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Design of robust multi-loop PI controller for improved disturbance rejection with constraint on minimum singular value

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Disturbance rejection performance optimization with constraints on robustness for a multivariable process is commonly encountered in industrial control applications. This paper presents the tuning of a multi-loop Proportional Integral (PI) controller method to enhance the performance of load disturbance rejection using evolutionary optimization. The proposed design methodology is formulated to minimize the load disturbance rejection response and the input control energy under the constraints of robust stability. The minimum singular value of multiplicative uncertainty is considered a multi-loop system robust stability indicator. Optimization is performed to achieve the same, or higher level than the most-explored Direct Synthesis (DS) based multi-loop PI controller, which is derived from a conventional criterion. Simulation analysis clearly proved that the proposed multi-loop PI controller tuning method gives better disturbance rejection, and either, the same or a higher level of robust stability when compared to the DS-based multi-loop PI controller.

Key words: multi-loop PI controller, multivariable system, decentralized control, disturbance rejection, robust stability

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1. Introduction

Almost all the control procedures consist of multiple inputs and multiple outputs (MIMO) [1]. Effective operation of MIMO procedures is highly challenging when compared to single-loop processes due to process and loop interactions [2]. The decentralized control is mostly employed in industrial processes when compared to multivariable centralized control, as the design, implementation, and maintenance are much simpler. Despite significant work on modernized multivariable controllers for MIMO processes, the multi-loop Proportional plus integral/Proportional plus integral and derivative (PI/PID) controllers continue as a standard controller for several processes due to their efficiency in performance, elementary structure, simple tuning process, and the capability to meet required specifications [3].

Many multi-loop controller design methods have been proposed, and they can be classified into the detuning method (BLT), sequential loop closing (SLC) method, relay auto-tuning method, independent loop method, and optimization method [4]. The single-loop model-based controller design methods such as DS and Internal Model Control (IMC) methods are extended to multi-loop systems [5–8]. In recent years, evolutionary algorithms are widely accepted methods to solve complex optimization problems. There is a variety of evolutionary algorithms such as the Genetic Algorithm (GA) [9, 10], Particle Swarm Optimization (PSO) [11–13], and Differential Evaluation (DE) [14, 15] which are available for process control optimization problems. Among the evolutionary computation techniques, the PSO algorithm has been popularly used to solve a variety of optimization problems [16, 17]. In many papers, due to a simple concept, quick convergence, and easy implementation PSO has been considered to tune the controller parameters.

Adequate performance of rejection in load disturbance is the main objective of a process control framework design. By employing $H\infty$ control, the robust stability and disturbance rejection of the controlled process is satisfied together [18]. In mixed $H_2/H\infty$ design of PID controller, the objective is to optimize PID controller parameters that enhance servo operation of the process by reducing the H_2 criterion and influencing the $H\infty$-norm constraint to accomplish performance with robustness. There are several numerical methods proposed in previous research works for Single-input single-output (SISO) and MIMO systems [19] and have formulated an $H\infty$ PID controller for reducing H_2 criterion like integral squared-error (ISE). Researchers have highlighted that; the Integral of Absolute Error (IAE) is a well-intentioned economic performance measure when compared to other performance measures. But, this performance measure is analytically intractable, therefore, quadratic performance measure like ISE is preferred by theoreticians.

The optimization problem stated in Astrom et al. 1998 [20] is used as a reference for the optimal input disturbance rejection problem. In their work, the optimization of the PI controller is performed for load disturbance rejection performance subject to given values of the maximum sensitivity function (Ms) and the maximum complementary sensitivity function (Mt) for the single-loop system. Such objectives give a trade-off for input disturbance rejection and robustness. This tuning method is applied to the multivariable processes with an index to measure the overall multivariable system robustness [21, 22]. In their work, the well-known Biggest Log Modulus Tuning (BLT) [23] is used as an additional robustness criterion. In several research works, the issues in controller design disturbance rejection are inherently accounted for the minimization of sensitivity function amplitude.

The robustness and performance should be efficiently compensated by an optimization problem. Also, the improved performance requires higher input energy. Hence a balanced optimization problem should also give importance to the input energy consumption. In this work, considering the importance of disturbance rejection, input energy consumption, and robust stability of multi-loop an efficient optimization problem is formulated.

The rest of the paper is structured as follows. Section 2 enumerates a brief description of the multi-loop PI control scheme. Section 3 mainly concentrates on the formulation of robust tuning of multi-loop PI Controller. Section 4 compares the performance of multi-loop PI controller with DS-based PI controller with multi-loop. Finally, Section 5 gives the conclusion.

2. Multi-loop PI control technique

The block diagram for a closed-loop process with a multi-loop is articulated in Fig. 1. This comprises a multivariable process $G_p(s)$ and a multi-loop controller $G_c(s)$. The main goal of multi-loop control is to stabilize the output Y at a specified reference R with the presence of disturbance D by manipulating U. Consider an $n \times n$ open-loop MIMO process, $G_p(s)$ which is specified by following general transfer function matrix.

$$G_p(s) = \begin{bmatrix} G_{p11}(s) & G_{p12}(s) & \dots & G_{p1n}(s) \\ G_{p21}(s) & G_{p22}(s) & \dots & G_{p2n}(s) \\ \vdots & \vdots & \ddots & \vdots \\ G_{pn1}(s) & G_{pn2}(s) & \dots & G_{pnn}(s) \end{bmatrix}. \tag{1}$$

Several techniques were employed to control the multivariable process, and the selection depends on design complexity and objectives. In this research work, a PI controller is chosen due to its simplicity, and its performance shall be improved by effective tuning. For a multi-system, n number of PI controllers with multi-loop $G_c(s)$ is executed for the operation of MIMO. The transfer function for the diagonal controller matrix is as follows,

$$G_c(s) = \begin{bmatrix} G_{c1}(s) & 0 & \dots & 0 \\ 0 & G_{c2}(s) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & G_{cn}(s) \end{bmatrix} \tag{2}$$

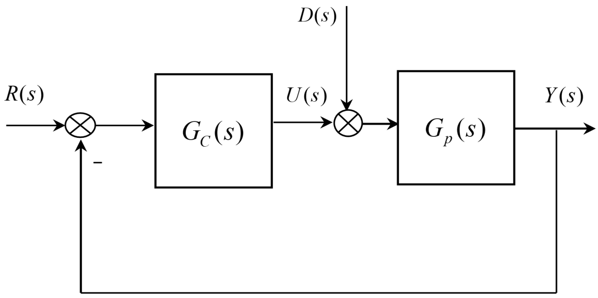

Figure 1: Multi-loop Closed Loop System

Equation (3) represents the architecture of the PI controller for each loop,

$$G_{ci}(s) = k_c \left(1 + \frac{1}{\tau_i s}\right), \tag{3}$$

where k_c and τ_i are the loop gain and integral time constant respectively. Therefore, the decision space is defined as $X = [k_{c1}, \tau_{i1}, \dots k_{cn}, \tau_{in}]$.

To attain better disturbance rejection performance, better tuning of each PI controller parameter is more important. The conventional PI controller model with multi-loop requires pairing of input and output. Hence, Relative Gain Array (RGA) evaluation is first performed to choose the correct input-output pair of the multi-loop process. In multi-loop systems, once the pairing is fixed, by tuning every PI controller with single-loop control, the performance is calculated. The proposed controller design is discussed in the subsequent section.

3. Robust tuning of multi-loop PI controller

Optimization methods have been employed for designing multi-loop control processes. A popular approach, evolutionary optimization is widely used for controller tuning of the multivariable process. For control applications, effective input rejection in disturbance is a significant issue. The process variable deviation from the desired reference for a long time will affect the final product quality. Hence, quick disturbance rejection is the primary focus of the multi-loop controller model.

In this study, the most preferred technique of multivariable system robust stability metric, i.e., the minimum singular value of multiplicative uncertainty is employed as a constraint for the optimization problem. The robustness and performance should be efficiently compensated by an optimization problem. Also, improved performance requires higher input energy. Hence, a balanced optimization problem should also give importance to input energy consumption. Considering the importance of input disturbance rejection, input energy consumption, and robust stability of multi-loop systems, the following simple and efficient optimization problem is formulated.

3.1. Multi-loop system robust stability

The closed-loop network is not much sensitive to process parameter variations/model uncertainty. The closed-loop network robustness is easily analyzed if the degree of process uncertainty is well-known. The stability of robustness is analyzed either under output or input with uncertainty in multiplicative nature. The most significant uncertainty method is the uncertainty in output multiplicative. In a closed-loop operation with uncertainty in output with multiplicative nature $[I + \Delta_0(s)]G_p(s)$, the upper bound of the robust stability is written as,

$$\gamma \leqslant \overline{\sigma}(\Delta_0), \tag{4}$$

$$\gamma \leqslant 1/\overline{\sigma}\left[\left(I + G_p(j\omega)G_c(j\omega)\right)^{-1} G_p(j\omega)G_c(j\omega)\right], \tag{5}$$

$$\gamma \leqslant \underline{\sigma}\left[I + \left(G_p(j\omega)G_c(j\omega)\right)^{-1}\right], \ \ \forall\, \omega \geqslant 0, \tag{6}$$

where $G_p(j\omega)G_c(j\omega)$ is invertible.

To achieve a robustly stable closed-loop process, the least singular value ($\underline{\sigma}$) of uncertainty in multiplicative nature should be equal to or lower than the selected γ, which explains the quantity of error in modeling over a presented nominal closed-loop system. It is considered to be the gain margin of MIMO processes. For comparing different controllers, the degree of robust stability (i.e., γ) will be held at the same level.

3.2. Optimization problem

Normally, it is challenging to define an optimal controller for a process, due to many issues like closed-loop performance, robustness, input usage, and noise sensitivity. The higher gain controller favors the performance with better output, but, the lower gain favors the other three objectives. The noise sensitivity issue is not treated in this formulation, since, this may be solved by a well-designed filter that is inserted in the feedback path. The multi-loop system performance for input rejection in disturbance performance is assessed by the following time domain performance measures.

3.2.1. Integral Absolute Error index

The function measure for the time domain, Integral Absolute Error (IAE) criterion is given as,

$$f_1(X) = \int_0^{T_f} |R(t) - Y(t)| \, \mathrm{d}t, \tag{7}$$

where T_f is a finite time, which was selected for the value of steady-state, and $\mathrm{d}t$ is the small-time interval, thus,

$$Y(s) = G_p(s) \left(I + G_p(s)G_c(s)\right)^{-1} D(s). \tag{8}$$

$D(s)$ is usually specified as a step type of input disturbance. The time domain response $Y(t)$ is attained by taking the Laplace transform inverse of Eq. (8).

3.2.2. Total Variation (TV)

TV of the manipulated variable is a decent measure of process input signal smoothness. TV is computed using Equation (9) as,

$$f_2(X) = \sum_{k=1}^{N} |U(k+1) - U(k)|, \tag{9}$$

where N is the discrete finite interval.

$$U(s) = G_p(s)G_c(s) \left(I + G_p(s)G_c(s)\right)^{-1} D(s). \tag{10}$$

The time domain response $U(t)$ is attained by taking Laplace transform inverse of Eq. (10), thereby, the discrete response $U(k)$ is obtained for the small sampling interval. TV is preferred rather than the other time domain metrics since it can offer a satisfactory result considering these generally conflicting time domain metrics.

3.2.3. Robust stability constraint

The degree of robust stability is indicated by the minimum singular value of multiplicative uncertainty (γ). In these works [24, 25], the multi-loop controller parameters were tuned by adjusting the controller parameters, such that the minimum singular value of multiplicative uncertainty (γ) should be the same or larger than that of other controllers. The higher value shows that the controller has good robustness compared to another controller. It should be noted that the controller parameters can be tuned to achieve the specified minimum singular value of multiplicative uncertainty (γ) value.

In this research work this robust stability indicator γ is adopted as a constraint. The multi-loop PI controller parameters are optimized such that the least singular value of (γ) must be the same as or higher than that of the benchmark value.

3.2.4. Optimization problem

The optimization problem is formulated to find the solution such that,

$$\begin{aligned}
&\min_{X} f(X) = (\alpha f_1(X) + \beta f_2(X)) \\
&\text{Sunject to} \\
&\gamma - \gamma_{\min} \geqslant 0 \\
&K_{c\,\min} \leqslant K_c \leqslant K_{c\,\max} \\
&\tau_{i\,\min} \geqslant \tau_i \geqslant \tau_{i\,\max}
\end{aligned} \tag{11}$$

where f is the objective function, α and β are the weighting coefficient. By setting these weighting coefficients, the user balances a compromise between the performance objectives. The values of α and β are chosen in the range of 0 to 1. In this work, two contradictory performance requirements such as IAE and TV are carefully transferred into a single objective function. The importance of the performance requirement can be varied by adjusting the weighting coefficients α and β. The parameter $\gamma_{\min}$ is the user-specified scalar, and it is chosen in the range of 0 to 1. $K_{c\,\min}$, $K_{c\,\max}$; $\tau_{i\,\min}$, $\tau_{i\,\max}$ are the highest and lowest values of the PI controller with multi-loop parameters, and it is chosen between 0 to 100. PI controller with multi-loop parameters is optimized for the objective function by using one of the methods called PSO.

3.2.5. Particle Swarm Optimization (PSO)

PSO is a population-dependent searching methodology concerning the co-operative manner of collection of animals, birds, or schools of fish. In a typical PSO system, the independent parameters called particles, alter their states with time, and a parameter set is associated with each particle. In multidimensional search space, these particles move around. During the move, according to its own experience, each particle adjusts its position. Considering the direction of swarm movement, the particle is explained by a group of particles, neighboring particles, and its experience.

In a search space, X_i and V_i show the i-th position of a particle and its corresponding velocity. The position of the best previous of i-th particle is recorded and exhibited as personal best $Pbest_i$. The best particle in the group among every particle is exhibited as $Gbest$. A velocity is valued owing to the vector of the personal best of all the particles and related to the vector of globally best-performing particle. With the best-performing updated vector, all particle velocity is added. Finally, in all the iterations, every particle vector is updated using Eq. (12).

$$V_i^{k+1} = \omega^k V_i^k + c_1 rand_1 \left(Pbest_i^k - X_i^k\right) + c_2 rand_2 \left(Gbest^k - X_i^k\right) \tag{12}$$

where ω^k is the weight of inertia at iteration k, c_1 and c_2 are the factors for acceleration, $rand_1$ and $rand_2$ are the random uniform numbers between 0 and 1.

Each move from the current position to the next one with modified velocity is given as,

$$X_i^{k+1} = X_i^k + V_i^{k+1}. \tag{13}$$

The given technique is repetitive until the functional evaluations with the highest number are achieved.

4. Simulation and comparison study

Three simulation illustrations were performed to demonstrate the performance of the developed model of PI controller with multi-loop in comparison with the DS-based PI controller with multi-loop. The PI controller with multi-loop is designed for optimal disturbance rejection response with minimum input energy under the constraint of robust stability using PSO evolutionary algorithm. In all three simulation examples, the following settings are used in the PSO optimization routine: the size of the swarm is 40, the highest number of functional evaluations is 1500, the problem dimension is 4, maximum velocity is 25 percent of the range of design variables, inertia weight is declining from 0.9 to 0.2 with respect to iteration, accelerate coefficient (c_1 and c_2) is 1, and the number of runs is 20. The best optimal solution is reported in subsequent tables for all the simulation examples. The weighting coefficients are considered as unity to deliver equal significance to the performance objectives.

The robustness constraint in this optimization problem is the least singular value of multiplicative uncertainty (γ), which should be either the same or higher than that of the DS-based PI controller with a multi-loop. In all the simulation examples, γ value of DS based PI controller with multi-loop is taken as $\gamma_{\min}$ value for the proposed optimization problem. The optimization problem is formulated to attain either the same or a higher value of $\gamma_{\min}$.

Example 1

The Wood and Berry distillation (WB) column plant has been analyzed by several authors, and the matrix of the transfer function in the WB column as exhibited in [14] is,

$$G_m(s) = \begin{bmatrix} \dfrac{12.8e^{-s}}{16.7s+1} & \dfrac{-18.9e^{-3s}}{21s+1} \\ \dfrac{6.6e^{-7s}}{10.9s+1} & \dfrac{-19.4e^{-3s}}{14.4s+1} \end{bmatrix}.$$

The multi-loop PI controller parameters are designed with a view towards the DS method [14] and are reported in Table 1, and it has to be noted that, this controller has γ value of 0.48. Hence, for the proposed optimization problem, $\gamma_{\min}$ value is taken as 0.48. The controller parameters of the multi-loop PI are attained by using PSO for the objective function, as expressed in Equation (11). The obtained multi-loop controller parameters employing optimization procedure are reported in Table 1. It should be noted that optimization achieves the same level of robustness as a DS-based multi-loop controller (see column 5). The convergence rate of fitness function performance is signified in Figure 2.

Table 1: Controller parameters, performance, and robustness measures for WB column

Tuning method	Loop	k_c	τ_i	γ	J_{IAE}	J_{TV}
PI-Optimal	1	1.0016	8.9321	0.48	89.2810	5.8994
	2	–0.054	3.0262			
PI-DS	1	0.75	10.07	0.48	133.4447	5.0542
	2	–0.08	7.98			

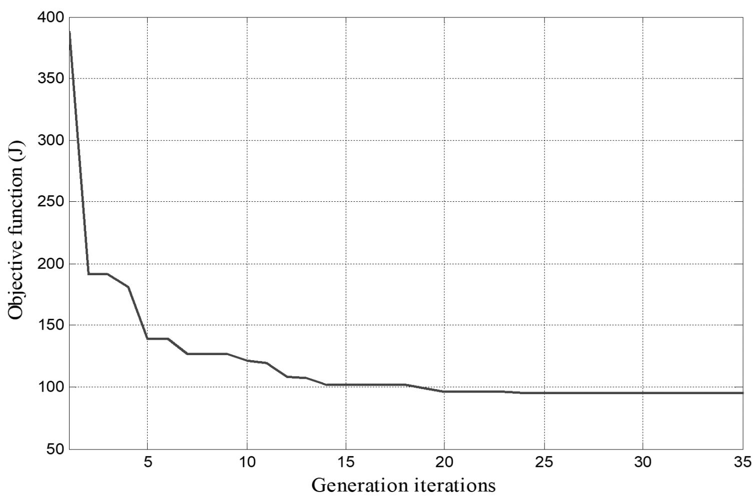

Figure 2: Convergence test for WB column

The frequency response of singular values of multiplicative uncertainty, as given in Equation (6) is computed using the MATLAB command 'sigma'. Based on the system dynamics the frequency to plot is determined. The singular values are used to plot the frequency response of the proposed and DS-based multi-loop PI controller and are exhibited in Figure 3. From Figure 3, it is inferred that both the schemes have the same value of minimum singular value ($\underline{\sigma}$) of multiplicative uncertainty, i.e. $\gamma = 0.48$.

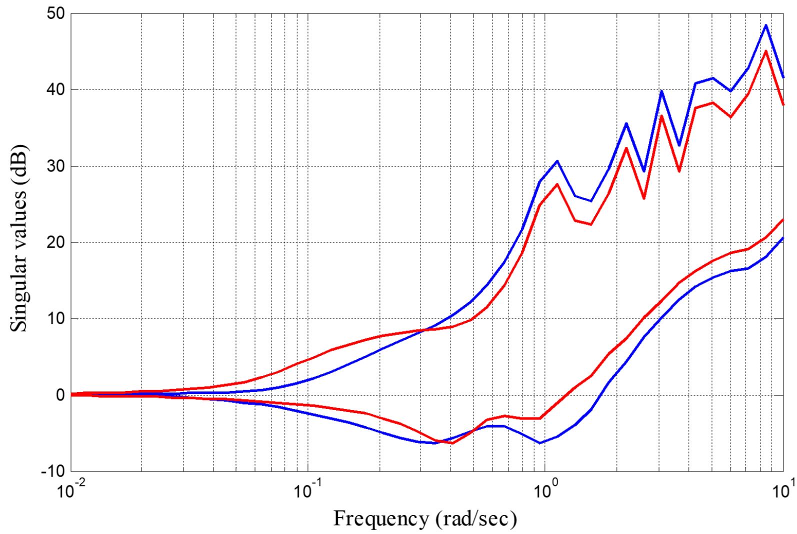

Figure 3: Plot for the singular value of multiplicative uncertainty for the WB column: —— Proposed multi-loop PI and —— DS based multi-loop PI

The functions with sensitivity and complementary sensitivity singular value plots are given in Figures 4 and 5. The highest singular value, i.e., peak value in the plot of singular value sensitivity function of the proposed multi-loop PI and DS-based multi-loop schemes are comparable. This shows that both schemes have equal sensitivity towards modeling errors.

The maximum singular value, i.e., the peak value in the singular value plot of the function with complementary sensitivity of proposed multi-loop PI and DS-based multi-loop schemes are almost equal. Hence, both schemes have equal sensitivity to sensor noise.

For inputs unit step change, the disturbance rejection operations are evaluated. The process output and the controller operations are given in Figures 6 and 7. The computed operation measures such as IAE and TV values of rejection in disturbance operations are presented in Table 1. The optimal PI controller with multi-loop exhibits improved disturbance rejection operation when compared to

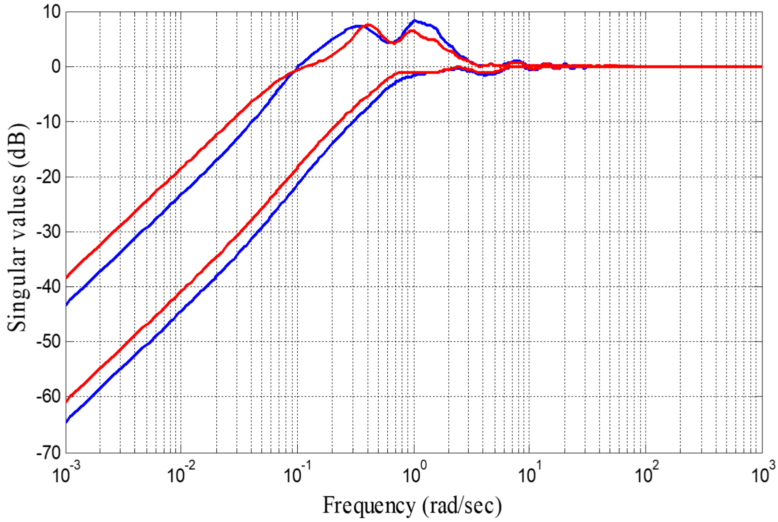

Figure 4: Plot for the singular value function with sensitivity for WB distillation column: ——— Proposed multi-loop PI and ——— DS based multi-loop PI

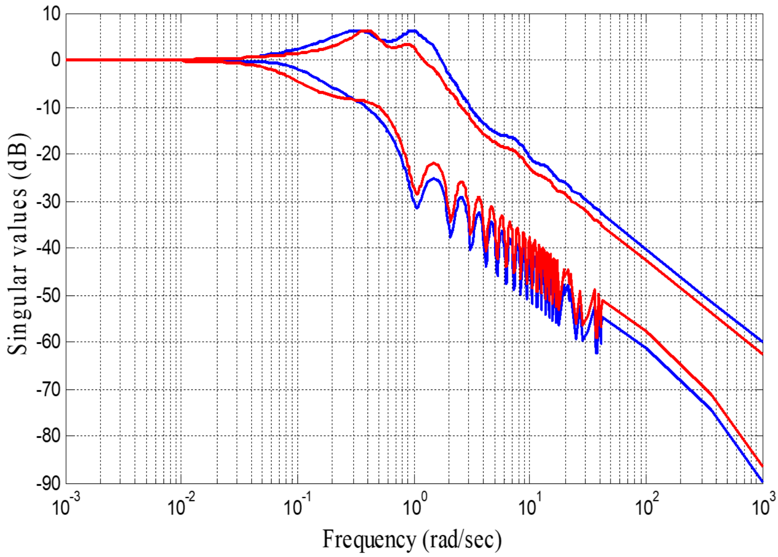

Figure 5: Plot for singular value complementary function with sensitivity for WB distillation column: ——— Proposed multi-loop PI and ——— DS based multi-loop PI

the DS-based multi-loop PI controller. It is noticed that both schemes have similar robustness levels. Regarding TV values, the optimal multi-loop PI controller has slightly higher input energy to achieve improved performance.

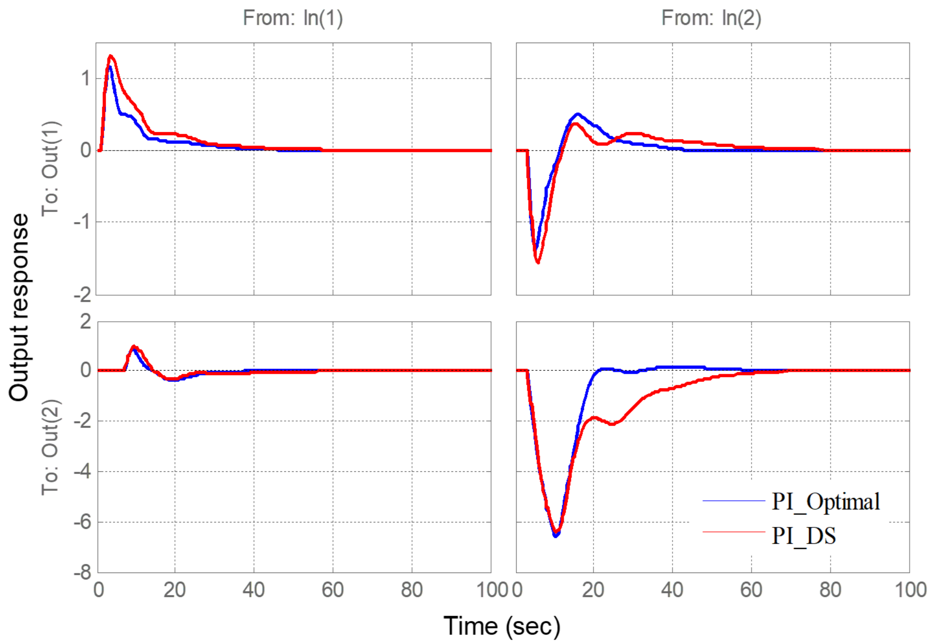

Figure 6: Step input disturbance response for the WB distillation column

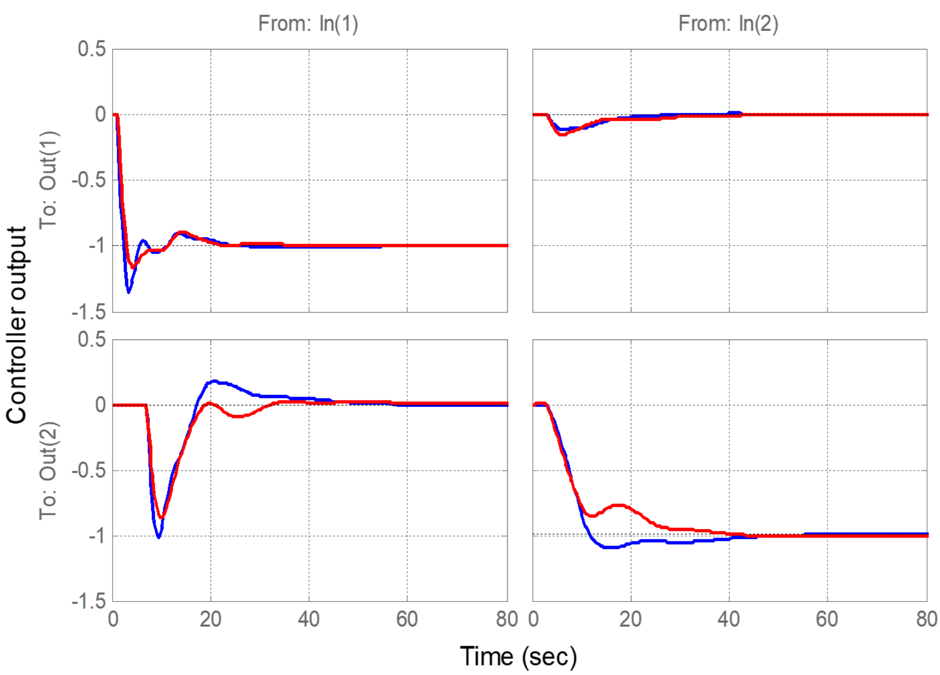

Figure 7: Controller output for the WB distillation column

Example 2

Numerous researchers have considered the stable Industrial Scale Polymerization (ISP) reactor process, and the ISP reactor transfer function matrix, and shall be represented as [12],

$$G_m(s) = \begin{bmatrix} \dfrac{22.89e^{-0.2s}}{4.572s+1} & \dfrac{-11.64e^{-0.4s}}{1.807s+1} \\ \dfrac{4.689e^{-0.2s}}{2.174s+1} & \dfrac{5.8e^{-0.4s}}{1.801s+1} \end{bmatrix}.$$

The PI controller with multi-loop parameters was developed based on the DS [12] and is reported in Table 2, and this controller has γ value of 0.57. So, for the proposed optimization problem, $\gamma_{\min}$ value is taken as 0.57. The obtained controller parameters are enlisted in Table 2. As similar to the previous example, optimization achieves a similar level of robustness to the DS-based multi-loop controller (see column 5). The fitness function convergence rate performance is exhibited in Figure 8. The plots with singular value frequency variations of the developed and DS PI controller with multi-loop are exhibited in Figure 9. From Figure 9, it is inferred that both the schemes possess the same value of the lowest singular value of multiplicative uncertainty, i.e. $\gamma = 0.57$.

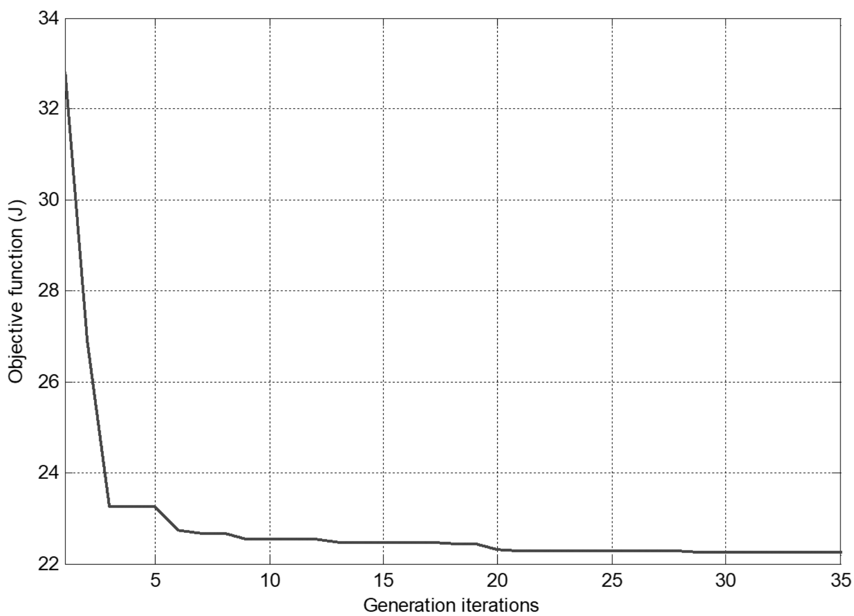

Figure 8: Convergence test for ISP reactor system

Table 2: Controller parameters, performance, and robustness measures for ISP reactor system

Tuning method	Loop	k_c	τ_t	γ	J_{IAE}	J_{TV}
PI-Optimal	1	0.85	1.99	0.57	13.21	5.33
	2	0.14	1.11			
PI-DS	1	0.43	3.95	0.57	24.33	4.52
	2	0.13	1.18			

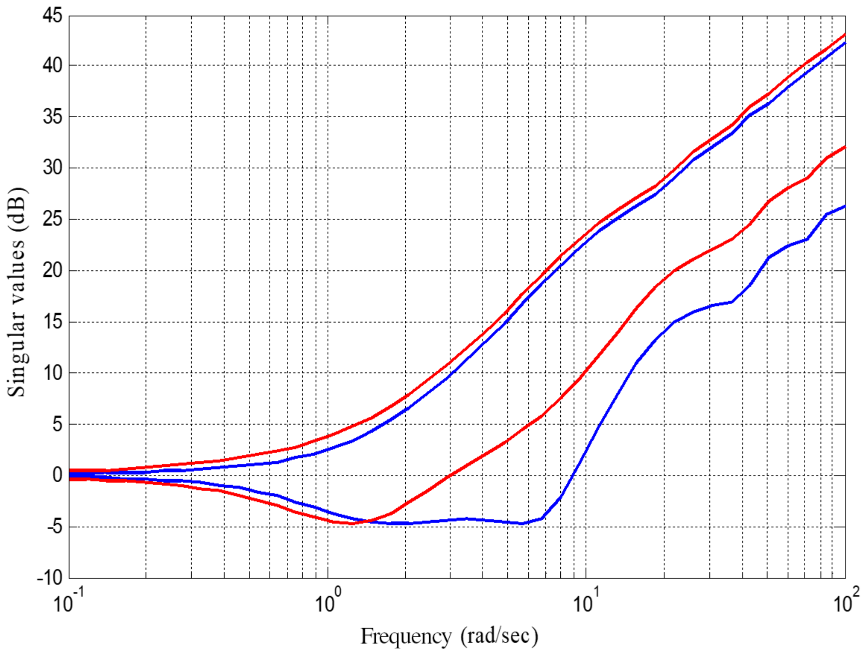

Figure 9: Plot for singular value multiplicative uncertainty for WB distillation column: —— Proposed multi-loop PI and —— DS based multi-loop PI

For input unit step change, the rejection in disturbance operations is calculated, and the performances are given in Figure 10. The evaluated IAE and TV range of rejection in disturbance operations are enlisted in Table 2. The operation, performance, and robustness measures clearly reveal that the optimal PI controller with multi-loop exhibits the highest rejection in disturbance response when compared to a DS-based PI controller with multi-loop of the same robustness level.

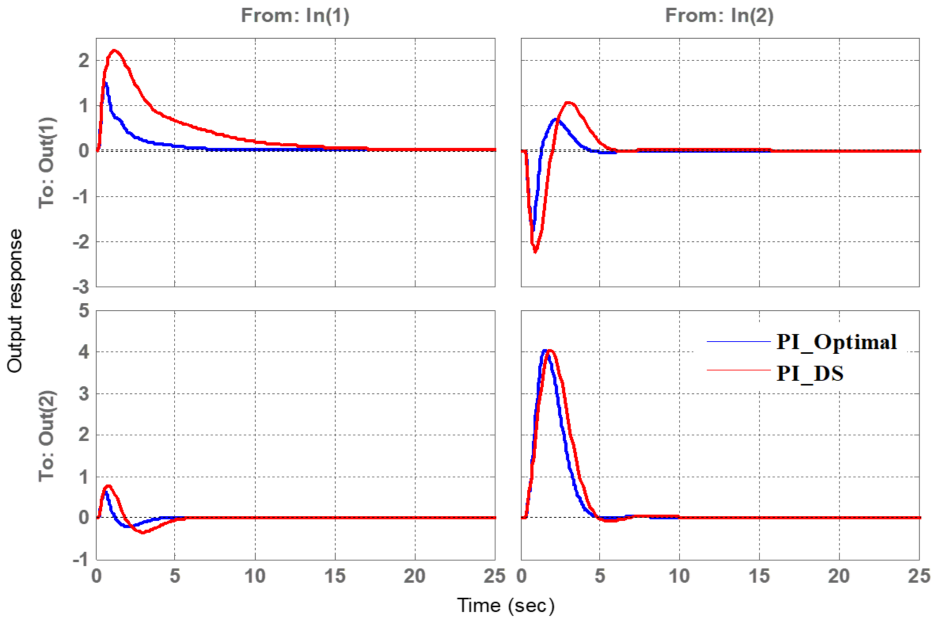

Figure 10: Step input disturbance performance for the ISP reactor

Example 3

The performance of the control scheme is demonstrated through examples, and there are more than two loops, namely the Orgunnaike and Ray (OR) distillation column. The matrix for the process transfer function as represented in [12] is as follows,

$$G_m(s) = \begin{bmatrix} \dfrac{0.66e^{-2.6s}}{6.7s+1} & \dfrac{-0.61e^{-3.5s}}{8.64s+1} & \dfrac{-0.0049e^{-s}}{9.06s+1} \\ \dfrac{1.11e^{-6.5s}}{3.25s+1} & \dfrac{-2.36e^{-3s}}{5s+1} & \dfrac{-0.01e^{-1.2s}}{7.09s+1} \\ \dfrac{-34.68e^{-9.2s}}{8.15s+1} & \dfrac{46.2e^{-9.4s}}{10.9s+1} & \dfrac{0.87(11.61s+1)e^{-1s}}{(3.89s+1)(18.8s+1)} \end{bmatrix}.$$

The PI controller with multi-loop parameters, which are modeled concerning DS [12] are reported in Table 3, and this controller has γ value of 0.035.

For the proposed optimization problem, $\gamma_{\min}$ value is taken as 0.035. The resultant controller parameters are given in Table 3. From the table, it could be concluded that the optimal multi-loop PI controller has a higher robustness level (see column 5). The performance of the fitness function convergence rate is

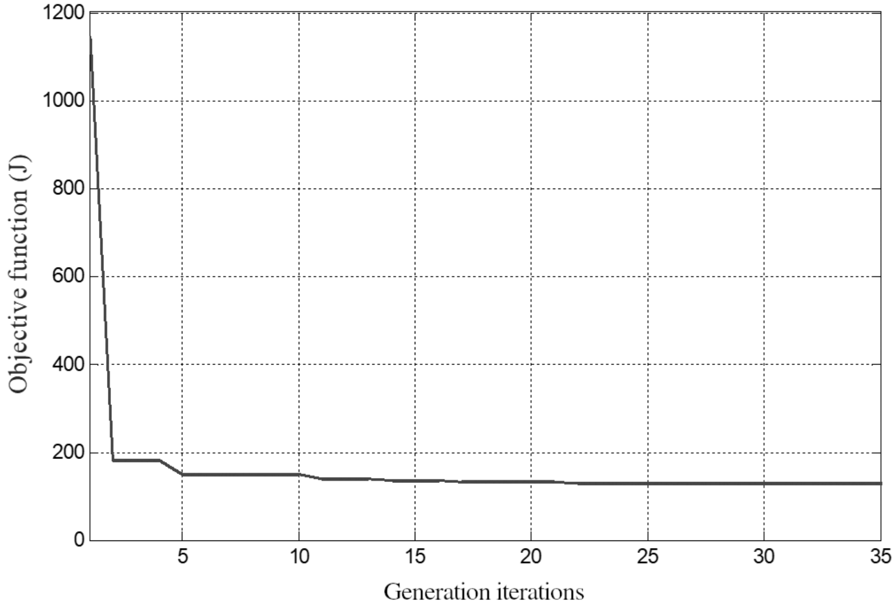

Figure 11: Convergence test for OR distillation column

elaborated in Figure 11. The frequency response of singular values plots of the proposed and DS PI controller with multi-loop are enumerated in Figure 12.

Table 3: Controller parameters, performance, and robustness measures for OR distillation column

Tuning method	Loop	k_c	τ_t	γ	J_{IAE}	J_{TV}
PI-Optimal	1	1.3478	11.4408	0.05	89.0532	39.1973
	2	0.1157	1.9680			
	3	7.9236	2.7136			
PI-DS	1	1.57	5.96	0.035	148.1442	55.7839
	2	0.31	4.81			
	3	6.10	9.6			

From Figure 12, it could be clearly inferred that the optimal multi-loop PI controller has a high value (γ = 0.05) of the least singular value of multiplicative uncertainty when compared to the DS-based multi-loop PI controller. This high value indicates that the proposed multi-loop PI controller has higher robust stability when compared to the DS-based multi-loop PI controller.

For input unit step change, the disturbance rejection performances were calculated, and the concerned operations are depicted in Figure 13. The evaluated

IAE and TV values of rejection in disturbance performances are tabulated in Table 3. Similar to previous examples, the developed optimal multi-loop PI controller exhibits increased rejection in disturbance when compared to the DS-based

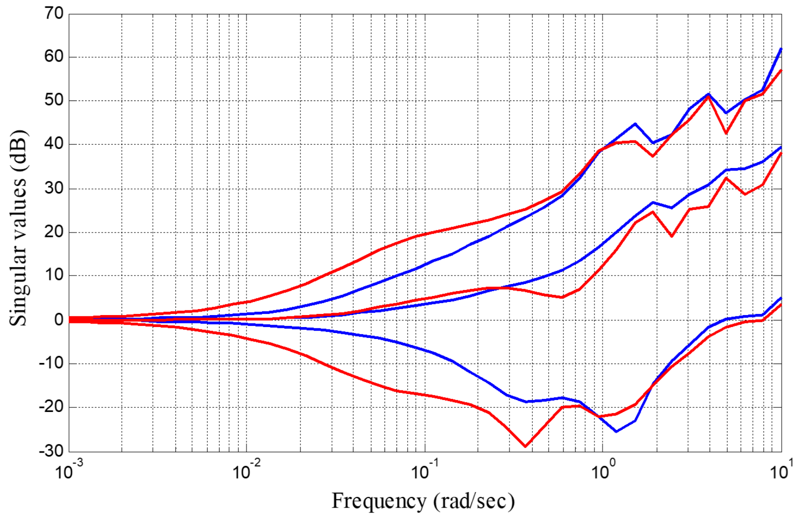

Figure 12: Singular value plot of multiplicative uncertainty for OR distillation column: ——— Proposed multi-loop PI and ——— DS based multi-loop PI

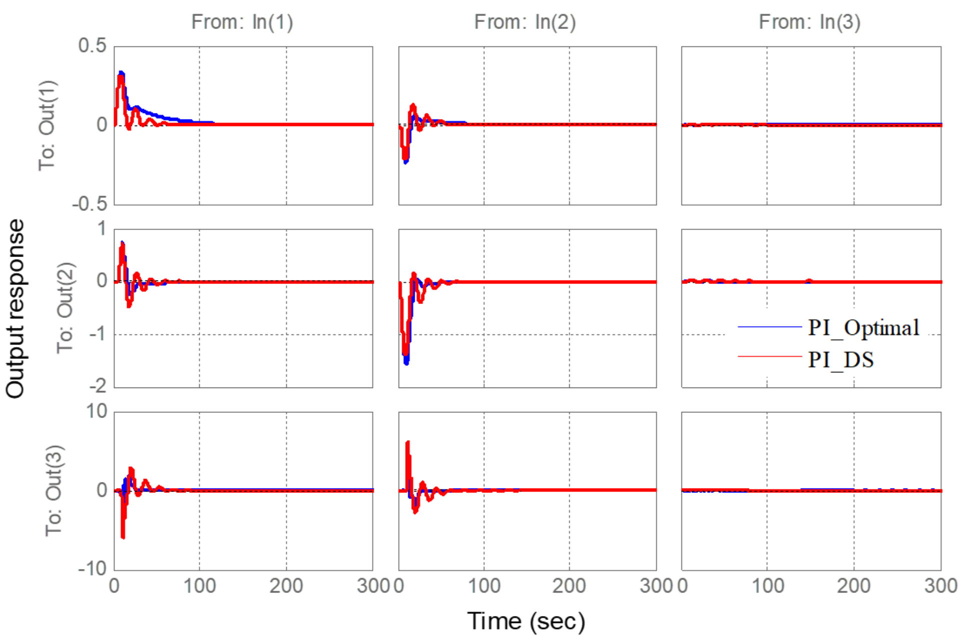

Figure 13: Step input disturbance response for the OR distillation column

multi-loop PI controller with a higher robustness level. Besides, it could be noted that the proposed optimal multi-loop PI controller exhibits a lower TV value, which indicates that, the proposed design gives an improved performance with less input energy for OR distillation column process.

To analyze the performance of multi-loop controllers under process parameter uncertainty, a simulation study is performed for the OR column process. The controllers function for perturbation in all three process parameters (20% high than that of the nominal value) is studied.

The sum of IAE and TV values are computed and is given in Table 4. For input unit step change, the disturbance rejection performances were evaluated for the perturbed process, and the performances are given in Figure 14. The IAE

Table 4: Perturbed Process Performance Comparison of Proposed and DS multi-loop PI Controller for OR column

	+20% perturbation	
	J_{IAE}	J_{TV}
PI-Optimal	185.9692	137.68
PI-DS	462.5303	157.739

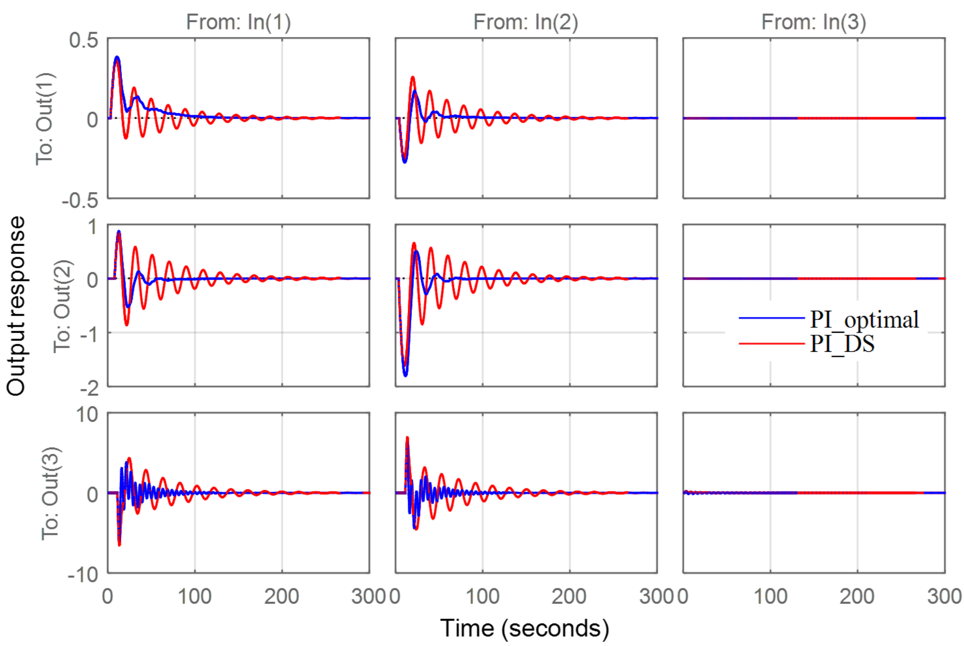

Figure 14: Perturbed process step input disturbance response for the OR distillation column

and TV values of the optimal multi-loop PI controller are found to be less than the DS-based multi-loop PI controller. This analysis reveals that the developed design controller performs better concerning process parameter uncertainty.

5. Conclusion

In this study, tuning of PI controller with multi-loop, for optimal performance is discussed. The controller parameters were optimized for improved disturbance rejection response, with minimum control energy under the constraint of robustness. The robust stability requirement and the improved disturbance rejection performance can be efficiently compensated by optimizing the controller with the right choice of the objective function. The least singular value of output multiplicative uncertainty is employed for the system stability robustness with closed-loop. Simulation analysis exhibits improved capability of the developed structure of PI controller with multi-loop, with an identical or higher level of robust stability when compared to the popularly referred DS-based multi-loop PI controller. The robustness analysis is also performed by uncertainty in perturbation of +20% in each three process parameters. The simulation outputs clearly reveal that the developed tuning methodology offers a better robust function in the presence of plant model mismatch.

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