Archives of Mining Sciences **47**, 1 (2002) 23–34

JAN SZLĄZAK*

NUMERICAL DETERMINATION OF VELOCITY FIELD OF AIRFLOW IN GOB

NUMERYCZNE WYZNACZENIE POLA PRĘDKOŚCI PRZEPŁYWU POWIETRZA W ZROBACH

In most Polish coal mines the extraction process is conducted by means of caving and a characteristic feature of excavated seams is a high risk of fire. The intensity of ventilation in the gob of a longwall system with caving is the main influence on the level of risk of such a fire hazard. The reasons for such spontaneous fires are: firstly of all, coal losses in the gob or coal from external sources — secondly the quantity of air supplied by the main fans and permeating that gob.

This article presents a mathematical model for the airflow in the caving zone in the context of a changeable co-efficient of gob permeability. A numerical method for solving equations describing airflow, which enables the creation of a computer programme for the determination of velocity and pressure in the caving zone, is also given. Such programmes have been designed for the most typical ventilation systems. The calculations of the distribution of air velocity and pressure may be of some help when selecting preventive measures to reduce fire risks.

Key words: ventilation, gob, spontaneous fires, airflow

W większości kopalń polskich eksploatacja prowadzona jest na zawał, a eksploatowane pokłady charakteryzują się dużym zagrożeniem pożarowym. Intensywność przewietrzania zrobów ścian zawałowych stanowi główny czynnik decydujący o wielkości zagrożenia pożarowego w zrobach. Przyczyną tych pożarów są przede wszystkim straty węgla w zrobach lub dostawanie się do nich węgla z pokładów pozabilansowych i przenikanie powietrza przez te zroby pod wpływem oddziaływania wentylatorów głównych.

W artykule przedstawiono model matematyczny przepływu powietrza przez zroby ścian zawałowych, przy uwzględnieniu zmiennego współczynnika przepuszczalności zrobów. Podano również metodę numeryczną rozwiązania równań opisujących przepływ powietrza, która umożliwiła opracowanie programu komputerowego dla określenia rozkładu prędkości i ciśnienia w strefie zawału. Programy takie zostały opracowane dla najbardziej typowych systemów przewietrzania. Obliczenia rozkładu prędkości i ciśnienia powietrza w zrobach mogą być pomocne przy doborze profilaktyki zwalczania zagrożenia pożarowego.

Słowa kluczowe: wentylacja, zroby ścian, pożary endogeniczne, przepływ powietrza

* WYDZIAŁ GÓRNICZY, AKADEMIA GÓRNICZO-HUTNICZA, 30-059 KRAKÓW, AL. MICKIEWICZA 30

1. Introduction

As higher coal seams become mined out, exploatation must begin at lower levels. As the depth of mining increases the natural hazards also become more serious. Associated risks (fire, rock-burst, higher temperatures and methane) are present in to a more severe extent in seams located at greater depths. In most Polish mines the extraction process is conducted by means of caving and a characteristic feature of excavated seams is a high risk of fire. The intensity of ventilation in the gob of a longwall system with caving is the main influence on the level of risk of such a fire hazard. With a low ventilation intensity the gob remains filled with non-reactive gases, preventing the spontaneous-combustion process of coal. However, a high ventilation intensity results in the removal of heat, which is the product of the oxidation processes occurring in the gob. In this way it is a preventive measure against the process of spontaneous ignition of the coal. Medium--intensity ventilation is the most problematic situation, when despite an adequate delivery of oxygen, there is not enough air-flow to provide effective cooling of the gob. Heat accumulation in the gob accelerates the process of self-combustion.

On the basis of statistical analysis it may be concluded that fires in the active gob of a longwall system, especially in seams where methane is present, are a high risk. The reasons for such spontaneous fires are: firstly, coal losses in the gob or coal from external sources getting into the gob and secondly, the quantity of air supplied by the main fans and permeating that gob.

2. Airflow in gob

As a result of strata dislocation a rubble heap and fracture zones are formed behind a longwall face. The caving area of a longwall is filled with roof rock mass forming a block-structure of different grain-size. The initial very loose packing of the grains in the face zone becomes tighter and tighter, as the distance from the longwall face increases (Litwiniszyn 1949; Szlązak, Szlązak 1987a). Little rock grains fill the gob interstices between bigger ones and the non-uniformly supported larger fragments crumble under the imposes stress of the rock mass, filling the gob tighter and tighter. Therefore for theoretical considerations concerning the airflow in the gob it may be assumed that the gob constitutes a real porous medium where a linear Darcy's law of filtration is valid.

An essential factor influencing the airflow in a porous medium is the type of gas flow and the way in which this airflow takes place. Therefore a vector of filtration velocity can be determined by means of the following formula:

$$v = -K \operatorname{grad} p \tag{1}$$

where:

$K = \dfrac{k(x)}{\mu}$ — filtration co-efficient of a porous medium,

v — filtration velocity,
p — pressure,
μ — co-efficient of absolute viscosity of air,
$k(x)$ — co-efficient of permeability.

In order to consider the gas flow in a porous medium the following should be known:

- an equation of continuity that can be expressed in the following form:

$$m\frac{d\rho}{dt}+\operatorname{div}\rho v=0 \tag{2}$$

- an equation of state in the form of isotherm:

$$\frac{p}{\rho}=\text{const}=\frac{p_o}{\rho_o} \tag{3}$$

where:

m — medium porosity,
ρ — air density.

Assuming steady air-flow, isothermal process and a flat area without any source of flow results in an elliptic partial differential equation of the second order over pressure area in the following form:

$$\frac{\partial}{\partial x}\left(K\frac{\partial p}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial p}{\partial y}\right)=0 \tag{4}$$

Most theoretical considerations, in mining literature, of the air-flow in gob have been based on this formula.

Co-efficient K in dependence (4) characterises a medium where airflow takes place. If the gas flow in a porous medium is taken into consideration, co-efficient K is a filtration co-efficient. In the case of an isotropic medium co-efficient K is a scalar function dependent on spatial co-ordinates. However in an anisotropic media this value characterises a medium but is dependent not only on spatial co-ordinates, but also on the direction considered; therefore it is a tensor of rank II.

On the basis of the considerations in work (Szlązak 2000) it can be inferred a co-efficient of gob permeability can be described by a dependence taking the following form:

$$k(x)=\frac{\mu}{r_0+ax^2}\quad [\text{m}^2] \tag{5}$$

for $0 \le x \le \frac{2}{3}\cdot l$

where:

l — signifies total length of face [m],

$r_0,\ a$ — empirical co-efficients dependent on mining and geological conditions of caving,

$k_0 = \frac{\mu}{r_0}$ — co-efficient of caving permeability behind the longwall face [m^2],

x — distance from the longwall face [m].

However for value $x = \frac{2}{3} l$ the lowest value of the co-efficient of gob permeability is obtained. After reaching the minimum the co-efficient of permeability increases until the longwall is reached. Along the length where the co-efficient of gob permeability increases its value can be determined from the following dependence:

$$k(x) = \frac{\mu}{r_0 + a\left(\frac{4}{3} l - x\right)^2} \quad [\mathrm{m}^2] \tag{6}$$

for $\frac{2}{3} l \le x \le l$.

However the values of co-efficients r_0, k_0, a and b are dependent on the bed separation resistance of the roof rock mass located over the seam under consideration The change in the co-efficient of caving permeability behind the longwall face was approximated by the following dependence:

$$k_0 = \frac{\mu}{6} 10^{-10} R_{rrs}^{1,74} \quad [\mathrm{m}^2] \tag{7}$$

and the change in co-efficient a from dependence:

$$a = 6 \cdot 10^9 R_{rrs}^{-1,74} \quad [\mathrm{Ns/m}^4] \tag{8}$$

where:

R_{rrs} — bed separation resistance of roof.

3. Solution to equations describing airflow in gob

The movement of air through the porous structure of gob is described by an elliptical partial differential equation in form (4), which is obtained by combining an equation of continuity and a mathematical record of Darcy's law.

Boundary conditions for equation (4) are usually defined as distributions of pressure in excavations adjoining the area under consideration. Therefore these are the conditions of the first kind. Solving equation (4) in a numerical way results in obtaining an area of pressure in the gob area. Further, on local values, the co-ordinates of the vector of linear air velocity in the area filtered can be calculated on the basis of Darcy's law:

$$u = K(x)\frac{\partial p}{\partial x}, \qquad v = K(x)\frac{\partial p}{\partial y} \tag{9}$$

where:

$v\{u.v\}$ — vector of velocity mentioned above.

If the permeability of medium k is not a function of pressure but only of spatial variables x and y, there is a possibility of obtaining an analytical solution to equation (4) with boundary conditions of the first kind. Experiences of calculations show that an analytical form of the solution expressed by a weakly convergent Fourier series is not particularly convenient to use, as it has does not have the character of a mathematical generalisation; therefore cannot be used for boundary conditions determined experimentally in mining conditions. Therefore the determination of an area of pressure in the gob on the basis of numerical solutions is justified.

The calculation algorithm suggested was obtained on the basis of the method of finite differences that can be physically interpreted in a clear graphical form (Fig. 1). The gob

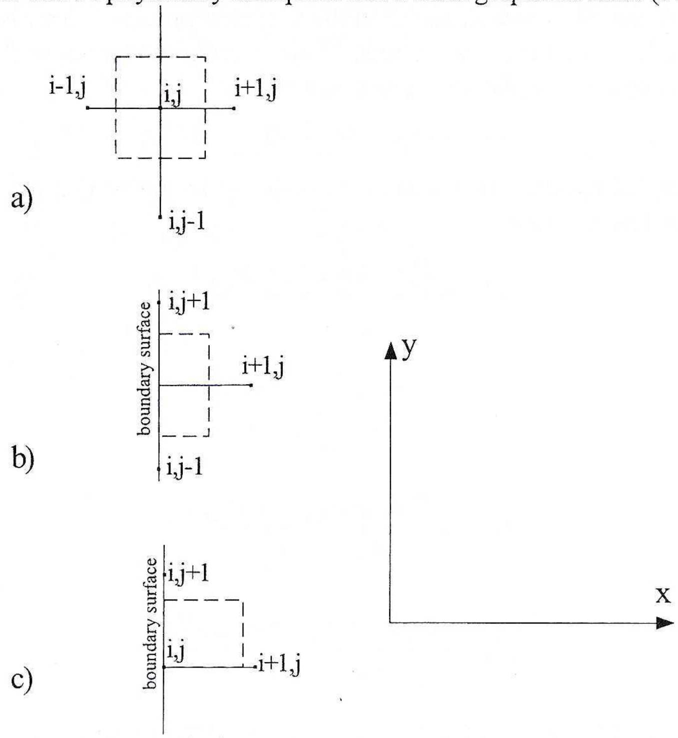

Fig. 1. Graphic interpretation of discrete balance equations
a — in the case of inside nodes; b — in the case of boundary nodes; c — in the case of corner boundary nodes

Rys. 1. Graficzna interpretacja dyskretnych równań bilansowych
a — w przypadku węzłów wewnętrznych; b — w przypadku węzłów brzegowych; c — w przypadku węzłów brzegowych narożnych

area under consideration is covered with a net of calculation nodes, that is points, where discrete values of an area of pressure $p(x, y)$ are calculated. In a general case of a heterogeneous net the distances between adjoining nodes (the so-called steps of numerical integration) can be different. The simplest form of algolithm that is charakterised by determined symetry of matrix of a system of equations is obtained in the case of homogenous calculation nets when discrete spatial variables are expressed by the following formulae:

$$x_i = (i-1)\Delta x \qquad \text{and} \qquad y_i = (j-1)\,\Delta y \qquad (i, j = 1, 2, 3, \ldots) \tag{10}$$

Fig. 1a presents the so-caled "differential star", which is equivalent to a discrete equation referring to control volume, determined by rectangle marked with a broken line. The sides of the rectangle, with *de facto* are areas of mass exchange, are drawn perpendicular to steps of integration in points determining the centres of these steps. The balance of air streams in the two-dimensional case considered (the length of the third edge of the rectangular prism is equal to unity) is recorded for this segment of space only. Because of the stationary character of the filtration process (no air is accumulated inside the control volume) the total volume of air flowing in and out of the rectangular prism is zero. Therefore the following is obtained:

$$Q_{x1} - Q_{x2} + Q_{y1} - Q_{y2} = 0 \tag{11}$$

where particular volumetric streams are expressed by formulae that are a differential interpretation of Darcy's law:

$$Q_{x1} = \frac{F_x k_{x1}(p_{i-1,j} - p_{i,j})}{(x_i - x_{i-1})} \tag{12}$$

$$Q_{x2} = \frac{F_x k_{x2}(p_{i,j} - p_{i-1,j})}{(x_{i+1} - x_i)} \tag{13}$$

$$Q_{y1} = \frac{F_y k_{y1}(p_{i,j-1} - p_{i,j})}{(y_{i-1} - y_j)} \tag{14}$$

$$Q_{y2} = \frac{F_y k_{y2}(p_{i,j} - p_{i,j+1})}{(y_{j+1} - y_j)} \tag{15}$$

The values of filtration co-efficients K are determined at mid-points between the intervals of integration and the differences in indices are the result of the heterogeneous character of this value inside the gob. The values of an area of mass exchange are respectively as follows:

$$F_x = \frac{(x_{i+1} - x_{i-1})a}{2} \qquad \text{and} \qquad F_y = \frac{(y_{j+1} - y_{j-1})a}{2} \tag{16}$$

where:

a — signifies airflow along the length of 1 m (a = 1 m).

By inserting definitions of streams (12—15) into the balance equation (11), after transformations, the final form of a single balance equation, which is equivalent to central node i, j.

$$A_{x1} p_{i-1,j} + A_{x2} p_{i+1,j} + A_{y1} p_{i,j+1} + A_{y2} p_{i,j+1} - Bp_{i,j} = 0 \tag{17}$$

where co-efficients A are the following functions of discrete variables i, j

$$A_{x1} = k_{x1} F_x (x_i - x_{i-1}) \tag{18}$$

$$A_{x2} = k_{x2} F_x (x_{i+1} - x_i) \tag{19}$$

$$A_{y1} = k_{y1} F_y (y_j - y_{j-1}) \tag{20}$$

$$A_{y2} = k_{y2} F_y (y_{j+1} - y_j) \tag{21}$$

and

$$B = A_{x1} + A_{x2} + A_{y1} + A_{y2} \tag{22}$$

Inserting boundary conditions is a separate problem. Taking numerical steps is particularly simple in the case of the conditions of the first kind. In the case of nodes of a net adjoining the boundary on which the values of pressure are defined (according to the definition of the condition mentioned) these values are inserted directly to a discrete equation also reducing the number of unknowns in this equation. In the case of the boundary condition, an equation of the second order in the general form emerges

$$\frac{\partial p}{\partial n} = 0 \tag{23}$$

where:

n — normal to the sides of an excavation , numerical values of pressures in boundary nodes are calculated.

Figures, respectively (1b) and (1c), present the balance of boundary and boundary corner nodes. As can be seen in the numerical stars presented, both the control values and areas of mass exchange perpendicular to the boundary are smaller than in the case of interior nodes. For node i, j presented in figure 1b, an equation of balance of air streams takes the following form:

$$-Q_{x2} + Q_{y1} - Q_{y2} = 0 \tag{24}$$

However in the case of the corner (figure 1c) only the following can be presented:

$$-Q_{x2} - Q_{y2} = 0 \tag{25}$$

Areas of mass exchange are equal to:

- in the case (1a) $F_x = \frac{(y_{j+1} - y_{j-1})a}{2}$, $F_y = \frac{x_2 a}{2}$,
- in the case (1b) $F_x = \frac{y_2 a}{2}$, $F_y = \frac{y_1 a}{2}$,

where:

a — signifies the flow along the length of 1 m (a = 1 m).

Air streams present in equations (24, 25) are defined on the basis of formulae (12—15), consequently obtaining equations in the form of (17), where some of the co-efficients A are equal to zero. This is because there is no mass flow in the areas marked with a solid line adjoining the boundary.

It can be proved that discrete equations identical to (17) are obtained as a result of replacing derivatives in a filtration equation by appropriate differential quotients; therefore it is a result of using a classic method of finite differences. It is proved (Beckenbach 1956; Rosenberg 1969) that the schemes used are convergent and stable irrespectively of how condensed a numerical net is. This means that the numerical values of pressures in particular nodes of net approach precise values (coming from an analytical solution) if the incremental steps of integration approach zero, that is the compression of a net is higher. However the conclusion presented above is completely theoretical. In practice, as computer calculations are not accurate enough (the so-called "cut-off error") and speed of convergence of the iterative process is slowing down, an excessive increase in the number of nodes can even lead to a fall in the effective accuracy of a numerical solution. If a net gets more compressed there is also a considerable increase in the computer calculation-time.

4. Solving a set of differential equations

As a result of presenting algebraic equations in form (17) for all the nodes of a numerical net, a set of equations, which in the case of a linear differential equation also has a linear character, is obtained. If boundary conditions of the first kind are defined on the boundary of the domain, equations (17) are only presented for its interior nodes, as on the boundaries the values of the function are defined *a priori*.

A set of differential equations (17) can be solved by means of precise methods (Gauss's Method of Elimination, Cholewski's Method) or by iterative methods (approximated). Due to the structure of the matrix of this set (pentagonal for a two-dimensional case and heptagonal for a three-dimensional case) the use of precise methods is not recommended as it takes up too much computer memory. Therefore single or multi-iterative point methods are used. Gauss-Seidel's relaxation method (Beckenbach 1956; Patankar 1980) is an example of a one-point iterative method. However it is not effective enough due to a relatively slow numerical concurrence (especially in the case of nets containing a large number of nodes). The procedure most frequently used is a multipoint iteration, which enables the value of a searched function along the whole line containing nodes to be improved (from the boundary to the opposite boundary). The improvement happens in two stages; in the first stage lines parallel to axis *OX* are served and in the second one — the ones parallel to axis *OY*. The sequence of steps to be taken in order to put the procedure mentioned above into practice can be expressed in the following way:

1. Assume *a priori* the values of area searched (in the case under consideration — of pressure) in particular nodes of a net treating them as the first approximation.
2. Treating the values of function in nodes on horizontals as unknowns, solve a set of linear equations for each of these lines using the values of functions in adjoining nodes along axis *OY* obtained from the previous iteration.
3. The steps described in point 2 should be repeated for verticals using the values of function in adjoining nodes along axis *OX* obtained in stage 2.
4. Check the absolute value of the maximum difference of function on a net obtained in the two subsequent iterations. If it is smaller than the positive value assumed, which shows the accuracy of the numerical solution, calculations should be ceased.

The procedure described, which is called the *Alternating Directions Method*, was formed by Peaceman — Ratchford (Beckenbach 1956). In the case of a matrix with a predominating main diagonal it is convergent, which in the case of differential equations describing the transport of mass, momentum and energy, is always fulfilled.

A set of equations leading to a simultaneous calculation of the values of rhe function in nodes belonging to a particular line is solved on the basis of algorithm TDMA (Tridiagonal Matrix Algorithm) suggested by Thomas (Beckenbach 1956; Patankar 1980; Roache 1978). This procedure, which is a special form of Gauss's method of elimination, is assigned for linear sets characterised by a tri-diagonal structure of matrix. It means that non-zero matrix elements are located only on its main diagonal and on diagonals located close to the diagonal mentioned above. The remaining matrix elements are equal to zero.

The algorithm mentioned above is very convenient when computer technology is used, it occupies little operating memory and is characterised by only a small increase in the rounding error.

5. Calculating air streams flowing into gob area

The air stream flowing into the gob is calculated on the basis of the following integral formula:

$$Q_L = h \int_0^L v_n d\xi \tag{26}$$

where:

h — height of zone of airflow,

L — length of excavation,

$\xi \in \{x,y\}$ — co-ordinate parallel to the axis of excavation,

v_n — velocity co-efficient parallel to the axis of excavation.

In order to calculate the value of the integral defined by formula (26), initially, the values of velocity co-efficients u and v are calculated by approximating spatial derivatives of pressure (formula (9)) by frontal differential quotients. Later, the definite integral (26) is solved numerically by means of the well-known method of trapezoids.

6. Calculation example

The considerations presented above allow a computer programme for the three most frequently used ventilation systems of longwalls to be written. This programme solves the problem of a stable, two-dimensional flow of a incompressible gas through a porous medium characterised by heterogeneous permeability. The programme developed allows for the calculation of the distribution of airflow velocity in the gob for three different ventilation systems of longwalls.

The results of calculation are presented on Fig. 2. The identical properties of gob rocks and similar geometrical parameters of faces were assumed in calculation.

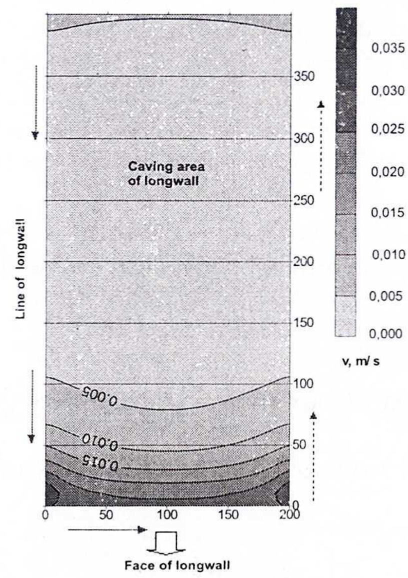

Fig. 2a. A map of isolines of air velocity in gob of longwall system ventilated by system U — advancing mining

Rys. 2a. Mapa izolinii prędkości powietrza w zrobach ścian zawałowych przewietrzanych systemach na U do granic

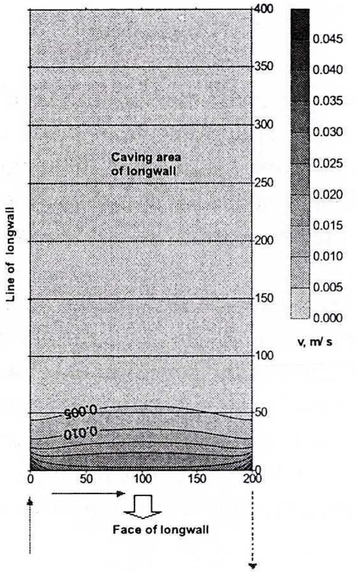

Fig. 2b. A map of isolines of air velocity in gob of longwall system ventilated by system U — retreat mining

Rys. 2b. Mapa izolinii prędkości powietrza w zrobach ścian zawałowych przewietrzanych systemach na *U* od granic

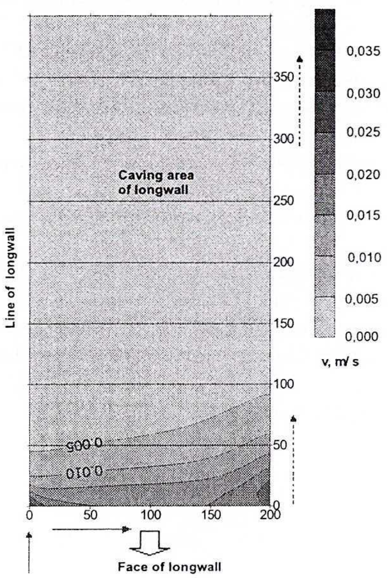

Fig. 2c. A map of isolines of air velocity in gob of longwall system ventilated by system Y — retreat mining

Rys. 2c. Mapa izolinii prędkości powietrza w zrobach ścian zawałowych przewietrzanych systemach na *Y* od granic

7. Conclusions

In most Polish mines mining is conducted by means of caving and excavated seams are characterised by a great fire risk. The intensity of ventilation in the gob of the longwall system is the main decisive factor about the magnitude of the fire hazard. The reasons for these fires are firstly, coal losses in the gob or coal from displaced seams getting into it and secondly, the permeation of air through the gob as a result of the work of the main fans.

The mathematical model (presented above) of the airflow in the caving zone, while a variable co-efficient of gob permeability is taken into consideration, and also a numerical method of solving it, allowed the development of a computer programme aiming at determination of the distribution of velocity and pressure in a caving zone. Such programmes were developed for the most typical ventilation systems (U — advancing mining, U — retreat mining and Y — retreat mining). The calculations of the distribution of air velocity and pressure in gob can help the selection preventive measures to reduce fire risks.

REFERENCES

Beckenbach E.F., 1956: Modern mathematics for the engineer. Mc Graw-Hill Book Company, Inc. New York.

Litwiniszyn J., 1949: Stationary Flows in Heterogeneously Unisotropic Mediums. Annales de la Societe Polonaise de Mathematique t. XXII.

Patankar S.V., 1980: Numerical heat transfer and fluid flow. Washington DC.

Roache P. J., 1976: Computational fluid dynamics. Albuquerque, Hermosa Publ.

Rosenberg D., 1969: Methods for the numerical solutions of partial differential equations. Americ. Elsev. Publ., New York.

Szlązak J., Szlązak N., 1987: Ucieczki powietrza przez zroby zawalowe scian przewietrzanych systemem C — do granic. Arch. of Min. Sc. vol. 32, i. 4 .

Szlązak J., Szlązak N., 1987a: Determination of aerodynamic drag of goafs of the breaking down longwalls. Arch. of Min. Sc. vol. 32.

Szlązak J., Szlązak N., 1987b: The air escapes through the caving goaf of the longwalls during the ventilation by C-system in the direction of the strike. Arch. of Min. Sc. vol. 32, i. 4.

Szlązak J., 2000: Airflow in the zone of caving in the light of theoretical and experimental research. Uczelniane Wydawnictwa Naukowo-Dydaktyczne, Dissertations Monographies no 90, Cracow.

REVIEW BY: PROF. DR HAB. INŻ. JANUSZ ROSZKOWSKI, KRAKÓW

Received: 26 June 2001