Mathematical modeling of inanimate natural phenomena

The Living Earth

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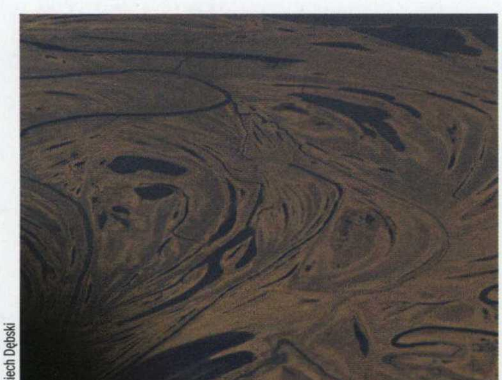
The ongoing climate change and major natural disasters we have been witnessing in recent years, such as the unprecedented floods in Poland and the eruption of the Eyjafjallajókull volcano in Iceland in 2010, encourage us to profoundly rethink how well we really understand our own planet

One of the difficulties faced by science in its attempts to comprehend natural events is their incredible complexity and the interactions between various physicochemical processes, frequently characterized by extreme time and energy scales. This complex, slowly evolving natural system - the Living Earth - is also subject to significant external influences and modified by its own inhabitants. Scientists face the double, incredibly intricate problem of unraveling the cause and effect relationships between various phenomena and processes on one hand, and developing mathematical descriptions of this extraordinarily complex reality on the other. Alfonso X of Castile said back in the 13th century, "If the Lord Almighty had consulted me before embarking on creation thus, I should have recommended something simpler." The complexity of the Earth and its systems has perplexed humankind ever since we began attempting to comprehend it.

Sources of life

What conditions were essential for life to have started and developed on Earth? The first factor to consider is energy requirements. Energy reaches the Earth's surface in two ways: the first is thermal energy originating from within the planet itself, which is mainly the sum of thermal energy dating back to the period when the Earth was formed as a planet plus the heat created by natural radioactive decay; the second is absorption of energy from the Sun. The incident solar energy reaching the Earth is approximately 340W/m2, of which around 30% is reflected into space, with the rest absorbed on the surface. The Sun - a giant thermonuclear reactor and our main energy source - is also one of the main threats faced by our planet. Solar wind, the ionized particles formed during thermonuclear processes in the Sun, would be capable of destroying all advanced biological structures in a very short time. Another element permitting the existence of life on Earth is our planet's water resources, forming a


Wojciech Dębski

A complex, braided system of meandering rivers (Eastern Siberia) - a major problem for accurate mathematical description of flow dynamics

Wojciech Dębski

Complicated geometry of the 15km fault plane where the ground shifted along around 15km in length, to depths of around 16km, during the Mj=7.3 magnitude earthquake in Kobe, Japan (1995)

hydrologic cycle connecting the atmosphere with the lithosphere. Once again the driving force behind this cycle is solar energy, which causes evaporation and draws water molecules up into the atmosphere. Water shortage is one of the greatest threats faced by today's human population.

Why Earth?

The answer to the question why Earth is able to support life is hidden deep beneath our feet. At depths of 2900 km and below is the outer core, the source of the Earth's magnetic field; it is this magnetic field forming the magnetosphere in the cosmic space around the planet that deflects the solar wind away from the surface, protecting life against the Sun's high-energy particles. Those particles that do manage to penetrate this protective layer end up near the Earth's polar regions, and form the extraordinary spectacle of the aurora as they are absorbed in the outer reaches of the atmosphere. As well as plasma particles, the Sun emits huge amounts of energy as electromagnetic radiation. The visible, infrared and near ultraviolet regions of this radiation are a blessing for life on Earth. However, the ultraviolet, X-ray and higher frequency radiation are lethal to all biological forms. The atmosphere steps in once again to eliminate this radiation: its specific chemical composition absorbs and converts this radiation, practically eliminating 99% of the energy at wavelengths harmful to life. The presence of the hot, liquid outer core causes very powerful convection currents in the Earth's mantle; this makes the rigid rock masses that form the crust and the upper mantle move against each other. The movement is slight on the global scale; in tectonically stable regions (such as Poland) it is on the order of a few millimeters, but it can reach a few centimeters or more per year in regions such as Japan, causing earthquakes, volcanic eruption and subsequent disasters. Similarly the atmosphere, in a state of a very delicate balance, responds violently to perturbations, leading not only to such natural disasters as typhoons and tornadoes but also to longer-term phenomena such as prolonged droughts or rainfall. Another very important element of this system in terms of its complexity is the climate, encompassing the statistics of numerous meteorological measurements. The Earth's climate is not naturally stable and fluctuates as a result of variation in the Earth's motion around the Sun, the Sun's activity, and the Earth's axial tilt. Humanity's probable effect on the global distribution of energy and mass (water and air), and the

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Turbulent flows, although visually impressive and very widely encountered, remain too complex for us to fully describe mathematically

resulting impact on natural climate trends, have been subject of much heated debate in recent years. Anthropopressure is, then, a relatively new yet extremely important element that needs to be considered in our attempts to elucidate the complex living system that is our planet.

Complexity of nature vs. mathematics

Polish cosmologist and theologian Michał Heller, in his fascinating journey through the history of physics and cosmology Podglądanie wszechświata [Spying on the Universe], notes that the role of experimentation boils down to inspiring, controlling, confirming, or refuting theoretical constructs, whereas insight into the studied field comes not from experimentation but rather from mathematical structures. Such structures allow us to construct mathematical models - simplified representations of reality. One very important issue when attempting such descriptions and analyses is the degree of complexity of those mathematical models. Unfortunately such models can only be used by top specialists. Modeling various processes constitutes a major challenge, while the models themselves allow us to predict various types of processes, for example to analyze how rivers "respond" to changes in water supply and debris. Studying those "responses" to flood flow is of particular significance. Floods cause a cascade of physical processes which change both the river stream itself and its bank regions; they can be regarded as major natural disturbances to the system. In spite of major efforts at several research centers, successful modeling of flood flows remains limited, as shown by our major shortcomings in understanding those processes. When constructing mathematical models and writing them in an abstract mathematical form, we frequently encounter significant problems with their structure. One example is flow equations. Those equations turn out to be extremely universal - they can be used to describe atmospheric conditions and ocean and river flow, albeit the latter on a rather different scale. On a geological timescale (millions of years), they are used to describe convection of matter in the Earth's mantle and the resulting phenomena such as movement of tectonic plates. Unfortunately there are major problems in the precision of the solutions. For example, solutions to chaos models are extremely sensitive to minute changes in initial conditions or model parameters. They explain the Butterfly Effect (a term coined by the meteorologist Edward Lorenz) – a theoretical example of a hurricane's formation being contingent on whether or not a distant butterfly had flapped its wings several weeks before. The mathematical nature of the applied equations (their nonlinearity) also explains the ongoing lack of success in formulating long-term weather forecasts. One excellent example is turbulence, a phenomenon ubiquitous on and near Earth, which actually results from a loss of stability. The phenomenon has kept many a scientist awake at night, as it continues to evade elucidation. Turbulence in the atmosphere, plasma, oceans, and rivers is characterized by interactions of different structures at a full range of sizes and durations. Those interactions generate speed fluctuations in all directions. Only the largest vortexes are created as a result of instability of averaged motion. Under favorable conditions they dissipate to form smaller ones and transfer some of their own energy to the newly created vortexes. As the scale of the vortexes decreases, the effect of friction increases and becomes significant, defining the size of the smallest vortexes. As a result turbulence shows overlapping vortexes over a continuous spectrum of scale. This fascinating phenomenon forms the basis of the processes of mixing and energy and mass exchange in water and air. It is also responsible for many minor anomalies in the Earth's magnetic field. Assessing the role played by uncertainty is very important in constructing mathematical models for interpreting and assessing past events, as well as research and forecasting.

Predicting the future

The environment in which we live and for which we are responsible is an extremely complex system of interconnected processes, usually nonlinear, with an uncommon range of spatial and energy scales. The system's changes over time, which we are now able to study using sophisticated observation techniques or attempt to reconstruct from available geological records, are extremely intricate. In effect, at our current levels of knowledge we are only able to describe and model natural processes with acceptable levels of accuracy in short bursts and at short time scales. However, the efforts in attempting to learn about the world surrounding us and describing it mathematically are yielding increasingly measurable effects. In many cases the development of observation techniques and application of modeling techniques allows us to minimize the human toll taken by sudden and violent natural disasters such as floods, hurricanes, tsunamis and, to a smaller degree, earthquakes. The latter remains out of our reach, although the experience of recent years show that humankind should eventually be able to achieve a state of uneasy symbiosis with earthquakes. ■

Further reading:

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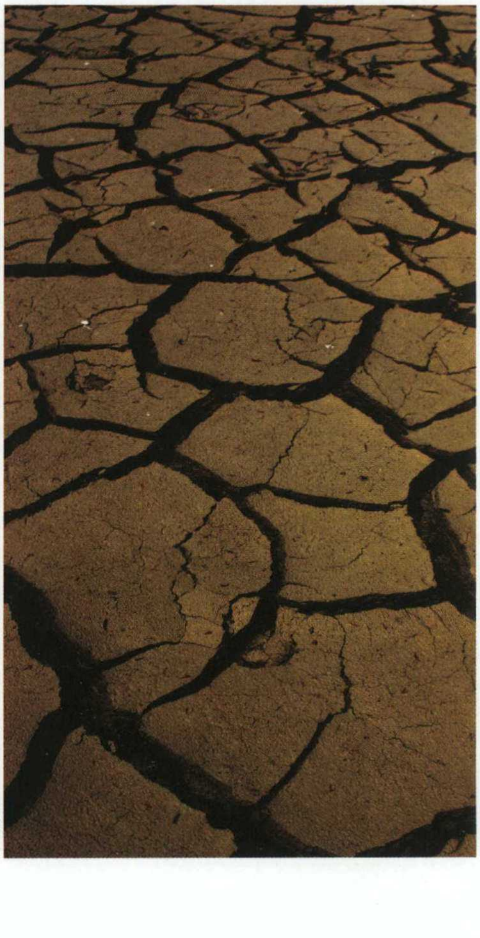

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The drying out of soil is an easily accessible example of the vast complexity of geophysical processes, which lead to intricate spatial structures