ARCHIVES OF METALLURGY

Volume 47 2002 Issue 2

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PLASTIC DEFORMATION OF LAYERED COMPOSITE MATERIAL WITH DIFFERENT FEATURES OF COMPONENTS UNDER COMPRESSION TEST

PLASTYCZNE ODKSZTAŁCENIE WARSTWOWYCH MATERIAŁÓW KOMPOZYTOWYCH O RÓŻNYCH CECHACH SKŁADNIKÓW W WARUNKACH TESTU ŚCISKANIA

Deformation of a composite material being compressed between two parallel plates is analyzed from the point of view of changing the relative thickness of particular layers during the process. The three layer composite consists of two different materials: one of them is assumed to be viscoplastic while the other one is perfectly plastic. Variation of the relative thickness of the different layers is investigated with respect to values of material constants, geometrical arrangements of the materials as well as the loading history.

W pracy przedstawiono analizę plastycznego odkształcenia mateiału kompozytowego w warunkach ściskania między dwoma równoległymi sztywnymi płytami z punktu widzania zmian relatywnej grubości poszczególnych warstw podczas procesu odkształlcenia. Rozważano kompozyt składający się z trzech symetrycznie położonych warstw z dwóch różnych materiałów: jeden z nich wykazuje własności lepko plastyczne, a drugi jest idealnie plastyczny. Zróżnicowanie względnych grubości różnych warstw przebadano z uwzględnieniem wartości stałych materiałowych, geometryczne aranżacji odkształcanych materiałów oraz historii i rodzaju obciążenia. Wykazano, że w zależności od wszystkich w/w czynników charakter warunków zachodzących na powierzchni kontaktu różnych materiałów może się zmieniać w trakcie odkształcenia od połączenia idealnego materiałów do poślizgu między nimi. Udowodniono, że zmiana relatywnej grubości poszczególnych warstw jest możliwa wyłącznie w przypadku wystąpienia poślizgu pomiędzy warstwami.

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1. Introduction

Compressing a strip between rough long parallel plates is one of the more famous tests in experimental methods. Such a test enables analysis of plastic deformability of the material. Theoretically such a problem was first formulated by Prandtl [1] for a rigid/perfectly plastic material. His solution was completed by Nadai (see for example [2 p. 234]. Kuznetsov in his paper [3] has investigated the compression of a non-homogeneous isotropic strip. The effect of the inertia terms has been studied by Najar [4]. Adams *et.al* [5] have extended the original problem for viscoplastic materials. The progressive compression of a strip was first considered by Collins and Meguid [6] for both isotropic and anisotropic strain-hardening materials. Other aspects of the compression of a three-layer strip have been investigated in [7–9].

A further generalization of Prandtl's problem the compression of a three-layer symmetric strip consisting of two different materials has been done in the paper [10]. One of the materials is assumed to be rigid/perfectly plastic and the other is rigid/viscoplastic. Qualitatively different solutions have been obtained depending on the combination of the layers: (a) the viscoplastic layer is between two rigid/perfectly plastic layers and (b) the rigid /perfectly plastic layer is between two viscoplastic layers. Instantaneous solutions of the problem have been found and discussed. However, it is very important not only to have information about the velocity, strain-rate and stress distributions within the composite, but also to have a control for variation of the thickness of the composite as well as the relative thickness of the different layers during the deformation. Moreover, it has been shown in paper [10] that various boundary conditions along the bi-material interface (sticking or sliding) can occur during the compression, depending on relationships between all mechanical, geometrical parameters of the problem at each considered moment of time. The object of the present study is to analyze the process of deformation itself depending on all the material parameters as well as on the deformation path.

2. Problem formulation and the preliminary analysis

Below we present the indispensable information from paper [10] necessary for further analysis. Let us consider a model issue of compressing three layers consisting of different materials situated symmetrically towards each other (See Fig.1). The plastic material is characterized by the appropriate maximal shear stress k whereas the viscoplastic material is described in accordance with the law proposed by Adams *et al.* [5]:

$$k^{(l)} = k_1\left[1+\left(\frac{\dot{\varepsilon}_i}{\dot{\varepsilon}_0}\right)^n\right], \tag{1}$$

where k_1, $\dot{\varepsilon}_0$ and n are certain material constants, while $\dot{\varepsilon}_i$ is the effective strain-rate calculated at the particular points in the area of the viscoplastic material [10] .

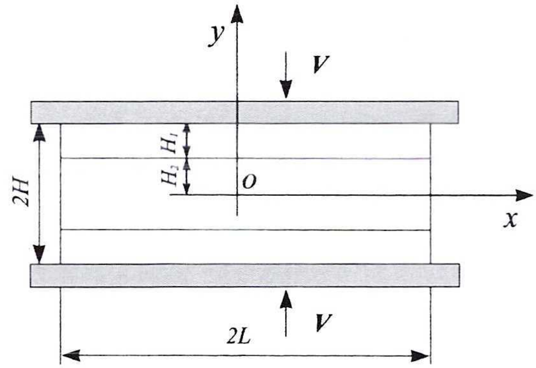


Fig. 1. The general scheme of the three layer composite under compression

At the outside boundary, the friction is defined by the Tresca's law with the friction factor m $(0 \leq m \leq 1)$. The thickness of the three-layer material is $2H$, the thickness of the internal layer is $2H_2$, and the relative thickness of this layer with respect to the overall thickness is denoted as $\tilde{H}_2 = H_2/H (0 < \tilde{H}_2 < 1)$. It is known that the upper and the lower boundaries of the composite are approaching each other at the velocity V. Note that the parameters H, V and $\tilde{H}$ generally speaking depend on time. Assuming that $H = H(t)$ (and what follows $V = V(t) = -H'(t)$) is known (depends on the loading applied), unknown relative thickness $\tilde{H}_2 = \tilde{H}_2(t)$ can be found from the solution to the problem. The following notation will also be used:

$$\dot{\varepsilon}_c(t) = -\frac{V(t)}{H(t)} = \frac{H'(t)}{H(t)} \tag{2}$$

which characterizes the overall strain-rate of the whole composite. In the conditions of the loading (unloading does not occur during the whole plastic deformation) this value is always non-positive.

In paper [10] it was shown that, in the case when the viscoplastic material is situated between the layers of the plastic material, there always exists a solution of the modeling problem corresponding to the perfect bonding between the materials in the three-layer composite. In such a case the distribution of the velocities in the direction of OY axis (Fig. 1) is as follows:

$$v_y(y) = -\frac{V}{H} y = \dot{\varepsilon}_c y \tag{3}$$

independent of whether the particle is in the plastic or viscoplastic material. Therefore the relative thickness of this kind of composite materials during deformation does not change.

However, this conclusion becomes to be false, in general, when the plastic material is between the layers of the viscoplastic one. In such a case there are three possibilities:

I. The first of them occurs when

$$\frac{k_0}{mk_1} \le \tilde{H}_2, \tag{4}$$

then the viscoplastic material is in the rigid state, and only the plastic material undergoes deformation with sliding along the bi-material interface. Thus we have: $H_2'(t) = H'(t) = -V(t)$ or $H_2(t) = H(t) - H_1$, where H_1 is a constant thickness of the viscoplastic material situated in the rigid state. In a period of time when condition (4) is satisfied, the relative thickness of the internal layer changes according to the law:

$$\tilde{H}_2(t) = 1 - \frac{H(0) - H_2(0)}{H(t)}. \tag{5}$$

Let us note for further considerations that function $\tilde{H}_2(t)$ from (5) can be defined as the solution of the differential equation:

$$\tilde{H}_2'(t) = \frac{H'(t)}{H(t)}(1 - \tilde{H}_2(t)), \tag{6}$$

with the initial condition:

$$\tilde{H}_2(0) = H_2(0)/H(0). \tag{7}$$

As results, under condition (4), the relative thickness $\tilde{H}_2(t)$ of the internal perfectly plastic layer does not depend on the history of the deformation represented by the function $\dot{\varepsilon}_c(t)$.

The remaining two cases occur when condition (4) is not true:

$$\frac{k_0}{mk_1} > \tilde{H}_2, \tag{8}$$

and depend on whether an auxiliary parameter

$$\zeta = \frac{V}{H}\frac{\delta}{\sqrt{1-m^2}}\left[\frac{k_0}{mk_1\tilde{H}_2} - 1\right]^{-1/n} - 1 = -\frac{\delta\dot{\varepsilon}_c}{\sqrt{1-m^2}}\left[\frac{k_0}{mk_1\tilde{H}_2} - 1\right]^{-1/n} - 1 \tag{9}$$

is greater or smaller than zero (see [10]). Note that this parameter essentially depends on the deformation history. We have also introduced here a new constant which is actually a material constant:

$$\delta = \frac{2}{\sqrt{3\dot{\varepsilon}_0}}. \tag{10}$$

Let us not, that the formula (9) always makes sense because the term in the square brackets is positive what follows from condition (8). Apart from that it should be underlined that the parameter ζ defined in (9) varies during the deformation depending on the relative thickness of the internal layer $\tilde{H}_2 = \tilde{H}_2(t)$ and on the overall strain-rate $\dot{\varepsilon}_c = \dot{\varepsilon}_c(t)$ $(\zeta = \zeta(\dot{\varepsilon}_c, \tilde{H}_2))$.

II. Let us assume that:

$$0 \leq \zeta < \infty, \tag{11}$$

where

$$\zeta \to \infty \qquad \text{as } \tilde{H}_2 \to \frac{k_0}{mk_1}, \qquad \text{and}$$

$$\zeta = 0 \qquad \text{as } \tilde{H}_2 = h_2 \equiv \frac{k_0}{mk_1}\left[1 + \left(-\frac{\delta\dot{\varepsilon}_c}{\sqrt{1-m^2}}\right)^n\right]^{-1}. \tag{12}$$

In other words, condition (11) is equivalent to

$$h_2(\dot{\varepsilon}_c) \leq \tilde{H}_2 < \frac{k_0}{mk_1}. \tag{13}$$

Then, as follows from [10], only the instantaneous solution which assumes sliding between the layers is possible. The velocities of the external and internal (bi-material) boundaries are written as:

$$v_y^{(l)}(H) = -V, \qquad v_y^{(l)}(H_2) = v_y^{(p)}(H_2) = -\frac{V(H_2+\zeta H)}{H(1+\zeta)}, \tag{14}$$

where the parameter $\zeta = \zeta(H(t), \tilde{H}_2(t))$ was described in (8).

To derive the equation describing how the relative thickness of the internal layer changes in time, let us consider the thickness of the particular layers in time momentums t and $t+\Delta t$. Some simple transformations lead to:

$$\tilde{H}_2(t+\Delta t) = \frac{(1+\zeta)\tilde{H}_2 + (\tilde{H}_2+\zeta)\dot{\varepsilon}_c\Delta t}{(1+\zeta)(1+\dot{\varepsilon}_c\Delta t)}. \tag{15}$$

This makes it possible to calculate the derivative of the sought-for function:

$$\dot{H}_2'(t) = \frac{H'}{H}(1-\tilde{H}_2(t))\frac{\zeta}{1+\zeta}. \tag{16}$$

This is an ordinary differential equation describing together with the initial condition (7) the functions $\tilde{H}_2 = \tilde{H}_2(t)$.

III. Let us consider the last of the possible cases:

$$\zeta \leq 0. \tag{17}$$

As it was shown in paper [10], under such an assumption there exists the instantaneous solution of the problem with sticking conditions along the bi-material interface. This solution leads to formula (3) which means that relative thickness of the materials does not change during that portion of deformation where condition (17) is satisfied.

In other words in this case:

$$\tilde{H}_2(t) = \tilde{H}_2(0). \tag{18}$$

We can rewrite (18) in the form of ordinary differential equation:

$$\tilde{H}_2'(t) = \frac{H'}{H}(1 - \tilde{H}_2(t)) \cdot 0 \tag{19}$$

with the initial condition (7) and assuming that additional condition (17) is fulfilled.

In paper [10] it was also shown that in the transitional position between the assumptions (11) and (17)

$$\zeta = 0, \tag{20}$$

conditions of sticking and sliding are satisfied at the same time so that the velocity jump towards OX axis equals zero, but, at the same time, the value of the shear stress in the plastic material assumes its greatest value at the interface of the layers.

3. Basic equation

The above considerations lead us to the statement that the sought-for function $\tilde{H}_2 = \tilde{H}_2(t)$ satisfies the differential equations (6), (16) and (19) depending on which of the conditions (4), (11) or (17) is satisfied at a given moment. This can be described by means of one differential equation:

$$\tilde{H}_2'(t) = -\frac{H'}{H}(\tilde{H}_2 - 1)\cdot g(\tilde{H}_2, -H'/H) = \dot{\varepsilon}_c(\tilde{H}_2 - 1)\cdot g(\tilde{H}_2, \dot{\varepsilon}_c), \tag{21}$$

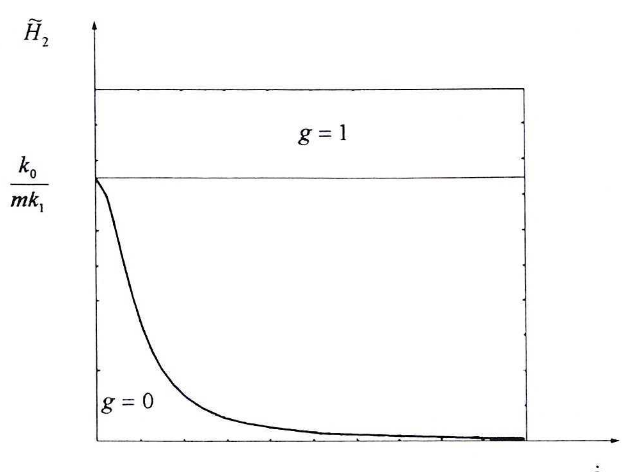


Fig. 2. Schematic distribution of the values of function g

with the initial condition (7). In this equation, g is a smooth and piece-wise differentiable function of its parameters described as follows:

$$g(\tilde{H}_2, \dot{\varepsilon}_c) = \begin{cases} 1, & \text{gdy } \tilde{H}_2 \geq k_0/(mk_1), \\ \zeta/(1+\zeta) & \text{gdy } 0 \leq \zeta(\tilde{H}_2, \dot{\varepsilon}_c) < \infty, \\ 0, & \text{gdy } \zeta(\tilde{H}_2, \dot{\varepsilon}_c) < 0. \end{cases} \tag{22}$$

Fig. 2 shows the distribution of the values of this function depending on the values of the arguments $\tilde{H}_2 = \tilde{H}_2(t)$ and $\dot{\varepsilon}_c = \dot{\varepsilon}_c(t)$.

4. Quantitative analysis of the Cauchy problem (21), (7).

Assuming that all the functions are known then an equivalent to the problem under consideration integral representation of the solution can be derived:

$$\ln(1 - \tilde{H}_2'(t)) = \int_0^t \frac{H'}{H} g(\tilde{H}_2, -H'/H)\,dt + \ln c.$$

Taking into account the initial condition (7) the formula can be written as:

$$\tilde{H}_2(t) - 1 = (\tilde{H}_2(0) - 1)\exp\left[\int_0^t \frac{H'}{H} g(\tilde{H}_2, -H'/H)\,dt\right]. \tag{23}$$

As function g is bounded ($0 \leq g \leq 1$), one can get the estimate at any time t regardless of the method of loading:

$$1 - \frac{H(0)[1 - \tilde{H}_2(0)]}{H(t)} \leq \tilde{H}_2(t) \leq \tilde{H}_2(0). \tag{24}$$

Here the upper bound corresponds to the process under perfect bonding of different materials during the whole process. The lower bound occurs if during the entire time of deformation the viscoplastic material remains in the rigid state and the only plastic material is deformed.

Let us assume that the process of deformation proceeds in such a way that the overall strain-rate takes a constant value ($a > 0$):

$$\dot{\varepsilon}_c(t) = const = -a \quad \Rightarrow \quad H(t) = \exp[-at]. \tag{25}$$

The solution of equation (23) depends on whether the initial value $\tilde{H}_2(0)$ is greater or smaller than $h_2(\dot{\varepsilon}_c)$ described in (12) and for which the parameter $\zeta = 0$. Namely if $\tilde{H}_2(0) \leq h_2(\dot{\varepsilon}_c)$ then the right hand side of the differential equation (21) equals to zero, therefore the value of the function does not change: $\tilde{H}_2(t) = \tilde{H}_2(0)$. As in this case the value of $h_2(\dot{\varepsilon}_c)$ is constant, this characteristic of deformation is retained throughout the process.

However, if $\tilde{H}_2(0) > h_2(\dot{\varepsilon}_c)$ then the variables in the differential equation (21) can be separated thus deriving:

$$\int_{\tilde{H}_2(0)}^{\tilde{H}_2(t)} \left[1 - \frac{\sqrt{1-m^2}}{a\delta} \left(\max\left\{ 0, \frac{k_0}{mk_1 \xi} - 1 \right\} \right)^{1/n} \right]^{-1} \frac{d\xi}{1-\xi} = \ln H(t) = -at. \tag{26}$$

Function $\tilde{H}_2(t)$ described in (26) is smooth and monotonically decreasing and satisfies the following inequality:

$$h_2(\dot{\varepsilon}_c) < \tilde{H}_2(t) \leq \tilde{H}(0), \tag{27}$$

with the lower bound never to be reached and the following estimate:

$$\tilde{H}_2(t) \to h_2(-a) \equiv \frac{k_0}{mk_1} \left[1 + \left(\frac{a\delta}{\sqrt{1-m^2}} \right)^n \right]^{-1}, \quad \text{when} \quad t \to \infty \ (H(t) \to 0). \tag{28}$$

Regarding $\dot{\varepsilon}_c(t)$ described in (2) as external control function which belongs to a certain functional space we can formulate the following mathematical problem:

Find such test function $\dot{\varepsilon}_c(t)$ belonging to the set S of continuous, piece-wise differentiable non-positive functions in the interval $0 \leq t \leq T$ so that the solution to the Cauchy problem (21), (7) satisfies the additional conditions: $\tilde{H}_2(T) = \tilde{H}_2^*$, $\Phi(\tilde{H}_2, \dot{\varepsilon}_c) \to \min$ where functional $\Phi(\tilde{H}_2, \dot{\varepsilon}_c)$ determines, for example, the total plastic energy performed during the process of compression.

5. Numerical simulations

In this part of the paper we will investigate the behavior of the relative thickness of the internal layer with respect to various parameters of the problems. All values of the parametes will be given in dimensionless form. Thus, everywhere later we assume that the initial thickness of the composite is equal to unit $H(0) = 1.0$. Also the time in some dimensionless parameter.

A) Effect of the strain-rate value.

Let us assume that that the process of deformation occurs at the overall constant strain-rate with different values of the parameter a (see (25)). The external friction factor in the calculations equaled $m = 0.5$, the viscoplastic factor $n = 0.5$, and the ratio of the shear stresses were $k_1/k_0 = 0.3$. The value of the parameter δ describing viscosity of the external material (see (10)) was equal to $\delta = 0.01$. The initial relative thickness of the internal plastic layer $\tilde{H}_2(0) = 0.8$ was chosen to satisfy condition (4) at the initial moment. Fig. 3a shows us given changing of the total composite thickness during the compression process from the initial unit value to the same final thickness reached at different time (what corresponds, due to (25), to the different values of the parameter a). Fig. 3b shows the corresponding variation of the relative thickness of the internal layer $\tilde{H}_2$.

According to the results obtained above, the way of loading does not matter as long as the relative thickness $\tilde{H}_2(t)$ satisfies condition (4) which in our case occurs when $\tilde{H}_2(t) \geq 0.6$. Then only the internal perfectly plastic material is being under deformation whereas the viscoplastic material remains in the rigid state. This result is visible from

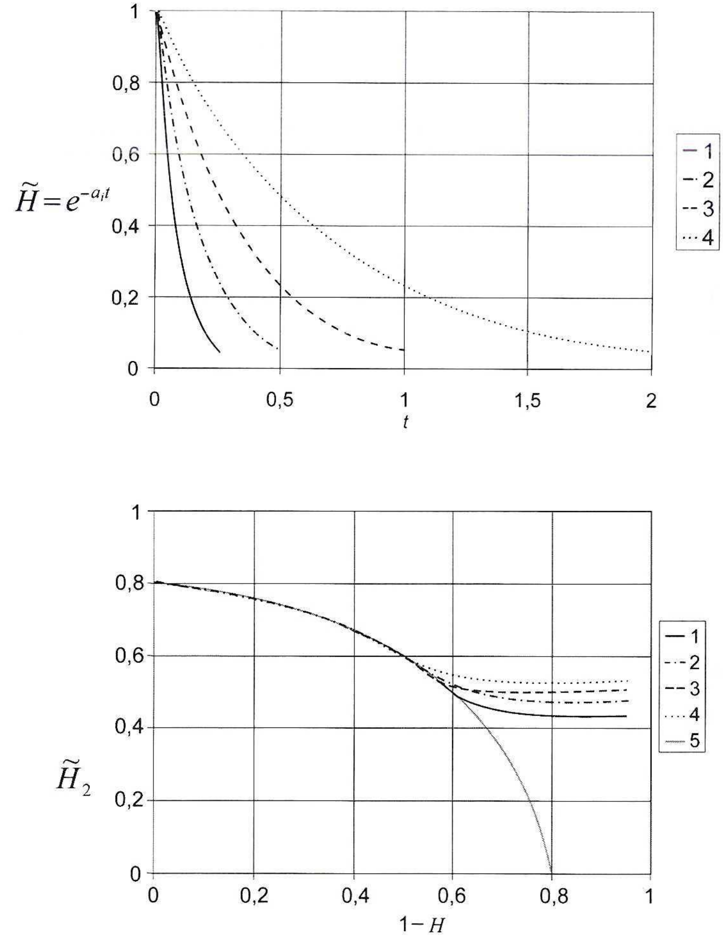


Fig. 3. (a) – given variations in overall thickness of the composite layer $H(t)$; (b) – behavior of the relative thickness of the plastic layer depending on different history of entire deformation. The lower dashed line 5 in Fig. 3b represents the lower bound estimate for $\tilde{H}_2$ obtained in (24)

Fig. 3b where all the lines coincides themselves at values $0.6 \leq \tilde{H}_2(t) \leq 0.8$. However, after exceeding these values, the constant a determining the total strain-rate $\dot{\varepsilon}_c(t)$ affects remarkably the final result (Fig. 3b.) The greater absolute value of the total strain-rate (the greater value of the parameter a) corresponds to the smaller relative thickness of the internal layer. The lower bound from the estimate (24) is drawn as the dashed line 5 which corresponds as has been mentioned above to the deformation the internal layer only while the external material is situated in the rigid state. The horizontal parts of the curves correspond to the sticking conditions between the different materials.

B) Effect of the value of the viscosity parameter δ.

Let us investigate now the influence of one of the most important viscosity parameter

$\dot{\varepsilon}_0$ from the relation (1) or what is equivalent, due to (10), to the parameter δ. Namely we consider four different values of this parameter $\delta = 10^{-1}$; 10^{-2}; 10^{-3}; 10^{-4}. As in the previous case, we still assume that the total strain-rate is a constant during the deformation (the respective curve 3 in Fig. 3a). Other values of the parameters were: the external friction factor equals $m = 0.5$ and the ratio of $k_1/k_0 = 0.3$. In Fig. 4 variation of the relative thickness of the internal layer $\tilde{H}_2$ with respect to the changing of the total thickness of the composite are presented.

At the beginning of the process, the initial relative thickness $\tilde{H}_2(0) = 0.8$ satisfies condition (4) and the loading history does not influence the relative thickness if $0.6 \leq \tilde{H}_2(t) \leq 0.8$. It is easy to note that diminishing δ to zero (which means that mechanical properties of the viscoplastic material approach those of the perfect-plastic one) eliminates the effects of viscosity as it could be expected. In the limited case $\delta = 0$ the external material is rigid first and the internal material is only deformed until its relative thickness reaches the value $\tilde{H}_2(t) = 0.6$, and then the simultaneous flow of the materials with the perfect layer bonding follows. Such kind of composite behavior has been reported in experimental [7] and theoretical [11] works for combinations of two perfectly plastic materials. On the other hand, the greater viscosity of the external

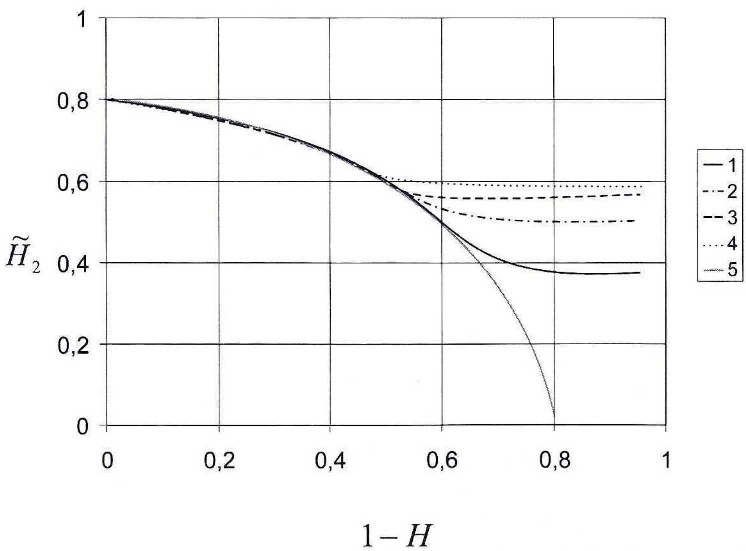


Fig. 4. Variation of the relative thickness of the plastic layer for different values of the viscosity parameter $\delta = 10^{-1}$; 10^{-2}; 10^{-3}; 10^{-4} (the curves 1, 2, 3, 4, respecively). The lower dotted line corresponds with the lower estimate for $\tilde{H}_2$ obtained from (24). Compression under given constant total strain-rate defined by the curve 3 in Fig. 3a

material leads to the smaller relative thickness of the internal layer at the end of the process with all the other parameters being equal. (For smaller values of δ the corresponding lines are higher situated.)

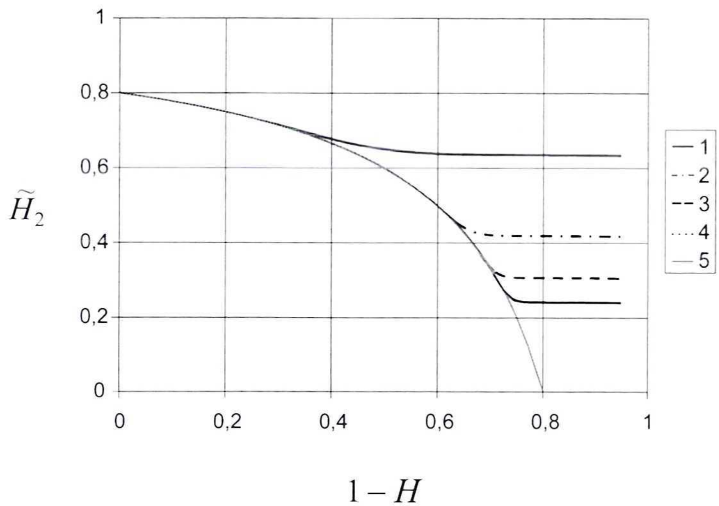


Fig. 5. Variation of the relative thickness of the plastic layer $\tilde{H}_2$ for different values of the friction parameter along the bi-material interface m = 0.4; 0.6; 0.8; 0.95 (the curves 1, 2, 3, 4, respecively). The total strain-rate is a constant value (defined by the curve 3 in Fig. 3a)

C). Effect of the friction factor m.

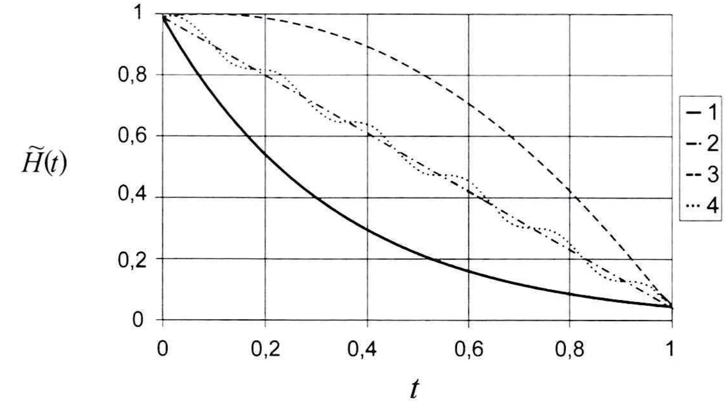
Let us still assume the same loading history as in the previous case (the curve 3 in Fig. 3a), but the friction parameter takes the values: $m = 0.4$; 0.6; 0.8; 0.95. The ratio $k_1/k_0 = 0.3$ and the viscosity parameter $\delta = 0.01$ were chosen. Fig. 5 shows corresponding results of numerical simulation of the relative thickness of the internal layer $\tilde{H}_2$ for each value of the friction factors m.

Now the relative thickness $\tilde{H}_2(t)$ satisfies condition (4) at various segments depending on the value of the friction factor m. It is easy to observe different points where the respective lines for different values of the friction factor start to depart from the lower boundary of the solution described in (24). Note that the greater value of the parameter m influences the greater segment at which the method of loading does not affect $\tilde{H}_2(t)$. The final value of $\tilde{H}_2(t)$ is the smallest for the greater value of m. (For the smaller values of m the corresponding curves are higher.)

D). Effect of the loading history.

Let us consider different ways of deformation such that the overall thickness of the composite changes to the same magnitude during the same periods of time (see Fig. 6a). Let us assume as before that the friction factor $m = 0.5$, and the ratio $k_1/k_0 = 0.3$. The value of the viscosity parameter δ is equal to $\delta = 0.01$. The initial thickness of the internal layer is $\tilde{H}_2(0) = 0.8$ and satisfies condition (4). Fig. 6b shows us the respective variation of the relative thickness of the internal layer with respect to the total thickness of the composite.

As in Figs. 3b and 4 the loading history does not influence the relative thickness $\tilde{H}_2$ within the interval $0.6 \le \tilde{H}_2(t) \le 0.8$. It is easy to conclude from Fig. 6b that this occurs



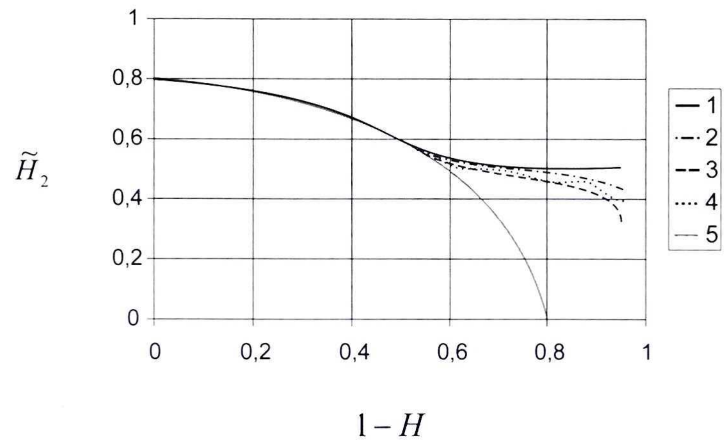


Fig. 6. (a) – various external loading history of deformation of the composite; (b) – the corresponding variation of the relative thickness of the internal plastic layer H_2

when $0.5 \leq H(t) \leq 1.0$. Note that, if the velocity of the external platen V is nonnegative during the entire process (V is equal to zero in particular points) there does not exist any unloading (see the curve 4 in Fig. 6a). In this special case character of the response of the relative thickness $\tilde{H}_2$ becomes to be more complicated than it was in all other cases with the strictly positive value of velocity. Namely, the regime of deformation in this case changes several times within the interval $\tilde{H}_2(t) < 0.6$ (sticking regime changes to the sliding regime then to the sticking one and vice versa – see the curve 4 in Fig. 6b). As it was shown above, such behavior is impossible for the loading under a constant total strain-rate.

To examine better this phenomenon we have considered various paths of deformation according to following relation defining a function oscillating around a straight line:

$$H(t) = 1 - \frac{1 - H(T)}{T}\left[t \pm \frac{T}{\pi l}\sin\frac{\pi l}{T}t\right]. \tag{29}$$

In Fig. 7a the curves for the different values of $l = 3$ and 6 as well as the different signs in equation (29) are presented. Here T is a moment of time at the end of deformation which is the common for all loading history due to the assumptions. Essential differences in the final relative thickness of the internal layer $\tilde{H}_2$ as well as in the variation of the quantity itself during the deformation indicate us that it is always necessary to accurately check the method of loading during experimental tests. This enables us to take into account the influence of the path of deformation on the final product.

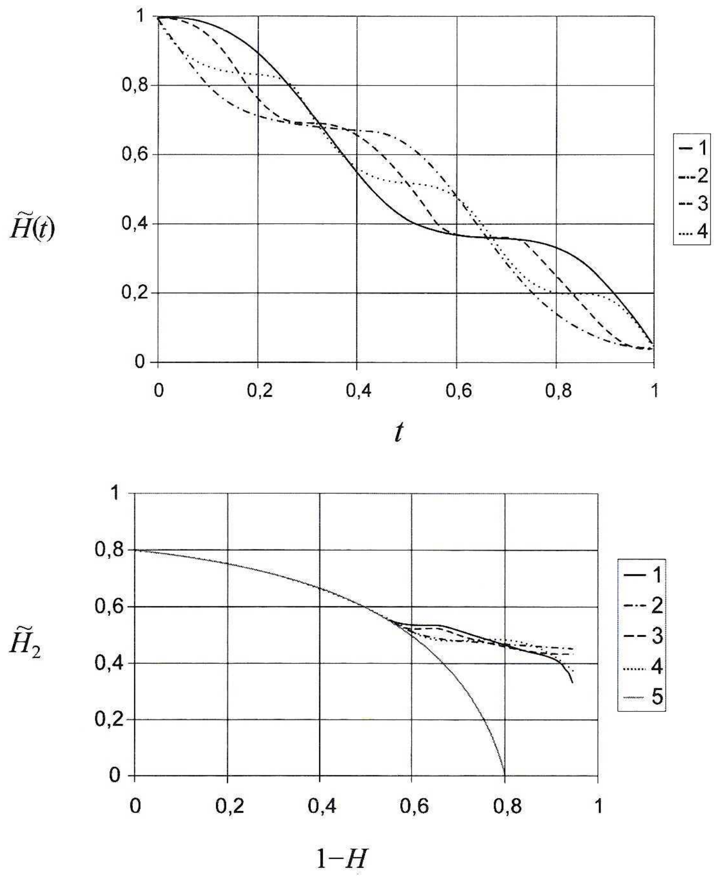
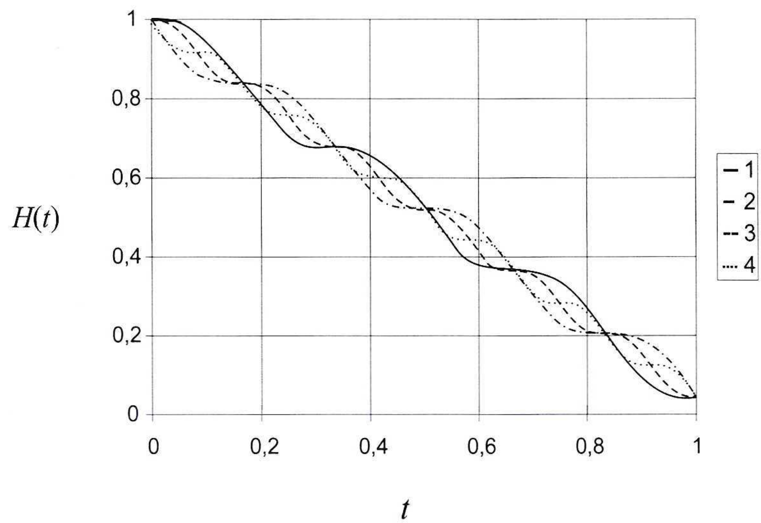


Fig. 7. a) Various deformation paths of the composite layer;
b) the respective response of the relative thickness of the plastic layer $\tilde{H}_2$

It would be interesting to learn whether there exists a convergence of results when the number of vacillations remarkably increases whereas the amplitude of vacillations decreases. Mathematical description (29) enables us to carry out such an analysis by means of increasing the number n in the expression. The fact that in such case the amplitude of oscillations is decreasing can be checked directly. In Fig. 8a the variation of the total thickness of the composite for the different values of l = 6 and 12 and for the different signs in equation (29) are presented.



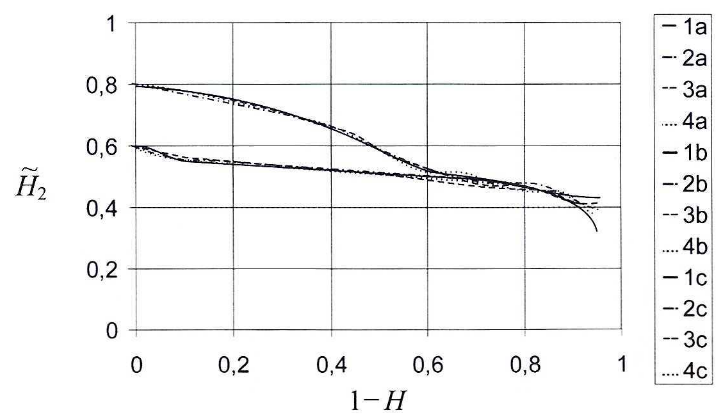


Fig. 8. (a) Various deformation paths of the composite layer; (b) the corresponding variation of the relative thickness of the plastic layer $\tilde{H}_2$ for different initial values

To investigate also the influence of the initial relative thickness of the internal layer let us consider three different values of $\tilde{H}_2(0) = 0.8,\ 0.6,\ 0.4$ (which means that the simultaneous deformation of the two materials would start at different stages of deformation). Namely, for the initial thickness $\tilde{H}_2(0) = 0.8$ the external material is in a rigid state at the beginning of the deformation, for $\tilde{H}_2(0) = 0.6$ both the materials become under deformation from the very beginning, but the sliding occurs along the bi-material interface and finally, if $\tilde{H}_2(0) = 0.4$ both the materials exhibit the plastic flow with the sticking conditions along the interface from the beginning.

As it follows from the Fig. 8b there is a good convergence to the limit type loading (straight line at this assumptions), but only when the global thickness of the composite does not become to be sufficiently small in comparison with the initial one. At this last stage of the deformation process not the initial thickness but mostly the loading history plays an important role.

6. Conclusions

The new solution of model problem concerning compression of three layers of two different materials has been proposed. Numerical simulations showing behaviour of all output process parameters have been presented with respect to various relationships of all input parameters.

Basing on comparison of the numerical results with experiment ones (e.g. [7,12]), the following conclusions can be drawn:

– geometrical parameters of the deformed materials and their arrangement (thickness ratio)

– mechanical features of the materials (plastic/viscoplastic behaviour with various values of the material constants)

– friction effect at the material/tool and material/material interfaces

have been taken into account in the proposed model in an adequate way, what allows to predict the main peculiarities of the process.

Thus, the relative thickness of the simultaneously deformed layers consisted of different materials changes during the compression process only in the case when sliding boundary conditions occurs along the bi-material interface. In the case of sticking boundary conditions, the proportional flow is only possible. However, what kind of friction regime (the type of the interface boundary conditions) is appropriated at each time moment depends in a complicated way on the instantaneous geometrical parameters as well as mechanical and friction parameters).

Among other, this makes us possible to extract information about the type of the interfacial boundary conditions analysing simple measured values such as relative thickness of the materials during the real process.

Acknowledgements

This work was carried out under Grant 7 T 08B 02818.

Paper presented at the 2nd Symposium "Integrated Study on Basics of Plasic Deformaiton of Metals", which was held in Łańcut on April 24–27, 2001.

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REVIEWED BY: ZDZISŁAW JASIEŃSKI

Received: 10 January 2002.