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Research paper

Application of a new theory of restraint factor after cracking of reinforced concrete members

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Abstract: A detailed tie model of cracking is proposed. The model is dedicated to both semi-massive RC (reinforcement concrete) members subjected to early-age imposed strains and non-massive members in which imposed strains occur after concrete hardening. As distinct from the currently applied European guidelines, the proposed model enables an analysis of crack width changes. These are a function of progressive imposed strain, material and geometry data, but also depend on the scale of cracking which determines the strain conditions of a member. Consequently, the new model takes account of not only the factors determining the cracking development but also the member relaxation effect that results from cracking. For this reason a new definition of restraint factor is proposed, which takes into account the range of cracking of a structural member, i.e. the number and width of cracks. Parametric analyses were performed of both the changes of the degree of restraint after cracking as well as the changes of crack width depending on the adopted type of aggregate, class of concrete and the coefficient of thermal expansion of concrete. These analyses indicate the potential benefits of the application of the presented model for both a more accurate interpretation of research and economical design of engineering structures.

Keywords: crack width, degree restraint, imposed deformation, standards, thermal and shrinkage cracking

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1. Introduction

So far, the cracking of RC structures caused by imposed strain in the period of concrete maturing has not been researched exhaustively. This is due to the multi-faceted complexity of the progressing thermo-physical processes taking place while concrete matures [1]. On the one hand, these processes are determined by intended activities such as design objectives [2], building technology [3] of the structure and concrete composition [4]. On the other hand, the degree and time sequence of imposed strain are heavily affected by atmospheric factors which are impossible to determine precisely. These include especially the variations of temperature of the surroundings, humidity, exposure to the sun, wind velocity and precipitation. All these factors affect at least one parameter necessary to perform calculations of the field of temperature or humidity. If these parameters are underestimated, the calculations will significantly deviate from both the actual increase of thermo-physical parameters of concrete as well as the degree of imposed strain. As a result the calculation model will not describe the cracking risk and cracking itself relevantly.

Various cases of cracking of engineering objects at the stage of their construction are given in Jędrzejewska et al. [5]. RC tanks, of which complete water tightness is required, and which frequently undergo cracking, are a special case [6–8]. While a reduction of shrinkage [9] or temperature [10] induced imposed strains may result in the avoidance of cracking or a reduction of the number of cracks, it does not lead to the reduction of cracks width. In Jędrzejewska et al. [5] a comprehensive analysis of cracking of individual structures is done following European guidelines (EN 1991-1 [11], Knauff et al. [12], EN 1992-3 [13], CIRIA 766 [14]). A significant role in these calculations is performed by the so-called restraint factor which defines the degree of restraint of the structure, i.e. the part of imposed strain which remains restrained and will result in the formation of undesirable stress in maturing concrete. This factor covers many problematic issues, and is the fastest path to performing engineering calculations or comparative scientific research analyses.

The restraint factor is commonly used in **engineering** analytical models. These models are assumed to enable relatively efficient and safe calculation of the effects from imposed strain. Restraint factor depends on a number of factors including: the geometry of the restrained member, the geometry of the restraining member, concrete maturity, rigidity and type of inter-element constraints, position in the structure, type of ground. In view of the above a precise definition of the restraint factor is a complex and multi-faceted matter. Therefore, owing to the knowledge of its value for certain types of structural members the designer avoids the necessity of doing a considerable portion of calculations. The approaches to the determination of the restraint factor used currently in various design standards and guidelines (prEN 1992-1-1 [15], EN 1992-3 [13], CIRIA C766 [14], ACI 207.2 [16], JCI 2008 [17], JSCE [18]) were described by, *inter alia*, Jędrzejewska and Zych [19]. Although the new method of calculating stresses from imposed strain, which is to be included in prEN 1992-1-1 being developed [15] is now one of the most extensive standard approaches, it is the guidelines necessary for the determination of the degree of restraint were treated in the prEN only marginally. One of the firmly established analytical methods of the determination of the restraint factor of uncracked members is the compensation plane method (CPM). It was proposed in 1985 [20] and is now a basis for a number of standard models (EN 1992-3 [13], JCI [14]). From the practical point of view the determination of this factor by a designer is most frequently reduced to reading its value in a table or a diagram, or applying a simple formula approximating the results obtained from analytical or numerical models. The restraint factor is also a parameter that **researchers** most often use in order to assess the favourable or unfavourable impacts affecting the restraint factor. Also, the changes in the restrained part of the imposed strain are assessed quantitatively, which is chiefly determined by the adopted design solutions. This enables, *inter alia*, an evaluation of more in-depth calculation analyses, and next, the development of more precise cracking risk models. Analytical scientific models are based on the CPM idea as often as the standard models (Al-Gburi [21], Nilsson [22]). An alternative solution to analytical models is to determine the restraint factor based on numerical models, owing to which other additional aspects affecting the degree of restraint including the particular layers of a structure (JSCE [18]), type of ground (Klemczak and Knoppik-Wróbel [23]), member cracking (Zych [24]) can be taken into account.

The significance of the restraint factor for both engineering and research purposes is therefore indisputable. However, it is commonly defined for an uncracked structure, which, in the author's opinion, greatly restricts its practical application. Owing to the assumption of a lack of impact of member cracking on the value of restraint factor it is proper to take into account its value only in the cracking risk calculation, while including it in the calculation of cracks width or the estimation of the degree of cracking is substantively questionable. The concept of the application of the restraint factor after structure cracking was originally presented by Kheder [25]. He used this factor in the calculations of cracks width in straight walls. Such an assumption is substantively justified but it implies a further challenge which is the necessity of performing more complex calculations. Another solution for arched walls, based on a numerical model, covering cracking in construction joints and cracking in the wall axis was proposed by Zych [24]. On the basis of these calculations diagrams of the changes in the degree of cracking after the cracking of arched walls were produced. The change of the degree of restraint of hardening concrete also after cracking was analysed by means of adjustable restraining frames (ARFs) by Schlicke et al. [26]. The restraint factor was defined as:

$$R_{ax} = \frac{1}{1 + \dfrac{E \cdot A_m}{L} + \dfrac{1}{k_F}} \tag{1.1}$$

where: $E \cdot A_m$ – mean axial stiffness of the specimen dependent on the actual modulus of elasticity of concrete and state of specimen's cracking, k_F – equivalent spring stiffness of the frame, L – length of the specimen.

Taking account of the equivalent stiffness of frame (k_F) and the stiffness of specimen ($E \cdot A_m$) Eq. (1.1) defines the restraint factor globally. This means that the effect of external restraints and that of concrete hardening as well as potential cracking manifested by first increase and next decrease of stiffness are taken into account. In practice, however, Schlicke et al. [26] employ an equation based on a measured change of tensile force (F_{ARF}) at the moment of cracking:

$$R_{ax}\left(t^{\mathrm{II}}\right)=\frac{F_{\mathrm{ARF}}\left(t^{\mathrm{I}}\right)}{F_{\mathrm{ARF}}\left(t^{\mathrm{II}}\right)}\cdot R_{ax}\left(t^{\mathrm{I}}\right)\leq 1 \tag{1.2}$$

where: t^{I} – time immediately prior to cracking, t^{II} – time immediately after cracking.

Cracking causes a decrease of the tensile force ($F_{\mathrm{ARF}}(t^{\mathrm{II}}) < F_{\mathrm{ARF}}(t^{\mathrm{I}})$), which following Eq. (1.2) will contribute to an increase rather than decrease of the degree of restraint. The same tendency is maintained in the theoretical Eq. (1.1), as both formulae take account only of changes of stiffness rather than the effect of compensation of additional cracking induced strains. Consequently, in Eq. (1.3), which describes changes of the tensile force resultant (F_{ARF}), the cracking induced strain is taken account of as a separate component ($\sum \Delta w/L$), while stiffness EA_m must, moreover, include the effect of cracking [26]:

$$F_{\mathrm{ARF}}=\left(\varepsilon_{0,p}+\frac{\sum w}{L}\right)\cdot EA_m(t)\cdot R_{ax} \tag{1.3}$$

where: $\varepsilon_{0,p}$ – imposed deformation during hardening of concrete, $\sum w$ – sum of occurring crack widths along the specimen.

The aim of the present paper is to propose a concept of an analytical model for the determination of the degree of restraint of tie members at various stages of their cracking. Unlike the existing models, the new definition of the restraint factor can be used for a more precise estimation of the structure's behaviour also after its cracking. This definition takes account of each episode of relaxation resulting from member cracking. Next, further parametric analyses of the degree of restraint after a member cracking, taking account of variable parameters including the class of concrete, coefficient of thermal expansion of concrete and modulus of elasticity of concrete will be presented. Another, parallel range of calculations will concern the changeable width of cracks taking account of the effect of member relaxation. In the future further research will focus on the improvement of the present model particularly in the aspect of the parameters of hardening concrete and the parameters of the member which restrains the strains of the analysed member.

2. Restraint factor and cracks width after member relaxation

The restraint factor after cracking $R^{\mathrm{cr}}(t)$, i.e. for $t > t_{\mathrm{cr}}$ is described by equation:

$$R^{\mathrm{cr}}(t)=\frac{\sigma_c(t)}{\left[R^{\mathrm{uncr}}(t)\sigma_{\mathrm{fix}}(t)\right]} \tag{2.1}$$

where: R^{uncr} – restraint factor before cracking, σ_c – actual stresses in concrete, σ_{fix} – stresses that would occur in a fully restrained element.

In Eq. (2.1) stress σ_c in concrete includes the effects of member cracking. $\sigma_c = R^{\mathrm{uncr}}\cdot\sigma_{\mathrm{fix}}$ in turn, indicates that there are no cracks in the member and what is important are only the parameters adopted for the determination of the degree of restraint prior to cracking (R^{uncr}). To define R^{uncr} the proportions between the rigidity of strain restrained and restraining members are taken into account.

Next, the decrease of the tensile force $\Delta N_{\text{rel}}(t_{\text{cr}})$, which is chiefly determined by the geometry and method of member reinforcement, i.e. factors that heavily affect the crack width, is expressed in a more compact form, i.e. the function of the degree of restraint after cracking:

$$\Delta N_{\text{rel}}(t_{\text{cr}}) = N^{\text{uncr}}(t_{\text{cr}})\left[1 - R^{\text{cr}}(t_{\text{cr}})\right] \tag{2.2}$$

where: $N^{\text{uncr}}(t)$ – tensile force in uncracked section.

Consequently, when the number of cracks increases, the criterion taking account of member relaxation should be employed in the form:

$$N_{\text{cr}}\left(t_{\text{cr}}^{i}\right) \leq R^{\text{uncr}} \cdot K_{c1} \cdot \varepsilon_{\text{free}}\left(t_{\text{cr}}^{i}\right) \cdot E_{\text{cm}}\left(t_{\text{cr}}^{i}\right) \cdot A_c\left(1 + \alpha_e\left(t_{\text{cr}}^{i}\right) \cdot \rho\right) - \sum_{i=1}^{n-1} \Delta N_{\text{rel}}^{i}\left(t_{\text{cr}}^{i}\right) \tag{2.3}$$

where: K_{c1} – coefficient for the effect of creep on stress relaxation at early-age, $\varepsilon_{\text{free}}(t)$ – free imposed strain, $E_{\text{cm}}(t)$ – modulus of elasticity of concrete, A_c – area of concrete, $\alpha_e(t)$ – ratio of moduli of elasticity of steel and concrete, ρ – reinforcement ratio, $\Delta N_{\text{rel}}(t_{\text{cr}})$ – tensile force decrease caused by cracking.

Thus defined decrease of tensile force (Eq. (2.2)) is a basis for the determination of the value of further imposed strain necessary for the formation of subsequent cracking of the member, which provides a basis for, *inter alia*, the determination of the final scale of member cracking. After each cracking event the relaxation results also in temporary decrease of cracks width until the restrained part of the imposed strain increases again. Further considering the fact that the tensile force drop and cracks width decrease are an interconnected phenomenon, it is necessary to solve a set of equations composed of two dependencies. One describes the actual width of the crack, taking account of a certain decrease of tensile force, while the other one defines the value of the decrease of this force due to the formation of a crack of a certain width [27]. The solution of this set of equations produced the dependencies (2.4), (2.5) describing, respectively: the basic component of cracks width dependent on the decrease of force expressed in the function of quantity and the width of early-age cracks, and the change in the tensile force due to the given number of cracks of width defined after Eq. (2.4):

$$\Delta_{\text{rel}}^{i}\left(t, t_{\text{cr}}, \Delta N_{\text{rel}}^{i}\right) = \frac{k \cdot \varepsilon_{\text{ctu}}(t_{\text{cr}}) + \dfrac{\Delta_{\text{rel}}^{i-1}(n-1)}{L}}{\dfrac{2\alpha_e(t_{\text{cr}}) \cdot \rho}{s_{r,\max} \cdot K_{c1}} + \dfrac{n}{L}} \tag{2.4}$$

$$\Delta N^{i}_{\text{rel}}(t_{\text{cr}},\Delta^{i}_{\text{rel}}) = \frac{A_c \cdot E_{\text{cm}}(t_{\text{cr}}) \cdot (1+\alpha_e(t_{\text{cr}})\cdot\rho)}{L} \cdot \left(\frac{k\cdot\varepsilon_{\text{ctu}}(t_{\text{cr}}) + \dfrac{\Delta^{i-1}_{\text{rel}}(n-1)}{L}}{\dfrac{2\alpha_e(t_{\text{cr}})\cdot\rho}{s_{r,\max}} + \dfrac{n}{L}} \cdot n - \Delta^{i-1}_{\text{rel}}\cdot(n-1) \right) \tag{2.5}$$

where: k – coefficient which allows for the effect of non-uniform self-equilibrating stresses, ε_{ctu} – tensile capacity of concrete, L – length of the element, $s_{r,\max}$ – maximum crack spacing, n – total number of cracks.

Dependencies (2.4), (2.5) concern a case of the occurrence of imposed strain over a very short period of time during which one crack follows the next one and each event is followed by a corresponding decrease of the tensile force. Consequently, identical parameters of concrete-reinforcement bonding around each crack are adopted because the properties of concrete remain unchanged, which may correspond to, for example, a case of sudden cooling of the member. The final crack width (as a sum of its basic component resulting from reinforcement elongation and the component resulting from concrete relaxation in the vicinity of the crack) is described by equation:

$$w_k(t) = \Delta_{\text{rel}}(t) + \frac{s_{r,\max}\cdot\sigma_c(t_{\text{cr}})}{2E_{\text{cm}}(t_{\text{cr}})\cdot K_{c1}} \tag{2.6}$$

where:

$$\sigma_c(t_{\text{cr}}) = \left[k f_{\text{ct,eff}}(t_{\text{cr}}) - \frac{\Delta N_{\text{rel}}(t_{\text{cr}})}{A_c\cdot\left(1+\alpha_e\left(t_{t_{\text{cr}}}\right)\cdot\rho\right)}\right]\cdot K_{c1} \tag{2.7}$$

where: $f_{\text{ct,eff}}$ – effective tensile strength of concrete.

Making Eq. (2.1), i.e. the relationship describing the restraint factor after cracking $R^{\text{cr}}(t)$) more specific, the following dependence is obtained:

$$\begin{aligned} R^{\text{cr}}\left(t^{i}_{\text{cr}}\right) &= 1 - \frac{\sum\limits_{i=1}^{n-1}\Delta N^{i}_{\text{rel}}\left(t^{i}_{\text{cr}}\right)}{R^{\text{uncr}}(t)\cdot K_{c1}\cdot\varepsilon_{\text{free}}\left(t^{i}_{\text{cr}}\right)\cdot E_{\text{cm}}(t^{i}_{\text{cr}})\cdot A_c\left[1+\alpha_t\left(t^{i}_{\text{cr}}\right)\cdot\rho\right]} \\ &= 1 - \frac{\sum\limits_{i=1}^{n-1}\Delta\varepsilon^{i}_{\text{rel}}\left(t^{i}_{\text{cr}}\right)}{R^{\text{uncr}}(t)\cdot K_{c1}\cdot\varepsilon_{\text{free}}\left(t^{i}_{\text{cr}}\right)} \end{aligned} \tag{2.8}$$

From Eq. (2.8) it follows that the maximum value $R^{\text{cr}} = 1.0$ will be valid for an uncracked section, still before member relaxation. This equation should be used together with Eq. (2.5) describing the impact of the number of cracks on member relaxation. In Eq. (2.8) the favourable effect of creep on the reduction of its value can be optionally included, which in real-life structures subjected to drying shrinkage should be treated merely as a theoretical issue. Creep, in turn, included in the calculation of cracking risk is commonly taken account of either by the effective modulus of elasticity after Bažant [28] or in the form presented here, i.e. coefficient K_{1c} (CIRIA C766 [14]).

In the present paper parametric analyses based on the new definition of restraint factor will be proposed. These analyses take account of the type of aggregate, class of concrete and the value of the coefficient of thermal expansion of concrete.

3. Parametric analysis

3.1. Impact of type of aggregate on R^{cr} and w_k

EN1992-1-1 [11] recommends that in the design of any structure it is necessary to take into account the impact of the type of aggregate on the modulus of elasticity of concrete. Its value is specified by multiplying the value fixed for concretes with quartztite aggregate (Table 3.1 in EN1992-1-1 [11]) by the coefficient relevant for the given type of aggregate: 0.9 for limestone aggregates, 0.7 for sand aggregates and 1.2 for basalt aggregates. In the case of imposed strains the higher modulus of elasticity will contribute to a reduction of tensile strain capacity of concrete (ε_{ctu}) and faster increase of stresses from changing strains. A question, however, arises as to what exactly impact will this have on both crack width changes and degree of restraint.

For the analysis the following were adopted: $R^{uncr} = 1.0$, thickness, height and length of member, respectively, of 0.6, 1.0 and 10 m, constant temperature 20°C of concrete hardening, concrete cover $c_{nom} = 40$ mm, 10 bars reinforcement along both edges $\varphi 20$ ($\rho = 1.05\%$), class of concrete C30/37, $t = 3$ days, $k_1 = 1.14$, as well as imposed strain range of up to 500 $\mu\varepsilon$. An adequately high value of the adopted strain covers nearly all practical cases of the amount of this strain. The calculations were done for concretes with the four above mentioned aggregates quoted in EN1992-1-1 [4]. Figure 1 illustrates changes of restraint factor R^{cr} with no effect of creep of concrete and taking account thereof. The restraint factor decreases stepwise after each cracking event. In the presentation of the

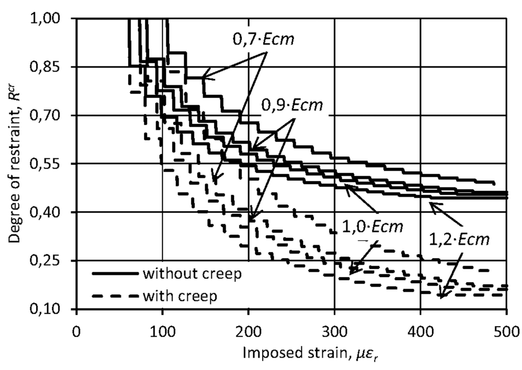


Fig. 1. Change of degree of restraint R^{cr} in the function of imposed strain for a members with different type of aggregate

results, however, they are presented as an approximation of minimal values (Fig. 2a), which facilitates a comparison of individual cases. In general it can be concluded that with an increase of imposed strain (ε_r) the degree of restraint (R^{cr}) due to cracking significantly decreases, and a reduction of its value caused by creep is determined by the value of imposed strain, and is about 50% its basic value over its significant range.

In Figs. 2a and 2b changes in the degree of restraint after cracking R^{cr} and changes in cracks widths w_k, are presented, respectively. The curves in Fig. 2b represent cracks widths immediately after each relaxation event of the member and immediately prior to the next cracking. This figure also shows the values calculated after EN1991-3 [13], which for a three-day concrete are not the upper limit of cracks width, which results from the proposed model of factor $k_1 = 1.14$ adopted after CIRIA C766 [14]. It turns out that the highest degree of restraint is observed in concretes with aggregates of lower modulus of elasticity, which also results in a slightly bigger cracks width. What is a favorable aspect of the application of such aggregates is an increase of tensile strain capacity of concrete (ε_{ctu}), which delays the time of cracking and ensures the possibility of development of further cracks in the entire range of the adopted imposed strains. The higher the modulus of elasticity the lower the degree of restraint after cracking. However, cracking of a member is initiated earlier and the imposed strains range for which reinforcement effectively restricts cracks width is smaller. The higher degree of restraint of concretes with a lower modulus of elasticity results from a higher ratio between reinforcement stiffness (in the section through crack) and the stiffness of the uncracked part of section.

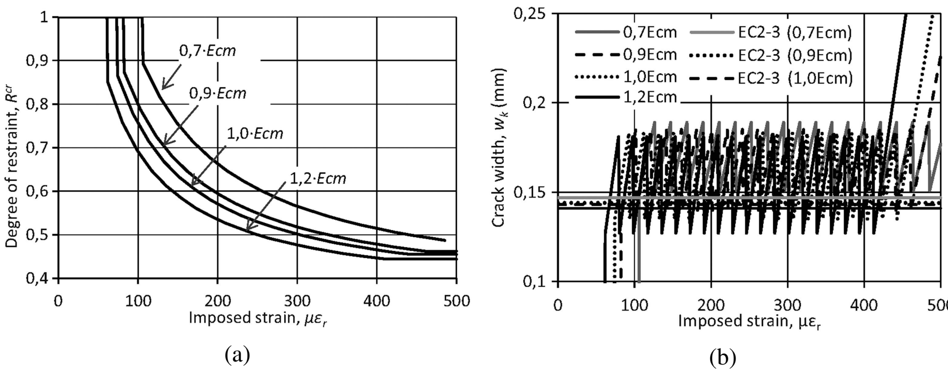
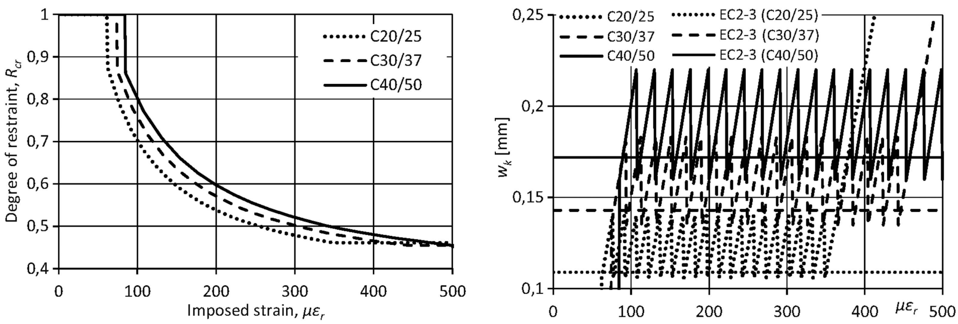


Fig. 2. a) Change of degree of strain R^{cr} for concrete class C30/37 with various aggregates, b) change of cracks width w_k for concrete class C30/37 with various aggregates

3.2. Impact of class of concrete on R^{cr} and w_k

In another analysis class of concrete: C20/25, C30/37, C40/50 with quartzite aggregate was adopted as the variable parameter. The other data for the calculations adopted as in the previous section of the paper. In Figs. 3a and 3b changes of the degree of restraint after cracking R^{cr} and changes in cracks widths w_k are presented respectively. From the calculations it can be concluded that the highest degree of restraint is found for the higher classes of concrete, which is mainly connected with the cracking initiation at higher imposed strains. Actually, the degree of restraint for structural members different only in the class of concrete for each cracking episode is very close to each other. An exception is a case of stabilized cracking as, for instance, for concrete class C20/25, when above strains 350 $\mu\varepsilon$ no further cracks are formed and the degree of restraint is constant. Thereby the class of concrete has a significant impact only on the range of imposed strains at the level of which reinforcement effectively restricts cracks width. However, the class of concrete does have a major impact on the crack width, which results from a higher strength of concrete and a bigger allowable tensile force in a member. In concrete class C40/50 the cracks widths are nearly twice bigger compared with concrete class C20/25. From the performed analysis it can be concluded that for design purposes it would be sufficient to specify the degree of restraint after cracking of a given member for one class of concrete with a varying reinforcement method and to recommend a limitation in the form of allowable imposed strains.



(a) (b)

Fig. 3. a) Change of degree of restraint R^{cr} for selected classes of concrete, b) change of cracks width w_k for selected classes of concrete

3.3. Impact of coefficient of thermal expansion of concrete on R^{cr} and w_k

Another variable parameter adopted was the coefficient of thermal expansion of concrete α_T. EN1992-1-1 [12] recommends the value of $1.0 \cdot 10^{-6}/°\mathrm{C}$. This coefficient, however, depends on the concrete mix composition and concrete maturity. Its value is several times higher in the initial hours of concrete curing (Bergström and Byfors [29]). According to CIRIA C766 [14], when no detailed data are available $\alpha_T = 1.2 \cdot 10^{-6}/°\mathrm{C}$ should be used. After general recommendations of CIRIA C766 [14] an aggregate of a lower coefficient of thermal expansion should be used.

For the purposes of the present parametric analysis the following values were adopted α_T: $0.8 \cdot 10^{-6}/°\mathrm{C}$, $1.0 \cdot 10^{-6}/°\mathrm{C}$, $1.2 \cdot 10^{-6}/°\mathrm{C}$. In Figs. 4a and 4b changes of the degree of restraint after cracking R^{cr} and changes of cracks widths w_k. are presented, respectively.

The calculations indicate that the highest degree of restraint is found in concretes of lower coefficient of thermal expansion, which, similar as in the case of variable class of concrete, is connected with cracking initiation at higher imposed strains. Actually, the degree of restraint for concretes of different coefficients of thermal expansion of concrete for each cracking event is identical.

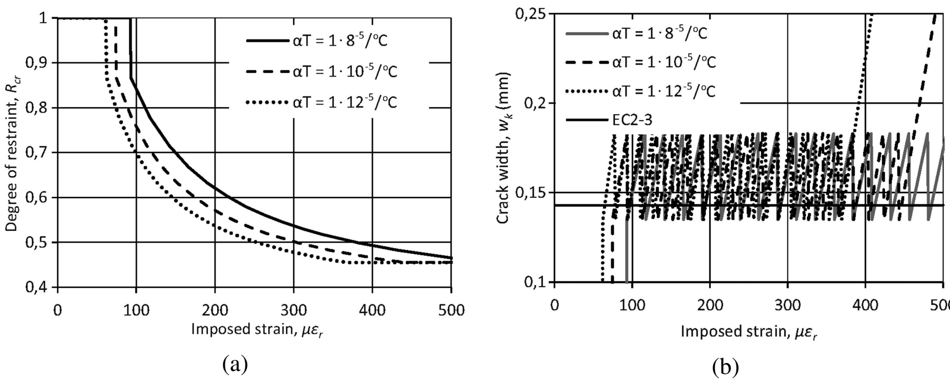


Fig. 4. a) Change of degree of restraint R^{cr} for selected coefficients of thermal expansion of concrete, b) change of cracks width for selected coefficients of thermal expansion of concrete

Again, an exception is when the level of strains above which no new cracks are formed has been reached. The coefficient of thermal expansion, however, does not affect changes of cracks width, only the level of cracking initiation and the level of strains to which reinforcement effectively restricts crack width. Thereby it can be concluded that, as in the case of various classes of concrete, in practice it would be reasonable to specify the degree of restraint after cracking only for one coefficient of thermal expansion of concrete, with additional information of the threshold strain that decides of the transition from non-stabilized to stabilized cracks spacing.

4. Summary of selected results

Actual and allowable number of cracks for all the analysed cases is tabulated in Table 1. On this basis two stages of member cracking have been distinguished. In the first stage cracks widths are effectively restricted by reinforcement so these widths depend mainly on the tensile strength of concrete and degree of member reinforcement. In the second stage cracking becomes stabilised and higher imposed strains result in cracks width increase independent of concrete strength and degree of reinforcement. In Table 1 strains causing primary cracking and an increase of strains causing subsequent cracking are also tabulated. On this basis a conclusion can be drawn of a “surplus” imposed strain which can be compensated only by the existing cracks rather than default subsequent ones, contributing to the widening of the former ones.

Table 1. Number of cracks in a member and cracking strains increase

Cases	Parameters: (type of aggregate; class of concrete; coefficients of thermal expansion)	Number of cracks in a member for ε_{imp} = 500 με vs. allowable number of cracks	Strain generating initial cracking (με)	Strain increase generating subsequent cracking (με)	Value of strains non-consumed by cracks (με)
1A	0.7 E_{cm}	19 < 20	106	21.0	0
1B	0.9 E_{cm}	20 = 20	83	20.0	–17
1C	1.0 E_{cm}	20 = 20	75	19.4	–37
1D	1.2 E_{cm}	20 = 20	62	18.4	–70
2A	C 20/25	20 = 20	62	15.1	–136
2B	C 30/37	20 = 20	75	19.4	–37
2C	C 40/50	19 < 20	85	23.0	0
3A	$1 \cdot 8^{-5}/°C$	17 < 20	93	24.2	0
3B	$1 \cdot 10^{-5}/°C$	20 = 20	75	19.4	–37
3C	$1 \cdot 12^{-5}/°C$	20 = 20	62	16.2	–114

Figure 5 illustrates the allowable levels of imposed strain, i.e. a limit between the stabilised and non-stabilised state of cracking to which the reinforcement will effectively restrict cracks width. The most unfavourable solution in the analysed cases is one in which aggregates of a higher modulus of elasticity (Fig. 5a) or in more precise analyses aggregates of a high coefficient of thermal expansion (Fig. 5c) are used. The use of concretes of lower classes is also a definitely worse solution (Fig. 5b).

Although the same degree of reinforcement of all the members and identical geometry of a member, *inter alia*, were adopted in the presented analyses, considerable differences in both the restraint factor and cracks width were obtained. In the case of cracks, their maximal values in the period of non-stabilised cracking range from 0.14 to 0.22 mm, while during stabilised cracking and strain of 500 με the cracks widths exceed 0.25 mm in practically each case.

In Fig. 6 envelopes of restraint factor for all the cases analysed in the present paper are shown. The largest differences between the extreme values are observed at lower levels of imposed strain. In general, the considerable differences in the restraint factor and cracks width for all the calculation cases indicate a great significance of the other, apparently less important design and executive objectives.

Since in the literature on the subject no studies are presented on the impact of the type of aggregate, class of concrete and coefficient of thermal expansion on the degree of restraint after cracking, the calculations are performed only qualitatively. In the future therefore quantification is necessary based on experimental verification.

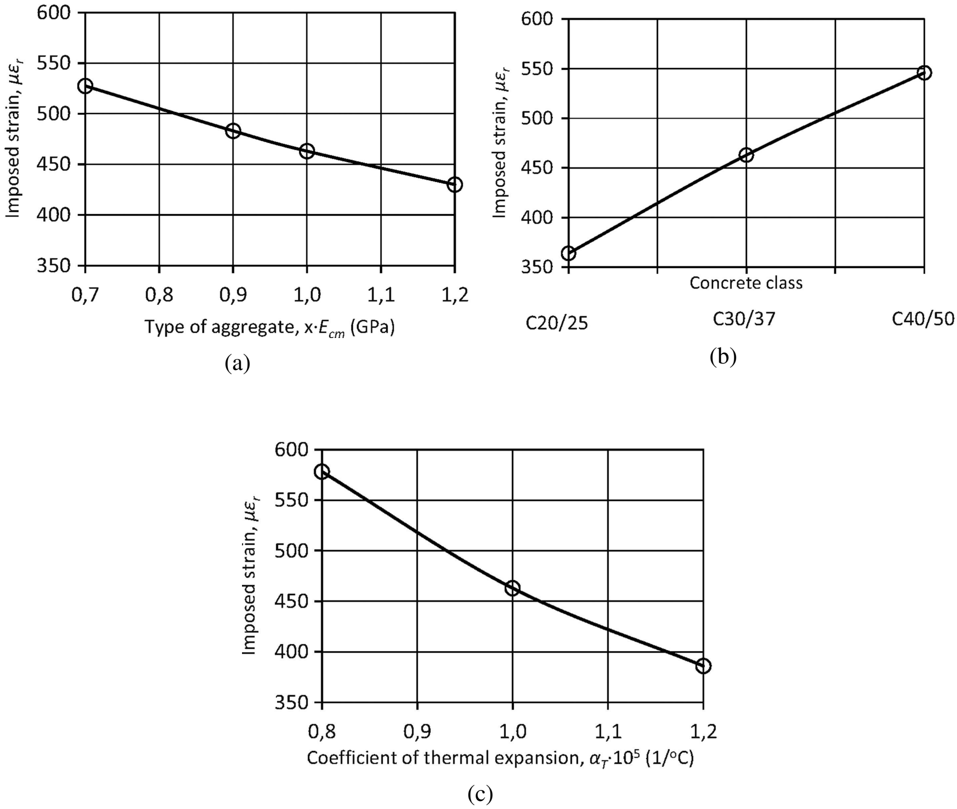


Fig. 5. Allowable range of restrained strains considering cracks width limitation by reinforcement for: a) different type of aggregate, b) different classes of concrete, c) different coefficients of thermal expansion of concrete

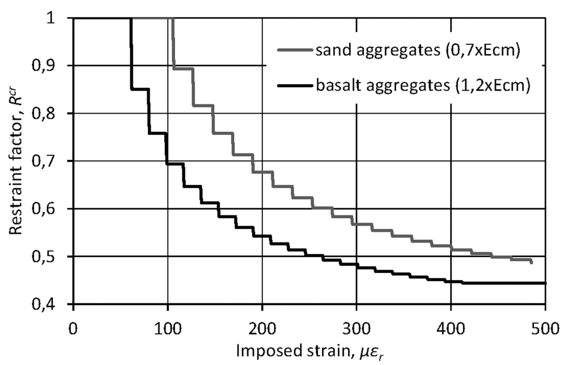


Fig. 6. Upper and lower envelopes of restraint factor for all analyzed cases

5. Conclusions

The proposed model enables a more precise analysis of cracking that result from the effect of imposed strain in members restrained at both ends. A considerable decrease of the degree of restraint is observed after subsequent cracking events of a member, whose values depend chiefly on the degree of reinforcement of the member and its cross-section.

The changeability of the degree of restraint R^{cr} of a member restrained at its ends in the function of: the degree of reinforcement, thickness of the member, class of concrete, type of aggregate and coefficient of thermal expansion of concrete is presented. Some of the introduced variables (e.g. class of concrete or coefficient of thermal expansion) proved not to impact significantly the degree of restraint. The highest effect on a change of its value is exerted by the type of aggregate, member dimensions and age of concrete.

The imposed strains limit to the level of which reinforcement will effectively restrict crack width can be increased primarily by: reduction of the coefficient of thermal expansion of concrete, an increase of the class of concrete, the application of an aggregate of lower modulus of elasticity and the application of a lower amount of reinforcement.

Owing to the presented new model it is possible to calculate the degree of restraint after cracking in a construction joint taking account of all cracks, with no distinction, however, whether cracking develops in a construction joint (frequently of a different degree of reinforcement) or in the member itself. For this reason this analytical model needs further development, which will enable an analysis cracking in stages of the construction joints themselves first, to be followed by an analysis of the cracking of the member, including higher maturity of concrete. Such a model will be highly applicable in the practice of the design of construction joints whose objective will be to compensate the imposed strain in order to limit or avoid cracking.

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Zastosowanie nowej teorii współczynnika skrępowania po zarysowaniu elementów żelbetowych

Słowa kluczowe: normy, odkształcenia wymuszone, stopień skrępowania, szerokość rysy, termiczne i skurczowe zarysowanie

Streszczenie:

W artykule przedstawiono uszczegółowioną propozycję modelu zarysowania elementów prętowych. Model ten dedykowany jest zarówno do średnio-masywnych elementów żelbetowych poddanych wczesnym odkształceniom wymuszonym oraz do niemasywnych elementów, w których odkształcenia wymuszone występują po stwardnieniu betonu. W odróżnieniu od obecnie stosowanych europejskich wytycznych zaproponowany model umożliwia przeprowadzenie analizy zmian szerokości rys. Zmiany te są funkcją postępującego odkształcenia wymuszonego, danych materiałowych i geometrycznych, ale zależą również od skali zarysowania, która determinuje warunki skrępowania elementu. Zdefiniowano spadek siły rozciągającej po zarysowaniu, który jest podstawą do wyznaczenia wielkości dalszego odkształcenia wymuszonego niezbędnego do powstania kolejnego zarysowania elementu. Daje to również podstawę do określenia finalnej skali zarysowania elementu. Odprężenie po każdorazowym zarysowaniu powoduje również okresowe zmniejszenie szerokości rys, do czasu pojawienia się kolejnego przyrostu skrępowanej części odkształcenia wymuszonego. W modelu uwzględniono, że spadek siły rozciągającej oraz spadek szerokości rysy jest zjawiskiem sprzężonym, przez co konieczne było rozwiązanie układu równań składającego się z dwóch zależności. Pierwsza opisuje aktualną szerokość rysy, z uwzględnieniem pewnego spadku siły rozciągającej. Druga z kolei definiuje wielkość spadku tej siły na skutek pojawienia się rys o pewnej szerokości. Rozwiązanie niniejszego układu równań doprowadziło do zależności opisujących odpowiednio: podstawową składową szerokości rys zależną od spadku siły wyrażonej w funkcji ilości i szerokości rys powstałych wcześniej oraz zmianę siły rozciągającej na skutek danej liczy rys o obliczonej szerokości. Zdefiniowane zależności dotyczą sytuacji wystąpienia odkształceń wymuszonych w bardzo krótkim okresie czasu, w którym powstaje jedna rysa zaraz po drugiej oraz mają miejsce odpowiadające im każdorazowe spadki siły rozciągającej. Tym samym przyjęto jednakowe parametry przyczepnościowe betonu do zbrojenia w otoczeniu każdej rysy, gdyż właściwości betonu pozostają niezmienne, co może przykładowo odpowiadać sytuacji nagłego ochłodzenia elementu. Finalną szerokość rysy obliczana jest jako suma podstawowej jej składowej wynikającej z wydłużenia zbrojenia oraz składowej wynikającej z odprężenia betonu w okolicach rysy. Tym samym nowy model uwzględnia nie tylko czynniki determinujące progres zarysowania, ale też efekt odprężenia elementu wynikający z zarysowania. Z tego względu zaproponowano nową definicję współczynnika skrępowania, która uwzględnia skalę zarysowania elementu konstrukcyjnego, tj. ilość i szerokości rys. Wykonano analizy parametryczne zarówno zmian stopnia skrępowania po zarysowaniu i zmian szerokości rys w zależności od przyjętego: rodzaju kruszywa, klasy betonu oraz współczynnika rozszerzalności termicznej betonu.

W obliczeniach przyjęto między innymi: grubość, wysokość i długość elementu, odpowiednio 0,6, 1,0 i 10 m, stałą temperaturę 20°C dojrzewania betonu, otulenie betonu $c_{\text{nom}} = 40$ mm, zbrojenie 10 φ 20 wzdłuż każdej krawędzi elementu ($\rho = 1,05\%$), czas zarysowania $t = 3$ dni, $k_1 = 1{,}14$ oraz zakres odkształceń wymuszonych do 500 $\mu\varepsilon$. Odpowiednio duża wartość przyjętych odkształceń obejmuje niemal wszystkie praktyczne przypadki wielkości tego odkształcenia. W pewnej grupie wprowadzonych zmiennych (jak np. klasy betonu, czy współczynnika rozszerzalności termicznej

betonu) okazało się, że stopień skrępowania nie ulega znaczącym zamianom. Największy wpływ na zmianę jego wartości ma rodzaj kruszywa, wymiary elementu oraz wiek betonu.

Pomimo przyjęcia w niniejszych analizach między innymi jednakowego stopnia zbrojenia wszystkich elementów oraz jednakowej geometrii elementu, otrzymano znaczne różnice zarówno we współczynniku skrępowania oraz szerokościach rys. W przypadku rys maksymalne ich wartości w okresie nieustabilizowanego zarysowania mieszczą się w zakresie od 0.14 do 0.22 mm. Podczas gdy dla ustabilizowanego zarysowania i odkształcenia 500 $\mu\varepsilon$ praktycznie w każdym przypadku szerokości rys przekraczają wartość 0.25 mm. W artykule zestawiono dla wszystkich przeanalizowanych przypadków rzeczywistą i dopuszczalną liczbę rys. Na tej podstawie rozróżniono dwa etapy zarysowania elementu. Pierwszy etap, na którym szerokości rys ograniczone są efektywnie przez zbrojenie, tym samym ich szerokości zależne są głównie od wytrzymałości betonu na rozciąganie i stopnia zbrojenia elementu. Drugi etap odpowiada ustabilizowanemu zarysowaniu, na którym większe odkształcenia wymuszone powodują niezależnie od wytrzymałości betonu oraz stopnia zbrojenia, przyrost szerokości rys. Ponadto zestawiono również: odkształcenia powodujące pierwsze zarysowanie oraz przyrost odkształcenia podającego kolejne zarysowania. Na tej podstawie można między innymi wnioskować o „nadwyżce” odkształcenia wymuszonego, która nie może być skompensowana przez kolejne rysy, a jedynie przez rysy już istniejące, przyczyniając się do zwiększenia ich szerokości. Zwiększenie limitu odkształceń wymuszonych, do którego poziomu zbrojenie efektywnie będzie ograniczać szerokość rysy można uzyskać przede wszystkim poprzez: zmniejszenie współczynnika rozszerzalności termicznej betonu, zwiększenie klasy betonu, zastosowanie kruszywa o niższym module sprężystość oraz zastosowane mniejszego stopnia zbrojenia.

W ogólności można stwierdzić, że przeprowadzone analizy parametryczne wskazują na możliwe korzyści z stosowana niniejszego modelu zarówno w zakresie dokładniejszej interpretacji wyników badań naukowych, jak również w zakresie ekonomiczniejszego projektowania konstrukcji inżynierskich. Ponadto zaprezentowany nowy model umożliwia obliczenie stopnia skrępowania po zarysowaniu z uwzględnieniem obecności wszystkich rys, jednak bez rozróżnienia czy zarysowanie występuje w połączeniu konstrukcyjnym (często o innym stopniu zbrojenia), czy w analizowanym elemencie. Z tego względu konieczne jest dalsze rozwijanie tego modelu analitycznego, który pozwoli na analizę etapowego zarysowania w pierwszej kolejności samych połączeń konstrukcyjnych, a następnie zarysowania elementu, z jednoczesnym uwzględnianiem większej dojrzałości betonu. Model taki będzie miał duże znaczenie praktyczne w zakresie projektowania połączeń konstrukcyjnych, których celem będzie kompensacja odkształcenia wymuszonego, aby ograniczyć lub uniknąć zarysowania.

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