ANALYSIS OF IMPACT OF ATMOSPHERIC ATTENUATION AND MEASUREMENT UNCERTAINTIES ON LASER HAZARD DISTANCES IN NAVIGABLE AIRSPACE FOR VISIBLE CW LASER RADIATION

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Abstract

This document provides a simplified solution to the problem of calculation of laser hazard distances defined in the Advisory Circular 70-1B by the U.S. Federal Aviation Administration regarding atmospheric attenuation (assuming its constant value) and measurement uncertainties. The calculation approaches and examples presented in this document do not specify the procedure that should be followed in the case of atmospheric attenuation, nor do they take into account the uncertainties associated with the measured parameters. The analysis presented in the article complements to some extent AC 70-1B and can be used by those who need such a simplified solution regarding illumination of landing or taking off aircrafts. The article presents a sample analysis for a typical laser pointer, where the necessary parameters of the laser beam along with the appropriate uncertainties were determined in accordance with the methods accredited by the Polish Centre for Accreditation while the appropriate laser hazard distances were calculated taking into account different atmospheric attenuation coefficients.

Keywords: aviation safety, laser hazard distances in navigable airspace, laser safety, laser hazard distance, laser illumination.

1. Introduction

Due to increasing access to laser radiation sources characterized by higher and higher radiation power, the number of incidents related to their use to illuminate taking off and landing aircrafts is also increasing [1, 2]. Such situations can interfere with law enforcement and rescue missions, and infrequently leads to a 'go-around' on a landing attempt. The problem was also highlighted by the *European Union Aviation Safety Agency* (EASA) [3]. In 2021, 9723 such incidents were reported to the US *Federal Aviation Administration* (FAA) [4].

On Feb. 14, 2022 a revised version of Advisory Circular 70-1B (AC 70-1B) was released by the U.S. FAA [5]. The document provides information for those proponents planning to conduct outdoor laser operations that may affect aircraft operations. Also, it explains why notification to the

FAA is necessary, how to notify the FAA of the planned laser operation, and what action the FAA will take to respond to such notifications. This document retained the same levels of exposure to radiation as in previous editions, which define appropriate eye hazard distances. However, it does not take into account the influence of attenuation of radiation by the atmosphere and measurement uncertainties that certainly accompany all kinds of measurements and calculations.

Examples and calculations presented by the FAA are simplified to the most common cases of a single circular beam, or many circular beams sharing the same aperture (beams that are coaxial and superimposed), which was first described in [6]. Moreover, the calculations are limited to beam diameter at the output aperture up to 1 cm. However, the FAA has always allowed proponents to propose a more complex, alternative analysis as long as it is based on ANSI Z136 or other established methods, and as long as the methods and calculations are documented.

The influence of the atmosphere on radiation becomes particularly important for propagation over long distances. The principal effects of the atmosphere on radiation are absorption which is mainly caused by air molecules, scattering due to aerosols (dust, water) and turbulence caused by small-scale temperature fluctuations. Usually absorption is constant, while scattering and turbulence can vary significantly over longer and shorter time, respectively. Atmospheric absorption and scattering are usually expressed in terms of the attenuation coefficient (extinction coefficient) which strongly depends on wavelength and atmospheric conditions and can be described with good accuracy by the well-known Lambert-Beer law. That is why the atmospheric absorption and scattering effects are simply called attenuation or extinction [7].

Atmospheric turbulence can also have a strong effect on a propagating radiation beam and strongly depends on the geometry of radiation emission, the propagation distance and atmospheric conditions. It includes *e.g.*, beam wandering (small-scale beam's position changes), beam divergence increasing and beam break-up (creating many small beams). In addition, there may also appear hot spots within the beam (scintillation) manifested by local high intensities in the beam, which may be greater than in the absence of turbulence. Scintillations are characterized by very fast changes, so they are more significant for single high-energy laser pulses than for cw or multiple pulse emissions where these effects average out [7].

Visible radiation is especially attenuated by fog, smoke, dust, rain, snow, or other airborne substances (mainly due to scattering). Even in the case of high visibility, the atmosphere can significantly reduce the intensity of the radiation propagating over long distances. Moreover, when laser beams propagate outdoors, atmospheric attenuation is also a function of altitude (decreases with altitude) and current weather conditions [8].

Having this in mind, laser safety experts have discussed the pros and cons of including atmospheric attenuation and turbulence in eye hazard calculations. They have agreed that in most situations that involve consumer misuse of lasers against aircraft at relatively low altitudes and at night, in relatively clear air, it is simplest and easiest to assume no attenuation and no atmospheric turbulence [8].

Calculating the effect that the atmosphere has on a laser beam that is directed outdoors into airspace is extremely complex and taking into account the agreement of laser safety experts, various organizations assisting the FAA in producing AC 70-1B, such as the SAE G-10, ANSI Z136, and ILDA, purposefully did not include atmospheric attenuation and turbulence into methods for calculating hazard distances. However, there may be situations where ignoring atmospheric attenuation and turbulence is insufficient [8]. Developing a comprehensive solution to the problem, taking into account both attenuation and turbulence, is not a trivial process. However, in some less complex situations, only attenuation can be considered with sufficient accuracy applying a simplified model of a constant value of the attenuation coefficient (Lambert-Beer law). These may include horizontal propagation (limited by curvature of the earth) or

propagation up to some altitude at a certain angle where atmospheric attenuation is averaged over a certain distance. Even though these assumptions are rather abstract and not confirmed by any collection of research papers they may occur to be very useful in the case of military applications. For example, one can imagine a military aircraft flying at a very low altitude just over the ground. Therefore, the presented considerations, even if they do not seem to have a concrete reflection in the current civilian environment, may turn out to be very useful in military applications and not only in the case of aircraft pilots.

The aim of this article is to present a method of calculating appropriate laser hazard distances defined in AC 70-1B for a single circular laser beam characterized by the specific wavelength, radiation power, divergence and t beam diameter at a given output aperture along with the measurement uncertainties, taking into account a constant value of attenuation of the atmosphere also accompanied by some uncertainty.

The article also shows the analysis of a typical laser pointer, where measurements of the parameters above were performed in accordance with the procedures accredited by the Polish Centre for Accreditation. The analysis presented in the article complements to some extent AC 70-1B and can be used by those who need such a simplified solution (for a constant value of atmospheric attenuation) in the case of some military applications or, for example, court proceedings regarding illumination of landing or taking off aircrafts.

2. Calculation of NOHD, SZED, CZED and LFED taking into account atmospheric attenuation

There are four laser hazard distances defined in the AC 70-1B document. These are:
– *Nominal Ocular Hazard Distance* (NOHD) – beyond this distance the beam is the *Maximum Permissible Exposure* (MPE) or less. The beam is an eye hazard from the laser source to this distance. The values of MPE depends on wavelength and time of exposure and are tabulated in well-known standards ANSI Z136.1 about safe use of lasers [9], and IEC 60825-1 about safety of laser products [10].
– *Sensitive Zone Exposure Distance* (SZED) – beyond this distance, the beam irradiance is 100 µW/cm^2 or less. Between the NOHD and the SZED, the beam can cause temporary flashblindness with resulting afterimages.
– *Critical Zone Exposure Distance* (CZED) – beyond this distance, the beam irradiance is 5 µW/cm^2 or less. Between the SZED and the CZED the beam may cause veiling (obscuring) glare.
– *"Laser-Free" Exposure Distance* (LFED) – beyond this distance the beam irradiance is 50 nW/cm^2 or less. Between the CZED and the LFED the beam may distract a pilot while beyond the LFED the beam is not expected to cause distraction.

The explanations for each of these distances were also presented in [5] and [11].

In the case of a single circular beam of a given wavelength, power, divergence and beam diameter at the output aperture, the mentioned distances can be determined using the following simply derived formulas, already described in [11] and [12].

$$\text{NOHD} = \frac{\sqrt{\dfrac{4P}{\pi MPE}} - w}{\tan \varnothing}, \tag{1}$$

$$\text{SZED} = \frac{\sqrt{\dfrac{4P \cdot VCF(\lambda)}{\pi EL_{SZED}}} - w}{\tan \varnothing}, \tag{2}$$

$$CZED = \frac{\sqrt{\dfrac{4P \cdot VCF(\lambda)}{\pi EL_{CZED}}} - w}{\tan \emptyset}, \tag{3}$$

$$LFED = \frac{\sqrt{\dfrac{4P \cdot VCF(\lambda)}{\pi EL_{LFED}}} - w}{\tan \emptyset}, \tag{4}$$

where: P – power of the beam [W], $\emptyset$ – divergence [radian], w – beam diameter at the output aperture [m], tan – tangent, $VCF(\lambda)$ – Visual Correction Factor that depends on the wavelength [no unit], MPE – exposure level defining NOHD (maximum permissible exposure) [W/m^2], EL_{SZED} – exposure level defining SZED [W/m^2], EL_{CZED} – exposure level defining CZED [W/m^2], EL_{LFED} – exposure level defining LFED [W/m^2].

The exposure levels are determined in AC 70-1B and are presented in Table 1. The values of $VCF(\lambda)$ for visible wavelengths in the range from 400 nm up to 700 nm with a 5 nm step are also determined in AC 70-1B. Some of these values are shown in Table 2. According to AC 70-1B, if the wavelengths fall between two table entries, the "conservative approach" can be applied and, thus, the more restrictive (larger) value of the two resulting $VCF(\lambda)$ should be used. So, for example, the value of $VCF(\lambda)$ for 532 nm wavelength is the same as for 535 nm wavelength.

Table 1. Exposure levels [5].

Distance	Exposure level	Value [W/m^2]
NOHD	*MPE*	26.0
SZED	EL_{SZED}	1
CZED	EL_{CZED}	$5 \cdot 10^{-2}$
LFED	EL_{LFED}	$5 \cdot 10^{-4}$

Table 2. $VCF(\lambda) V(\lambda)$ values $V(\lambda)$ for several wavelengths [5].

Wavelength [nm]	400	445	532	555	635	700
VCF(λ)	0.0004	0.0305	0.9073	1.0	0.2202	0.0041

Thus, to calculate the above-mentioned distances the missing parameters are the wavelength, power, divergence, and the diameter of the beam at the output aperture which should be measured for a given laser radiation source.

The presented equations (1)–(4) do not take into account atmospheric attenuation which will certainly contribute to the reduction of the distances. According to the well-known Lambert-Beer law the radiation power P that reaches the given distance x is described by the following equation:

$$P = P_0 e^{-\gamma x}, \tag{5}$$

where: P_0 – output power of the radiation source measured at the output aperture [W], γ – atmospheric attenuation coefficient (extinction coefficient) [m^{-1}].

One could derive the final equation for calculating laser hazard distances by inserting the (5) into (1)–(4) and equating x with the appropriate laser hazard distance. However, such equations would not be analytically solvable, which means that it is not possible to derive an equation where

an appropriate laser hazard distance was only on one side of the equation while the rest of the parameters on the other side. So, a slightly different approach should be taken. One effective solution may be to insert (5) into (1)–(4) and calculate laser hazard distances in function of x with a sufficiently small step, *e.g.*, 0.1 m and then find the values of the laser hazard distances that are equal or close (from the side where the values are higher to keep the safety) to the value of x.

The extinction coefficient may vary from around 0.06 km^{-1} in exceptionally clear conditions through 0.7 km^{-1} in medium haze to around 10 km^{-1} in moderate fog [7]. While in the range of visible radiation it may vary by not more than 20%.

In Figs 1 and 2 the laser hazard distances calculated for a laser beam characterized by the wavelength of 532 nm, the power of 100 mW, the divergence of 1 mrad and the beam diameter at the output aperture of 1 mm, assuming extinction coefficient equal to 0.06 km^{-1} are presented. The intersections of the straight blue line (representing the function $f(x) = x$) with the curves representing appropriate laser hazard distances in function of x show the places where the calculated laser hazard distances are equal to x which, in turn, are the wanted values of appropriate laser hazard distances. It is shown that the calculated NOHD, SZED, CZED and LFED distances are equal to about 68.8 m, 335.5 m, 1454.1 m and 10944.8 m, while in the absence of the atmospheric attenuation they would be about 69.0 m, 338.9 m, 1519.0 m and 15199.1 m, respectively.
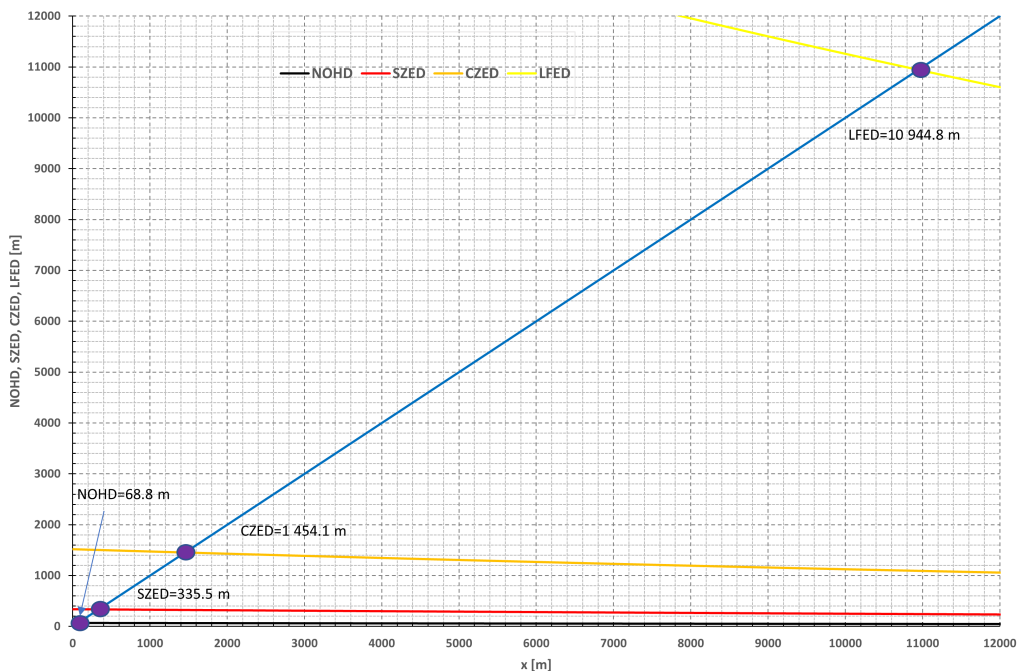

Fig. 1. Laser hazard distances calculated for a laser beam characterized by the wavelength of 532 nm, power of 100 mW, divergence of 1 mrad, and beam diameter at the output aperture of 1 mm, assuming extinction coefficient equal to 0.06 km^{-1}.

The calculation results of laser hazard distances for the laser beam described above and for different extinction coefficients are presented in Table 3. As can be expected, the highest reduction of the distance with increasing extinction coefficient is for the longest laser hazard distances. The values of LFED decreases almost 24 times when the extinction coefficient increases from 0 to 10.
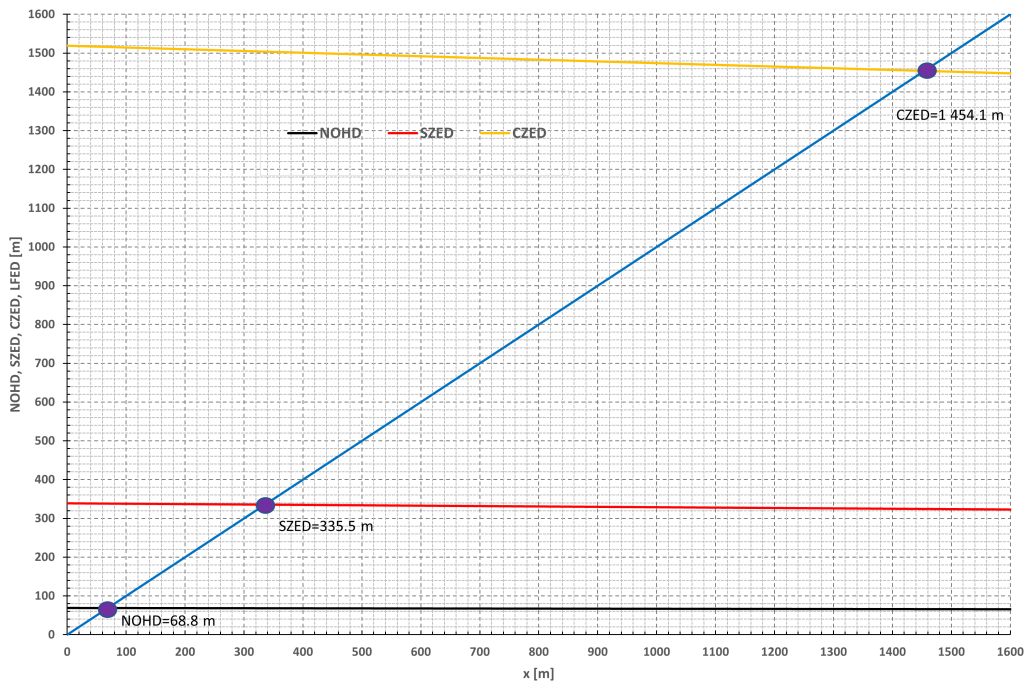
Fig. 2. Same as the previous figure, but zoomed in 7.5× on the distance scale, to better show the relationships of the *NOHD*, *SZED*, and *CZED*.

Table 3. Laser hazard distances for different extinction coefficients assuming a laser beam characterized by the wavelength of 532 nm, power of 100 mW, divergence of 1 mrad and beam diameter at the output aperture of 1 mm.

Distance [m]	Extinction coefficient [km^{-1}]				
	0	0.06	0.7	1	10
NOHD	69.0	68.8	67.3	66.7	52.8
SZED	338.9	335.5	304.5	292.6	155.4
CZED	1519.0	1454.1	1051.1	946.1	314.6
LFED	15199.1	10944.8	3892.0	3148.4	634.9

3. Analysis of the uncertainties

One of the most important factors that should be taken into account during all measurements and calculations are the accompanying uncertainties. One could say that the result given without identifying the accompanying uncertainty is questionable. Nevertheless, in many publications the results are usually given without any uncertainty. In the case of calculating laser hazard distances, specifying uncertainty seems to be particularly important as in many cases it has a direct or indirect impact on human safety and health. To determine the uncertainty of the calculations the Gauss' law of error propagation cannot be used because, following (1)–(4) and (5), laser hazard distances and *P* are not independent. However, according to [13], when the variables affecting the

uncertainty of a function are not independent, this uncertainty is not higher than the simple sum of products of absolute values of partial derivatives of this function with respect to appropriate variables and uncertainties of these variables. For a function $f(z, y)$, that depends on variables z and y, this approach can be expressed by the following equation:

$$\Delta f(z, y) \le \left| \frac{\partial f}{\partial z} \right| \Delta z + \left| \frac{\partial f}{\partial y} \right| \Delta y, \tag{6}$$

where: $\Delta f(z, y)$ – uncertainty of the $f(z, y)$ function [unit depends on the function], Δz – uncertainty of z [unit depends on variable z], Δy – uncertainty of y [unit depends on variable y], $\frac{\partial f}{\partial z}$ – partial derivative of $f(z, y)$ with respect to z [unit depends on the function], $\frac{\partial f}{\partial y}$ – partial derivative of $f(z, y)$ with respect to y [unit depends on the function].

In (1)–(5) parameters P_0, Ø and w are usually measured (by a user or the manufacturer) for a specific laser source and are characterized by some uncertainties. The other parameters such as $VCF(\lambda)$, MPE, EL_{SZED}, EL_{CZED} and EL_{LFED} are tabulated in AC 70-1B and are not subject to any uncertainties. $VCF(\lambda)$ is determined with a 5 nm step and, according to the guidelines included in AC 70-1B, if the wavelength (which is also usually measured) falls between two table entries, the most restricted case should be assumed and the highest value of $VCF(\lambda)$ should be used. This approach can be extended to the wavelength which is also usually characterized by some uncertainty and some bandwidth. If the uncertainty with the bandwidth takes up parts of two bands between three table entries, the highest value of the three should be used. The extinction coefficient is also characterized by some uncertainty based on the measurements or some kind of simulation and calculation applying specialized software. The last parameter that is characterized by some uncertainty is x that is equated with the appropriate laser hazard distance. This uncertainty determines the uncertainty of the radiation power P according to (5) which, in turn, determines the uncertainty of the calculated laser hazard distance.

Following the approach presented in (6), partial derivatives of appropriate laser hazard distances (described by (1)–(4)) with respect to the appropriate parameter can be presented in the following way:

$$\frac{\partial NOHD}{\partial P} = \sqrt{\frac{1}{P\pi MPE(\tan Ø)^2}}, \tag{7a}$$

$$\frac{\partial NOHD}{\partial w} = -\frac{1}{\tan Ø}, \tag{7b}$$

$$\frac{\partial NOHD}{\partial Ø} = -\frac{\sqrt{\frac{4P}{\pi MPE}} - w}{(\sin Ø)^2}, \tag{7c}$$

$$\frac{\partial SZED}{\partial P} = \sqrt{\frac{VCF(\lambda)}{P\pi MPE_{SZED}(\tan Ø)^2}}, \tag{8a}$$

$$\frac{\partial SZED}{\partial w} = -\frac{1}{\tan Ø}, \tag{8b}$$

$$\frac{\partial SZED}{\partial Ø} = -\frac{\sqrt{\frac{4PVCF(\lambda)}{\pi MPE_{SZED}}} - w}{(\sin Ø)^2}, \tag{8c}$$

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$$\frac{\partial CZED}{\partial P} = \sqrt{\frac{VCF(\lambda)}{P\pi MPE_{CZED}(\tan\varnothing)^2}}, \tag{9a}$$

$$\frac{\partial CZED}{\partial w} = -\frac{1}{\tan\varnothing}, \tag{9b}$$

$$\frac{\partial CZED}{\partial\varnothing} = -\frac{\sqrt{\dfrac{4PVCF(\lambda)}{\pi MPE_{CZED}}} - w}{(\sin\varnothing)^2}, \tag{9c}$$

$$\frac{\partial LFED}{\partial P} = \sqrt{\frac{VCF(\lambda)}{P\pi MPE_{LFED}(\tan\varnothing)^2}}, \tag{10a}$$

$$\frac{\partial LFED}{\partial w} = -\frac{1}{\tan\varnothing}, \tag{10b}$$

$$\frac{\partial LFED}{\partial\varnothing} = -\frac{\sqrt{\dfrac{4PVCF(\lambda)}{\pi MPE_{LFED}}} - w}{(\sin\varnothing)^2}. \tag{10c}$$

Then, the total uncertainties for different laser hazard distances can be estimated according to the following equations:

$$uc_{NOHD} \leq \left|\frac{\partial NOHD}{\partial P}\right| \cdot uP + \left|\frac{\partial NOHD}{\partial w}\right| \cdot uw + \left|\frac{\partial NOHD}{\partial\varnothing}\right| \cdot u\varnothing, \tag{11}$$

$$uc_{SZED} \leq \left|\frac{\partial SZED}{\partial P}\right| \cdot uP + \left|\frac{\partial SZED}{\partial w}\right| \cdot uw + \left|\frac{\partial SZED}{\partial\varnothing}\right| \cdot u\varnothing, \tag{12}$$

$$uc_{CZED} \leq \left|\frac{\partial CZED}{\partial P}\right| \cdot uP + \left|\frac{\partial CZED}{\partial w}\right| \cdot uw + \left|\frac{\partial CZED}{\partial\varnothing}\right| \cdot u\varnothing, \tag{13}$$

$$uc_{LFED} \leq \left|\frac{\partial LFED}{\partial P}\right| \cdot uP + \left|\frac{\partial LFED}{\partial w}\right| \cdot uw + \left|\frac{\partial LFED}{\partial\varnothing}\right| \cdot u\varnothing, \tag{14}$$

where: uP – uncertainty of the power at the appropriate distance [W], uw – uncertainty of the measured beam diameter at the output aperture [m], $u\varnothing$ – uncertainty of the measured divergence [radian].

Radiation power P presented in (7a)–(8a) should be determined according to (5), where x should be replaced with the previously calculated appropriate laser hazard distance. Moreover, power uncertainty uP presented in (11)–(14) should be determined according to the following formula (applying (6)):

$$uP \leq \left|\frac{\partial P}{\partial P_0}\right| \cdot uP_0 + \left|\frac{\partial P}{\partial\gamma}\right| \cdot u\gamma + \left|\frac{\partial P}{\partial x}\right| \cdot ux, \tag{15}$$

where:

$$\frac{\partial P}{\partial P_0} = e^{-\gamma x}, \tag{16}$$

$$\frac{\partial P}{\partial \gamma} = -xP_0 e^{-\gamma x}, \tag{17}$$

$$\frac{\partial P}{\partial x} = -\gamma P_0 e^{-\gamma x}, \tag{18}$$

uP_0 –uncertainty of the measured power at the output aperture [W], $u\gamma$ – uncertainty of the extinction coefficient [m^{-1}], ux – uncertainty of the distance [m].

Thus, the uncertainties of laser hazard distances can be calculated by inserting (15) into (11)–(14), replacing x with the previously calculated appropriate laser hazard distance and equating ux with the uncertainty of the appropriate laser hazard distance. As can be seen, it can be analytically solvable giving the expressions for the uncertainty of different laser hazard distances as follows:

$$uc_{NOHD} \leq \frac{\left|\frac{\partial NOHD}{\partial P}\right| \cdot \left(\left|\frac{\partial P}{\partial P_0}\right| \cdot uP_0 + \left|\frac{\partial P}{\partial \gamma}\right| \cdot u\gamma\right) + \left|\frac{\partial NOHD}{\partial w}\right| \cdot uw + \left|\frac{\partial NOHD}{\partial \emptyset}\right| \cdot u\emptyset}{1 - \left|\frac{\partial NOHD}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|}, \tag{19}$$

$$uc_{SZED} \leq \frac{\left|\frac{\partial SZED}{\partial P}\right| \cdot \left(\left|\frac{\partial P}{\partial P_0}\right| \cdot uP_0 + \left|\frac{\partial P}{\partial \gamma}\right| \cdot u\gamma\right) + \left|\frac{\partial SZED}{\partial w}\right| \cdot uw + \left|\frac{\partial SZED}{\partial \emptyset}\right| \cdot u\emptyset}{1 - \left|\frac{\partial SZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|}, \tag{20}$$

$$uc_{CZED} \leq \frac{\left|\frac{\partial CZED}{\partial P}\right| \cdot \left(\left|\frac{\partial P}{\partial P_0}\right| \cdot uP_0 + \left|\frac{\partial P}{\partial \gamma}\right| \cdot u\gamma\right) + \left|\frac{\partial CZED}{\partial w}\right| \cdot uw + \left|\frac{\partial CZED}{\partial \emptyset}\right| \cdot u\emptyset}{1 - \left|\frac{\partial CZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|}, \tag{21}$$

$$uc_{LFED} \leq \frac{\left|\frac{\partial LFED}{\partial P}\right| \cdot \left(\left|\frac{\partial P}{\partial P_0}\right| \cdot uP_0 + \left|\frac{\partial P}{\partial \gamma}\right| \cdot u\gamma\right) + \left|\frac{\partial LFED}{\partial w}\right| \cdot uw + \left|\frac{\partial LFED}{\partial \emptyset}\right| \cdot u\emptyset}{1 - \left|\frac{\partial LFED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|}. \tag{22}$$

Equations (19)–(22) are valid when the denominator is higher than zero. Otherwise, the minority sign should be replaced with the majority sign.

In Table 4 the maximum uncertainties of laser hazard distances calculated for a sample laser beam characterized by the wavelength of (532 ± 1) nm with negligible bandwidth, power of (100 ± 1) mW, divergence of (1 ± 0.01) mrad, beam diameter at the output aperture of (1 ± 0.01) mm, and for different extinction coefficients are presented. For calculating the uncertainty of the extinction coefficient equal to 1% was assumed.

Analysing Tables 3 and 4, one can see that the maximum uncertainty of *CZED* for the extinction coefficient of 10 km^{-1} and the maximum uncertainties of *LFED* for the extinction coefficient of 0.7 km^{-1}, 1 km^{-1} and 10 km^{-1} were calculated to be infinitely large. It is caused by the fact that in these cases the denominator in (21) and (22) is below zero and the minority sign is replaced with the majority sign.

Analyzing (19)–(22) one can see that when the products $\left|\frac{\partial NOHD}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$, $\left|\frac{\partial SZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$, $\left|\frac{\partial CZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$ and $\left|\frac{\partial LFED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$ are smaller than 1 the maximum uncertainties reach certain

Table 4. Maximum uncertainties of laser hazard distances calculated for different extinction coefficients assuming a laser beam characterized by the wavelength of (532 ± 1) nm, power of (100 ± 1) mW, divergence of (1 ± 0.01) mrad, and beam diameter at the output aperture of (1 ± 0.01) mm.

Distance [m]	Extinction coefficient [km^{-1}]				
	0 ± 0	0.06 ± 0.0006	0.7 ± 0.007	1 ± 0.01	10 ± 0.1
uc_{NOHD}	1.0	1.1	1.1	1.1	1.3
uc_{SZED}	5.1	5.1	5.5	5.7	16.3
uc_{CZED}	22.8	23.5	31.1	35.5	∞
uc_{LFED}	228.0	298.0	∞	∞	∞

values. When these products approach 1, the uncertainties approach infinity. While they are equal to 1, the uncertainties are equal to infinity. In case they are higher than 1, the minority sigh should be replaced with the majority sign, and again, the maximum uncertainties are equal to infinity. The products depend on the extinction coefficient, output power measured at the output aperture, divergence as well as exposure level and the determined laser hazard distance. Thus, there are some configurations of these parameters when the maximum uncertainty of the appropriate laser hazard distance is equal to infinity making the calculated laser hazard distance questionable.

4. Case study

The investigated laser device was a typical laser pointer commonly available on the market. The device was powered by batteries. Thanks to the adjustable optics, it was possible to vary the beam divergence to some extent.

In accordance with the procedures accredited by the Polish Centre for Accreditation, measurements of the wavelength of the emitted radiation, the power, the beam divergence and the beam diameter at the output aperture were carried out. The measurements were made for cw stable operating conditions. The values of the measured parameters with their total uncertainties (including uncertainty type A and type B), not the extended uncertainty at the confidence level of 95%, are presented in Table 5.

Table 5. Values of the measured parameters with their total uncertainties.

Parameter	Value	Uncertainty
Peak emission wavelength [nm]	532.1	0.3
FWHM bandwidth [nm]	0.6	0.6
Power of radiation measured at the output aperture [mW]	111	3
Minimal divergence at the 1/e level [mrad]	0.44	0.01
Maximum divergence at the 1/e level [mrad]	11.09	0.06
Beam diameter at the output aperture at the 1/e level [mm]	1.20	0.05

The peak emission wavelength with its uncertainty, along with the FWHM bandwidth and its uncertainty, fall between two table entries determined in the AC 70-1B document. These

are 530 nm which refers to $VCF(\lambda) = 0.8621$ and 535 nm which refers to $VCF(\lambda) = 0.9073$. Following the guidelines in AC 70-1B, the higher value of the two should be used to calculate the distances. In Tables 6 and 7 the calculated laser hazard distances and their maximum uncertainties for different extinction coefficients are presented. For calculating the uncertainty of the extinction coefficient equal to 1% was assumed. As can be seen from Table 7, some maximum uncertainties, especially for higher extinction coefficients and smaller divergence are equal to infinity. In these situations, the products $\left|\frac{\partial SZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$, $\left|\frac{\partial CZED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$ and $\left|\frac{\partial LFED}{\partial P}\right| \cdot \left|\frac{\partial P}{\partial x}\right|$ are higher than 1.

Table 6. Laser hazard distances calculated for the investigated laser for different extinction coefficients.

Distance [m]	Divergence [mrad]	Extinction coefficient [km^{-1}]				
		0 ± 0	0.06 ± 0.0006	0.7 ± 0.007	1 ± 0.01	10 ± 0.1
NOHD	0.44	164.8	164.0	155.9	152.5	99.3
SZED		811.1	792.0	646.4	600.2	241.2
CZED		3636.9	3294.4	1881.3	1618.0	427.2
LFED		36393.3	19982.5	5432.9	4279.9	770.6
NOHD	11.09	6.5	6.5	6.5	6.5	6.3
SZED		32.2	32.1	31.8	31.7	28.0
CZED		144.3	143.7	137.5	134.9	91.4
LFED		1443.9	1385.1	1012.9	914.1	308.7

Table 7. Maximum uncertainties of laser hazard distances calculated for the investigated laser for different extinction coefficients.

Uncertainty [m]	Divergence [mrad]	Extinction coefficient [km^{-1}]				
		0 ± 0	0.06 ± 0.0006	0.7 ± 0.007	1 ± 0.01	10 ± 0.1
uc_{NOHD}	0.44	6.1	6.1	6.2	6.3	8.7
uc_{SZED}		29.5	29.8	32.4	33.9	∞
uc_{CZED}		132.0	136.3	237.0	379.1	∞
uc_{LFED}		1319.1	2108.0	∞	∞	∞
uc_{NOHD}	11.09	0.1	0.1	0.1	0.1	0.1
uc_{SZED}		0.6	0.6	0.6	0.6	0.7
uc_{CZED}		2.7	2.7	2.8	2.8	4.0
uc_{LFED}		27.3	28.0	35.3	39.6	∞

As an example, the calculated laser hazard distances for the investigated laser and for the divergence of (0.44 ± 0.01) mrad and the extinction coefficient of (1 ± 0.01) km^{-1} are presented in Fig. 3.
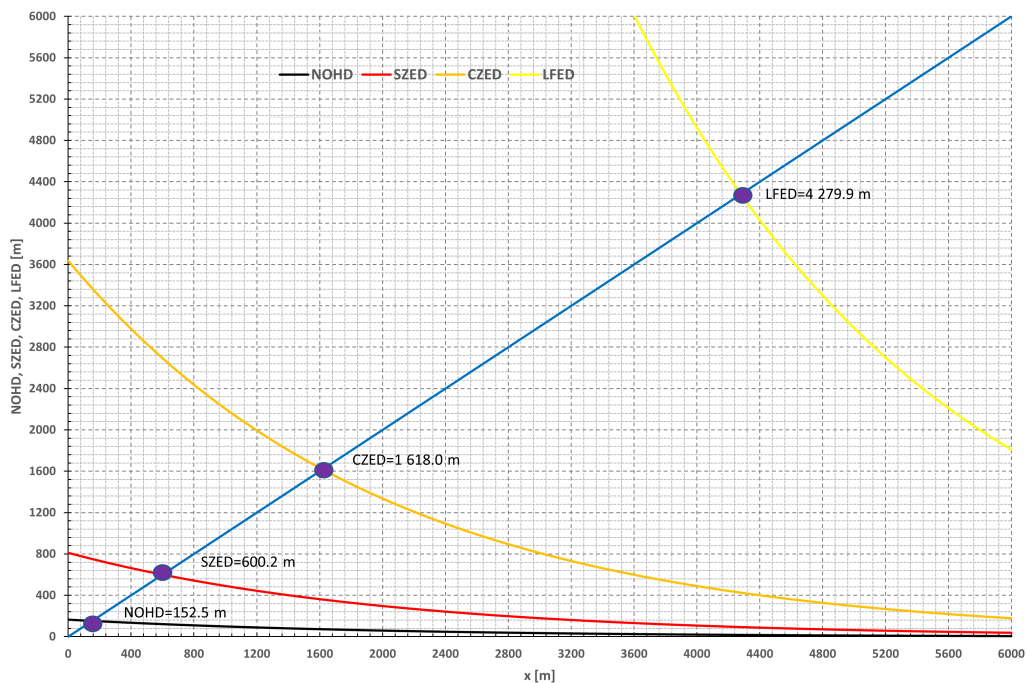
Fig. 3. Laser hazard distances calculated for the investigated laser and for the divergence of (0.44 ± 0.01) mrad and the extinction coefficient of (1 ± 0.01) km^{-1}.

5. Conclusions and additional considerations

Proposed calculations of laser hazard distances present a simplified solution to the problem of calculation of laser hazard distances defined in Advisory Circular 70-1B by the U.S. Federal Aviation Administration regarding atmospheric attenuation and measurement uncertainties. The analysis presented in the article complements to some extent AC 70-1B and can be used by those who need such a simplified solution regarding illumination of landing or taking off aircrafts.

Analysis of the calculation of uncertainties has showed that when the product of the absolute values of partial derivative of the appropriate laser hazard distance with respect to the power and partial derivative of the power with respect to x (which is equated with the appropriate laser hazard distance) is smaller than 1, the maximum uncertainty reaches a certain value. When this product approaches 1, the uncertainty approaches infinity. And when it is equal to 1 or higher, the uncertainty is equal to infinity. The product depends on the extinction coefficient, output power measured at the output aperture, divergence as well as the exposure level and the determined laser hazard distance. Thus, there are some configurations of these parameters when the maximum uncertainty of the appropriate laser hazard distance is equal to infinity making the calculated laser hazard distance questionable.

In all the calculations the total uncertainties were used taking into account the uncertainty of type A and that of type B. Thus, to present the final calculated extended uncertainty at the confidence level of 95%, the calculated total uncertainty should be multiplied by a factor 2.

The proposed way of determining the distances as well as uncertainties is shown only for a single circular laser beam, however, with some additional calculations it can be extended

to multi-beam radiation sources with coaxial and superimposed laser beams characterized by different power, different divergences in two perpendicular planes (elliptical beams) and different diameter at the output aperture. Secondly, the intensity distribution in the cross-section of the beam was assumed to be Gaussian. Analysis of other types of intensity distribution could bring some interesting new results. Additionally, the results presented in the article refer only to *continuous wave* (cw) laser sources. Analysis that concerns single pulse or repetitively pulsed sources would certainly deliver some very useful results. In the case of the *NOHD*, it can be achieved by applying appropriate values of *MPE* for the right exposure time according to the standards [9, 10]. In the case of the other distances, it is not so simple, however, there are some guidelines in AC 70-1B that can be followed. The mentioned limitations may be the subject of further work on the problem of illumination of aircraft pilots with laser radiation.

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