

2. Mathematical description

The specificity of tensegrity lies in the fact that the self-stress states stabilize the existing infinitesimal mechanisms. It should be noted that the self-stress states also occur in geometrically invariable structures. In the paper, the finite elements method is used [8–12]. In a global coordinate system $(x,\ y,\ z)$, a finite element e is described by a Young's modulus E^e, a cross-sectional area A^e, a length L^e and by a compatibility matrix $\mathbf{B}^e$ $(\in \mathbb{R}^{1\times 6})$ [11, 13]:

$$\mathbf{B}^e = \begin{bmatrix} -\mathrm{c}_x & -\mathrm{c}_y & -\mathrm{c}_z & \mathrm{c}_x & \mathrm{c}_y & \mathrm{c}_z \end{bmatrix} \tag{2.1}$$

where: c_i – directional cosines: $\mathrm{c}_x = \dfrac{x_j - x_i}{L^e}$, $\mathrm{c}_y = \dfrac{y_j - y_i}{L^e}$, $\mathrm{c}_z = \dfrac{z_j - z_i}{L^e}$.

The analysis is provided for n-element space truss ($e = 1, 2, \ldots, n$) described by the elasticity matrix $\mathbf{E}$ $(\in \mathbb{R}^{n\times n})$ = diag $\begin{bmatrix} \dfrac{E^1A^1}{L^1} & \dfrac{E^2A^2}{L^2} & \ldots & \dfrac{E^nA^n}{L^n} \end{bmatrix}$ with m – degrees of freedom $\mathbf{q}$ $\left(\in \mathbb{R}^{m\times 1}\right) = [q_1 \ \ q_2 \ \ \ldots \ \ q_m]^T$. The compatibility matrix $\mathbf{B}$ $(\in \mathbb{R}^{n\times m})$ for tensegrity structures is determined using the finite element formalism [8–10, 14–20]: $\mathbf{B} = \left[\mathbf{B}^1\mathbf{C}^1 \ \ \mathbf{B}^2\mathbf{C}^2 \ \ \ldots \ \ \mathbf{B}^n\mathbf{C}^n\right]^T$, where: $\mathbf{C}^e$ $\left(\in \mathbb{R}^{6\times m}\right)$ – a Boolean matrix.

The complete analysis of tensegrity structures is a two-stage process. The first stage is a qualitative analysis, which includes the identification of self-stress states and infinitesimal mechanisms. The second stage, so-called quantitative analysis, focuses on the behavior of tensegrities under external loads.

2.1. Qualitative analysis

The qualitative analysis can be done through the singular value decomposition of the compatibility matrix $\mathbf{B}$ – Eq. (2.1): $\mathbf{B} = \mathbf{Y}\,\mathbf{N}\,\mathbf{X}^T$, where: $\mathbf{Y}$ $(\in \mathbb{R}^{n\times n}) = \left[\mathbf{y}_1 \ \ \mathbf{y}_2 \ \ \ldots \ \ \mathbf{y}_n\right]$, $\mathbf{X}$ $(\in \mathbb{R}^{m\times m}) = \left[\mathbf{x}_1 \ \ \mathbf{x}_2 \ \ \ldots \ \ \mathbf{x}_m\right]$ – orthogonal matrices, $\mathbf{N}$ $(\in \mathbb{R}^{n\times m})$ – a rectangular diagonal matrix [11, 21–28]. The orthogonal matrices $\mathbf{Y}$ and $\mathbf{X}$ as well as matrix $\mathbf{N}$ are related to the eigenvectors and eigenvalues of the following problems:

$$\left(\mathbf{B}\mathbf{B}^T - \mu\mathbf{I}\right)\mathbf{y} = \mathbf{0}, \qquad \left(\mathbf{B}^T\mathbf{B} - \lambda\mathbf{I}\right)\mathbf{x} = \mathbf{0} \tag{2.2}$$

where: μ, λ – eigenvalues of the respective matrix.

The existence of self-stress states and infinitesimal mechanisms depends on the existence of zero eigenvalues μ_i = and $\lambda_i = 0$, respectively. The self-stress state is considered as an eigenvector $\mathbf{y}_i = \mathbf{S}(\mu_i = 0)$ related to zero eigenvalue of the matrix Eq. $(2.2)_1$, whereas the mechanism is understood as an eigenvector $\mathbf{x}_i = \mathbf{q}(\lambda_i = 0)$ related to zero eigenvalue of the matrix Eq. $(2.2)_2$. The identification of the self-stress state $\mathbf{S}$ enables the formation of the geometric stiffness matrix $\mathbf{K}_G(\mathbf{S})$ $(\in \mathbb{R}^{m\times m})$. In order to identify whether the mechanism is infinitesimal or finite, the spectral analysis of the stiffness matrix with the regard of the effect of self-equilibrated forces should be provided: $(\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}) - \sigma\mathbf{I})\,\mathbf{z} = \mathbf{0}$, where: $\mathbf{K}_L = \mathbf{B}^T\mathbf{E}\mathbf{B}$ – the linear stiffness matrix. If all eigenvalues σ are positive, the identified mechanism is infinitesimal and the structure is stable. Zero eigenvalues are related to finite mechanisms, whereas a negative eigenvalue represents instability of the structure.

The qualitative analysis leads to the classification of the structures to the one of four classes [11, 28]. The classification is based on the identification of six tensegrity features, i.e. the structures are trusses (T) with at least one existing self-stress state (S) and infinitesimal mechanism (M), their elements form a discontinuous set of compressed elements (D)

which is contained within a continuous net of tensile elements (I), tensile elements are cables with zero compression rigidity (C).

Generally, double-layer tensegrity grids are built from basic tensegrity modules connected in a contiguous configuration (struts are connected to each other) or a non-contiguous configuration (maintaining a discontinuous arrangement of compressed elements). Modules can be connected edge-to-edge, node–node or strut–cable. The double-layered tensegrity grids considered in this work do not satisfy the requirements of the feature (D) because of the method of connecting modules (node–node). The analyzed grids are classified as structures with tensegrity features of class 1 or structures with tensegrity features of class 2. In the first case, grids are featured by five characteristics T, S, I, C and M, whereas in the second – the mechanisms (M) are not identified. This classification is very important due to the occurrence of immanent tensegrity features what affects the behavior of structures under external actions.

2.2. Quantitative analysis

In the case of classical lattice structures, quantitative analysis can be carried out assuming small displacements, i.e. a linear geometric model. However, this approach is inappropriate for tensegrity systems. The nonlinear analysis (third order theory) assuming the hypothesis of large displacements is used:

$$\left[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S}) + \mathbf{K}_{N,NL}(\mathbf{q})\right] = \mathbf{P} \tag{2.3}$$

where: $\mathbf{P}$ – the load vector, $\mathbf{K}_{N,NL}(\mathbf{q})$ – the non-linear displacement stiffness matrix.

Additionally, in order to illustrate the influence of external loads on the stiffening of considered structures, the quasi-linear approach (second order theory) is used as well:

$$[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S})] = \mathbf{P} \tag{2.4}$$

The explicit forms of stiffness matrices $\mathbf{K}_L$, $\mathbf{K}_G$ $(\mathbf{S})$, $\mathbf{K}_{N,NL}(\mathbf{q})$ can be found for example in [12]. The qualitative analysis is parametric because the normal forces $\mathbf{N}$ are determined as a function of the initial prestress forces S: $\mathbf{N} = \mathbf{y}_i S$, where: $\mathbf{y}_i$ – the normalised vector of the self-stress state determined in the qualitative analysis.

The quantitative analysis leads to the determination of the impact of initial prestress level S on the behavior of structures under static load. The consideration contains the determination of the minimum ($S_{\min}$) and maximum ($S_{\max}$) initial prestress level, the assessment of the influence of initial prestress level on the displacements $\mathbf{q}$ and the assessment of the influence of the initial prestress level on the effort of the structure $W_{\max} = N_{\max}/N_{Rd}$ (where: $N_{\max}$ is the maximum normal force and N_{Rd} is the load-bearing capacity).

3. Results

In this paper, the qualitative and quantitative analyses of double-layered tensegrity grids are performed. The structures built with modified Simplex modules are considered. The modified Simplex module (Fig. 1a) consists of twelve elements ($n = 12$), i.e, three struts and nine cables, six nodes ($w = 6$) and, in contrast to the normal module (Fig. 1b), the top surface of the module is inscribed into the bottom one. This modification allows the easy connection of single units into multi-module structures. Because of the above mentioned reasons, the modified module is chosen to build the considered structures, which can be used as deployable footbridges. Four grids consisting of six – MS6 (Fig. 2a), ten – MS10 (Fig. 2b), fourteen – MS14 (Fig. 2c) and eighteen – MS18 (Fig. 2d) modules are taken into account. The conducted considerations help to understand the behavior of the structure built with n modules (Fig. 2e). In all figures, the cables are marked in red (bottom), green (top) and blue (diagonal), whereas the struts – in black. The different colors of cables correspond to the different values of the self-stress state.

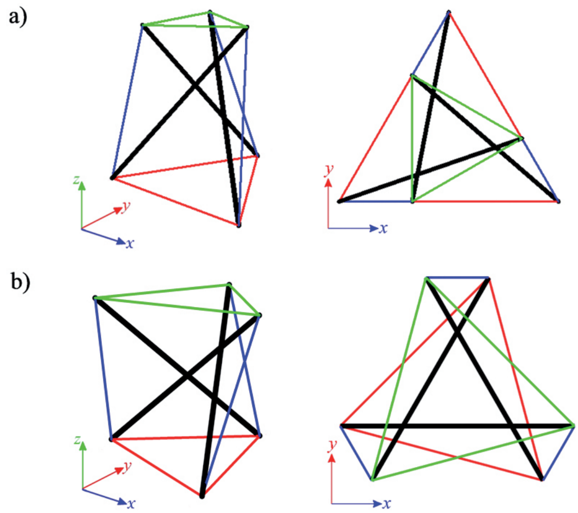


Fig. 1. Single Simplex module: a) modified, b) normal

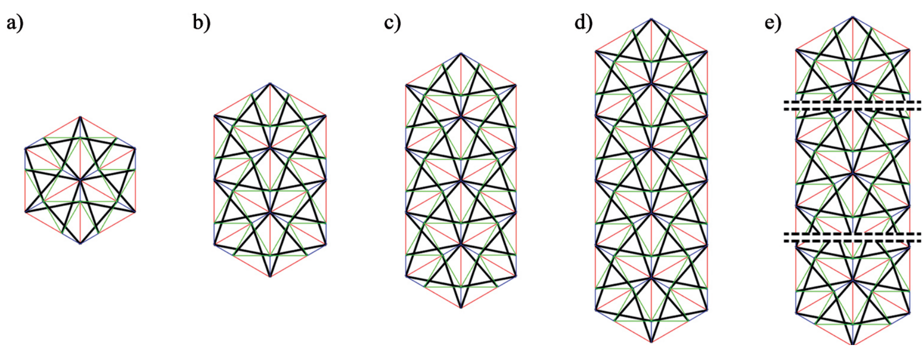


Fig. 2. Top view of double-layered tensegrity grids models: a) MS6, b) MS10, c) MS14, d) MS18, e) MSn

The parametric analysis, including the influence of the level of initial prestress and the change of support conditions on maximum displacement of nodes in the z direction and the effort of the structure W_{max} is investigated. Three support conditions are considered:

– model MSn–1 – grid simply supported in three bottom nodes (Fig. 3a),
– model MSn–2 – grid simply supported in four bottom nodes (Fig. 3b),
– model MSn–3 – grid simply supported in all bottom boundary nodes (Fig. 3b).

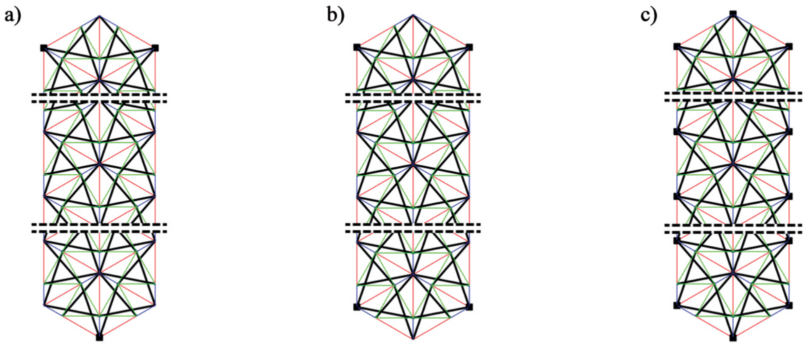


Fig. 3. Support conditions: a) MSn–1, b) MSn–2, c) MSn–3

The considerations are of cognitive nature; therefore all models are loaded with the concentrated vertical forces $P_z = -1$ kN applied to all top nodes (Fig. 4). The design solution of the Halfen DETAN Rod System is used and, for the adopted case of load, the following characteristics are assumed:

– Young modulus: $E = 210$ GPa and density: $\rho = 7860$ kg/m^3,
– cables: made of rods, steel S460N, diameter $\phi = 20$ mm, load-bearing capacity: $N_{Rd} = 110.2$ kN,
– struts: made of hot-finished circular hollow section, steel: S355J2, diameter: $\phi = 76.1$ mm, thickness: $t = 2.9$ mm, load-bearing capacity: $N_{Rd} = 203.5$ kN.

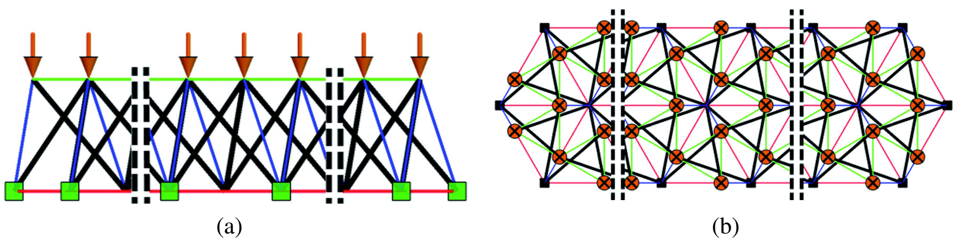


Fig. 4. The model of the load for MSn: a) front view, b) top view

The quasi-linear (II order theory) and non-linear analyses (III order theory) are carried out. For calculation, a procedure in the Mathematica environment was created.

3.1. Qualitative analysis

Firstly, the qualitative analysis was performed for the simple module with twelve degrees of freedom ($m = 12$) (the blocked displacements are q_1, q_3, q_5, q_6, q_7, q_9) thus the number of elements and the number of degrees of freedom are equal ($n = m = 12$). The compatibility matrix $\mathbf{B}$ $\left(\in \mathbb{R}^{12\times12}\right)$ is square; therefore the matrices $\mathbf{BB}^T$ and $\mathbf{B}^T\mathbf{B}$ are equal. There is one zero eigenvalue in both matrices, thus one self-stress state (Fig. 5) and one mechanism is identified. All eigenvalues of the matrix $[\mathbf{K}_L + \mathbf{K}_G(\mathbf{S})]$ are positive so the identified mechanism is infinitesimal and the stability of the structure is ensured. The single modified Simplex module is characterized by all tensegrity features, i.e.T, S, M, I, C and D, it means that it can be classified as the ideal tensegrity.

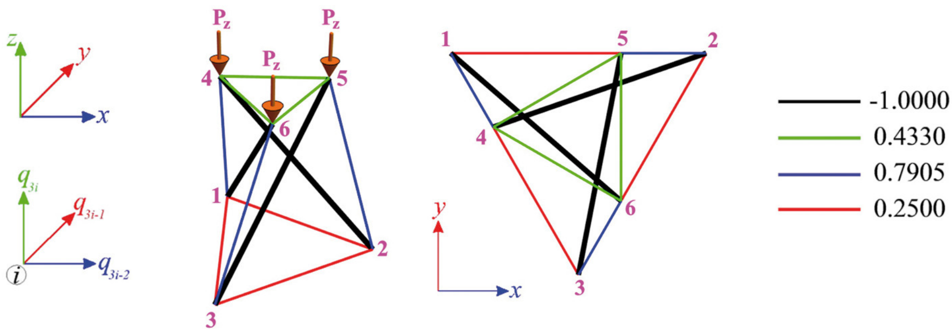


Fig. 5. Normalized self-stress state of the single modified Simplex module

The summarized results of the qualitative analysis of the double-layered tensegrity grids are presented in Table 1. All models are characterized by the following features: T, S, I and C, however, due to the connection of the modules in the node-node system, in all multi module structures the discontinuity of struts is not preserved (D).

In case of six-module double-layered grids (MS6), all models (MS6–1, MS6–2, MS6–3) are characterized by the presence of the mechanism (M) and they are classified as structures with tensegrity features of class 1. Interestingly, for structures built with ten ($n = 10$), fourteen ($n = 14$) and eighteen ($n = 18$) modules, the mechanism is present only in models MSn–1 and MSn–2 and they are included into structures with tensegrity features of class 1. The models MSn–3 lack mechanisms and are classified as structure with tensegrity features of class 2.

For all models a lot of the self-stress states were identified (from 13 to 63). Unfortunately, none of them identifies correctly the type of elements (that is, what is a strut and what is a cable). So, in the quantitative analyses the normalized self-stress state for the single modified Simplex module is taken into account (Fig. 3). The complete analysis of the single module is contained in [13, 29, 30].

Table 1. Results of the qualitative analysis of the double layer tensegrity grids

Model	No. of nodes (w)	No. of elements (n)	No. of degrees of freedom (m)	No. of mechanisms (M)	No. of self-stress states (S)	Structure with tensegrity features of:
MS6–1	19	60	48	1	13	class 1
MS6–2			45	1	16	
MS6–3			39	1	22	
MS10–1	29	98	78	1	21	class 1
MS10–2			75	1	24	
MS10–3			53	0	35	class 2
MS14–1	39	136	108	1	29	class 1
MS14–2			105	1	32	
MS14–3			87	0	49	class 2
MS18–1	49	174	138	1	37	class 1
MS18–2			174	1	40	
MS18–3			111	0	63	class 2

3.2. Quantitative analysis

The range of the prestress forces S must be determined in the way that the lowest level of initial prestress $S_{\min}$ must ensure an appropriate identification of the type of element (cables or struts), whereas maximum $S_{\max}$ cannot cause the exceedance of the load-bearing capacity of elements. Generally, if the structure is classified as one with tensegrity features of class 1, the minimum level of self-stress state $S_{\min}$ increases with the number of modules used in given model (Table 2). If the structure is categorized as one with tensegrity features of class 2, the minimum level of self-stress stays at low level, i.e. $S_{\min} = 0.01$ kN. The maximum level of prestress $S_{\max}$ is the same for all models and is equal to $S_{\max} = 60$ kN, so the maximum load-bearing capacity ratio differentiate between 87% (MS6–3) and 95% (MS18–1).

Table 2. Influence the support conditions MSn–i ($n = 6, 10, 14, 18$; $i = 1, 2, 3$) on the values of the minimum level of prestress $S_{\min}$

	MS6			MS10			MS14			MS18		
i	1	2	3	1	2	3	1	2	3	1	2	3
$S_{\min}$ [kN]	1	1	0.01	6.5	6	2	17.5	15.5	2	31.6	29	2

Maximum displacements in z direction q_z are shown in Fig. 6. Table 3 contains the comparison between displacements calculated using second (II) and third (III) order theory for three levels of self-stress: minimum – $S_{\min}$, intermediate – chosen as S_{int} = 40 kN and maximum – $S_{\max}$. The most significant difference between the quasi-linear and non-linear approach can be observed for the structure built with six modified Simplex module for the lowest level of self-stress $S_{\min}$. However, the results converge quickly and the difference decreases significantly. Six-module double-layered grid is also the least sensitive for the change of support conditions. For the rest of the models, static scheme affects the behavior of the structure relevantly. The lowest displacement is obtained for models MSn–3, while the highest – for models MSn–1.

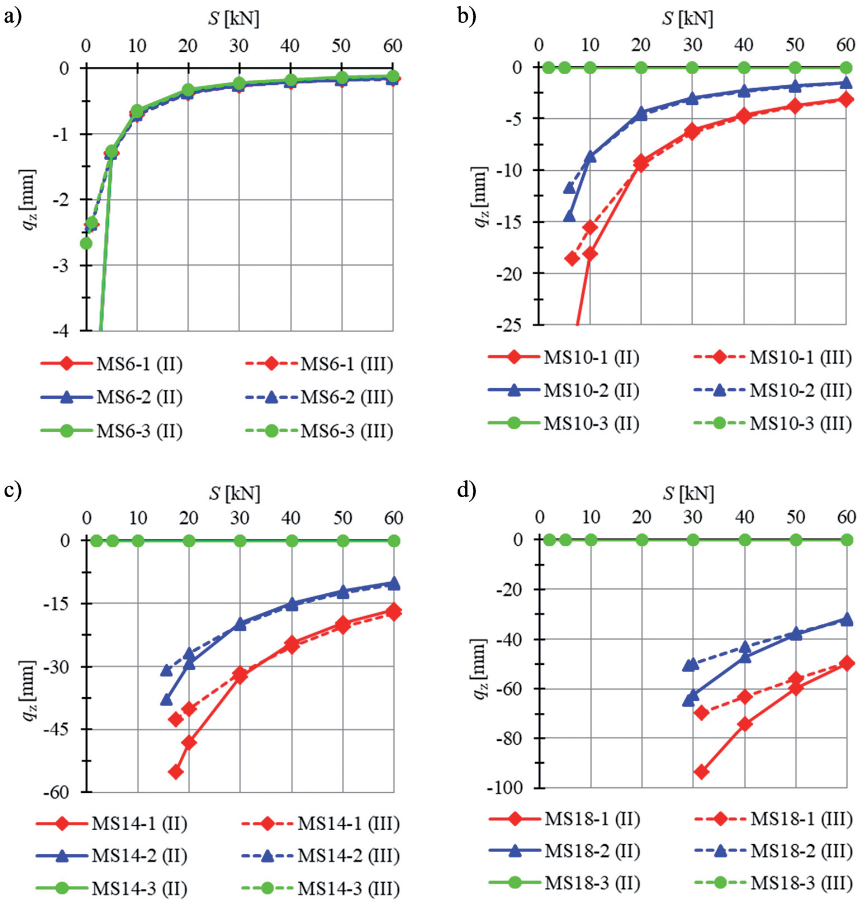
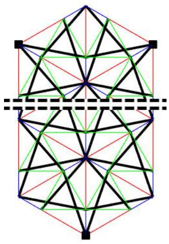
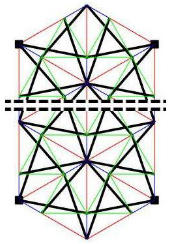
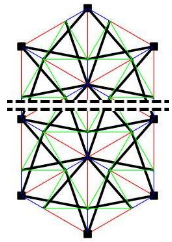


Fig. 6. Influence of the initial prestress on the maximum displacement q_z for models: a) MS6, b) MS10, c) MS14, d) MS18

The influence of the initial prestress level on the effort of the structures $W_{\max}$ is presented in Fig. 7. Comparing results attained for various support conditions, there is no significance difference for models MS6–i (Fig. 7a) and MS10–i (Fig. 7b). For these models, the relation between the level of initial prestress and the effort of the structures is

Table 3. Comparison of the maximum displacement q_z [mm] calculated using the second (II) and third (III) order theory for models MS$n - i$ (n = 6, 10, 14, 18; i = 1, 2, 3)

i	1			2			3		
									
	II	III	error [%]	II	III	error [%]	II	III	error [%]
n	6								
S_{min}	−6.22	−2.38	161.3	−6.22	−2.38	161.2	−614.8	−2.67	$2.3 \cdot 10^4$
S_{int}	−0.21	−0.21	1.47	−0.21	−0.21	1.5	−0.17	−0.17	1.8
S_{max}	−0.15	−0.16	0.9	−0.15	−0.16	0.9	−0.12	−0.12	1.1
n	10								
S_{min}	−27.80	−18.54	49.6	−14.43	−11.64	24.0			
S_{int}	−4.60	−4.81	4.4	−2.24	−2.33	3.7	−0.02	−0.02	0.0
S_{max}	−3.10	−3.21	3.2	−1.52	−1.56	2.6			
n	14								
S_{min}	−55.09	−42.53	29.5	−37.75	−30.75	22.8			
S_{int}	−24.3	−25.1	3.1	−14.80	−15.45	4.2	−0.03	−0.03	0.0
S_{max}	−16.39	−17.27	5.1	−9.96	−10.41	4.4			
n	18								
S_{min}	−93.5	−69.5	34.5	−64.8	−50.6	28.1			
S_{int}	−74.2	−63.0	17.6	−47.2	−43.1	9.5	−0.03	−0.03	0.0
S_{max}	−49.8	−49.5	0.6	−31.7	−32.4	1.9			

almost linear for all considered static schemes ($I = 1, 2, 3$). For other models, i.e. MS14–i (Fig. 7c) and MS18–i (Fig. 7d) this relation remains linear only for MSn–3, which is not characterized by mechanisms. For models MSn–1 and MSn–2, the effort is not linearly dependent on the initial prestress level. Similarly to the displacements, the effort of the structures varies when the support conditions change. Models MSn–3 are characterized by the lowest effort, while models MSn–1 – the highest. Additionally, with the increase of the level initial prestress, the difference between the efforts obtained for different support conditions decreases.

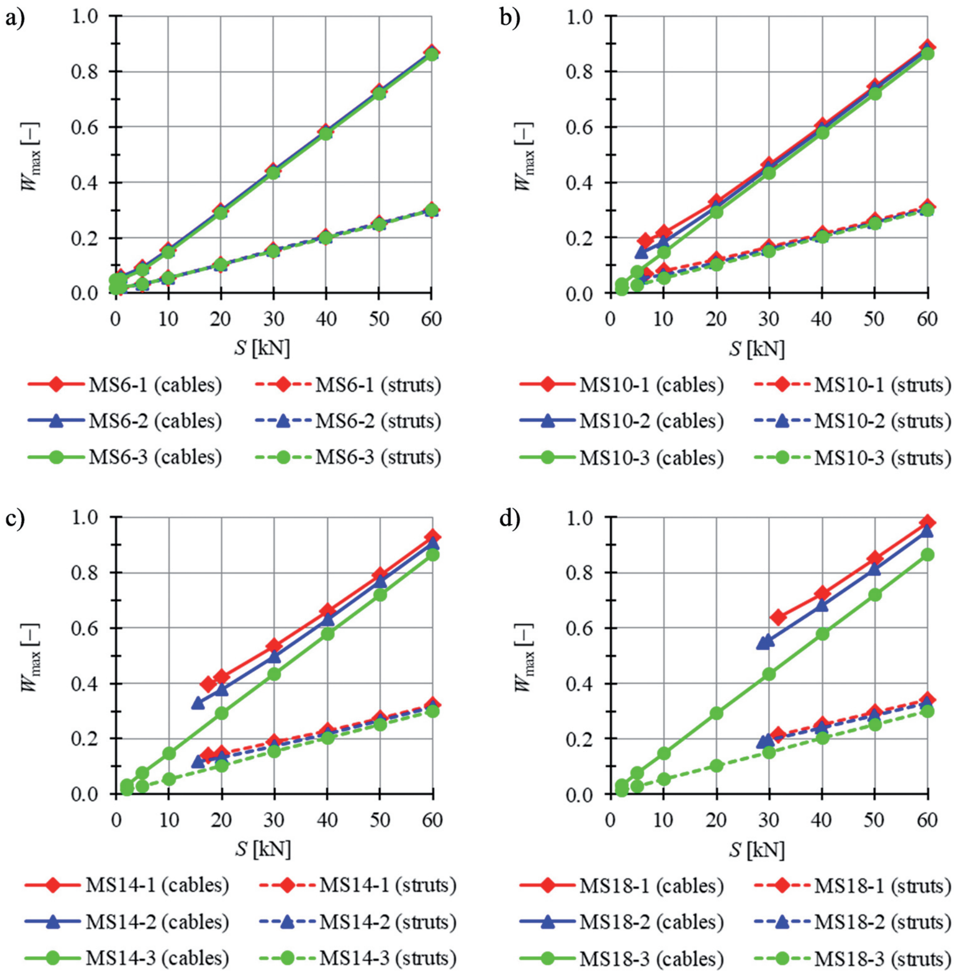


Fig. 7. Influence of the initial prestress on the effort of structure W_{max} for models: a) MS6, b) MS10, c) MS14, d) MS18

4. Conclusions

In this paper, the static behavior of double-layered tensegrity grids built with the modified Simplex modules is considered. Structures consisting of various number of modules with different support conditions are considered. Due to the specific characteristics, these grids can be used, for example, as deployable footbridges. For such structures, the possibility of the control of the behavior is very important. Additionally, in case of deployable structures, the influence of the change of support conditions is an important point to consider. The aim of the work is to prove that control of the behavior of tensegrity grids is possible.

The first step is the qualitative analysis. It is leads to identification the immanent tensegrity properties, like the self-stress states and infinitesimal mechanisms, next, to the classification of structures. Analyzed grids are classified as structures with tensegrity features of class 1 or class 2. In the first case, structures are featured by mechanisms, whereas in second they are not.

The second step is the quantitative analysis. It is the parametric analysis, which includes the influence of the level of initial prestress and the change of support conditions on the behavior of structures under static load. In particular, the influence on displacements and effort of structure is analyzed. In the case of structures characterized by the presence of the mechanism (structures with tensegrity features of class 1), the control of static parameters is possible. Due to the occurrence of the other immanent tensegrity feature, the existence of self-stress state, these structures are stable. Their stiffness is found to depend not only on the geometry and material properties but also on the initial prestress level and the external load. The load, causing displacements in accordance with the form of the infinitesimal mechanism, causes additional prestress of the structure – additional tensile forces are generated in cables and additional compressive forces are generated in struts. The rise of the initial prestress causes the decrease of the nodal displacements and the reduction of the impact of the geometrical nonlinearity. Additionally, the impact of the nonlinearity depends on the minimal level of self-stress. If the minimal level is low, the influence of the nonlinearity is significant so only non-linear approach (third order theory) provides appropriate results. If the minimum level of self-stress is higher, the impact of the nonlinearity declines and quasi-linear approach (second order theory) can be used. The minimal level of self-stress state decreases when more modules are used to build the grid. It results in the decline of the impact of the nonlinearity. If the structure lacks mechanisms, i.e. it is classified as one with tensegrity features of class 2, the control of static parameters in not possible.

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Zastosowanie immanentnych właściwości tensegrity do kontroli zachowania dwuwarstwowych kratownic

Słowa kluczowe: dwuwarstwowe kratownice tensegrity, mechanizm infinitezymalny, stan samonaprężenia, analiza jakościowa, analiza ilościowa

Streszczenie:

W pracy analizowano statyczne zachowanie się dwuwarstwowych kratownic typu tensegrity. Z uwagi na występowanie charakterystycznych cech, takich jak stany samonaprężenia i mechanizmy infinitezymalne, konstrukcje te mogą być stosowane jako rozkładalne. W takim przypadku bardzo ważna jest możliwość kontrolowania zachowania się konstrukcji. Głównym celem pracy jest wykazanie, że taka kontrola jest możliwa w przypadku struktur tensegrity, które charakteryzują się występowaniem mechanizmów. Sztywność takich struktur zależy nie tylko od geometrii i właściwości materiału, ale także od poziomu wstępnego sprężenia i od obciążenia zewnętrznego. W przypadku, gdy mechanizmy nie występują, konstrukcje są niewrażliwe na poziom wstępnego sprężenia. Występowanie mechanizmów można kontrolować poprzez zmianę warunków podparcia konstrukcji.

W pracy rozważane były rozkładalne kładki zbudowane ze zmodyfikowanych modułów Simplex. Rozpatrzono konstrukcje o różnych warunkach podparcia składające się z różnej liczby modułów. Analiza struktur tensegrity jest dwuetapowa. Pierwszym etapem jest analiza jakościowa, która polega na identyfikacji immanentnych własności tensegrity, takich jak stany samonaprężenia i mechanizmy infinitezymalne. Na tej podstawie konstrukcje są klasyfikowane jako struktury o cechach tensegrity klasy 1 lub klasy 2. W pierwszym przypadku struktury charakteryzują się występowaniem mechanizmów, natomiast w drugim nie. Drugim etapem jest analiza ilościowa. Jest to analiza parametryczna, która obejmuje wpływ poziomu wstępnego sprężenia oraz zmiany warunków podparcia na zachowanie konstrukcji pod obciążeniem statycznym. W szczególności analizowany jest wpływ na przemieszczenia i wytężenie konstrukcji.

W przypadku konstrukcji charakteryzujących się obecnością mechanizmu (konstrukcje o cechach tensegrity klasy 1) możliwa jest kontrola parametrów statycznych. Ze względu na występowanie drugiej immanentnej cechy tensegrity, czyli stanu samonaprężenia, struktury te są stabilne. Stwierdzono, że obciążenie, powodując przemieszczenia zgodnie z postacią mechanizmu infinitezymalnego, powoduje występowanie dodatkowego sprężenia konstrukcji – generowane są dodatkowe siły rozciągające w cięgnach i dodatkowe siły ściskające w zastrzałach. Wzrost poziomu wstępnego sprężenia powoduje zmniejszenie przemieszczeń i zmniejszenie wpływu nieliniowości geometrycznej. Wpływ nieliniowości zależy również od minimalnego poziomu stanu samonaprężenia. Jeśli minimalny poziom jest niski, wpływ nieliniowości jest znaczny, więc tylko podejście nieliniowe (teoria trzeciego

rzędu) daje odpowiednie wyniki. Jeśli minimalny poziom samonaprężenia jest wyższy, wpływ nieliniowości maleje i można zastosować podejście quasi-liniowe (teoria drugiego rzędu). Minimalny poziom samonaprężenia zmniejsza się, gdy konstrukcja składa się z większej liczby modułów. Jeżeli konstrukcja nie charakteryzuje się występowaniem mechanizmów, tj. została sklasyfikowana jako konstrukcja z cechami tensegrity klasy 2, kontrola parametrów statycznych nie jest możliwa.

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