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Assessment of heat and mass transfer processes in vapor bubbles under conditions of metastable equilibrium of liquids

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Abstract This paper proposes a mathematical model that allows expanding the scope of research into the mechanism of heat transfer during explosive boiling, cavitation and boiling of multicomponent liquids, identifying the most influential factors and optimizing technological processes. The proposed model takes into account the processes of heat accumulation in the high-boiling part of liquid mixtures (for example, emulsions) and the use of this energy in the process of boiling their thermolabile part, as well as for superheating the resulting steam in steam bubbles. This effect can also be used to evaluate the effects of liquid boiling in thermodynamically unstable regions of liquid media.

Keywords: Heat and mass transfer; Gas-vapour bubble; Superheated liquids; Mathematical model

Nomenclature

A	–	work
a	–	thermal diffusivity coefficient, m^2/s
B	–	universal gas constant, $\mathrm{J/(mol\,K)}$
b	–	thickness, m
c	–	heat capacity, $\mathrm{J/(kg\,K)}$
L	–	heat of vaporization, J/kg

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M – molecular mass
p – pressure, Pa
p_l – liquid pressure, Pa
p_s – steam pressure, Pa
p_1 – outer pressure, Pa
Q – heat, J
q – specific heat flux, W/m^2
R – radius of the bubble, m
R_1 – outer radius, m
r – radius, m
S – surface area, m^2
T – temperature, °C
T_l – liquid temperature, °C
T_s – steam temperature, °C
u – velocity, m/s
u_r – radial velocity of the liquid at the boundary with the bubble, m/s
V – volume of the bubble, m^3

Greek symbols

α – heat transfer coefficient, W/(m^{2}K)
λ – coefficient of thermal conductivity, W/(m K)
μ – dynamic viscosity coefficient, Pa·s
ρ – density, kg/m^3
σ – surface tension, N/m
τ – time, s

Subscripts

0 – initial value
b – buble
g – gas
i – inside
l – liquid phase
r – radial coordinate
s – steam
ν – vapour

1 Introduction

Vapour-gas mixtures are formed in many technological processes, both in the intermediate stage of the implementation of these processes [1–9], for example, evaporation, degassing of water, distillation of petroleum products, surface cleaning, mixing colloidal solutions, foaming, etc., and as the main technological method, for example, homogenization of emulsions, fuel, cavitation, etc. [10–16].

The processes of spontaneous boiling of highly superheated liquids under conditions when the liquid suddenly finds itself in a non-equilibrium metastable state have attracted increased attention from researchers. This interest is determined, on the one hand, by the complexity of the observed phenomena and processes, which are determined by many difficult-to-control factors. On the other hand, understanding the specifics of heat and mass transfer processes in thermodynamically nonequilibrium regions and studying the kinetics of decomposition of a metastable state can contribute to the creation of new high-intensity technologies in various industries [1–5]. In particular, the phenomenon of explosive boiling of superheated liquid was studied theoretically and experimentally in relation to the problem of obtaining finely dispersed water-fuel emulsions [17,18].

The metastable state occurs in phases of various natures, and the existence of the metastable state is due to the fact that the liquid overheating limit is not reached. Heterogeneous nucleation and the process of nonequilibrium vaporization usually occur during irreversible nonequilibrium interphase mass transfer of metastable liquids through adiabatic channels. Many works are devoted to this class of problems [19–21], the purpose of which was the development of mathematical models or experimental studies of critical flows of a two-phase mixture in channels taking into account irreversible nonequilibrium interphase mass transfer. A change in pressure in a liquid is accompanied by vaporization. We consider a class of problems not related to the critical flow of a mixture, although our model can also be applied to assess such phenomena. The physical processes that are studied in our work are associated with the study of the phenomena of heat and mass transfer between vapor-gas bubbles and liquid at the moment of violation of thermodynamic equilibrium. For example, in the case of a sharp decrease in pressure, as in the above articles, but not in critical flow regimes. In this case, the temperature and pressure of the vapor in the bubbles and the temperature of the boundary layer of the liquid will constantly change. In this case, heat and mass flows can be directed both from liquid to vapor and vice versa, depending on the amount of accumulated energy in these phases [15–17].

The class of problems closest to ours is considered in [22–25]. Unlike flash boiling models, we take into account the influence of bubbles inside the droplet on the heat transfer process. The classical model of a homogeneous nucleon is complemented by a model of bubble growth (Rayleigh). In the cited works, a uniform droplet temperature was assumed, and the droplet energy balance equation was associated with submodels of bubble

formation, growth, and explosion. For single-component fuels, the boiling point can be used to determine whether the liquid fuel reaches the boiling point. However, for multicomponent fuels, evaporation and boiling do not depend only on the boiling point of the components. In general, the process of evaporation of multicomponent droplets is quite complex. Changing the temperature of liquid and vapor or the thermophysical properties of the medium will significantly affect the evaporation regime. Modern models usually focus solely on the evaporation process or the boiling process. The transition between evaporation and boiling is not fully understood. Existing boiling evaporation models used empirical formulas to calculate internal heat transfer and did not adequately consider the effect of the presence of bubbles in the liquid on the evaporation rate. Moreover, most existing boiling evaporation models are designed only for single-component droplets. However, in real environments, such as fuel, many components are present, and their properties vary significantly. In addition, the processes of bubble dynamics at the stage of their development or collapse are interesting from the point of view of the possibility of intensifying heat and mass transfer processes, and in this sense, our task was also to take into account in the model the phenomena of accumulation of thermal energy in a liquid and assess the influence of this phenomenon on the dynamics of vapor bubbles, which also makes it possible to develop technologies for the use of metastable liquids.

The difficulty of directly studying these processes, due to the small size of the droplets and the high intensity of heat and mass transfer, has led to the widespread use of mathematical modelling methods that make it possible to optimize these technological processes.

As a rule, known methods for calculating heat- and mass-transfer processes of this type only approximately take into account the temperature regime of the liquid and phase-transition processes occurring around gas-vapour bubbles. Taking into account transient processes in a liquid will significantly improve the accuracy of the calculations and expand the scope of application of such mathematical models. That is why research in this direction is extremely relevant.

In [17,18], the conditions for the growth of the vapour phase in an emulsion between the surface of a water drop and the continuous phase during heating were considered. A distinctive feature of the boiling of a liquid located in the volume of another is the presence of a continuous phase, which in this case will affect heat exchange processes. In this case, the formation and growth of vapour bubbles will occur at the interface between the com-

ponents of the mixture due to the reduced interfacial tension. The higher boiling phase will accumulate thermal energy, which can be used as an additional heat source when the thermolabile component (water) boils. These features of boiling emulsions were considered in works [26–32].

To increase the accuracy of modelling heat transfer processes in a gas-vapour bubble, it is necessary to take into account the heat transfer to the liquid medium. Several authors assume the liquid temperature to be constant: when calculating the swelling of materials [33–36], when determining the thermodynamic characteristics of steam [37–39] and cavitation bubbles [40, 41].

In [42,43], the authors consider the heat transfer layer of the liquid to be so thin that the curvature of the bubble surface can be ignored. However, such assumptions are possible only for a very limited group of problems. In the study [44] for cavitation bubbles and in [45] for vapour bubbles, an analytical solution to the problem of unstable thermal conductivity in a layer of liquid surrounding an oscillating bubble is presented. As a result of the boundary conditions introduced by the authors, a solution was obtained in which the heat transfer layer of the liquid is equal to the radius of the bubble and does not depend on the thermophysical characteristics of the liquid. A review of the literature sources shows that phase transition processes in the liquid surrounding a gas bubble have not been sufficiently studied. A change in the phase state of the liquid leads to a significant change in the processes of heat and mass transfer at the boundary. To expand the boundaries of mathematical modelling of a gas-vapour bubble, it is necessary to take into account the phase transition processes of heat transfer in the liquid that surrounds the gas-vapor bubble. The peculiarity of this problem is the movement of the bubble wall, the velocity of which at certain times can reach several tens of metres per second.

Most methods for assessing heat and mass transfer processes in two-phase media were aimed at studying the dynamics of the growth of vapour bubbles that arise either in the bulk of a liquid or on a superheated surface. A feature of the boiling of emulsion media is that the formation of a vapour phase will occur at the interface between liquids if one liquid (for example, oil) is a surfactant or a surfactant is present in the emulsion (in multicomponent emulsions), which is based on the reduction of interfacial tension at the interface between the dispersion medium (water) and the dispersed phase (oil). Experimental data [31, 46–48] indicate that the process of formation of a new phase (steam) is initiated by internal thermal effects on the surface of oil particles. With a sharp release of pressure, the preheated

emulsion will find itself in a state of supersaturation of the thermolabile aqueous phase (water is overheated relative to the saturation temperature at a given pressure), i.e. containing excess heat. This excess heat is spent on the work of forming a vapour layer and further vaporisation.

The purpose of this paper is to create a mathematical model of heat transfer in a liquid that surrounds an oscillating gas-vapour bubble. This mathematical model must take into account the mobility of the bubble wall, phase transitions in the layers of the liquid, and changes in its thermophysical characteristics.

2 Research methods

Analysis of literature data shows that today, despite the large number of publications concerning theoretical studies of the dynamics of vapour bubbles, there are no mathematical models that would provide equally accurate prediction of the behaviour of an individual bubble, and, moreover, bubble clusters, in any given interval operating parameters corresponding to real conditions. The imperfection of existing models is primarily due to the fact that such phenomena as acoustic and hydrodynamic cavitation, volumetric condensation, boiling at low superheats, explosive boiling, bubble growth in subcooled liquid traditionally considered and studied as fundamentally different branches of thermophysics. However, from a physical point of view, this is one and the same phenomenon, which represents the process of relaxation of the "bubble-liquid" system tending to a thermodynamic equilibrium state. The behaviour of the bubble in these processes is subject to the same laws and must be described by the same equations, but with different initial conditions. Therefore, a mathematical model of bubble dynamics, taking into account the totality of all the main determining factors, should equally describe any of these phenomena. Such a model cannot contain specific assumptions that are valid only for one of the phenomena, since there are no objective grounds for establishing a clear boundary, and the question of where cavitation ends and boiling begins has no physical meaning.

When studying physical processes in which a dynamically developing bubble plays a decisive role, it is useful to have a physically reliable and, at the same time, not cumbersome mathematical model, which, without additional restrictions and simplifications, would equally well predict the behaviour of a vapour bubble under any initial conditions.

2.1 Basic provisions of the model

When developing a model of the dynamics of a single bubble, the following assumptions were made:

- the bubble has a spherical shape,
- the liquid is viscous, but incompressible,
- inside the bubble there is vapour, the amount of which is determined by the mass transfer of the bubble with the surrounding liquid,
- the vapour inside the bubble is considered as a real gas.

The validity of the first of these assumptions is confirmed by an analysis of known experimental data, from which it follows that, in relation to the class of problems considered in this work, the bubble retains a spherical shape at all stages of its development. This assumption is generally accepted in all known models, with the exception of special cases of bubble growth or destruction near solid surfaces, which is beyond the scope of this study.

The second assumption is due to the desire to avoid unnecessary complexity of the problem and an unjustified increase in the cost of computer time when performing calculations. The compressibility factor of a liquid becomes important when describing the shape and propagation of the pressure pulse emitted in a liquid by a bubble at the moment of its explosion.

The fourth assumption significantly increases the reliability of the model, especially when considering dynamic effects in the phenomena of cavitation or explosive boiling.

Let us formulate the problem statement as follows. A bubble with radius R_0, which contains, in the general case, in addition to steam, an extraneous noncondensable gas with a partial pressure p_0, is in equilibrium with the liquid at temperature $T_0 = \text{const}$. The equilibrium conditions for a bubble in a liquid are determined by the following equation equation that can be obtained by analyzing the equilibrium equations of a vapor bubble presented in [49]:

$$dA = pdV + \sigma dS = \left(p + \sigma \frac{dS}{dV}\right) dV. \tag{1}$$

The general condition for thermodynamic equilibrium in a one-component two-phase disperse system can be written as

$$T_s - T_l = 0\,, \qquad p_s - p_l - \frac{2\sigma}{R} = 0. \tag{2}$$

The $2\sigma/R$ term in the equation can be interpreted as some additional pressure due to the action of surface forces. It follows that in heterogeneous disperse systems, where at least one of the phases is not macroscopic, the condition of mechanical equilibrium between the phases takes the form: $p_s = p_l + 2\sigma/R$.

2.2 Equation of motion of the bubble boundary

According to the accepted assumption of spherical symmetry, the fluid flow in the vicinity of the bubble, caused by the movement of its surface, will be irrotational, and the potential of the velocity field potential $\varphi(r,\tau)$, as well as the velocity of the fluid velocity $u(r,\tau)$, depends only on the radial coordinate r and on time τ. Solving the Laplace equation $\Delta\varphi = 0$ in spherical coordinates relative to $\varphi(r,\tau)$ and taking into account that the velocity of the liquid $u(r,\tau)$ at a distance from the centre of the bubble at any time is related to the potential at this point by the relation $u(r,\tau) = \partial(r,\tau)/\partial r$, we obtain the continuity equation of the form $ur^2 = f(\tau)$, where $f(\tau)$ is an arbitrary function of time, the current value of which is the same for the entire volume of liquid. If the radial velocity of the liquid at the boundary with the bubble is known $u_r(\tau) = u(r,\tau)$, we can write that $f(\tau) = u_r(\tau)r^2$ and represent the continuity equation as $ur^2 = u_r r^2$. The values of the fluid velocity and the velocity field potential at an arbitrary point of the fluid at time τ are determined by the expressions

$$u(r,\tau) = \frac{u_r R^2}{r^2}\,, \qquad \varphi(r,\,\tau) = -\frac{u_r R^2}{r}\,. \tag{3}$$

The movement of a viscous fluid obeys the Navier-Stokes equation, which in vector form has the form:

$$\frac{du}{d\tau} = -\frac{\operatorname{grad} p_l}{\rho_l} + \nu\frac{\operatorname{grad}\operatorname{div} u}{3} + \nu\Delta u. \tag{4}$$

For an inviscid fluid ($\nu = 0$), the last two terms on the right side of (4) disappear and the Navier-Stokes equation is reduced to the Euler equation of motion for an ideal fluid. If the fluid is viscous but incompressible ($\operatorname{div} u = 0$), the second term is excluded from the right side of (4).

The assumption of the irrotational nature of liquid flow in the vicinity of a bubble allows us to use the Euler equation to describe the flow of viscous liquids. Indeed, for any potential flow $\Delta\varphi = 0$ and $u = \operatorname{grad}\varphi$. From the vector analysis, it follows that $\Delta u = \operatorname{grad}\Delta\varphi$. Therefore, the viscosity term

$\nu\Delta u$ can be excluded from the right-hand side of (4), even if $\nu \neq 0$. The result obtained leads to the paradoxical conclusion that the flow of a viscous fluid obeys the equation of motion of an ideal fluid, since both terms in (4), containing viscosity ν, become identically equal to zero. The first term is excluded due to the a priori assumption of the incompressibility of the fluid, and the second term is eliminated due to the assumption of the irrotational nature of the flow.

Since the flow velocity depends only on the coordinate r, derivatives of velocity in perpendicular directions are zero. Thus, radial flows do not have any tangential stresses, and there is no internal friction.

Using the rules of vector analysis, Euler's equation [50] in this form

$$\frac{du}{d\tau} + (u\nabla)\, u + \frac{\operatorname{grad} p_l}{\rho_l} = 0 \tag{5}$$

can be reduced to the well-known Lagrange-Cauchy integral [51]:

$$\frac{\partial \varphi}{\partial \tau} + \frac{u^2}{2} + \frac{p}{\rho_l} = f(\tau), \tag{6}$$

which connects the velocity of an unsteady flow at an arbitrary point r with the pressure at this point. The function $\varphi(\tau)$ in (6) represents an arbitrary time function determined from the boundary conditions, and the value of this function is the same throughout the fluid flow.

To transform Eq. (6), we substitute into it the expressions obtained above for potential and velocity:

$$u(r) = \frac{u_r R^2}{r^2}, \quad \varphi(r) = -\frac{u_r R^2}{r}, \quad \frac{\partial \varphi}{\partial \tau} = -\frac{1}{r}\left(\frac{du_r}{d\tau} R^2 + 2R u_r \frac{dr}{d\tau}\right). \tag{7}$$

After transformations, Eq. (6) will take the form

$$-\frac{1}{r}\left(\frac{du_r}{d\tau} R^2 + 2R u_r \frac{dr}{d\tau}\right) + \frac{u_r^2 R^4}{2r^4} + \frac{p_l}{\rho_l} = f(\tau). \tag{8}$$

Due to the fact that the function $f(\tau)$ in (6) does not depend on spatial coordinates, it can be determined using any of two boundary conditions: at an infinite distance from the bubble or at the boundary with the bubble. When $r \to \infty$, the pressure p_l takes on the value p_∞, equal to the external pressure applied to the liquid. Substituting this value p_l into (8), we find the value $f(\tau) = p_\infty/\rho_l$. Obviously, pressure p_∞ can be any given function

of time, including a constant value. Substituting the value $f(\tau) = p_\infty/\rho_l$ into (8), we obtain

$$\frac{p_l(r,\tau) - p_\infty}{\rho_l} + \frac{u_r^2 R^4}{2r^4} - \frac{1}{r}\left(\frac{du_r}{d\tau}R^2 + 2Ru_r\frac{dr}{d\tau}\right) = 0. \tag{9}$$

This equation makes it possible to calculate the pressure field $p_l(r,\tau)$ in the vicinity of the bubble for any moment of time, if the current values of its radius $r(\tau)$ and the fluid velocity u_r at its surface are known.

To determine the velocity of movement of the bubble boundary $u_r(\tau)$ during its expansion or compression, you can use another boundary condition for pressure by substituting in (9) instead of $p_l(r,\tau)$ the value of the liquid pressure at the surface of the bubble $p_l(r,\tau) = p_r(\tau)$, which we agree to consider for now as a known function of time. As a result, we obtain a differential equation, the solution of which makes it possible to find the unknown function $u_r(\tau)$:

$$\frac{du_r}{d\tau} = \frac{p_r - p_\infty - p_l u_r\left(2\dfrac{dr}{d\tau} - \dfrac{u_r}{2}\right)}{\rho_l R}. \tag{10}$$

If we assume that $dr/d\tau = u_r$, Eq. (10) becomes simpler. The assumption $dr/d\tau = u_r$ is strictly justified only in the absence of phase transitions on the surface of the bubble. It will be shown below that even with intense evaporation or condensation on the interphase surface, the difference between these values is small, except in extreme cases.

Also important is the equation of motion for the case when the bubble is not in an infinite volume of liquid but in the center of a limited spherical volume with an outer radius R_1, for example, inside a drop of liquid. Then, substituting in (8) the corresponding boundary conditions for pressure: $p_l(R_1) = p_1$ and $p_l(R) = p_1$, we obtain instead of (10) an equation of the form

$$\frac{du_r}{d\tau} = \frac{p_r - p_1 - 1.5\rho_l\left(\dfrac{dr}{d\tau}\right)^2}{\rho_l R} + \frac{u_r^2\left[1 + \dfrac{R}{R_1} + \left(\dfrac{R}{R_1}\right)^2\right]}{2R_1}. \tag{11}$$

Equation (11) can be used to analyse explosive destruction of droplets of highly superheated liquid in a metastable state.

2.3 Basic equations of the model

The model contains a system of differential equations with the corresponding initial conditions. The liquid temperature T_{0l} is known. The initial pressure in the liquid is also a known parameter, the subsequent change of which over a given period of time $\Delta\tau$ to a given value p_l determines the further behaviour of the bubble. Under thermodynamic equilibrium conditions, the values of p_{0l} and T_{0l} determine the initial conditions for the temperature, density, and pressure of the vapour and gas inside the bubble. If only vapour is present in the bubble, the initial radius of the bubble R_0 is uniquely determined from the condition $p_l = p_{0l} + 2\sigma/R_0$. When analyzing the growth process of the vapor nuclei, the initial gas content has virtually no effect on the subsequent behaviour of the growing bubble.

The following are the basic equations of the bubble model.

Change in the velocity of radial movement of the liquid near the surface of the bubble:

$$\frac{du_r}{d\tau} = \frac{p_r - p_\infty - 1.5\rho_l u_r^2 - \dfrac{2\sigma}{R} - \dfrac{4\mu_l u_r}{R}}{\rho_l R}\,. \tag{12}$$

Changing the radius of the buble [29,31]:

$$\frac{dr}{d\tau} = u_r + \frac{j}{\rho_l}\,. \tag{13}$$

where j is the transfer of mass through the interfacial surface.

Changing the temperature of the vapour-gas medium in the buble [29,31]:

$$\frac{dT_b}{d\tau} = \frac{3}{(\rho_g c_g + \rho_l c_l)}\left(q - jc_l T_b - p_g \frac{dr}{d\tau}\right)\frac{1}{R}\,. \tag{14}$$

Change in the amount of heat transferred to the bubble:

$$\frac{dQ_l}{d\tau} = -4\pi R^2\,(jL - g)\,, \tag{15}$$

where g is the rate of heat transfer through the interfacial surface, and L is the heat of vaporization (condensation).

Initial conditions:

$$u_r(0) = u_{R_0}\,, \qquad R(0) = R_0\,, \qquad Q_l(0) = Q_{ol}\,. \tag{16}$$

Additional equations:

- gas pressure in the bubble [18]:

$$p_g = \frac{B\rho_g T_g}{M_g - \rho_g b_g} - \frac{a_g \rho_g^2}{M_g^2}\,. \tag{17}$$

- vapour pressure in the bubble:

$$p_r = \frac{B\rho_\nu T_b}{M_\nu - \rho_\nu b_\nu} - \frac{a_\nu \rho_\nu^2}{M_\nu^2}\,, \tag{18}$$

- pressure of the vapour-gas medium in the buble:

$$p_{\nu g} = p_g + p_r\,, \tag{19}$$

- interfacial mass transfer rate [29, 31]:

$$j = 0.25\alpha\left(\rho_l u_s - \rho_r u_\nu\right), \tag{20}$$

- interfacial heat transfer rate [29, 31]:

$$g = 025\left(\rho_l u_\nu c_l - \rho_g u_g c_g\right)\left(T_s - T_{\nu g}\right) + j c_l T_s\,, \tag{21}$$

- velocity of molecular kinetic movement of vapour and gas molecules:

$$u_\nu(T_b) = \left(\frac{8BT_{\nu g}}{\pi M_\nu}\right)^{\frac{1}{2}}, \quad u_\nu(T_s) = \left(\frac{8BT_l}{\pi M_\nu}\right)^{\frac{1}{2}}, \quad u_g(T_b) = \left(\frac{8BT_{vg}}{\pi M_g}\right)^{\frac{1}{2}}. \tag{22}$$

Heat balance equation:

$$\left(T_0 - T_s\right)\lambda_l\left(\frac{2}{b} + \frac{1}{R}\right) = -j\left(T_s\right)L\left(T_s\right) - q\left(T_s\right). \tag{23}$$

$$b = 2R\left(\sqrt{1 + \frac{3Q_l}{4\pi R^3 \rho_l c_l\left(T_s - T_l\right)}} - 1\right). \tag{24}$$

3 Bubble growth under boiling conditions of highly superheated liquids

3.1 Change the radius of the bubble

Figure 1 shows a comparison of experimental [51,52] and calculated dependencies $R = f(\tau)$. The good quantitative agreement between the results of calculations and experiments was unexpected, considering the difference in the thermophysical properties of the objects being studied. The authors' own model adequately describes the kinetics of bubble growth since this model was developed based on this experiment and is intended for its analysis. As noted by the authors themselves [52], "the radius of the bubble is the only variable available for direct measurements."

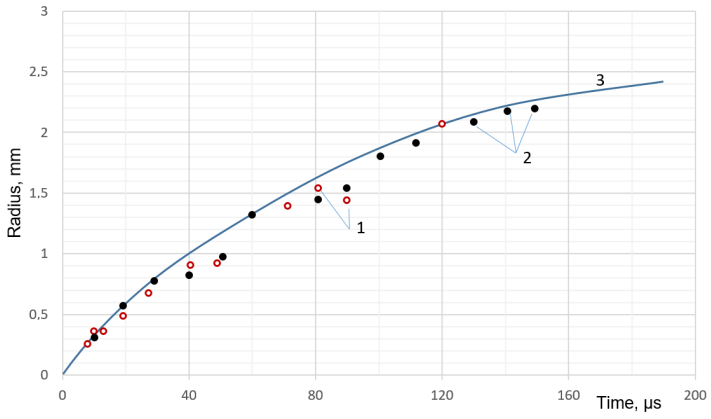

Figure 1: Changes in the radius of a vapour bubble during the explosive boiling of a liquid drop (comparison of experimental results *1* and *2*, according to [51] and [52], respectively, with calculations using our model – curve *3*).

3.2 Bubble growth rate

The experimental values of the bubble growth rate (Fig. 2) were determined in [52] by numerical differentiation with respect to the time of the averaged radius change curve, which is presented in Fig. 1. The period of accelerated bubble growth, which, according to the authors, lasts approximately 3 μs, which could not be recorded in the experiment. According to the authors, the difference between the experimental data on $dr/d\tau$ and the calculation

results using their model is 30%. As can be seen in Fig. 2, our model predicts the growth rate with high accuracy.

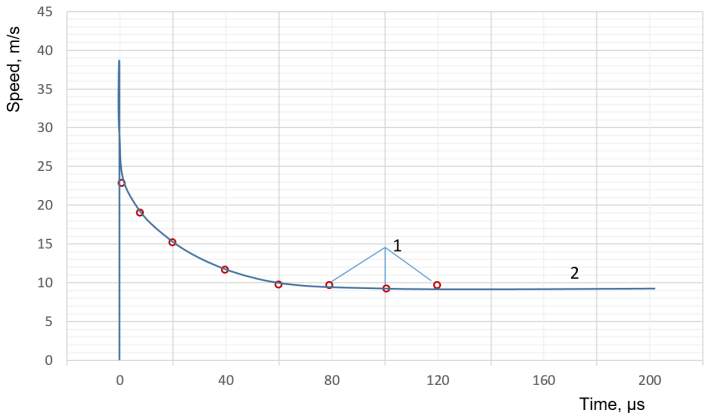

Figure 2: The speed of radial motion of the wall of a bubble growing during explosive boiling of a liquid drop: comparison of experimental results (points *1*) [52] with calculated data using the model (curve *2*).

3.3 Transfer of mass of the bubble with the surrounding liquid

Figure3 shows the experimental data [52] on the change in vapor mass in a droplet during the explosive boiling process, confirming thatwithin the time $\Delta\tau \approx 180$ μs, the entire mass of the droplet transformed into vapor. The experimental data points in this figure were derived from an analysis of the $R = f(\tau)$ dependence in Fig. 1 and subjective assumptions about the mass transfer rate, which are incorporated into their model equation. As can be seen in Fig. 3, calculated curve *2* differs from the experimental data (red points on curve *2*) in that the authors [53] employed the calculated dependence $R = f(\tau)$ from Fig. 1. The $m = f(\tau)$ dependence calculated using our model (Fig. 3, curve *3*) indicates that when the first pressure peak appears (at $\Delta\tau \approx 180$ μs), no more than 8% of the droplet mass has transformed into the vapor phase.

With a sharp drop in pressure, the growth of the vapour bubble occurs in a monotonic regime. In this case, the vapour pressure in the bubble should theoretically decrease. However, because under these conditions the phenomenon of evaporation of water into the bubble also occurs, the pressure in the bubble remains at the same level or can increase and during

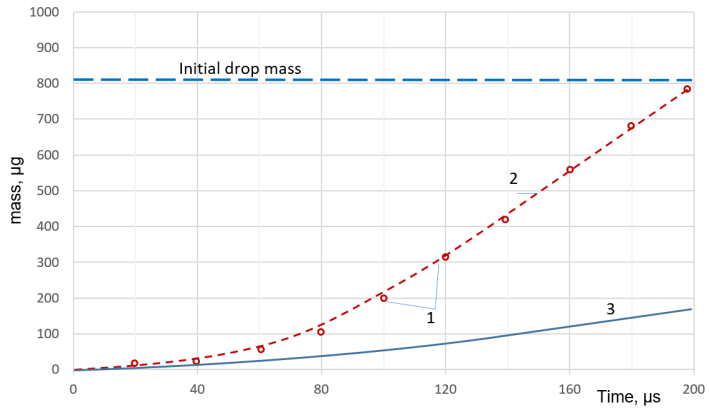

Figure 3: Change in the mass of vapour in a bubble as a result of the evaporation of liquid from a drop into the bubble during the explosive boiling of the drop.

the growth process the condition $p_l > p_{0l}$ is satisfied. This condition is confirmed by all known data on the growth of bubbles under boiling conditions at low and medium superheats.

It is known that in the absence of mass transfer of a bubble with the surrounding liquid, for example, in the case when the bubble contains only noncondensable gas, the release of pressure leads to damped oscillations of such a bubble [18, 52]. Obviously, the experimentally recorded pattern of pressure fluctuations shown in Fig. 4 led the authors [52] to the conclusion

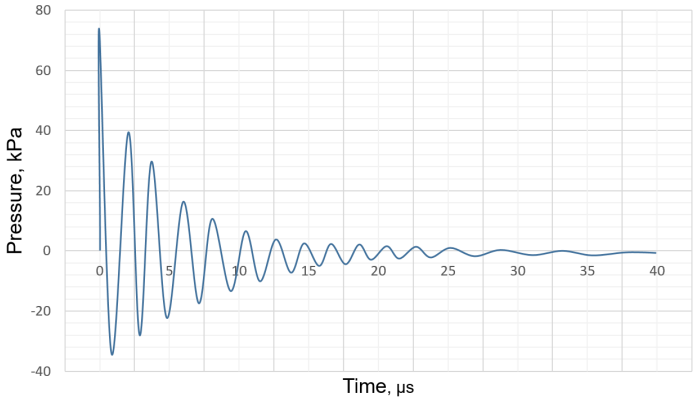

Figure 4: Change in the pressure in the liquid at a distance of 6 mm from the centre of the vapor bubble during the explosive boiling of a drop until its destruction [52].

that at $\tau > 180$ μs the droplet has completely evaporated and the bubble behaves like a 'gas bubble'.

In reality, at very high overheating, which leads to the effect of explosive boiling, the dynamic processes in the inertial stage of growth proceed so intensely that they do not have time to be compensated for by evaporation into the bubble. The thermophysical parameters of the vapour inside the bubble (T_{ib}, p, ρ) and the liquid adjacent to it (T_l, ρ_l, μ_l, σ, L) change significantly during the process, and in this case the vapour cannot even be approximately considered an ideal gas. All these factors, which, as a rule, are not taken into account in known models, can have a decisive influence on the specific nature of the boiling of highly superheated liquids.

Figure5 shows the dependences $R = f(\tau)$ and $m = f(\tau)$ calculated using our model in the time interval from 0 to 20 μs. The bubble actually performs damped oscillations, but at the same time mass transfer of the bubble with the surrounding liquid continues. Since no additions were introduced to the model to take into account the specifics of boiling under conditions of severe overheating, this behaviour of the bubble apparently reflects the characteristic features of the phenomenon under study. From Fig. 5 it follows that complete evaporation of a drop in a bubble is completed only 20 μs after the start of the explosion, which is 120 times higher than the value obtained by the authors of [52] when analyzing the experiment.

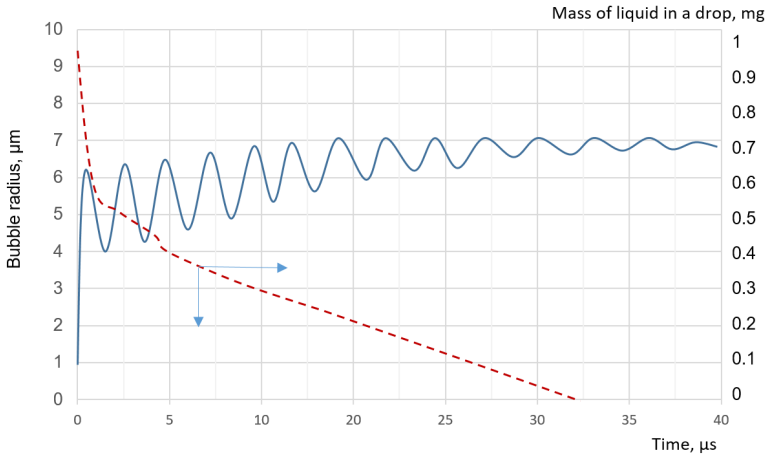

Figure 5: Change in the mass of the droplet $m = f(\tau)$ due to evaporation into the bubble and variation in the size of the vapour bubble $R = f(\tau)$ during the explosive boiling of the droplet: solid line – $m = f(\tau)$; dashed line – $R = f(\tau)$.

The idea of a monotonic distribution of liquid temperature within the thermal layer, adopted in existing models, is quite justified for describing the process of bubble growth, but for oscillating bubbles such a simplification can introduce a significant error, especially when estimating peak temperatures and pressures at the stage of maximum compression of the bubble. Since the period of bubble oscillation is significantly less than the characteristic relaxation time of the temperature field in the liquid, the liquid layers adjacent to the surface do not have time to heat up and cool so quickly and the temperature distribution in the vicinity of the oscillating bubble cannot remain monotonic. Figure5 shows the change in the radius of the bubble; similar to these changes, the pressure and temperature of the steam also change. The temperature distribution in the vicinity of a spherical particle also changes at different times, after the particle, intensely heating up (due to internal sources) and giving off heat to adjacent layers of liquid, then begins to cool (due to internal heat sinks), while taking away heat from adjacent layers. This temperature distribution also occurs in the vicinity of a pulsating bubble. Taking into account the decisive influence of heat and mass transfer processes on the dynamics of the bubble, the need to correctly take into account the temperature field in the vicinity of the bubble is obvious. If in early works on hydrodynamic and acoustic cavitation the assumption of the adiabatic nature of bubble collapse was used as the main simplifying assumption, then subsequent studies showed that a reliable description of phenomena in cavitation processes is impossible without accurately taking into account interphase heat and mass transfer. Thus, taking into account heat transfer in oscillating and collapsing bubbles leads to a decrease in the thermodynamic parameters of the vapor in the bubble by almost an order of magnitude compared to adiabatic collapse.

An analysis of works on bubble dynamics shows that when modeling the behavior of bubbles, regardless of the applied orientation of the model, the most important thing is an adequate representation of interfacial heat and mass transfer, the thermodynamic state of the vapor phase, as well as the temperature field in the vicinity of the bubble.

In our model, it is assumed that the temperature of the surface of the bubble on the liquid side is different from the temperature of the gas phase, which is equivalent to the assumption that there is a temperature jump on the wall of the bubble on the gas phase side. Due to the temperature difference, vapor and gas molecules, when in contact with the surface of a liquid, completely exchange energy with it, regardless of whether the molecule condenses on the surface or is reflected back. In other words,

the coefficient of thermal accommodation is taken equal to unity, which corresponds to the real mechanism of heat transfer.

With a nonequilibrium nature of phase transitions, the temperature at the interface always undergoes a jump, the magnitude of which determines the intensity of the mass flow from the bubble wall into the gas phase during evaporation or condensation of the vapor.

Analytical solutions to the problem of bubble growth or collapse in the cited works were obtained under the assumption that the bubble size changes monotonically. This assumption is acceptable when analyzing boiling processes, but is not justified when studying bubble oscillation processes. To assess the heat exchange between the liquid and the bubble, it is necessary to know the temperature of the liquid directly at the interface. This temperature is decisive when calculating the rate of mass and heat transfer according to the equations of our model and is included in these equations as an unknown parameter. Therefore, our model is supplemented with an independent equation to determine this temperature.

Table 1 shows the errors in the calculated values of the dynamic characteristics of the bubble with the experimental data proposed in the studies of Shepherd *et al.* and Lesin *et al.* [51, 52].

Table 1: Forecasting errors.

References	Change in bubble radius over time, %	Velocity of movement of the bubble boundary, %	Duration of the evaporation proces, %	Change in mass over time, %
[51]	17	30	–	–
[52]	8	5	8	12

The calculated data are quite difficult to verify if we estimate the velocity of movement of the phase boundary. Previous studies, such as [18, 51–54], have presented high-speed video data of vapor bubble growth that qualitatively align with our calculations. Therefore, we compared our calculated data with the data given in [28, 55–57]. The results of the comparative analysis showed good agreement between our calculated data and the data given in the cited works.

The mathematical model we propose takes into account the temperature jump at the interface between vapor and liquid in a bubble, the oscillation of thermodynamic parameters in the vapor and in the liquid phase, and therefore more accurately describes the transient processes of bubble growth

or collapse. At this stage, we do not take into account the dynamic effects that neighboring bubbles or clusters create. It is obvious that they affect the values of thermodynamic parameters and the processes of heat and mass transfer in bubbles. But the developed model will be the basis for further modeling and is a very convenient tool for performing such an analysis. Thus, we assume that, based on the developed model, we will be able to improve the methodology for predicting the thermodynamic situation not only in single bubbles, but also in clusters.

4 Conclusions

Currently, there are no mathematical models capable of predicting with equally high accuracy the behavior of a bubble in boiling processes over a wide range of changes in operating parameters. Well-known models are developed based on the assumption of the quasi-equilibrium nature of the heat and mass transfer process and the monotonic distribution of temperature in the vapor phase and in the liquid. This article shows that currently existing methods for studying the dynamics of bubbles are not able to provide information necessary from a practical point of view regarding the dynamic and kinematic effects accompanying the phenomena of explosive boiling of highly superheated liquids due to the above assumptions.

The presence of a temperature gradient at the phase contact boundary corresponds to real processes of heat and mass transfer between vapor and liquid, and makes it possible to take into account the effects of energy accumulation in the phases and take into account the influence of these effects on the intensity of heat and mass transfer. On the other hand, taking into account the accumulation effect explains the reason for the discrepancies in the data on the duration of water evaporation, which in calculations may be less or more than measured experimentally. The intensification of heat and mass transfer processes depends on the degree of fluid superheating, and in such problems the data discrepancy can reach 180% (for the available analyzed works).The proposed mathematical model of the dynamics of a single bubble describes, with a minimum number of limiting assumptions, the behaviour of bubbles over the entire temperature range of the existence of the liquid phase. The model does not contain empirical coefficients or any fitting parameters and includes into consideration all physical factors that control the behaviour of the bubble, without limiting the degree of influence of a particular factor at various stages of the process.

An example of using the model in relation to the analysis of well-known experiments in the study of the dynamics of the growth of steam and vapour-gas bubbles in superheated liquids showed the ability of this model to predict the quantitative results of experiments with higher accuracy than currently used mathematical models.

Using the model to analyze experimental data allows us to obtain a large amount of additional information and qualitatively new data regarding the laws of boiling processes. The approach we propose can be used in the analysis of various phenomena and effects associated with the behavior of bubbles that are observed in experiments and do not find a clear physical explanation.

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