# Optimization programming tools supporting supply chain management

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**Abstract.** The issue of transportation is a particular type of mathematical programming that facilitates searching for and determining an optimal distribution network, considering the set of suppliers and recipients. This paper uses a numerical example to present a solution to a transport problem utilizing classical computation methods, i.e., the northwest corner, the least cost in a matrix, and the VAM approximation method. The objective of the paper was to develop tools in the form of algorithms that would then be implemented in three various computing environments (R, GNU Octave, and Matlab) that allow us to optimize transport costs within an assumed supply network. The model involved determining decision variables and indicating limiting conditions. Furthermore, the authors interpreted and visualized the obtained results. The implementation of the proposed solution enables users to determine an optimal transport plan for individually defined criteria.

**Key words:** mathematical programming; optimization; supply chain.

## 1. INTRODUCTION

Transport is an integral part of the logistics system. The systematic expansion of military equipment with vehicles that exhibit increased load capacity, capable of moving over varying terrain, as well as the reconstruction and development of the road network clearly indicate the increasing importance of road transport [1]. An innovative optimization method based on objective functions and design constraints for the optimal formation of virtual enterprises was described in [2].

A constraint satisfaction problem, the solution of which facilitates evaluating the behaviours of a system of concurrently interacting cyclic processes was presented in [3]. The current military road transport is used to move not only people and weapons but also all kinds of cargo, over both short and long distances. In times of peace, military cargo freight using road transport is characterized by high stability. Better operational organization of road transport facilitates reducing costs associated with the operation, better arrangement of the cargo space, or limiting the number of people required to implement a task. In turn, during combat operations, intensively volatile tactical situations mean that the established supply route may be changed. In military activity, especially when the time to make a decision is limited, determining an optimal solution requires organizational effort from commanders. The ability to utilize economic theories to solve transport problems for individually defined criteria gains particular importance [4]. The fuel consumption of vehicles powered by internal combustion engines constitutes an essential issue in forming the development trends

of the automotive industry and is an important factor in the economics of transport [5].

The problem of optimizing the delivery time within the assumed distribution network, allowing the company to develop and optimally plan the transport of goods with a short shelf life was considered in [6]. A genetic algorithm to find the control of the reservation limit to minimize maximum regret was proposed in [7]. A new optimization tool that supports supply chain management in the multi-criteria aspect using the genetic algorithm was proposed in [8]. The evaluation of the effectiveness of supply chain management was conducted based on the global function of the criterion. Three examples of actions in favour of transportation issues in a metropolitan area, demonstrating the functional and strategic role of mobility, were proposed in [9]. A probabilistic approach to determining a modal split was described in [10]. Analysis with the use of a logistic regression model was conducted for travel within the Warsaw transport system. In the paper, a new method was developed to find the initial basic feasible solution to the transportation problem using the arithmetic mean [11]. A new modification to Vogel's approximation method was introduced in [12] to find an initial feasible basic solution to the transportation problem. The article [13] proposed a novel Karagul–Sahin approximation method used. The solutions obtained using the proposed method were as good as the solutions using the VAM method.

Due to the development of computer technologies and the growing demand for quantitative IT tools, the market range of operational test methods available has been significantly expanded over recent years [14–17]. Many tools that facilitate the use of programming techniques, loops, and the generation of new user functions are available with the purpose of solving transport problems [18]. The development of technology and

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the increasing interest in new mathematical computation methods that facilitate the solution of transport problems affect the rational management of a modern fleet of means of transport. The source literature repeatedly addressed problems related to the reduction of the computational complexity of transportation algorithms using, e.g., the C++ programming language, the Matlab environment, the GNU Octave software, or the R programming language. However, it must be acknowledged that they require the user to have a thorough knowledge of the functions available and be proficient in their use.

### 1.1. Application of the C++ programming language

A new heuristic total opportunity-cost method to determine the basic feasible solution to the transportation problem was proposed in [19]. In [20], the author analyzed several variants of Vogel's approximation method, taking into account the concept of the total opportunity-cost method. Using the C++ programming language Turbo, it was found that the VAM method combined with the total opportunity-cost method results in an optimal or close to optimal solution. Implementation of the C++ programming language was also described in [21]. Computations show that the Juman & Hoque method in as many as 16 problems considered resulted in a minimum transport cost being determined. In [22], a transportation problem was solved using the row minimum method, the least cost in the matrix, the northwest corner, and the VAM methods. The calculations were conducted using object-oriented programming. Due to the need to execute a number of iterations leading to a solution to the problem in question, the C++ programming language was used. The practical application of C++ to find the optimal solution using the northwest corner rule, the least cost rule, and the Vogel approximation method was presented in [23].

### 1.2. Application of the Matlab environment

Matlab is a commercial programming language, which facilitates algorithm implementation, matrix manipulations, user interface creation, function plotting, and interfacing with programs developed in other languages. There are many solutions to the transportation problem in the source literature that use the Matlab environment. The problem of transporting chemical substances using the northwest corner method was analyzed in the paper [24]. In [25], the authors proposed a solution based on Vogel's approximation method and the modified distribution method that would facilitate the computation of different numerical problems. In [26], the validity of the use of Matlab software was proven not only due to the reduction of time required to perform the calculations but also due to the high efficiency of the algorithm used. Algorithm descriptions based on the minimum cost method, the northwest corner method, the MODI method, and the VAM method were presented in [27]. The proposed approach emphasizes the dominance of computer programming methods over classical analytical methods. More and more companies have adopted the cost of logistics distribution as one of the important indicators affecting the development of companies [28]. The quantitative analysis, overall analysis, and individual analysis on the supply characteristics of 402 suppliers using Matlab software were conducted in [29].

The use of a program written in Matlab proposed in [30] allows us to effectively manage vehicles and determine an optimal distribution plan within the supply chain.

### 1.3. Application of GNU Octave

GNU Octave is a free software suite licensed under the General Public License. The learning curve for GNU Octave is quite flat owing to its simple and intuitive syntax, which is largely compatible with Matlab [31]. GNU Octave is a high-level programming language primarily intended for scientific computing, generating new user functions, or executing complex numerical computations. The optimal solution to determine the transportation problem using GNU Octave programming was studied in [32]. The implementation of the proposed solution shortens the calculation algorithm and allows users to determine the optimal distribution plan for material resources for individually determined criteria. The authors of [33] presented the problem of staff allocation and then solved it using GNU Octave. The evaluation criterion was the minimization of worker safety risks. Linear programming is one of the most widely applied optimization models that involve finding efficient algorithms for an optimal solution. It is possible to use linear programming for the minimization of production costs, as well as for the maximization of profits. The application of GNU Octave software to optimize the cost-benefit criterion in production processes was presented in the article [34]. Integer programming using GNU Octave software to solve the complex combinatorial optimization problem was described in [35]. The proposed algorithm facilitated the calculation of feasible schedules in a brief time and cost-benefit analysis of different staffing levels. Various stochastic methods for combinatorial optimization can also be used to support the decision-making process. Stochastic optimization refers to the maximization or minimization of the objective function, in the presence of randomness within the search process. The authors of [36] applied a modified form of the honey bees mating optimization algorithm to design a water distribution network. All computations have been performed in GNU Octave software.

### 1.4. Application of R

The R programming language allows one to use programming techniques, generate new user functions, or perform complex numerical calculations. In addition, R is a powerful, easy-to-implement, and open-source programming language oriented at data science and analysis. An innovative method based on the particle swarm optimization algorithm was proposed in [37]. The combination of the Markov chain and Bayesian networks method was used to estimate the probability of risk factors associated with the investment strategy. The R programming language was used to solve the problem. Dynamic languages such as R can be used to process large data sets. Using the dynamic language R as an optimization target, the authors of [38] presented an optimization to reduce the memory consumption of R programs. An innovative algorithm for finding the least-cost areal mapping between paths using the R language was described in [39]. The least cost subnetwork was searched using the dynamic programming approach. R has a default-install

set of functionalities that can be easily expanded by the use of several thousand packages as well as user-written scripts. The application of R for the estimation of nonlinear parameters and minimization of essentially smooth functions was discussed in the article [40]. A genetic algorithm to identify the optimal routes for the pairs of seller-customer in the same group was included in [41]. The R programming language was used to test the effectiveness of the proposed algorithm. The travelling salesman problem is one of the combinatorial optimization problems and has wide application in various fields of science and technology. In [42] a new algorithm was proposed to solve the problem of travelling salespeople. The variable neighbourhood search algorithm implemented in R and a stochastic approach to finding the optimal solution were applied. There are many algorithms discussed in the source literature that deal with multi-objective optimization, most of them based on evolutionary techniques. In [43], two analytical examples selected from the literature were used to access the robustness of multi-objective optimization frameworks. The advantage of using a proposed algorithm implemented in the R language was reducing the computational cost of complex optimization tasks.

A conducted source literature review indicates that while the use of modern IT tools and programming languages that enable solving transport problems was the subject matter of many studies, the majority of publications required programming skills, familiarity with available functions, and proficiency in their application. Furthermore, the suggested solution often prevented one from determining the time required to perform computations. As a result, the objective characterized by the originality of this article set by its authors was to develop solutions in the form of scripts implemented in Matlab (ver. R2020b, 64-bit), GNU Octave (ver. 3.4.3, 64-bit), and R (ver. 2022.07.0, 64-bit), which would allow optimizing transport costs within an assumed supply network. The proposed solutions can be easily implemented by users who are not fluent in programming techniques since they eliminate the need to apply loops and user-generated functions. In addition, they not only ensure the determination of an optimal solution to the problem in question but also display the total computation time, which can be crucial when the time to make a decision is limited.

## 2. MATHEMATICAL MODEL

The optimization problem is also called the mathematical programming problem, and it involves finding the best solution in terms of the adopted decision criterion. The optimization issue can be reduced to searching for a set of permissible solutions, for which the function that represents a specific criterion and is described within this set reaches its extreme. Many linear programming problems can be solved after they are reduced to optimization problems. One of them is a transport problem that involves organizing a distribution network for a certain commodity from established suppliers to known recipients. Each of the recipients places a request and the resources of each supplier are also known. It is also assumed that supply and demand are equal. The problem requires determining the number of goods

from each supplier to each recipient to be delivered so that the total transport cost is as low as possible. The cost of transporting one unit of commodity between suppliers and recipients is known. The total transport cost is equal to the transport cost for all routes. The model adopted the following designations:

$M \in \mathbb{N}_+$ – number of suppliers,

$N \in \mathbb{N}_+$ – number of recipients,

$a_i \in \mathbb{R}_+$ – resource of the $i$-th supplier, $i = 1,\ldots,M$,

$b_j \in \mathbb{R}_+$ – demand of the $j$-th recipient, $j = 1,\ldots,N$,

$C = \left[c_{ij}\right]_{M \times N},\ c_{ij} \in \mathbb{R}_+$ – transport cost for one unit of the commodity from the $i$-th supplier to the $j$-th recipient,

$X = \left[x_{ij}\right]_{M \times N},\ x_{ij} \in \mathbb{R}_+^0$ – number of units to be transported from the $i$-th supplier to the $j$-th recipient,

$L \in \mathbb{R}_+$ – total transportation cost.

From the perspective of conducted analyses, the mathematical model that presents the essence of the solved problem is based on the following equations and assumptions [32]:

a) each supplier sends as many commodities as there are in its resources

$$\sum_{j=1}^{N} x_{ij} = a_i, \tag{1}$$

b) each recipient receives as many commodities as requested

$$\sum_{i=1}^{M} x_{ij} = b_j, \tag{2}$$

c) demand equals supply

$$\sum_{i=1}^{N} a_i = \sum_{j=1}^{M} b_j, \tag{3}$$

d) transport cost for one unit of commodity

$$L(X,C) = \sum_{i=1}^{M} \sum_{j=1}^{N} x_{ij} c_{ij}, \tag{4}$$

e) the total commodity transport cost is to be as low as possible

$$L(X^*) = \min_{X} L(X). \tag{5}$$

The transport problem is completely determined by the number of recipients $N$ and suppliers $M$. Please note that the number of all decision variables in the problem is $M \cdot N$, while the number of constraints is equal to $M + N$. In addition, the decision variables found under limiting conditions are only accompanied by a coefficient of 0 or 1. The balanced transport problem under consideration was solved based on the northwest corner, the least cost in the matrix, and Vogel's approximation (VAM) methods.

## 3. METHODOLOGY

### 3.1. The Northwest corner method

The northwest corner method involves searching a transport table and assigning specific values to variables, each time for routes in the left top corner of the cost matrix. In the case of the

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problem in question, the authors assumed that transport would be conducted using a general-purpose, large-capacity JELCZ 325 vehicle, for which the unit operating cost index per operating unit is PLN 6. Transport costs were calculated based on Order No. 40 of the Chief of the Armed Forces Support Inspectorate dated 17 February 2020. Numerical data on the route, demand, and costs of transport between suppliers and recipients are listed in Table 1.

**Table 1**
Data for numerical problem

| | | Suppliers | | | | | |
|---|---|---|---|---|---|---|---|
| | | S1 | S2 | S3 | S4 | S5 | Demand |
| Recipients | R1 | 432 | 1715 | 1092 | 765 | 1104 | 46 |
| | R2 | 858 | 2246 | 376 | 1357 | 1697 | 37 |
| | R3 | 1209 | 2764 | 494 | 1968 | 2308 | 53 |
| | R4 | 1450 | 938 | 1968 | 463 | 117 | 49 |
| | Supply | 36 | 29 | 23 | 42 | 55 | |

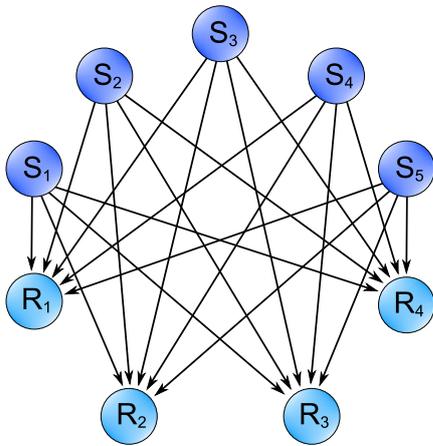The transportation model was presented by the network shown in Fig. 1.



**Fig. 1.** Network structure of transportation problem

Determining a permissible solution using the northwest corner method involves indicating an element in the left corner of the cost matrix and establishing corresponding supply and demand. The next stage requires selecting the lower value and entering a selected value in the field corresponding to the left corner first cell, as shown in Table 2.

Next, the entered number has to be subtracted from the supply and demand. Then check for which value the 0 was obtained. In a situation where demand takes the zero value, the other fields in a given line should be filled with zeros. The supply in the problem under consideration is 0. A permissible solution presented in Table 3 was determined by following the procedure described in the steps below.

**Table 2**
Results table for the northwest corner method

| | | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
| | | 36 | 29 | 23 | 42 | 55 |
| R1 | 46 | 36 | | | | |
| R2 | 37 | | | | | |
| R3 | 53 | | | | | |
| R4 | 49 | | | | | |

**Table 3**
A feasible solution for the northwest corner method

| | | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | 0 |
| R1 | 0 | 36 | 10 | 0 | 0 | 0 |
| R2 | 0 | 0 | 19 | 18 | 0 | 0 |
| R3 | 0 | 0 | 0 | 5 | 42 | 6 |
| R4 | 0 | 0 | 0 | 0 | 0 | 49 |

A permissible solution is degenerate when the number of baseline elements (different from zero) is equal to $M + N - 1$. Considering the obtained baseline solution, the sum of products of baseline elements and the corresponding transport costs were determined according to equation (4) in order to calculate transport costs with the northwest corner method. The total transport cost calculated with the northwest corner method is PLN 186 851.

### 3.2. The least cost in the matrix method

The first step to determine a permissible solution using the least cost method in the matrix is to select the least value among all in the cost matrix. Next, it is necessary to determine the lower value of supply and demand that corresponds to this cell. The results are shown in Table 4.

**Table 4**
Table of results for the least cost in the matrix method

| | | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
| | | 36 | 29 | 23 | 42 | 55 |
| R1 | 46 | | | | | |
| R2 | 37 | | | | | |
| R3 | 53 | | | | | |
| R4 | 49 | | | | | 117 |

The minimum element in the transport cost matrix is 117, with its supply at 55 and demand equal to 49. In the next iteration, both supply and demand were reduced by 49, which led to

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0 for demand. This resulted in filling the other field of the fourth line with 0 as well. Table 5 shows the results after applying the presented procedure.

**Table 5**
Results of the first iteration

|  |  | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
|  |  | 36 | 29 | 23 | 42 | 6 |
| R1 | 46 |  |  |  |  |  |
| R2 | 37 |  |  |  |  |  |
| R3 | 53 |  |  |  |  |  |
| R4 | 0 | 0 | 0 | 0 | 0 | 49 |

Proceeding in an analogous manner, considering the cells filled with 0, resulted in obtaining a degenerate permissible solution. Table 6 illustrates the result of the calculations performed.

**Table 6**
A feasible solution for the least cost in the matrix method

|  |  | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
|  |  | 0 | 0 | 0 | 0 | 0 |
| R1 | 0 | 36 | 0 | 0 | 10 | 0 |
| R2 | 0 | 0 | 0 | 23 | 14 | 0 |
| R3 | 0 | 0 | 29 | 0 | 18 | 6 |
| R4 | 0 | 0 | 0 | 0 | 0 | 49 |

The obtained permissible solution is also in degenerate form since it has 8 baseline elements. In order to calculate transport costs with the least cost in the matrix method, the authors once again determine the sum of products of baseline elements and costs listed in Table 1, with the result being PLN 186 009. It should be noted that the transport cost calculated with the least cost in the matrix method is lower than the value determined with the northwest corner method, which means that the acceptable solution in Table 6 is better.

### 3.3. Vogel's approximation method (VAM)

The VAM involves calculating the so-called indices, understood as the difference between the cheapest and second lowest cost in each line and column. Next, an element with the highest value should be determined among the calculated indices. If the highest difference in transport costs corresponds to the column, select the lowest cost in the column in question. If the highest index corresponds to a line, this will require selecting the minimum element in the analyzed line. Table 7 illustrates the result of the calculations performed.

The maximum transport cost difference in the problem in question corresponds to the column and amounts to 987. The

**Table 7**
The result of computations conducted for the VAM

|  |  | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
|  |  | 426 | 777 | 118 | 302 | 987 |
| R1 | 333 | 432 | 1715 | 1092 | 765 | 1104 |
| R2 | 482 | 858 | 2246 | 376 | 1357 | 1697 |
| R3 | 715 | 1209 | 2764 | 494 | 1968 | 2308 |
| R4 | 346 | 1450 | 938 | 1968 | 463 | 117 |

minimum element in the analyzed column is 117, with a demand of 49 and a supply of 55. The lowest value of the two should be entered in the cell with a cost value of 117, while the entered number should be subtracted from both supply and demand. The 0 obtained for demand leads to the need to fill other cells in the line in question with zeros. The consequences of applied transformations are illustrated in Table 8.

**Table 8**
Results table for the VAM

|  |  | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
|  |  | 36 | 29 | 23 | 42 | 6 |
| R1 | 46 |  |  |  |  |  |
| R2 | 37 |  |  |  |  |  |
| R3 | 53 |  |  |  |  |  |
| R4 | 0 | 0 | 0 | 0 | 0 | 49 |

Further computations should not include the lines or columns filled with 0. A permissible solution presented in Table 9 was determined by following the procedure described in the following steps.

**Table 9**
A feasible solution for the VAM

|  |  | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
|  |  | 0 | 0 | 0 | 0 | 0 |
| R1 | 0 | 0 | 0 | 0 | 40 | 6 |
| R2 | 0 | 6 | 29 | 0 | 2 | 0 |
| R3 | 0 | 30 | 0 | 23 | 0 | 0 |
| R4 | 0 | 0 | 0 | 0 | 0 | 49 |

The determined solution has 8 baseline elements; therefore, the requirement with respect to a degenerate permissible solution has been satisfied. The total transport costs calculated as the sum of products of the baseline solution and the costs resulting from Table 1 for the VAM method are PLN 163 585. Table 10 lists the permissible solutions obtained depending on the applied method.

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## 3.4. Karagul–Sahin approximation method

The Karagul–Sahin method is an iterative method consisting of 5 steps:

1. Determine the coefficients $r_{ij} = \dfrac{b_j}{a_i}$ and $r_{ji} = \dfrac{a_i}{b_j}$.
2. Determine the matrices:
   a) weighted A transport costs (i.e., the products $r_{ij}$ and the transport cost matrix),
   b) weighted B transport costs (i.e., the products $r_{ji}$ and the transport cost matrix).
3. Identify the minimum weighted transportation cost in matrices A and B and then identify the largest possible assignment corresponding to the value of supply or demand on the route (considering the limiting conditions).
4. If all elements of matrices A and B have been analyzed – then the end of the algorithm. Otherwise, return to step 3.
5. Determine the value of the objective function.

Table 10 summarizes the results of the calculations performed.

#### Table 10
The determined values of the coefficients $r_{ij}$, $r_{ji}$ and the values of the individual cells of the matrix A and B

| $r_{ij}$ | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| S1 | 1.28 | 1.03 | 1.47 | 1.36 |
| S2 | 1.59 | 1.28 | 1.83 | 1.69 |
| S3 | 2.00 | 1.61 | 2.30 | 2.13 |
| S4 | 1.10 | 0.88 | 1.26 | 1.17 |
| S5 | 0.84 | 0.67 | 0.96 | 0.89 |

| Matrix A | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| S1 | 552.00 | 881.83 | 1779.92 | 1973.61 |
| S2 | 2720.34 | 2865.59 | 5051.45 | 1584.90 |
| S3 | 2184.00 | 604.87 | 1138.35 | 4192.70 |
| S4 | 837.86 | 1195.45 | 2483.43 | 540.17 |
| S5 | 923.35 | 1141.62 | 2224.07 | 104.24 |

| $r_{ji}$ | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| S1 | 0.78 | 0.97 | 0.68 | 0.73 |
| S2 | 0.63 | 0.78 | 0.55 | 0.59 |
| S3 | 0.50 | 0.62 | 0.43 | 0.47 |
| S4 | 0.91 | 1.14 | 0.79 | 0.86 |
| S5 | 1.20 | 1.49 | 1.04 | 1.12 |

| Matrix B | R1 | R2 | R3 | R4 |
|---|---|---|---|---|
| S1 | 338.09 | 834.81 | 821.21 | 1065.31 |
| S2 | 1081.20 | 1760.38 | 1512.38 | 555.14 |
| S3 | 546.00 | 233.73 | 214.38 | 923.76 |
| S4 | 698.48 | 1540.38 | 1559.55 | 396.86 |
| S5 | 1320.00 | 2522.57 | 2395.09 | 131.33 |

The first minimum weighted transportation cost in matrix A is the value of 104.24, while in matrix B it is the value of 131.33. The largest possible assignment corresponding to the value of 104.24 is the number 49 and therefore this value was entered into the cell corresponding to the intersection of S5 and R4, as illustrated in Table 11.

#### Table 11
Table for results

| | | R1 | R2 | R3 | R4 |
|---|---|---|---|---|---|
| | | 46 | 37 | 53 | 49 |
| S1 | 36 | | | | 0 |
| S2 | 29 | | | | 0 |
| S3 | 23 | 0 | 0 | 23 | 0 |
| S4 | 42 | | | | 0 |
| S5 | 55 | | | | 49 |

The next minimum weighted transport cost in matrix A is the number 552 while in matrix B it is the number 214.38. The smaller of the two is 214.38 for which the largest possible assignment is the number 23, so this value was entered in the field corresponding to the intersection of cells S3 and R3. Following the procedure presented, the admissible solution presented in Table 12 was obtained.

#### Table 12
Permissible solution determined by the Karagul–Sahin method

| | | R1 | R2 | R3 | R4 |
|---|---|---|---|---|---|
| | | 46 | 37 | 53 | 49 |
| S1 | 36 | 0 | 6 | 30 | 0 |
| S2 | 29 | 0 | 29 | 0 | 0 |
| S3 | 23 | 0 | 0 | 23 | 0 |
| S4 | 42 | 42 | 0 | 0 | 0 |
| S5 | 55 | 4 | 2 | 0 | 49 |

Considering the degenerate baseline solution obtained, the sum of the products of the baseline elements and the corresponding transportation costs was determined according to formula (4) to calculate the transport costs using the Karagul–Sahin method. The total transport cost is PLN 163 587.

## 3.5. Maximum difference extreme difference method

The following steps can be detailed in the maximum difference method [44]:

1. Determine the difference between the two largest values in each row of the cost matrix.
2. Determine the difference between the maximum and minimum values of the cost in each column of the cost matrix.

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3. Identify the maximum among the values calculated in steps 1 and 2 and then find the corresponding minimum transportation cost.
4. If the maximum difference corresponds to two or more cells, then select the cost cell located in the upper left corner of the cost matrix.

   For the indicated cell, find the smaller of the corresponding values of supply and demand, and then reduce demand and supply by the entered value, completing the remaining cells with 0 values, depending on whether 0 was obtained in the row or in the column.
5. Repeat steps 1–4 until all supply and demand cells are equal to 0.
6. Determine the value of the objective function.

   Table 13 illustrates the results of applying the described treatment scheme.

### Table 13

The result of implementing the method of maximum difference

| | | R1 | R2 | R3 | R4 | | |
|---|---|---|---|---|---|---|---|
| | | 0 | 0 | 0 | 0 | | |
| S1 | 0 | 0 | 6 | 30 | 0 | | |
| S2 | 0 | 0 | 29 | 0 | 0 | | |
| S3 | 0 | 0 | 0 | 23 | 0 | | |
| S4 | 0 | 42 | 0 | 0 | 0 | | |
| S5 | 0 | 4 | 2 | 0 | 49 | | |
| Difference values determined for rows (maximum difference) | | | | | | | |
| 241 | 241 | 592 | – | – | – | – | |
| 518 | 518 | 531 | 531 | 531 | 531 | **2246** | |
| 876 | – | – | – | – | – | – | |
| 611 | 611 | 592 | 592 | – | – | – | |
| 611 | 611 | 593 | 593 | 593 | 593 | 1697 | **1697** |
| Difference values determined for columns (extreme difference) | | | | | | | |
| 1283 | 1870 | **2270** | 1851 | | | | |
| 1283 | 1388 | **1555** | 1333 | | | | |
| 1283 | **1388** | – | 1333 | | | | |
| **950** | 889 | – | 821 | | | | |
| 611 | 549 | – | **821** | | | | |
| **611** | 549 | – | – | | | | |
| – | 549 | – | – | | | | |
| | **1697** | | | | | | |

Taking into account the base solution (the solution is degenerate because the number of base elements is $M + N - 1$), in order to calculate the transport costs by the described method, the sum of the products of the base elements and the corresponding transport costs was determined according to formula (4). The total transportation cost is 163 587 PLN. Table 14 presents a summary of the admissible solutions depending on the method used and a graphical interpretation of the base solutions is presented in Fig. 2.
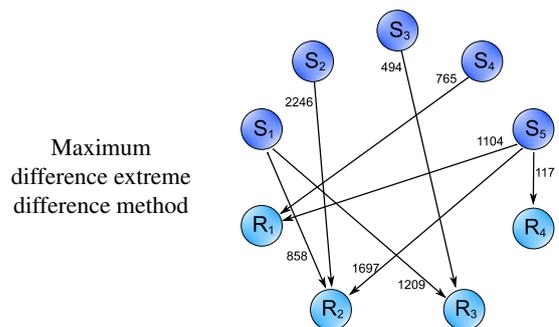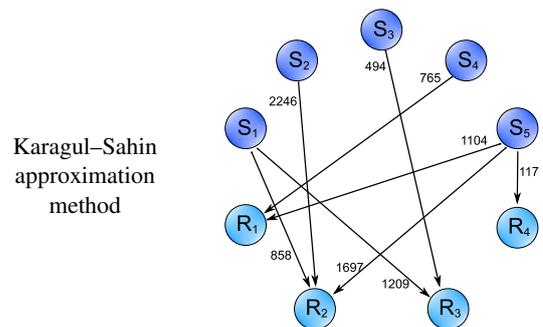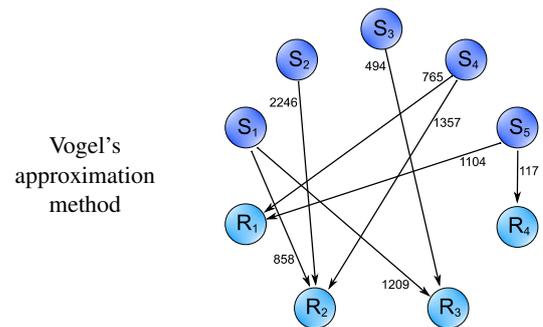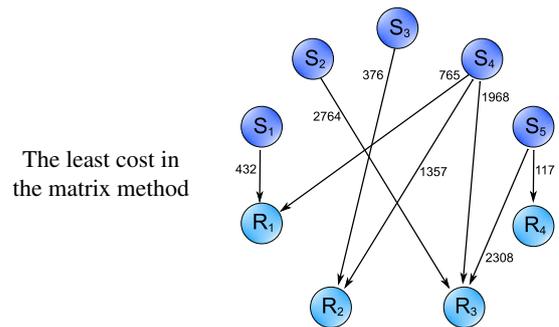


**Fig. 2.** The graphical interpretation of the base solutions

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### Table 14
Obtained permissible solutions

| | | S1 | S2 | S3 | S4 | S5 |
|---|---|---|---|---|---|---|
| The northwest corner method | R1 | 0 | 0 | 0 | 40 | 6 |
| | R2 | 6 | 29 | 0 | 2 | 0 |
| | R3 | 30 | 0 | 23 | 0 | 0 |
| | R4 | 0 | 0 | 0 | 0 | 49 |
| The least cost in the matrix method | R1 | 36 | 0 | 0 | 10 | 0 |
| | R2 | 0 | 0 | 23 | 14 | 0 |
| | R3 | 0 | 29 | 0 | 18 | 6 |
| | R4 | 0 | 0 | 0 | 0 | 49 |
| Vogel's approximation method | R1 | 0 | 0 | 0 | 40 | 6 |
| | R2 | 6 | 29 | 0 | 2 | 0 |
| | R3 | 30 | 0 | 23 | 0 | 0 |
| | R4 | 0 | 0 | 0 | 0 | 49 |
| Karagul–Sahin approximation method | R1 | 0 | 0 | 0 | 42 | 4 |
| | R2 | 6 | 29 | 0 | 0 | 2 |
| | R3 | 30 | 0 | 23 | 0 | 0 |
| | R4 | 0 | 0 | 0 | 0 | 49 |
| Maximum difference extreme difference method | R1 | 0 | 0 | 0 | 42 | 4 |
| | R2 | 6 | 29 | 0 | 0 | 2 |
| | R3 | 30 | 0 | 23 | 0 | 0 |
| | R4 | 0 | 0 | 0 | 0 | 49 |

### Table 15
The results of applied transformations

| Indirect costs | | | | | | |
|---|---|---|---|---|---|---|
| $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | | |
| **432** | **1715** | **−155** | **1319** | **1659** | | |
| 432 | 1715 | −155 | 1319 | 1659 | $Y_1$ | **0** |
| 963 | 2246 | 376 | 1850 | 2190 | $Y_2$ | **531** |
| 1081 | 2364 | 494 | 1968 | 2308 | $Y_3$ | **649** |
| −1110 | 173 | −1697 | −223 | 117 | $Y_4$ | **−1542** |

| Transportation costs | | | | |
|---|---|---|---|---|
| 432 | 1715 | 1092 | 765 | 1104 |
| 858 | 2246 | 376 | 1357 | 1697 |
| 1209 | 2764 | 494 | 1968 | 2308 |
| 1450 | 938 | 1968 | 463 | 117 |

### Table 16
The values of optimality indices for the northwest corner method

| $X_1$ | $X_2$ | $X_3$ | $X_4$ | $X_5$ | | |
|---|---|---|---|---|---|---|
| **432** | **1715** | **−155** | **1319** | **1659** | | |
| 0 | 0 | −1247 | 554 | 555 | $Y_1$ | **0** |
| 105 | 0 | 0 | 493 | 493 | $Y_2$ | **531** |
| −128 | −400 | 0 | 0 | 0 | $Y_3$ | **649** |
| −2560 | −765 | −3665 | −686 | 0 | $Y_4$ | **−1542** |

## 4. RESULTS

### 4.1. Optimization of the basic feasible solution (the potential method)

The potential method aims to optimize the permissible solution obtained. Based on the input data and the solution obtained with the northwest corner method, the authors developed a table for the results (i.e., indirect costs), so that the elements corresponding to the supply and demand values remain empty. The transport costs in the cells corresponding to the elements of the baseline solution were filled with the values in Table 1. It was assumed that $Y_1 = 0$. A transport cost equal to 432 corresponds to potential $Y_1$. In order to determine potential $X_1$, the authors calculated the difference between the cost and potential $Y_1$, according to the dependence: $X_1 = 432 − Y_1$. Next, in the line corresponding to potential $Y_1$, the authors indicated a next transport cost, which allowed us to determine potential $X_2$ understood as: $X_2 = 1715 − Y_1$. The other potentials, $X_i$, where $i = 1,\ldots,M$ and $Y_j$ where $j = 1,\ldots,N$ were determined similarly. The cells constituting zero elements of the permissible solutions were filled with the sums of potential $X_i + Y_j$. The results of applied transformations are illustrated in Table 15.

Table 16 shows the optimality indices that are the difference between indirect costs and costs arising from Table 1.

The presence of positive values among optimality indices means that the obtained permissible solution is not optimal. Obtaining a new solution with a lower transport cost requires building a cycle in such a manner that the maximum optimality index is the first element of a positive cycle. Next, it is required to indicate in the line with the positive cycle element an element having an equivalent in the column. The described procedure should be repeated until the cycle is fully closed. The minimum value among all negative cycle elements should be indicated. Next, the positive cycle elements should be increased and the negative cycle element reduced by this number. Table 17 illustrates the constructed cycle and a new permissible solution.

### Table 17
The results of applied transformations

| Cycle | | | | | New feasible solution | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 36 | 10 (−) | 0 | 0 | 0 (+) | 36 | 4 | 0 | 0 | 6 |
| 0 | 19 (+) | 18 (−) | 0 | 0 | 0 | 25 | 12 | 0 | 0 |
| 0 | 0 | 5 (+) | 42 | 6 (−) | 0 | 0 | 11 | 42 | 0 |
| 0 | 0 | 0 | 0 | 49 | 0 | 0 | 0 | 0 | 49 |

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The transport cost in the new baseline (degenerate) solution is lower than initially and amounts to PLN 183 521. A new solution was obtained after repeating the procedure, leading to determining optimality indices and cycle construction five times. Table 18 illustrates the ultimate results and calculated optimization indicators.

**Table 18**
The new feasible solution and optimization indicators

| New feasible solution | | | | | Optimization indicators | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 40 | 6 | −166 | −61 | −1541 | 0 | 0 |
| 6 | 29 | 0 | 2 | 0 | 0 | 0 | −233 | 0 | −1 |
| 30 | 0 | 23 | 0 | 0 | 0 | −167 | 0 | −260 | −261 |
| 0 | 0 | 0 | 0 | 49 | −2171 | −271 | −3404 | −685 | 0 |

The fact that all optimality indices are non-positive clearly indicates that the obtained solution is optimal. The entire cargo transport cost is PLN 163 585. The transport costs for each method and the result of the potential method that leads to an optimal solution are presented in Tables 19 and 20.

**Table 19**
Transport costs for the basic solution

| Method | Transport costs for basic solution (PLN) |
|---|---|
| The northwest corner method | 186 851 |
| The least cost in the matrix method | 186 009 |
| Vogel's approximation method | 163 585 |
| Karagul–Sahin approximation method | 163 587 |
| Maximum difference extreme difference method | 163 587 |

When comparing the results obtained, it should be noted that the objective function values for individual permissible solutions vary depending on the selected method. Each of the methods in question satisfied the condition in terms of the permissible solution degeneracy. The northwest corner method, despite the simplicity of the computation, turned out to be the least accurate, since it required applying the potential method five times. In the case of the least cost element in the matrix method, a lower transport cost was determined for the baseline solution than in the northwest corner method, however, it also required fivefold optimization of the permissible solution. The Karagul–Sahin method and the method of maximum differences provided the same admissible solution, which was not optimal because it required applying the method of potentials once. Only the VAM method ensured that a minimum objective function value was obtained, hence, determining an optimal commodity transport plan from the supplier to the recipients already in the first step. The transport costs after subsequent steps of improvement are illustrated in Fig. 3.

**Table 20**
Comparison of solutions obtained

| Transport costs after the next iterations of the potential method (PLN) | The northwest corner method | The least cost in the matrix method | Karagul–Sahin approximation method and maximum difference extreme difference method | Vogel's approximation method |
|---|---|---|---|---|
| 1st improvement of the solution | 183 521 | 177 135 | 163 587 | 163 585 |
| 2nd improvement of the solution | 181 305 | 174 665 | 163 585 | |
| 3rd improvement of the solution | 165 969 | 174 238 | | |
| 4th improvement of the solution | 164 983 | 164 581 | | |
| 5th improvement of the solution | 163 585 | 163 585 | | |



The Vogel's Approximation Method
The Karagul-Sahin Approximation Method
The Maximum Difference Extreme Difference Method
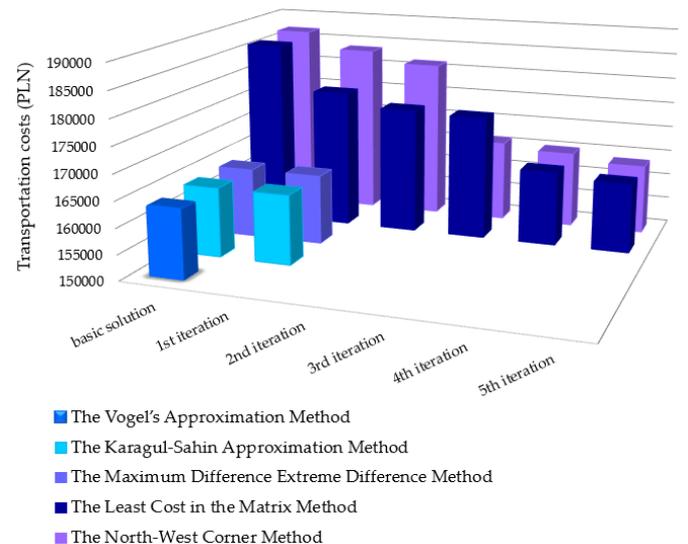The Least Cost in the Matrix Method
The North-West Corner Method

**Fig. 3.** The transport costs after subsequent steps of improvement

### 4.2. Reduction of computational complexity of the transportation problem using programming languages

As is well known, the travelling salesman problem is NP-difficult, which means that algorithms with polynomial computational complexity that solve this problem are not known. Therefore, in practical applications, it is necessary to use polynomial approximation algorithms. One possible algorithm is to check the possible Hamiltonian cycles and choose the shortest cycle. The disadvantage of this method is that the number of cycles for an $n$-vertex graph is $(n-1)!/2$. The problem of computational complexity has been the subject of research by many scientists, who have so far failed to find the best algorithm in

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terms of computational complexity. As examples, we can point to the Held–Karp algorithm, which has a complexity of $n^2 2^n$, the nearest neighbour algorithm with a complexity of $n^2$, the greedy algorithm with a complexity of $n^2 \log(n)$ or the RNN algorithm (repetitive NN algorithm) with a complexity of $n^3$.

### 4.2.1. Application of the Matlab environment

Considering the reduction in time necessary to perform the enormous calculations, the transport problem can be easily solved using the source code proposed in this article, which has been implemented in the Matlab command window (Appendix A). The R2020b (64-bit) version was used in the computations. The application of the built-in 'linprog' function ensured an optimal solution, consistent with the result of the VAM method. The total computation time determined with the 'tic (); toc' function was 0.021 s.

### 4.2.2. Application of the GNU Octave

A source code based on the 'glpk' function available in GNU Octave was developed to determine an optimal solution to the transport problem (Appendix B). In turn, determining the computation time was possible due to the application of the 'tic (); toc' function, which resulted in an optimal solution obtained after 0.354 s. GNU Octave v.3.4.3 (64-bit) was used in the investigation.

### 4.2.3. Application of the R

With optimal supply chain management in mind, the last of the solutions proposed in this article was to use the 'lp.transport' function called in R (Appendix C). The 'Sys.time()' function was used to determine the total computation time. As a result of the conducted analysis, it was concluded that the total time required to determine the optimal solution in R was only 0.013 s. The computations were performed using R v.2022.07.0 (64-bit).

Figure 4 summarizes the total time required to perform the calculations for each of the proposed scripts.
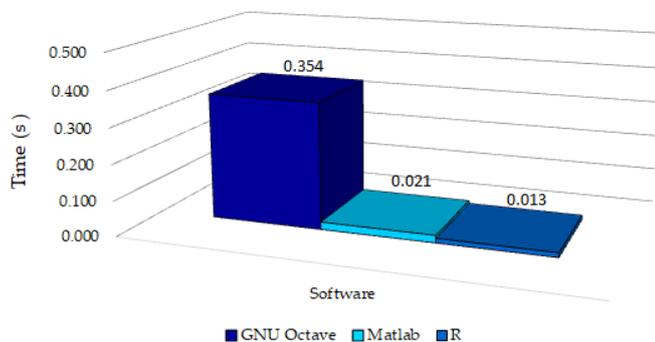


**Fig. 4.** The total time required to perform the calculations

The longest time of calculations ensuring the determination of an optimal solution to the problem in the questions has been recorded when using GNU Octave. Please note that implementing a script called in Matlab or R significantly reduced the computation time relative to GNU Octave. The fastest solution to the problem in question, hence, the lowest computation complexity was obtained using R. Table 21 lists similarities and differences resulting from the application of R, GNU Octave, and Matlab.

## 5. SUMMARY AND CONCLUSIONS

The transport and traffic subsystem are crucial elements of a logistics system. Its main task is to ensure the correct and timely transfer of people and goods. The development of technology and the increasing interest in new mathematical computation methods that allow one to solve transport problems are factors that support rational supply chain management. The application of computer technologies and the growing demand for IT optimization tools were the reason behind expanding the market offer in the field of operational studies over the recent years. Programming languages are increasingly being used to support the decision-making process. It should be stressed that decision-making time may be crucial in supply chain management. As a result, the objective of this article was to develop solutions in the form of scripts implemented in GNU Octave, Matlab and R, which enable minimizing transport costs within supply networks, while minimizing the time required for computations.

The solutions proposed in this publication can be easily implemented by users not fluent in programming techniques, since they eliminate the need to apply loops and user-generated functions. The paper used the north-west corner, the least cost in the matrix and the VAM methods. In turn, the potential method was used to optimize individual baseline solutions. Classical accounting methods were characterised by high computing complexity and, hence, were very time-consuming. The analysis conducted allowed one to determine an optimal solution with a total transport cost of PLN 163 585.0. The north-west corner method and the least cost in the matrix method required a five-fold application of the potential method, while VAM ensured determining an optimal solution in the first step.

To determine the optimal solution to the transport problem, the authors used a built-in 'glpk' function within the script implemented in GNU Octave. The total computation time was 0.354 s. However, determining the most favourable transport plan with the use of the Matlab environment involved the application of the 'linprog' function, which is an element of the Optimization Toolbox suite that employs the so-called simple-dual algorithm simultaneously solving the current and dual problems. The optimal solution to the problem in question was found after 0.021 s. The last of the proposed scripts has an implemented 'lp.transport' function, which was then called in the R environment. As a result of the computations performed, it was concluded that the total time required to determine an optimal solution in R was 0.013 s. The quickest solution to the problem in question, and thus, the lowest computational complexity, was obtained when using R. Furthermore, R, like GNU Octave, is free, unlike the Matlab computing environment, which is commercial. The developed scripts do not enable determining individual permissible solutions resulting from the potential method, while allowing one to determine an optimal

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**Table 21**

Comparison of GNU Octave, Matlab, and R

| Number | GNU Octave | Matlab | R |
|---|---|---|---|
| 1 | Object-oriented programming language | Interactive language, visualization tool, and computing environment | Numerically-oriented programming language |
| 2 | Most numerical experiments are compatible with Matlab | Matlab is compatible with GNU Octave but not R | R codes can be integrated with other programming languages |
| 3 | Can be used to solve linear and non-linear problems numerically | Can be used to solve linear and non-linear problems numerically | Can be used to simplify complex mathematical problems |
| 4 | Can run on Windows, macOS, and Linux | Can run on Windows, macOS, and Linux | Can run on Windows, macOS, and Linux |
| 5 | GNU Octave is slower than Matlab | Matlab is faster than GNU Octave | R is faster than Matlab |
| 6 | GNU Octave is not a commercial product. GNU Octave is open-source | Matlab is a commercial product of MathWorks | R is not a commercial product. R is open-source |
| 7 | GNU Octave offers online support to programmers, as well as providing tutorials, documentation, examples, blog, answers, and community support | Matlab offers online support to programmers, as well as providing tutorials, documentation, examples, blog, answers, and community support | R offers online support to programmers, as well as providing tutorials, documentation, examples, blog, answers, and community support |
| 8 | Easy to learn syntax | Easy to learn syntax | Relatively complex syntax |
| 9 | GNU Octave is a programming language used for numerical computations | Matlab is a programming language used for technical computations | R is a programming language used for statistical analysis |
| 10 | GNU Octave supports graphics for 2D and 3D data visualization | Matlab supports graphics for 2D and 3D data visualization | R supports graphics for 2D and 3D data visualization |
| 11 | GNU Octave is not widely used within the industry | Matlab is widely used within the industry | GNU Octave is not widely used within the industry |
| 12 | GNU Octave has proper support for auto-increment and assignment operators | Matlab does not have support for auto-increment and assignment operators | R does not have an increment operator in the base language, but it can be implemented in the 'Hmisc' package |
| 13 | Empty files can be loaded without any hassle | Empty files cannot be loaded | Empty files can be loaded without any hassle |
| 14 | Commands used to print the result on the screen in GNU Octave are 'print' and 'fprintf' | Commands used to print the result on the screen in Matlab is only 'fprintf' | Commands used to print the result on the screen in R are 'print', 'message', and 'cat' |
| 15 | It consumes less memory than Matlab | It consumes more memory than GNU Octave | It consumes less memory than GNU Octave |
| 16 | It was written in C, C++, and Fortran | It was written in C, C++, and Java | It was written in C, C++, and Fortran |

distribution network over a noticeably brief time, which may be crucial when the decision time is limited.

Every programming language has its advantages and disadvantages. R and GNU Octave are both open-source programming languages oriented towards data science and analysis. Anyone can work with R and GNU Octave without any license or fee. Matlab, GNU Octave, and R provide a wide range of applications. GNU Octave is mainly used for numerical computations, Matlab for technical computations, and R for statistical analysis. It should be emphasized that Matlab is compatible with GNU Octave but not with R. The increasing computing power of computers provides the ability to solve increasingly complex decision-making problems. Therefore, further research will be directed at extending the proposed solutions with the possibility of solving multi-criteria decision problems.

**APPENDIX A**

Application of the Matlab environment

```
tic ()
f = [423; 858; 1209; 1450; 1715; 2246; 2764;
938; 1092; 376; 494; 1968; 765; 1357; 1968; 463;
1104; 1697; 2308; 117]; % A matrix containing
```

```
the objective function coefficients
(the transport costs)
A=[]; % Linear inequality constraints, specified
as a real matrix
b=[]; % Linear inequality constraints, specified
as a real vector
beq=[36; 29; 23; 42; 55; 46; 37; 53; 49];
% Linear equality constraints, specified as
a real vector
Aeq=[
  1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
  0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0;
  0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0;
  0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0;
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1;
  1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0;
  0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0;
  0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0;
  0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1];
lb=[0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0];
% Lower bounds, specified as a real vector or
real array
ub=[]; % Upper bounds, specified as a real
vector or real array
[x, fval] = linprog(f,A,b,Aeq,beq,lb,ub)
toc
```

## APPENDIX B
Application of the GNU Octave
```
tic()
C = [423 858 1209 1450; 1715 2246 2764 938;
1092 376 494 1968; 765 1357 1968 463; 1104
1697 2308 117]; % A matrix containing
the objective function coefficients
(the transportation costs)
s = [36 29 23 42 55]'; % Supply
d = [46 37 53 49]'; % Demand
A = [
  1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0;
  0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0;
  0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0 0 0 0 0;
  0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1 0 0 0 0;
  0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 1 1 1 1;
  1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0;
  0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0;
  0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0;
  0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1 0 0 0 1]; % A
matrix containing the constraints coefficients
c = C'(:);
b = [s;d]; % A column array containing the
right-hand side value for each constraint in
the constraint matrix
lb = [0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0]';
% An array containing the lower bound on each
of the variables
```

```
ub = []; % An array containing the upper
bound on each of the variables. If ub is not
supplied, the default upper bound is assumed to
be infinite
ctype = "UUUUULLLL"; % An array of characters
containing the sense of each constraint in the
constraint matrix. Each element of the array may
be one of the following values: "U" <=; "S" =;
"L" >=
vartype = "CCCCCCCCCCCCCCCCCCCC"; % A column
array containing the types of the variables.
"C" - A continuous variable; "I" - An integer
variable
sense = 1; % If sense is equal 1, the problem
is a minimization; If sense is equal -1, the
problem is a maximization
[xopt,fopt] = glpk(c,A,b,lb,ub,ctype,vartype,1);
xopt = reshape(xopt,4,5)';
fopt % The optimum value of the objective
function
xopt % The value of the decision variables at
the optimum
toc
tic; toc
```

## APPENDIX C
Application of the R
```
start_time <- Sys.time()
cost=c(432,858,1209,1450,1715,2246,2764,938,
1092,376,494,1968,765,1357,1968,463, 1104,1697,
2308,117)
cost_matrix <- matrix(cost,nrow = 5, ncol = 4,
byrow = TRUE) # Matrix of costs; ij-th element
is the cost of transporting one item from source
i to destination j
row.rhs <- c(36,29,23,42,55) # Vector of numeric
values for the right-hand sides of the row
constraints
col.rhs <- c(46,37,53,49) # Vector of numeric
values for the right-hand sides of the column
constraints
row.signs <- c("=","=","=","=","=") # Vector of
character strings giving the direction of the
row constraints
col.signs <- c("=","=","=","=") # Vector of
character strings giving the direction of the
column constraints
sol <-
lp.transport(cost_matrix, direction="min",
row.signs, row.rhs, col.signs, col.rhs)
sol
sol$solution # The optimal solution
sol$objval # The objective function
end_time <- Sys.time()
end_time - start_time
```

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