10.24425/acs.2024.149649

Archives of Control Sciences
Volume 34(LXX), 2024
No. 1*, pages 5–22*

On observer compensator design for non-autonomous control semi-linear evolution equations

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This paper investigates the Luenberger observer design problem for non-autonomous control semilinear evolution equations with disturbances in Banach spaces. Then, the practical stabilization problem of the system is solved, yielding a compensator based on the Luenberger observer by using integral inequalities of the Gronwall type. Sufficient conditions of the controller and observer problem are satisfied, we show that the proposed controller with estimated state feedback from the proposed practical Luenberger observer will achieve global practical stabilization. We develop novel ideas and techniques, which present the further development of mathematical control theory. Furthermore, an example is given to show the applicability of our theoretical results.

Key words: compensator design, non-autonomous control semilinear evolution equations, practical stabilization, practical Luenberger observer

1. Introduction

Control theory treats itself with the basic theoretical principles underlying the analysis of feedback and the design of control systems. It differs from the more classical study of systems in its emphasis on inputs and outputs. In this theory, stabilizability and detectability are the qualitative control problems that play an important role in the systems. The theory was introduced by Curtain and Zwart [12] for autonomous infinite-dimensional systems. On its development, the theory can be generalized into stabilizability and detectability of the

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The author wish to thank the editor and the anonymous reviewers for their valuable and careful comments.

Received 10.05.2023. Revised 30.01.2024.

non-autonomous control systems, see [6] and the reference therein. The theory of compensator design is a simple extension of the finite-dimensional theory and has been used as a starting point in many control designs for distributed parameter systems, see [2, 23]. Alternative direct state-space finite-dimensional compensator designs can be found in [5, 7]. For extensions to systems with unbounded input and output operators (see [8–10]), and for a comparison of various finite-dimensional control designs (see [11]). On the other hand, the problem of compensator design for autonomous infinite-dimensional linear systems can be solved in [12], but if the system contains some nonlinearities, This problem has been used as a starting point in many control designs for semilinear evolution equations whose autonomous linear nominal system, and the nonlinearities satisfies conditions, see [1, 14, 15, 17]. In [1], the stabilization problem around a desired equilibrium profile of a class of infinite-dimensional semilinear systems is resolved, yielding a compensator based on a Luenberger-like observer. Designing observer-based controllers has investigated a certain class of dynamical systems, see, for instance, [3, 20, 21, 26]. In finite dimensions, one simple way of designing a compensator is to first construct a state feedback stabilizer and an observer for the system and then combine the two to design a compensator using the feedback of the observer instead of the state. This is the so-called separation principle, see [4, 16, 18, 19]. There are almost no results on observers and compensators of control infinite-dimensional systems with the associated nominal system being linear that depends on the time. The evolution operators and their neighboring areas have expanded into an abstract theory that has become a necessary discipline in differential equations and functional analysis. In [6], the authors investigated the stability property for evolution operators in Banach spaces. Also, in [28] Sutrima et.al obtained some necessary and sufficient conditions for uniform exponential stability of the evolution operator. In recent years, the theory of robust stability analysis of partial differential equations (PDEs) has been extensively studied by many researchers in the qualitative theory of control dynamical systems, see [12, 22, 25]. In various situations, it is difficult to design a feedback controller ensuring exponential stabilization for infinite-dimensional systems. For instance, one can only ensure that system trajectories approach a neighborhood of the origin. For this aim, a more general stability called practical stability ([24]) is investigated. This general stability concept has been considered an interesting topic for further investigation of nonlinear differential equations. But a lot of differential equations do not possess the exact solution. Under this case, integral inequalities are significant for investigating the boundedness, stability, and asymptotic behavior of solutions to dynamical systems. In [13], the authors gave a new integral inequality and studied the existence, uniqueness, and stability properties of solutions of ordinary differential equations (ODEs).

Motivated by the preceding discussion, this paper investigates a novel procedure for constructing stabilizing compensators for a class of non-autonomous semilinear evolution equations with disturbances in Banach spaces by using an estimated feedback controller. A Luenberger-like observer-based controller synthesis based on integral inequalities of the Gronwall type guarantees exponential convergence of states and the estimation error to the neighborhood of the origin. We show, how under some assumptions of stabilizability and detectability of the linear non-autonomous control systems, we can construct a stabilizing feedback law and a Luenberger observer.

The remainder of the paper is organized as follows. Basic definitions and some preliminary results are presented in Section 2. Section 4 provides an example to illustrate the effectiveness of theoretical results. Conclusions are drawn in Section 5.

2. Preliminaries

Throughout this paper, let X, U and Y be Banach spaces endowed with norms $\|\cdot\|_X$, $\|\cdot\|_U$ and $\|\cdot\|_Y$, respectively. $\mathbb{R}_+$ denotes the set of all non-negative real numbers. For linear normed spaces X, Y let $L(X,Y)$ be the space of bounded linear operators from X to Y and $L(X) := L(X,X)$. A norm in these spaces we denote by $\|\cdot\|$. $C(X,Y)$ denotes the space of all continuous functions from X to Y.

Also, we define $L^p(\mathbb{R}_+, \mathbb{R}_+)$ as the set of functions positive and integrable with p-th power on $\mathbb{R}_+$ where $p \geqslant 1$. $L_2(0,\infty)$ denotes the space of square integrable functions on $(0,\infty)$.

Consider the following non-autonomous control semilinear evolution equation with disturbances:

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}(t)x(t) + \mathcal{B}(t)u(t) + \Upsilon(t, x(t), u(t), d(t)), \quad t \geqslant t_0\,, \\ y(t) &= \mathcal{C}(t)x(t), \quad x(t_0) = x_0\,, \end{aligned} \tag{1}$$

where t is the time, t_0 is the initial time, x_0 is the initial condition, $x(t) \in X$ is the system state, $u \in C(\mathbb{R}_+, U)$ is the control input, $y(t) \in Y$ is the measured output and $d(t) \in \mathcal{D}$ is a measurable, locally essentially bounded disturbance equipped with the norm $\|d\|_\infty = \operatorname*{ess\,sup}_{t \geqslant t_0} \|d(t)\|$. X, U, $\mathcal{D}$ and Y are assumed to be complex Banach spaces. The input and output operators are

$$\mathcal{B} \in C(\mathbb{R}_+, L(U,H)), \quad \text{with} \quad \sup_{t \geqslant 0} \|\mathcal{B}(t)\| < \infty.$$

and

$$C \in C(\mathbb{R}_+, L(H,Y)), \quad \text{with} \quad \sup_{t\geqslant 0} \|C(t)\| < \infty.$$

The nonlinearities $\Upsilon\colon X\times U\times \mathcal{D} \to X$ be nonlinear operator with $\Upsilon(t,0,u,d) = 0$, for all $t \geqslant 0$, all $u \in U$, and all $d \in \mathcal{D}$. Also,

$$(\mathcal{A}(t)\colon D(\mathcal{A}(t)) \subset X \to X)_{t\geqslant t_0}$$

is a family of linear and generally unbounded operators depending on time where the domain $D(\mathcal{A}(t)) = D$, independent of t of the operator $\mathcal{A}(t)$ is assumed to be dense in X for all $t \geqslant 0$ and generates a strongly continuous evolution family $(\Gamma(t,s))_{t\geqslant s\geqslant 0}$, that is, for all $t \geqslant s \geqslant t_0 \geqslant 0$ there exists a bounded linear operator $\Gamma(t,s)\colon X \to X$ satisfying the following properties:

(*i*) $\Gamma(s,s) = I$, $\Gamma(t,s) = \Gamma(t,r)\Gamma(r,s)$ for all $t \geqslant r \geqslant s \geqslant t_0$.

(*ii*) $(t,s) \mapsto \Gamma(t,s)$ is strongly continuous for $t \geqslant s \geqslant t_0$.

(*iii*) For all $t \geqslant s \geqslant t_0$ and all $v \in D(\mathcal{A}(s))$, we have

$$\frac{\partial}{\partial t}\Gamma(t,s)v = \mathcal{A}(t)\Gamma(t,s)v$$

and

$$\frac{\partial}{\partial s}\Gamma(t,s)v = -\Gamma(t,s)\mathcal{A}(s)v.$$

We consider mild solutions of (1), i.e. solutions of the integral form

$$\begin{aligned}\phi(t,t_0,x_0,u,d) = \Gamma(t,t_0)x_0 + \int_{t_0}^{t} \Gamma(t,s)\big[\mathcal{B}(s)u(s)\\ + \Upsilon(t,x(s),u(s),d(s))\big]\,\mathrm{d}s \end{aligned} \tag{2}$$

belonging to the class $C([t_0,t_m],X)$ for some $t_m > t_0$, where t_m is the maximal existence time of the solution corresponding to (t_0,x_0,u,d).

Definition 1. *We call $\Upsilon\colon \mathbb{R}_+ \times X \times \mathcal{D} \to X$ locally Lipschitz continuous in x, uniformly in t, u, and d on bounded intervals if for every $\widetilde{t} \geqslant 0$ and constant $r \geqslant 0$, there is a constant $N(r,\widetilde{t})$, such that*

$$\|\Upsilon(t,x,u,d) - \Upsilon(t,y,u,d)\|_X \leqslant N(r,\widetilde{t})\|x-y\|_X$$

holds for all $x,y \in X$, with $\|x\|_X, \|y\|_X \leqslant r$, all $d \in \mathcal{D}$ with $\|d\| \leqslant r$, all $u \in U$ with $\|u\|_U \leqslant r$, and all $t \in [0,\widetilde{t}]$.

The following assumption will be needed throughout the paper:

($\mathcal{H}_1$) The nonlinearity $\Upsilon(\cdot,\cdot,\cdot,\cdot)$ is continuous in t, u and d and locally Lipschitz continuous in x, uniformly in t, u, and d on bounded intervals.

Well-posedness. Since Assumption ($\mathcal{H}_1$) and $\mathcal{B}$ is continuous, then it follows from [27, Theorem 1.4] that for every initial condition $x_0 \in X$, every input $u \in C(\mathbb{R}_+, U)$, and every disturbance $d \in \mathcal{D}$, the system (1) has a unique mild solution that satisfies the integral equation (2).

Definition 2. *We say that a control system (1) is forward complete (FC) if for every $(t_0, x_0, u, d) \in \mathbb{R}_+ \times X \times U \times \mathcal{D}$ and for all $t \geqslant t_0$, the value $\phi(t, t_0, x_0, u, d) \in X$ is well-defined.*

The corresponding system without nonlinearities is described by

$$\begin{aligned} \dot{x}(t) &= \mathcal{A}(t)x(t) + \mathcal{B}(t)u(t), \quad t \geqslant t_0, \\ y(t) &= C(t)x(t), \quad x(t_0) = x_0, \end{aligned} \tag{3}$$

The mild solutions of (3) is of the integral form

$$x(t) = \Gamma(t, t_0)x_0 + \int_{t_0}^{t} \Gamma(t, s)\mathcal{B}(s)u(s)\,\mathrm{d}s, \tag{4}$$

belonging to the class $C([t_0, \infty), X)$.

Remark 1. *In the autonomous case, where $\Gamma(\theta, \tau) = S(\theta - \tau)$, $\theta \geqslant \tau$, is given by a strongly continuous semigroup on X generated by $\mathcal{A}$ and the operators $\mathcal{B}(t) = \mathcal{B}$ and $C(t) = C$ are both independents of t, the mild control system (4) has the form*

$$x(t) = S(t)x(0) + \int_{0}^{t} S(t - s)\mathcal{B}(s)u(s)\,\mathrm{d}s.$$

Let us define a uniformly exponentially stable evolution family is similar to the one given for semigroups.

Definition 3. *[6, Def 3.4 p. 60] The strongly continuous evolution family $\{\Gamma(t, t_0)\}_{t \geqslant t_0 \geqslant 0}$ is called uniformly exponentially stable if there exist $c, \omega > 0$, such that*

$$\|\Gamma(t, t_0)\| \leqslant ce^{-\omega(t - t_0)} \textit{ holds for all } t_0 \geqslant 0 \textit{ and all } t \geqslant t_0.$$

For the convenience of the reader, we recall the definitions of stabilizability and detectability in a non-autonomous infinite-dimensional setting, see [6, Def 5.2 p. 133] for details.

Definition 4.

(i) *The non-autonomous system (3) is said to be stabilizable if there exists a feedback operator* $\mathcal{K} \in C(\mathbb{R}_+, L(X,U))$, *with* $\sup_{t\geqslant 0} \|\mathcal{K}(t)\| < \infty$ *and a corresponding uniformly exponentially stable evolution family* $(\Gamma_{\mathcal{B}\mathcal{K}}(\theta,\tau)_{\theta\geqslant\tau\geqslant 0}$, *such that*

$$\Gamma_{\mathcal{B}\mathcal{K}}(\theta,\tau) = \Gamma(\theta,\tau)x + \int_{\tau}^{\theta} \Gamma(\theta,s)\mathcal{B}(s)\mathcal{K}(s)\Gamma_{\mathcal{B}\mathcal{K}}(s,\tau)x\,\mathrm{d}s. \tag{5}$$

(ii) *The non-autonomous system (3) is said to be detectable if there exists a feedback operator* $\mathcal{F} \in C(\mathbb{R}_+, L(X,U))$, *with* $\sup_{t\geqslant 0} \|\mathcal{F}(t)\| < \infty$ *and a corresponding uniformly exponentially stable evolution family* $(\Gamma_{\mathcal{F}C}(\theta,\tau)_{\theta\geqslant\tau\geqslant 0}$, *such that*

$$\Gamma_{\mathcal{F}C}(\theta,\tau) = \Gamma(\theta,\tau)x + \int_{\tau}^{\theta} \Gamma_{\mathcal{F}C}(\theta,s)\mathcal{F}(s)C(s)\Gamma(s,\tau)x\,\mathrm{d}s. \tag{6}$$

Remark 2. *In the case* $\mathcal{A}(t) = \mathcal{A}$, $\mathcal{B}(t) = \mathcal{B}$ *and* $C(t) = C$, *we have the system (3) is stabilizable and detectable if there exist operators* $\mathcal{K} \in L(X,U)$ *and* $\mathcal{F} \in L(Y,X)$, *such that the semigroups generated by* $\mathcal{A}+\mathcal{B}\mathcal{K}$ *and* $\mathcal{A}+\mathcal{F}C$ *are uniformly exponentially stable, see [12, Def 5.2.1 p. 227].*

Definition 5. *A forward complete system*

$$\dot{x}(t) = F(t,x(t),d(t)), \qquad t \geqslant t_0 \geqslant 0, \quad x(t) \in X, \quad d(t) \in \mathcal{D},$$

$F: \mathbb{R}_+ \times X \times \mathcal{D} \to X$ *is a nonlinear operator that is called globally practically uniformly exponentially stable if there exist positive scalars* ω, c, ρ, *such that for all* $(t_0, x_0, d) \in \mathbb{R}_+ \times X \times \mathcal{D}$ *and all* $t \geqslant t_0$,

$$\|x(t)\|_X \leqslant c\|x(t_0)\|_X e^{-\omega(t-t_0)} + \rho. \tag{7}$$

Remark 3. *Uniform practical exponential stability given in (7) are said with growth constants* c, ω, *which it is similar to the definition introduced by [16] in the case of finite-dimensional systems. The inequality (7) indicates that the trajectory will be ultimately bounded, that is the solution is bounded and approaches toward a neighborhood of the origin for sufficiently large* t.

Definition 6. *A forward complete system (1) is called practically stabilizable if there exists a continuous feedback control* $u: [t_0,\infty) \to U$, *such that the solution of the system (1) with* $u(t)$ *is globally practically uniformly exponentially stable.*

The below lemma of a generalization of the Gronwall-type inequality is used as a tool for the proof of the main results.

Lemma 1. *[13, Lemma 2]*
Let φ, γ and ψ be non-negative piecewise continuous functions on $\mathbb{R}_+$ for which the following inequality holds

$$\varphi(t) \leqslant a + \int_{t_0}^{t} \Big[\varphi(s)\gamma(s) + \psi(s)\Big] \mathrm{d}s, \quad \forall\, t \geqslant t_0 \geqslant 0,$$

where a is a non-negative constant. Then,

$$\varphi(t) \leqslant \left(a + \int_{t_0}^{t} \psi(s)\mathrm{d}s\right) e^{\int_{t_0}^{t} \gamma(s)\mathrm{d}s}, \quad \forall t \geqslant t_0 \geqslant 0.$$

3. Main results

3.1. Practical stabilization

We aim to design a feedback controller, such that system (1) is globally practically uniformly exponentially stable in Banach spaces using a generalization of the Gronwall-type inequality which is different to the semilinear system without disturbance where $\mathcal{A}(t) = A$ is the generator of C_0-semigroup, $\mathcal{B}(t) = \mathcal{B} \in L(U, X)$ and $C(t) = C \in L(X, Y)$ in [15, Theorem 3.1]. To make the control objective feasible, the following assumptions are posed on the system (1):

($\mathcal{H}_2$) There exist ω and ξ piecewise continuous functions, positives and satisfying for all $t \geqslant 0$, all $x, y \in X$, all $u \in U$, and all $d \in \mathcal{D}$,

$$\|\Upsilon(t, x, u, d) - \Upsilon(t, y, u, d)\|_X \leqslant \omega(t)\|x - y\|_X + \xi(t), \tag{8}$$

where $\omega \in L^1(\mathbb{R}_+, \mathbb{R}_+)$ and

$$\text{either} \quad \xi \in L^p(\mathbb{R}_+, \mathbb{R}_+), \quad \text{for some} \quad p \in [1, \infty) \quad \text{or} \quad \lim_{t \to \infty} \xi(t) = 0. \tag{9}$$

($\mathcal{H}_3$) The non-autonomous system (3) is stabilizable, there exists a feedback operator $\mathcal{K} \in C(\mathbb{R}_+, L(X, U))$, with $\sup_{t \geqslant 0} \|\mathcal{K}(t)\| < \infty$ and nonnegative constants c_1 and λ_1, such that

$$\|\Gamma_{\mathcal{B}\mathcal{K}}(t, t_0)\| \leqslant c_1 e^{-\lambda_1(t - t_0)}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0, \tag{10}$$

where $\Gamma_{\mathcal{B}\mathcal{K}}$ is an evolution operator given by (5).

It is indicate in [13, Lemma 3], that if the function $\xi(t)$ satisfies (9), then

$$\lim_{t\to\infty} e^{-\lambda t}\int_0^t e^{\lambda s}\xi(s)\,\mathrm{d}s = 0,$$

where λ is a positive constant.

Next, we are interested in a suitable feedback controller of the form:

$$u(t) = \mathcal{K}(t)x, \tag{11}$$

where $\mathcal{K}(t)$ is a known operator given by $(\mathcal{H}_3)$.

Theorem 1. *Let assumptions $(\mathcal{H}_1)$, $(\mathcal{H}_2)$ and $(\mathcal{H}_3)$ be satisfied. Then, the non-autonomous infinite-dimensional closed-loop system (1)–(11) is globally uniformly practically exponentially stable.*

Proof. Since B is continuous and u is continuous, and Υ satisfies $(\mathcal{H}_1)$ and $(\mathcal{H}_2)$, then according to the results of Pazy [27, Theorem 1.4], there is a unique maximal global mild solution $x(\cdot) = \phi(\cdot, t_0, x_0, u, d) \in C([t_0,\infty), X)$ of system (1) for any data $(t_0, x_0, u, d) \in \mathbb{R}_+ \times X \times U \times \mathcal{D}$.

This mild solution of the closed-loop system is given by:

$$x(t) = \Gamma_{\mathcal{BK}}(t,t_0)x_0 + \int_{t_0}^t \Gamma_{\mathcal{BK}}(t,s)\Upsilon(s,x(s),u(s),d(s))\,\mathrm{d}s. \tag{12}$$

Then, the solution (12) satisfies the following norm:

$$\begin{aligned}\|x(t)\|_X \leqslant{}& \|\Gamma_{\mathcal{BK}}(t,t_0)\|\,\|x_0\|_X \\ &+ \int_{t_0}^t \|\Gamma_{\mathcal{BK}}(t,s)\|\,\|\Upsilon(s,x(s),u(s),d(s))\|_X\,\mathrm{d}s.\end{aligned} \tag{13}$$

Using $(\mathcal{H}_3)$, inequality (13) gives

$$\nu(t) \leqslant c_1\nu(t_0) + \int_{t_0}^t \left[c_1\omega(s)\nu(s) + c_1 e^{\lambda_1 s}\xi(s)\right]\mathrm{d}s,$$

where

$$\nu(t) = e^{\lambda_1 t}\|x(t)\|_X. \tag{14}$$

Utilizing Lemma 1, we find for all $t_0 \geqslant 0$, and all $t \geqslant t_0$,

$$v(t) \leqslant \left(c_1 v(t_0) + \int_{t_0}^{t} c_1 e^{\lambda_1 s} \xi(s) \mathrm{d}s \right) e^{c_1 \int_{t_0}^{t} \omega(s) \mathrm{d}s}.$$

Thus, by (14), we have the estimation

$$\|x(t)\|_X \leqslant c_1 \|x_0\|_X e^{-\lambda_1 (t-t_0)} e^{c_1 \int_{t_0}^{t} \omega(s) \mathrm{d}s} + c_1 e^{-\lambda_1 t} \left(\int_{t_0}^{t} e^{\lambda_1 s} \xi(s) \mathrm{d}s \right) e^{c_1 \int_{t_0}^{t} \omega(s) \mathrm{d}s}.$$

Using $\omega \in L^1(\mathbb{R}_+, \mathbb{R}_+)$, one can get

$$\|x(t)\|_X \leqslant c_1 \|x_0\|_X e^{-\lambda_1 (t-t_0)} e^{c_1 \int_{t_0}^{\infty} \omega(s) \mathrm{d}s} + c_1 e^{-\lambda_1 t} \left(\int_{0}^{t} e^{\lambda_1 s} \xi(s) \mathrm{d}s \right) e^{c_1 \int_{t_0}^{\infty} \omega(s) \mathrm{d}s}.$$

Since

$$\lim_{t \to \infty} e^{-\lambda_1 t} \int_{0}^{t} e^{\lambda_1 s} \xi(s) \mathrm{d}s = 0,$$

then there exists $\kappa > 0$, such that

$$\|x(t)\|_X \leqslant c_1 e^{c_1 \int_{t_0}^{\infty} \omega(s) \mathrm{d}s} \|x_0\|_X e^{-\lambda_1 (t-t_0)} + c_1 \kappa e^{c_1 \int_{t_0}^{\infty} \omega(s) \mathrm{d}s}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0.$$

Therefore, the non-autonomous infinite-dimensional closed-loop system (1)–(11) is globally uniformly practically exponentially stable. □

3.2. Practical Luenberger observer design

In the previous subsection, we considered the problem of stabilizing by state feedback (11). This assumes that one can measure the whole state, which is not possible for an infinite-dimensional system. The problem that is naturally arises how to stabilize the system using only partial information about the state. A fundamental question is how to design a compensator. One answer we present here is to utilize the measurement (partial information to estimate the full state (the construction of an observer) and to use feedback on the estimated state. To deal with the problem of estimating the full state, the following assumption is posed on the system (1):

($\mathcal{H}_4$) The non-autonomous system (3) is detectable, there exists a feedback operator $\mathcal{F} \in C(\mathbb{R}_+, L(X, U))$, with $\sup_{t \geqslant 0} \|\mathcal{F}(t)\| < \infty$ and there exist nonnegative constants c_2 and λ_2, such that

$$\|\Gamma_{\mathcal{F}C}(t,t_0)\| \leqslant c_2 e^{-\lambda_2(t-t_0)}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0, \tag{15}$$

where $\Gamma_{\mathcal{F}C}$ is an evolution operator given by (6).

Consider the following Luenberger observer:

$$\begin{aligned} \dot{\hat{x}}(t) &= \mathcal{A}(t)\hat{x}(t) + \mathcal{B}(t)u(t) + \Upsilon(t,\hat{x}(t),u(t),d(t)) \\ &\quad + \mathcal{F}(t)(\hat{y}(t) - y(t)), \quad t \geqslant 0, \\ \hat{y}(t) &= C(t)\hat{x}(t), \end{aligned} \tag{16}$$

where $\hat{x}$ is the Luenberger observer with output injection $\mathcal{F} \in C(\mathbb{R}_+, L(X,U))$, with $\sup\limits_{t\geqslant 0}\|\mathcal{F}(t)\| < \infty$ and $y(t) = C(t)x(t)$.

Define estimation error e as $e = \hat{x} - x$, which is commanded by

$$\begin{aligned} \dot{e}(t) = \dot{\hat{x}}(t) - \dot{x}(t) &= (\mathcal{A}(t) + \mathcal{F}(t)C(t))\, e(t) + \Upsilon(t,\hat{x}(t),u(t),d(t)) \\ &- \Upsilon(t,x(t),u(t),d(t)), \end{aligned} \tag{17}$$

where $e_0 = \hat{x}_0 - x_0$.

The next result gives sufficient conditions under which the state estimation error is globally uniformly practically exponentially stable, the so-called practical exponential observer.

Theorem 2. *Let assumptions $(\mathcal{H}_1)$, $(\mathcal{H}_2)$ and $(\mathcal{H}_4)$ be satisfied. Then, the system (16) is a practical exponential Luenberger observer for the system (1).*

Proof. First, note that equation (17) has a unique global mild solution on $[t_0,\infty)$ by applying the results of Pazy [27] for every initial state estimation $e_0 \in X$, every initial time $t_0 \geqslant 0$, and every disturbance $d \in \mathcal{D}$.

This mild solution of the system (17) is given by:

$$e(t) = \Gamma_{\mathcal{F}C}(t,t_0)e_0 + \int_{t_0}^{t} \Gamma_{\mathcal{F}C}(t,s)\widetilde{\Upsilon}(t,e(s),u(s),d(s))\,\mathrm{d}s, \tag{18}$$

where

$$\widetilde{\Upsilon}(t,e,u,d) = \Upsilon(t,\hat{x},u,d) - \Upsilon(t,x,u,d)$$

and $e_0 = \hat{x}_0 - x_0$. Then,

$$\begin{aligned} \|e(t)\|_X &\leqslant \left\|\Gamma_{\mathcal{F}C}(t,t_0)\right\| \|e_0\|_X \\ &\quad + \int_{t_0}^{t} \left\|\Gamma_{\mathcal{F}C}(t,s)\right\| \left\|\widetilde{\Upsilon}(t,e(s),u(s),d(s))\right\|_X \mathrm{d}s. \end{aligned} \tag{19}$$

From $(\mathcal{H}_4)$, inequality (19) becomes

$$\|e(t)\|_X \leqslant c_2\|e_0\|_X e^{-\lambda_2(t-t_0)} + c_2 \int_{t_0}^{t} e^{-\lambda_2(t-s)} \left[\omega(s)\|e(s)\|_X + \xi(s)\right] \mathrm{d}s, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0.$$

Dividing both sides by $e^{\lambda_2 t}$, one can get

$$\|e(t)\|_X e^{\lambda_2 t} \leqslant c_2\|e_0\|_X e^{\lambda_2 t_0} + c_2 \int_{t_0}^{t} e^{\lambda_2 s} \left[\omega(s)\|e(s)\|_X + \xi(s)\right] \mathrm{d}s, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0.$$

Using Lemma 1, we obtain

$$\|e(t)\|_X e^{\lambda_2 t} \leqslant \left(c_2\|e_0\|_X e^{\lambda_2 t_0} + c_2 \int_{t_0}^{t} e^{\lambda_2 s}\xi(s)\,\mathrm{d}s\right) e^{c_2 \int_{t_0}^{t} \omega(s)\,\mathrm{d}s}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0.$$

One gets,

$$\|e(t)\| \leqslant c_2\|e_0\| e^{-\lambda_2(t-t_0)} e^{c_2 \int_0^\infty \omega(s)\,\mathrm{d}s} + \left(c_2 \int_{t_0}^{t} e^{-\lambda_2(t-s)}\xi(s)\,\mathrm{d}s\right) e^{c_2 \int_0^\infty \omega(s)\,\mathrm{d}s}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0.$$

Since

$$\lim_{t\to\infty} e^{-\lambda_2 t} \int_0^t e^{\lambda_2 s}\xi(s)\,\mathrm{d}s = 0,$$

then, there exists $\widehat{\kappa} > 0$, such that

$$\|e(t)\|_X \leqslant c_2 e^{c_2 \int_0^\infty \omega(s)\,\mathrm{d}s}\|e_0\|_X e^{-\lambda_2(t-t_0)} + c_2\widehat{\kappa} e^{c_2 \int_0^\infty \omega(s)\,\mathrm{d}s}, \quad \forall t_0 \geqslant 0, \quad \forall t \geqslant t_0. \tag{20}$$

Thus, the error equation (17) is globally uniformly practically exponentially stable. Hence, the system (16) is a global uniform practical exponential Luenberger observer for the system (1). □

3.3. The compensator design

We observe that the practical Luenberger observer (16) gives a good estimate of the state of (1) provided that the non-autonomous system (3) is detectable.

If we knew the state $x(t)$, then in order to practical stabilize the system we would apply the feedback (11). But, we only have partial information of the state $x(t)$ through the measurement $y(t) = C(t)x(t)$. In the following theorem, we shall show that the feedback

$$u(t) = \mathcal{K}(t)\hat{x}(t) \tag{21}$$

based on the estimated state has the same effect, provided that the estimation error converges toward a neighborhood of the origin, as $t \longrightarrow \infty$.

Theorem 3. *Assume that $(\mathcal{H}_1)$, $(\mathcal{H}_2)$, $(\mathcal{H}_3)$ and $(\mathcal{H}_4)$ hold.*
If $\mathcal{K} \in C(\mathbb{R}_+, L(X,U))$, with $\sup\limits_{t\geqslant 0} \|\mathcal{K}(t)\| < \infty$ and $\mathcal{F} \in C(\mathbb{R}_+, L(X,U))$, with $\sup\limits_{t\geqslant 0} \|\mathcal{F}(t)\| < \infty$ are such that $\Gamma_{\mathcal{B}\mathcal{K}}(\theta,\tau)_{\theta\geqslant\tau\geqslant 0}$, and $\Gamma_{\mathcal{F}C}(\theta,\tau)_{\theta\geqslant\tau\geqslant 0}$, are uniformly exponentially stable evolution family given by (5) and (6). Then, the controller (21), where $\hat{x}$ is the practical Luenberger observer with output injection $\mathcal{F}(t)$, practically stabilizes the closed-loop system. The stabilizing compensator is given by

$$\begin{aligned} \dot{\hat{x}}(t) &= \big(\mathcal{A}(t) + \mathcal{F}(t)C(t)\big)\hat{x}(t) + \mathcal{B}(t)u(t) \\ &\quad + \Upsilon(t,\hat{x}(t),u(t),d(t)) - \mathcal{F}(t)y(t), \\ u(t) &= \mathcal{K}(t)\hat{x}(t). \end{aligned} \tag{22}$$

Proof. Combining the abstract differential equations, we see that the closed-loop system is given by the dynamics of the extended state $x_e(t) = (\hat{x}(t), e(t))^T \in X \times X$,

$$\dot{x}_e(t) = A(t)x_e(t) + \widetilde{F}(t,\hat{x}(t),u(t),d(t)), \tag{23}$$

where

$$A(t) = \begin{pmatrix} \mathcal{A}(t) + \mathcal{B}(t)\mathcal{K}(t) & \mathcal{F}(t)C(t) \\ 0 & \mathcal{A}(t) + \mathcal{F}(t)C(t) \end{pmatrix}$$

and

$$\widetilde{F}(t,x,u,d) = \begin{pmatrix} \Upsilon(t,\hat{x},u,d) \\ \Upsilon(t,\hat{x},u,d) - \Upsilon(t,\hat{x}-e,u,d) \end{pmatrix}.$$

Let $x_e(t) = (\hat{x}(t), e(t))$ be the solution of the system (23) with the initial condition $x_e^0(t) = (\hat{x}_0, e_0) = (\hat{x}(t_0), e(t_0)) \in X \times X$ and disturbance $d \in \mathcal{D}$.

We can see that equation (23) has a unique mild solution $x_e(t)$ which is defined on $[t_0, \infty)$.

Since the component $e(t)$ satisfies the estimation (20), due of the non-autonomous system (3) is detectable with $(\mathcal{H}_2)$ (Proposition 2), it suffices to demonstrate that the component $\hat{x}(t)$ has the same property.

The solution of the closed-loop system:

$$\dot{\hat{x}}(t) = \big(\mathcal{A}(t) + \mathcal{B}(t)\mathcal{K}(t)\big)\hat{x}(t) + \Upsilon(t, \hat{x}, u, d) + \mathcal{F}(t)C(t)e$$

is given by:

$$\hat{x}(t) = \Gamma_{\mathcal{BK}}(t, t_0)\hat{x}_0 + \int_{t_0}^{t} \Gamma_{\mathcal{BK}}(t, s)\big(\Upsilon(t, \hat{x}, u, d) + \mathcal{F}(t)C(t)e(t)\big)\,\mathrm{d}s.$$

Thus,

$$\begin{aligned}\|\hat{x}(t)\|_X \leqslant \|\Gamma_{\mathcal{BK}}(t, t_0)\|\|\hat{x}_0\|_X + \int_{t_0}^{t} \|\Gamma_{\mathcal{BK}}(t, s)\|\Big(\|\Upsilon(t, \hat{x}, u, d)\|_X \\ + \ \|\mathcal{F}(t)C(t)\|\|e(s)\|_X\Big)\,\mathrm{d}s.\end{aligned}$$

Using assumptions $(\mathcal{H}_2)$ and $(\mathcal{H}_3)$, we obtain

$$\begin{aligned}\|\hat{x}(t)\|_X \leqslant c_1\,\|\hat{x}_0\|_X\, e^{-\lambda_1(t-t_0)} + c_1 \int_{t_0}^{t} e^{-\lambda_1(t-s)}\Big(\omega(s)\|\hat{x}(t)\|_X + \xi(s) \\ + \ \|\mathcal{F}(t)C(t)\|\|e(s)\|_X\Big)\,\mathrm{d}s. \qquad (24)\end{aligned}$$

From the proof of Proposition 2, one has for all $t_0 \geqslant 0$ and all $t \geqslant t_0$,

$$\begin{aligned}\|e(t)\|_X \leqslant c_2\|e_0\|_X e^{-\lambda_2(t-t_0)} e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s} \\ + \left(c_2 \int_{t_0}^{t} e^{-\lambda_2(t-s)}\xi(s)\,\mathrm{d}s\right) e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s}. \qquad (25)\end{aligned}$$

Hence, from (24) and (25), one gets that

$$\begin{aligned}\|\hat{x}(t)\|_X \leqslant c_1\|\hat{x}_0\|_X e^{-\lambda_1(t-t_0)} + c_1 \int_{t_0}^{t} e^{-\lambda_1(t-s)}\Bigg(\omega(s)\|\hat{x}(t)\|_X + \xi(s) \\ + \widetilde{c}\ell\left[c_2\|e_0\|_X e^{-\lambda_2(s-t_0)} e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s} + \left(c_2\int_{t_0}^{t} e^{-\lambda_2(s-\tau)}\xi(\tau)\,\mathrm{d}\tau\right) e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s}\right]\Bigg)\,\mathrm{d}s,\end{aligned}$$

where $\widetilde{c} = \sup_{t\geqslant 0} \|C(t)\|$ and $\ell = \sup_{t\geqslant 0} \|\mathcal{F}(t)\|$. Let,

$$\hat{\xi}(s) = \xi(s) + \widetilde{c}\ell \left[c_2\|e_0\|_X e^{-\lambda_2(s-t_0)} e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s} + \left(c_2 \int_{t_0}^{t} e^{-\lambda_2(s-\tau)}\xi(\tau)\,\mathrm{d}\tau \right) e^{c_2\int_0^\infty \omega(s)\,\mathrm{d}s} \right], \quad s \geqslant 0.$$

It yields,

$$\|\hat{x}(t)\|_X \leqslant c_1\|\hat{x}_0\|_X e^{-\lambda_1(t-t_0)} + c_1 \int_{t_0}^{t} e^{\lambda_1(t-s)} \left[\omega(s)\|\hat{x}(t)\|_X + \hat{\xi}(s)\right] \mathrm{d}s.$$

Applying Lemma 1, we get

$$\|\hat{x}(t)\|_X e^{\lambda_1 t} \leqslant \left(c_1\|\hat{x}_0\|_X e^{\lambda_1 t_0} + c_1 \int_{t_0}^{t} e^{\lambda_1 s}\hat{\xi}(s)\,\mathrm{d}s \right) e^{c_1\int_{t_0}^{t} \omega(s)\,\mathrm{d}s}.$$

It follows that,

$$\|\hat{x}(t)\|_X \leqslant c_1 e^{c_1\int_{t_0}^\infty \omega(s)\,\mathrm{d}s}\|\hat{x}_0\|_X e^{-\lambda_1(t-t_0)} + c_1 e^{c_1\int_{t_0}^\infty \omega(s)\,\mathrm{d}s} \int_{t_0}^{t} e^{-\lambda_1(t-s)}\hat{\xi}(s)\,\mathrm{d}s.$$

From $(\mathcal{H}_2)$, since

$$\lim_{t\to\infty} \hat{\xi}(t) = 0,$$

then

$$\lim_{t\to\infty} e^{-\lambda_1 t} \int_0^t e^{\lambda_1 s}\hat{\xi}(s)\,\mathrm{d}s = 0.$$

Hence, there exists $\sigma > 0$, such that

$$\|\hat{x}(t)\|_X \leqslant c_1 e^{c_1\int_{t_0}^\infty \omega(s)\,\mathrm{d}s}\|\hat{x}_0\|_X e^{-\lambda_1(t-t_0)} + c_1\sigma e^{c_1\int_{t_0}^\infty \omega(s)\,\mathrm{d}s}.$$

Therefore, the cascade system (23) is globally uniformly practically exponentially stable. □

Remark 4. *One can see that the theorem 3 generalizes the one given in [15, Theorem 3.3] for a class of control semilinear evolution equations in Hilbert* *spaces without disturbances where the associated nominal part is an autonomous linear system.*

4. Example

In this section, we present an example to illustrate the effectiveness and advantages of the main results.

Example 1. Let X be the space of all bounded continuous real function on $[0, \infty)$ with the supremum norm and $Y = U = L_2(0, \infty)$.
Consider a class of control system in the form (1) on the Banach space X with

$$\mathcal{A}(t)x(\zeta) = 2t\left(\frac{dx}{d\zeta}\right),$$

with domain

$$D(\mathcal{A}(t)) = D = \left\{x \in X/\frac{dx}{d\zeta} \in X, x_0 = x_0\right\},$$

$\mathcal{B}(t) = I$, $d(t) \in [0, 1)$,

$$C(t) = \begin{cases} -2t^{\frac{1}{2}}I & \text{if } 0 \leqslant t < 1, \\ I & \text{if } t \geqslant 1, \end{cases}$$

where I is the identity operator, and

$$\Upsilon(t, x(t), u(t), d(t)) = \frac{d(t)\sin(u(t))}{1+t^2}x(t) + \frac{\sin(x(t))(1+t)}{1+t^2}.$$

From [28], the operator $\mathcal{A}(t)$ generates a strongly continuous evolution family $(\Gamma(t, s))_{t \geqslant s \geqslant 0}$, of the form:

$$(\Gamma(t, s)x)(\zeta) = x(\zeta + s^2 + 2st), \quad \zeta, t, s \geqslant 0,$$

for all $x \in X$.
We choose a stabilizing feedback

$$u(t) = \mathcal{K}(t)x(t), \tag{26}$$

with

$$\mathcal{K}(t) = \begin{cases} -2tx(\zeta) & \text{if } 0 \leqslant \zeta + t < 1, \\ -x(\zeta) & \text{if } \zeta + t \geqslant 1, \end{cases}$$

for all $x \in D$.

We see that the operator $\mathcal{K} \in C(\mathbb{R}_+, L(X,U))$, with $\sup_{t\geqslant 0} \|\mathcal{K}(t)\| < \infty$ and a corresponding uniformly exponentially stable evolution family $(\Gamma_{\mathcal{BK}}(\theta,\tau)_{\theta\geqslant\tau\geqslant 0}$, is defined by

$$(\Gamma_{\mathcal{BK}}(t,s)x)(\zeta) = \begin{cases} e^{-(s^2+2st)}x(\zeta+s^2+2st), & \text{if } 0 \leqslant \zeta+s^2+2st < 1, \\ e^{-s}x(\zeta+s^2+2st) & \text{if } \zeta+s^2+2st \geqslant 1, \end{cases}$$

for all $t, s, \zeta \geqslant 0$, and all $x \in X$.

Moreover, we choose a stabilizing output injection such that

$$\mathcal{F}(t) = \begin{cases} t^{\frac{1}{2}} & \text{if } 0 \leqslant t < 1, \\ -I & \text{if } t \geqslant 1. \end{cases}$$

It is easy to verify that $\mathcal{A}(t)+\mathcal{F}(t)C(t)$ generates a strongly continuous evolution uniformly exponentially stable family

$$(\Gamma_{\mathcal{F}C}(t,s)x)(\zeta) = (\Gamma_{\mathcal{BK}}(t,s)x)(\zeta),$$

for all $t, s, \zeta \geqslant 0$, and all $x \in X$.

One can see that Assumption $(\mathcal{H}_2)$ is verified with $\omega(t) = \dfrac{1}{1+t^2}$ and $\xi(t) = \dfrac{2+2t}{1+t^2}$, in particular $\omega \in L^1(\mathbb{R}_+, \mathbb{R}_+)$ and $\xi \in L^p(\mathbb{R}_+, \mathbb{R}_+)$ for all $p \in (1,\infty)$.

Hence, all hypotheses of Theorem 3 are satisfied. We conclude that the stabilizing compensator is given by equation (22).

5. Conclusion

We have expanded the theory of Luenberger observers and stabilizing compensators to a class of non-autonomous control semilinear evolution equations with disturbances in Banach spaces. We have demonstrated, how under the assumptions of stabilizability and detectability of the linear non-autonomous control systems, we estimate practically exponentially the state while having a practical exponential convergence of the estimation error. Our approach is based on integral inequality. An illustrative example is given to indicate significant improvements and the application of the results.

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