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OPTIMAL CONTROL OF HIGH PRESSURE GAS NETWORKS BY TWO DIFFERENT METHODS

OPTYMALNE STEROWANIE SIECIĄ GAZOWĄ WYSOKIEGO CIŚNIENIA Z WYKORZYSTANIEM DWÓCH RÓŻNYCH METOD

The aim of the paper is to present two different methods implemented for optimal control of high pressure gas networks. The problem is of large scale and is highly non-linear in both objective function and constraints. The first method is based on the Lagrangian dual formulation. The second method uses hierachical systems theory. Discrete state equation for the case in which output pressure are treated as elements of the control vector has been formulated. Elements of the state vector are non-outlet node pressures and flows through units. In both cases algorithms for optimal control over periods up to a day have been developed. Both methods assume that the overall objective is to minimise the total cost of operating the compressors over the entire control period. This objective may be expressed as a time integral; the objective function used is a discrete approximation to this integral, a non-linear function. Some other published methods are briefly described also. Results of investigations are included.

Key words: optimal control, gas networks, non-linear programming, hierarchical systems theory.

W artykule omówiono dwa różne algorytmy optymalnego sterowania siecią gazową wysokociśnieniową, których celem jest minimalizacja kosztów przesyłu gazu. Koszty prowadzenia ruchu (koszty eksploatacji) sieci wysokociśnieniowej są zależne głównie od parametrów pracy sprężarek zainstalowanych w stacjach przetłocznych. Algorytmy omówione w artykule dotyczą minimalizacji kosztów ekploatacji sieci wysokociśnieniowych, przy założeniu że mamy do czynienia z nieustalonym przepływem gazu w gazociągach. Nieustalony przepływ gazu w rurociągu opisany został za pomocą liniowego równania dyfuzji. Jako kryterium optymalizacji przyjęto energię zużytą na sprężanie gazu we wszystkich tłoczniach, wyrażoną jako funkcję ciśnień ssania, tłoczenia i ilości przepływającego gazu przez tłocznię, obliczaną w określonym przedziale czasu. Występujące w opisie funkcji celu wielkości funkcyjne: ciśnienia tłoczenia, ciśnienia ssania oraz przepływy

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przez stacje przetłoczne i przepływy w punktach zasilających są związane równaniami ruchu gazu w sieci przy nieustalonych warunkach przepływu.

W pierwszym przypadku, optymalne sterowanie siecią gazową wysokociśnieniową potraktowano jako zadanie nieliniowego programowania z ograniczeniami. Wykorzystano dualną funkcję Lagrange'a. Algorytm optymalnego sterowania siecią w ujęciu dualnym jest realizowany w następujących krokach:

1. oszacowanie wartości współrzędnych wektora mnożników Lagrange'a,
2. dla obliczonych wartości wektora mnożników, obliczanie minimum rozszerzonej funkcji Lagrange'a.
3. sprawdzenie kryterium zbieżności i ewentualne powtórzenie kroków 1 i 2.

Ad 1. Do szacowania wartości wektora mnożników Lagrange'a przy rozwiązywaniu zadania dualnego optymalizacji w przypadku ograniczeń nierównościowych wykorzystano metodę płaszczyzn tnących.

Ad 2. W algorytmie optymalizacji wykorzystano metodę minimalizacji funkcji przy ograniczeniach nierównościowych w oparciu o metodę rzutowanego gradientu Rosena z numeryczną estymacją gradientu funkcji. Metoda rzutowanego gradientu dla przypadku minimalizacji funkcji z liniowymi ograniczeniami jest iteracyjną metodą poszukiwania minimum funkcji wielu zmiennych.

W drugim przypadku zastosowano metodę hierarchiczną. System został zdekomponowany na podsystemy. Ograniczenia zostały uwzględnione, wprowadzając funkcję rozszerzonego Lagrangianu. Szukano minimum funkcji celu dla każdego podsystemu. Żeby znaleźć globalne optimum, wprowadzono wektor zmiennych koordynacyjnych. Podsystemy traktują zmienne koordynacyjne jako znane dane wejściowe, które pozostają stałe aż do chwili, gdy warstwa koordynacji nie zmodyfikuje tych wartości. Algorytm optymalizacji opiera się na rozwiązywaniu na każdym poziomie czasowym równań stanu, równań sprzężonych oraz gradientu Hamiltonianu.

Obydwa algorytmy były badane przy wykorzystaniu fragmentów rzeczywistych sieci. Pierwsza sieć składała się z 23 węzłów, 19 rur, 3 stacji przetłocznych, 2 zbiorników zasilających oraz 1 źródła. Druga sieć to 64 węzły, 58 rur, 4 stacje przetłoczne, 3 zbiorniki zasilające, 1 stacja redukcyjna oraz 1 źródło. W przypadku stosowania metody hierarchicznej sieci badane zostały zdekomponowane odpowiednio na trzy i cztery podsystemy. Poprawność otrzymanych wyników sprawdzono, symulując sieć przy wartościach wektora sterowania otrzymanych w wyniku optymalizacji. Badania wykazały, że metoda pierwsza (dualna funkcja Lagrange'a) jest znacznie szybsza (ponad trzy razy). W przypadku algorytmu wykorzystującego dualną funkcję Lagrange'a czas obliczeń wynosił odpowiednio 6 i 10 sekund na 1 godzinę czasu rzeczywistego (komputer Pentium II).

Słowa kluczowe: sterowanie optymalne, sieci gazowe, nieliniowe programowanie, teoria systemów hierarchicznych.

1. Introduction

The growth of the national transmission system is accompanied by increasing opportunities for more efficient management. Central Control, whose main task is the overall management of the national system has, as the number of compressors increased, recognized the rising importance of fuel usage.

Gas compressor stations form a major part of the operational plant on each Transmission System. Their function is to restore the gas pressure reduction caused by frictional pressure losses. The compressors are driven mostly by gas turbines which use natural gas as fuel, taken directly from the transmission pipelines.

The compressor unit comprises three main components, a gas generator, a power turbine and a centrifugal gas compressor. The maximum shaft powers of the units range from 5.5 MW to more than 20 MW, the associated fuel consumptions are 2.5 to 5.5 MSCFD equivalent to 8.600 to 19.000 pounds per day fuel cost.

The value of compressor fuel used in United Kingdom is currently in excess of 30 million pounds per annum; this represents 80% of the total energy costs used by British Gas. [Fairbain, 1986].

According to AGA sources, the operating cost of running the compressor stations represents anywhere between 25% and 50% of the total company's operating budget.

Minimizing this fuel usage is a major objective in the control of gas transmission costs. Although the fuel usage of compressors is significant and should be optimized there are other objectives. Above all, Central Control must operate the system so that gas is supplied where needed, in the quantities needed and at the appropriate pressure. So there would be no point in minimizing compressor fuel usage if we than failed to supply the gas. In a nutshell this is the basic problem of running the transmission system; security of supply versus costs. The security needed must be carefully judged. It is usually expressed as a margin of pressure above the minimum essential at any offtake.

The brings us to the topic of linepack. This is the volume of gas held in the transmission system. Whenever pressures exceed the minimum some of this gas can be used as storage. This can have massive benefits. Linepack also helps to accommodate the unanticipated variations of demand.

There are many aspects to optimisation of gas networks. Optimisation can mean searching, according to a certain objective function, for optimal design parameters, optimal structures for development or optimal parameters for operation of networks. The cost of operating network when gas is at high pressure is determined mainly by mode of operation of compressors. For low and medium pressure gas networks, minimisation of operating costs means leakage reduction by optimising nodal pressures. This work is concerned with the minimisation of operating costs for high pressure gas networks under transient conditions. Depending on the character of gas flow in the system we distinguish steady and unsteady states.

1.1. Steady-state optimisation of large gas networks

The steady-state in a gas network is described by systems of algebraic non-linear equations. In steady-state problems, since loads and supplies are not functions of time, an algorithm for optimisation determines, once and for all, the structure of the network (i. e. the number of sources, compressors, values and regulators called-units which must be on). In addition, the algorithm must determine the optimal parameters of the operation, namely nodal pressures and flows through branches (pipes). For these reasons, the problem of optimisation is formulated in [Wilson, 1988] as a mixed integer problem. Each unit operates subject to a set of linear and non-linear equality and inequality constraints. By linearising the flow equation, non-linear constraints and the objective function, the problem of steady-state optimisation can be expressed as a mixed-integer linear programming one in the form:

$$\begin{aligned} &\min f(\boldsymbol{x}) = \boldsymbol{c}^T \cdot \boldsymbol{x} \\ &\text{subject to } \boldsymbol{A} \cdot \boldsymbol{x} \leqslant \boldsymbol{b} \\ &\boldsymbol{x} \geqslant 0 \end{aligned} \tag{1}$$

where some components of vector $\boldsymbol{x}$ can take integer values (only 0 or 1) and the rest continuous values. Alternatively, assuming that the structure is known, an algorithm for steady-state optimisation of large gas networks is described in [O s i a d a c z, 1988]. In this case, the problem of optimisation has been treated without simplification, i. e. as a non-linear problem with non-linear constraints. In the first case the problem has been solved using the Branch and Bound method. In the second case, the chosen method at each iteration minimises a quadratic approximation to the Lagrangian function subject to linear approximations to the constraints. A line search procedure utilising the 'watchdog technique' is used to force convergence when the initial values of the variables are far from the solution. In both cases the problem of optimisation has been solved without the necessity of using hierarchical systems theory.

Described in [L u o n g o, 1989] the optimisation strategy consists of two levels. The first level involves optimising the system with suction/discharge pressures as variables. Dynamic programming is used at this level. Important part of this process is the determination of how many compressors should be switch-on. The higher level optimiser is a search method with the system flow rates as independent variables. Combination of Nelder and Mead's and Simplex methods, plus a sequential exploration around a final point gives good results under all situations to be encountered.

GASOPT; a code for the steady-state optimisation of natural gas transmission networks can optimise high pressure gas network of arbitraty configuration. Three objective functions can be taken into account:

- operating fuel (cost minimisation),
- operating margin (revenue minus cost) — maximise,
- total throughput (maximise).

CISMA-SINTRA [A n g l a r d, 1988] has developed a large control system for the transsiberian Rusian gas pipeline. The paper concentrates on the optimisation methods developed for fuel use and steady-state planification. The pipeline control as a whole is solved using hierarchical structure. Four different levels have been specified.

- level 1 is called the 'National level',
- level 2 is called the 'Regional level',
- level 3 is called the 'Local level',
- level 4 is called the 'Remote terminal units level'.

The functions at different levels are the following:

- Level 1.

National planning according to level 0 (represents the national dispatching centre for all gas pipelines in the Russia) national boundary conditions:
◊ steady-state optimisation (criterion — minimisation of power consumption per unit flow rate),
◊ transient simulation (seven days maximum).

- Level 2.

Regional planning according to level 1 boundary conditions:
◊ steady-state optimisation using the following objective functions,
◊ maximise flow rate at the output of the region,
◊ minimise power consumption per unit flow rate,
◊ maximise discharge pressures at selected compressor stations,
◊ minimise the number of compressor units operating,
◊ maximise gas inventory in the industrial region,
◊ maximise delivery flow rate to specified users,
◊ minimise electric power consumption,
◊ minimise gas consumption by turbines.
Each one of the eight basic objective functions has an associated weight α_1. The objective function to minimise is:

$$f = \alpha_1 \cdot f_1 + \alpha_2 \cdot f_2 + \ldots + \alpha_8 \cdot f_8 . \quad (2)$$

The goal of the optimisation is to produce a profile of pressures, flows, and temperatures which minimises the function f.

- Level 3.

Compressors, compressor stations and pipeline parameters identification.

- Level 4.

Local equipment control.
Data collection.
At level 1 dynamic programming method is used for optimisation.
At level 2 dynamic programming and modified reduced gradient methods are used.

The purpose of the steady-state optimiser described in [Percell et al., 1987] is to find pressures, flows, temperatures, and compressors station configurations (i. e., choice of compressors units to be on), given fixed demands and resources for the network, which are optimal with respect to a chosen objective function. The following objective function were used:
◊ minimisation the amount of fuel which is consumed by the gas turbine drivers,
◊ maximisation throughput (total delivered flow),
◊ maximisation line pack.
The Generalised Reduced Gradient algorithm was used to solved above problems.

1.2. Dynamic optimisation of large gas networks

The transient optimisation is more difficult mathematically than the steady-state, but can achieve higher savings.

Dynamic optimisation requires the use of distributed-parameter models: a partial differential equation or a system of such equations. The form of these equations varies with the assumptions made as regards the conditions of operation of the gas pipeline. The necessity of taking into account transient effects, makes the problem of dynamic optimisation very complicated from a computational point of view. For such a problem the optimal parameters of the operation of the system (structure and pressures and flows) are functions of time.

Described in [Osiadacz et al., 1994] an algorithm for the optimal control of a gas network, with any configuration is based upon the Lagrangian dual problem.

Transient flow through a pipe is described by the linear diffusion equation. The cost of running a compressor is generally assumed to be proportional to the integral of horsepower over the control interval. Constraints are imposed on compressor operation including minimum and maximum values for flow, pressure, and compressor ratio. Constraints on pressures and flows are imposed on delivery points and sources. Each compressor imposes boundary conditions on the sending and receiving nodes of pipes incident to it. To minimise the following objective function

$$I = \sum_{j=1}^{M} \int_{t_0}^{t_f} \mathrm{HP}_j(t) \cdot dt, \tag{3}$$

where:

M — the number of operating compressors,
HP — the horsepower for the j^{th} compressor,
t_0, t_f — beginning and final time for function evaluation

subject to constraints, the Lagrangian dual problem [Bazaraa et al., 1993] has been used. The cutting plane method for solving the dual problem optimises a function that approximates the dual function at each iteration.

Scientific Software-Intercomp has developed a transient gas optimisation algorithm [Mantri et al., 1985] which minimises the cost of transporting natural gas over time periods. The objective function is formulated as:

$$f = \alpha_1 \cdot \int_{t_0}^{t_f} \sum_{j=1}^{J} \mathrm{CHP}_j(\mathrm{t}) \cdot \mathrm{dt} + \alpha_2 \cdot \sum_{j=1}^{J} \sum_{i=1}^{I} \mathrm{CSC}_{j,i}, \tag{4}$$

where:

α_1, α_2 — objective term weights,
J — number of compressor stations,
I — number of compressor units per station,
CHP — cost of horsepower,
CSC — cost of compressor unit status change.

Operating constraints include constraints on stations and on individual compressor units. On station level constraints are imposed on suction and discharge pressures, and maximum discharge temperatures. Constraints imposed on individual compressor are sufficient to ensure a feasible operating profile at all times during the solution.

Checks are made for surge and chock lines for each centrifugal compressor, and for cylinder capacity for each reciprocating compressor. The algorithm comprises two major sets of routines; the pipeline simulator and the optimiser. The network simulator calculates the data required to evaluate the objective function. The optimiser carries out a optimisation process by decoupling the problem into two parts; network optimisation, and the individual station optimisation. The Generalised Reduced Gradient method applies to the first problem. Dynamic Programming applies to the choice of units within each station.

The purpose of the work in [D u p o n t et al., 1987] was to develop a mathematical programming approach to reducing fuel costs in long gas transmission lines with time — varying demands. The 450 miles of a transmission system with 75 — mile station spacing was studied. Daily the delivered load swings from about 73% to 125% of average. The compressors simulated, were polytropic centrifugal machines with typical fuel efficiency versus flow relationships. The compressor station were controlled by a discharge pressure control valve. The inlet of the 450 — miles section was assumed to receive gas at a fixed pressure. In this case descent method together with Lagrange Multipliers was used to solve given non-linear problem. An algorithm described in [F u r e y, 1993] has been developed for dynamic optimal control of complex gas networks, and has been tested on realistic problems. The method is based on Sequential Quadratic Programming and takes account of the structure of the pipeflow equations by means of a reduced gradient technique which eliminates most of the variables from the quadratic subproblems. The letter involve only simple bound constraints, which are handled efficiently by a conjugate gradient-active set algorithm. Trust region techniques permit use of the exact Hessian, preserving sparsity. More general constraints are handled at an outer level by a truncated augmented Lagrangian method.

In the Ph. D. thesis [M a r q u e s, 1985] a general on-line optimisation scheme for the operation of gas pipeline networks is presented. It was built around the dynamic simulator GANESI (Technical University of Munich). This simulator is used to predict the behaviour of the network when it is subject to any operating policy. A successive quadratic programming optimiser is used to compute the optimal of such operating policies. In order to correct for modelling errors, demand forecast errors and errors in the initial conditions, the on-line measurements are used in a state estimator and the optimisation is carried out over a moving horizon. The optimisation algorithm has been tested in simulations on networks up to seven compressor stations.

For a gas network with several tens of pipes and over ten units it is very difficult to optimise as a whole. This case requires decomposition of the system into subsystems forming a hierarchical structure. Until now, only in one case has hierarchical systems theory been applied successfully to dynamic optimisation of a high pressure gas network. In [L a r s o n et al., 1971] the network is first decomposed into subsystems, each consisting of a single compressor and the pipeline network directly driven by it. Transient flow through a pipe is described by the non-linear diffusion model:

$$\frac{\partial^2 p^2}{\partial x^2} = \alpha \cdot \frac{\partial p^2}{\partial t}, \tag{5}$$

where:

$a = 4fQ/(DAc^2)$,
f — Fanning friction factor,
Q — volume flow,
D — diameter of pipe,
A — cross-section area of pipe,
c — speed of sound in the gas,
p — pressure

which is solved using an implicit numerical scheme. The cost of running a compressor is generally assumed to be proportional to the integral of horsepower over the control interval

$$I_j = \int_{t_0}^{t_f} A_j \cdot G_j(t) \cdot \left\{ \left(\frac{p_{dj}(t)}{p_{sj}(t)} \right)^{R_j} - B_j \right\} \cdot dt, \tag{6}$$

where:

p_d — discharge pressure for j^{th} compressor,
p_s — suction pressure for j^{th} compressor,
Q_j — flow through j^{th} compressor,
A_j, B_j, R_j — constants for the j^{th} compressor.

Constraints are imposed on compressor operation include minimum and maximum values for flow, pressure, and compressor ratio. Constraints on pressures and flows are imposed on delivery points and sources. Each compressor imposes boundary conditions on the sending and receiving nodes of pipes incident to it. Assuming that suction pressure p_{sj} is kept constant, then for $R_j < 1$ the criterion function is concave in $p_{dj}(t)$. The task of the co-ordination level is to select p_{sj}, $j = 1, 2, ..., N$ (N — number of subsystems).

Some of the assumptions made by authors are questionable. At the upper level they use an heuristic that states minimum energy consumption for the network is achieved when the suction and discharge pressures of each compressor are as close as possible, consistent with constraints. At the lower level they assume that the suction pressures of all compressors are constant. The validity of these heuristics is not supported by any evidence and the results of investigations. In [M a r q u e s, 1985] has been proved that the assumptions are generally not true. The proposed method in [L a r s o n et al., 1971] can be used (and has been applied by the authors) only to a system which has few branches and no loops (a tree-structured gas transmission system). A transmission system heavily branched with many loops cannot be split into subsystems, each of them being supplied only from one compressor (compressor station).

2. Objective function

In our case, the goal of optimisation is to minimise the following expression

$$I = \sum_{j=1}^{j} \int_{t_0}^{t_f} A_j q_j(t) \left\{ \left[\frac{p_{dj}(t)}{p_{sj}(t)} \right]^{R_j} - 1 \right\} dt, \tag{7}$$

where:

J — the number of operating compressors,
p_d — discharge pressure for j-th compressor,
p_s — suction pressure for j-th compressor,
q_j — flow through j-th compressor,
A_j, R_j — constants for the j-th compressor,
t_0, t_f — beginning and final time for function evaluation.

The cost of running a compressor is generally assumed to be proportional to the integral of horsepower over the control interval.

3. Gas network model

Transient flow through a pipe is described by the following linear diffusion equation:

$$\frac{\partial p}{\partial t} = \frac{c^2}{A} \frac{1}{\lambda} \frac{\partial^2 p}{\partial x^2}, \tag{8}$$

where:

$$\lambda = \lambda(p, M) = \frac{2fc^2 |M|}{DA^2 p}$$

$$M = -\frac{1}{\lambda} \frac{\partial p}{\partial x}$$

M — mass flow,
f — Fanning friction factor,
Q — volume flow,
D — diameter of pipe,
A — cross-section area of pipe,
c — speed of sound in the gas,
p — pressure.

Structure of the high pressure gas network was represented by directed graph $G_g = (V, E)$ which consists of a set of nodes V and another set E whose elements are called branches. The nodes represent the pipe connections while the branches represent the pipes themselves, a definite direction being assigned to each pipe.

For network analysis, it was necessary to select the following nodes and branches:

- supply nodes; sources and supplying storages $V_z \subset V$,
- pressure nodes; nodes at which constraints on the pressures were imposed $V_w \subset V$,
- units; $E_p \subset E$; $E_p \equiv (V_s, V_d)$; V_s — set of suction nodes, V_d — set of discharge nodes,
- control nodes; $V_c \subset \{V_z \cup V_d)$,
- units of the system; $U \equiv V_z \cup E_p$.

A mathematical model of the dynamic properties of a gas network has been elaborated using above equation and based upon the generalisation of the idea of a node including grid points along pipes, multi-junctions and pipe-ends being treated as off-takes with demand equal to zero.

For the whole network the following equation can be written [O s i a d a c z, 1987]:

$$\boldsymbol{H} \cdot \boldsymbol{p}^{k+1} = \boldsymbol{R} \cdot \boldsymbol{p}^{k} - \boldsymbol{L} \tag{9}$$

$\dim \boldsymbol{H} = n \times n$, $\dim \boldsymbol{R} = n \times n$, $\dim \boldsymbol{L} = n \times 1$,

n — number of nodes.

Matrices $\boldsymbol{H}$ and $\boldsymbol{R}$ are sparse and symmetric. Adding in sources, compressors (compressor stations), regulators and valves, and assuming the flow through each unit is a positive demand at the inlet node and a negative demand at the outlet, Eq. (9) can be written as:

$$\boldsymbol{H} \cdot \boldsymbol{p}^{k+1} = -\boldsymbol{K} \cdot \boldsymbol{q}^{k} + \boldsymbol{R} \cdot \boldsymbol{p}^{k} - \boldsymbol{L} \tag{10}$$

$\dim \boldsymbol{K} = n \times u$ (u — number of units)

$$k_{ij} = \begin{cases} +1, \text{ if the } j\text{-th unit has its inlet at node } i \\ -1, \text{ if the } j\text{-th unit has its outlet at node } i \\ 0, \text{ otherwise} \end{cases}$$

$\dim \boldsymbol{q} = u \times 1$

where:

$\boldsymbol{L}$ — vector of nodal loads,

$\boldsymbol{q}$ — vector of flows through units.

Writing equation (10) in the form:

$$\boldsymbol{H}_1 \cdot \boldsymbol{p}_1^{k+1} + \boldsymbol{H}_2 \cdot \boldsymbol{p}_2^{k+1} + \boldsymbol{K} \cdot \boldsymbol{q}^{k} = \boldsymbol{R}_1 \cdot \boldsymbol{p}_1^{k} + \boldsymbol{R}_2 \cdot \boldsymbol{p}_2^{k} - \boldsymbol{L}, \tag{11}$$

where:

$\boldsymbol{p}_1$ — the vector of non-outlet node pressures,

$\boldsymbol{p}_2$ — the vector of outlet node pressures,

$\dim \boldsymbol{p}_1 = (n-u) \times 1$, $\dim \boldsymbol{p}_2 = u \times 1$

finally we have:

$$[\boldsymbol{H}_1 | \boldsymbol{K}] \cdot \left[\frac{\boldsymbol{p}_1^{k+1}}{\boldsymbol{q}^{k+1}}\right] = [\boldsymbol{R}_1 | \boldsymbol{0}] \cdot \left[\frac{\boldsymbol{p}_1^{k}}{\boldsymbol{q}^{k}}\right] + (\boldsymbol{R}_2 - \boldsymbol{H}_2) \cdot \boldsymbol{p}_2^{k} - \boldsymbol{L}^{k} \tag{12}$$

$$[H_1 | K] = A_1, [R_1 | 0] = B_1, (R_2 - H_2) = C_1$$

$$\dim A_1 = n \times n, \ \dim B_1 = n \times n, \ \dim C_1 = n \times u$$

or

$$x(k+1) = A(k) \cdot x(k) + B(k) \cdot m(k) + C(k) \cdot z(k), \tag{13}$$

where:

$$x = \begin{bmatrix} p_1 \\ \hline q \end{bmatrix}, \ m = p_2, \ z = L$$

$$A = A_1^{-1} \cdot B_1, \ B = A_1^{-1} \cdot C_1, \ C = -A_1^{-1}.$$

Equation (13) is a discrete state equation for the case in which output pressures are treated as elements of the control vector. Elements of the state vector are non-outlet node pressures and flows through units.

4. Operational constraints

The following constraints were imposed on the system:

C1 — maximum source flow

$$q_j \leqslant q_j^{\max},$$

where: $j \in V_z$,

C2a — maximum compressor ratio

$$\frac{p_{d_j}}{p_{s_j}} \leqslant \varepsilon_j^{\max},$$

C2b — minimum compressor ratio

$$\frac{p_{dj}}{p_{sj}} \geqslant \varepsilon_j^{\min},$$

where: $j \in E_p$,

C3 — minimum pressure at selected nodes

$$p_j \geqslant p_j^{\min},$$

where: $j \in V_w$,

C4 — maximum horsepower of the compressor

$$W_j = A_j q_j \left\{ \left[\frac{p_{d_j}}{p_{s_j}} \right]^{R_j} - 1 \right\} \leqslant W_j^{\max}$$

where: $j \in V_w$,

C5 — range o the pressure variations at control nodes

$$p_{zj}^{\min} \leqslant p_{z_j} \leqslant p_{zj}^{\max}$$

where: $j \in V_c$,

C6 — line pack

$$\boldsymbol{p}^0 = \boldsymbol{p}^{N_T},$$

where:

$\boldsymbol{p}^0$ — pressure (vector) at $t = t_0$,
$\boldsymbol{p}^{N_T}$ — pressure (vector) at $t = t_f$.

Constraints C1–C6 together with state equation (Eq. (10)) form complete set of constraints imposed on the high pressure gas network.

5. Sequential Lagrangian algorithm

Problem of optimal control of high pressure gas networks under transient conditions treated as a non-linear problem with non-linear constraints can be formulated as follows:

$$\min f(\boldsymbol{x}, \boldsymbol{y}) \tag{14}$$

subject to: $\boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \leqslant \boldsymbol{0}$
and $\boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y}) = \boldsymbol{0}$,

where:

$$\boldsymbol{x} \in X \subset \boldsymbol{R}^n; X = \{\boldsymbol{x} : \boldsymbol{x}^{\min} \leqslant \boldsymbol{x}^{\max}\}; \boldsymbol{y} \in \boldsymbol{R}^n$$

The Lagrangian Dual Problem is the following:

$$\max q(\boldsymbol{u}, \boldsymbol{v}) \tag{15}$$

subject to: $\boldsymbol{u} \geqslant \boldsymbol{0}$,

where:

$$\theta(\boldsymbol{u}, \boldsymbol{v}) = \inf\{f(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{v}^T \boldsymbol{h}(\boldsymbol{x}, \boldsymbol{y})\}.$$

Since, equality constraints are taken into account by the state equation, the following simplified Lagrange function is used:

$$\Theta(\boldsymbol{u}) = \inf\{f(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y})\}. \tag{16}$$

Discretization of the objective function yields:

$$I(\boldsymbol{p}_2) = \sum_{n=1}^{N_T} \sum_{j=1}^{N_P} A_j q_j^n \left[\left(\frac{p_{d_j}}{p_{s_j}}\right)^{R_j} - 1\right]. \tag{17}$$

Formulated above problem of optimisation of high pressure gas network contains much more constraints than independent variables.

If the numbers of discrete stages and control variables are equal to N_T and N_C respectively, then:

$$N = N_T \times N_C, \tag{18}$$

where: N — is the number of independent variables. Thus, the number of constraints (C1 – C6) is equal to:

$$M = N_T \times (N_z + 3 \times N_p + N_w + 2 \times N_c) + N_n, \tag{19}$$

where:

N_p — the number of compressors which are on,
N_z — the number of supply nodes,
N_w — the number of nodes at which constraints on the pressure were imposed,
N_n — the number of nodes of the network.

The state equations (Eq. (10)) contain N_T systems of N_n equations.
For given value of $\boldsymbol{u}$, algorithm calculates minimum of the function:

$$\varphi(\boldsymbol{p}_2) = f(\boldsymbol{p}_1, \boldsymbol{p}_2) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{p}_1, \boldsymbol{p}_2) \tag{20}$$

subject to:

$$\boldsymbol{p}_2^{\min} \leqslant \boldsymbol{p}_2 \leqslant \boldsymbol{p}_2^{\max} \tag{21}$$

and

$$\boldsymbol{h}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \boldsymbol{0}, \tag{22}$$

where:

$$\boldsymbol{g}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \begin{bmatrix} q_j - q_j^{\min} & ; j \in V_z \\ \dfrac{p_{d_j}}{p_{s_j}} - \varepsilon_j^{\max} & ; j \in E_p \\ \varepsilon_j^{\min} - \dfrac{p_{d_j}}{p_{s_j}} & ; j \in E_p \\ p_j^{\min} - p_j & ; j \in V_w \\ A_j q_j \left[\left(\dfrac{p_{d_j}}{p_{s_j}} \right)^{R_j} - 1 \right] - W_j^{\max} & ; j \in E_p \end{bmatrix}.$$

Constraint $\boldsymbol{h}(\boldsymbol{p}_1, \boldsymbol{p}_2) = \boldsymbol{0}$ is represented by state equation.
Final values (for $t = t_f$) of nodal pressures are obtained by solving set of linear equations for each discrete stage of the process (Eq. (24)).
The following conditions is taking into account:

$$\left\| \boldsymbol{p}^{N_T} - \boldsymbol{p}^0 \right\| \leqslant \varepsilon \tag{23}$$

$$\left[\begin{array}{l} H_1^1 p_1^1 + Kq^1 = R^1 \left[\frac{p_1^0}{p_2^0}\right] - H_2^1 p_2^1 - L^1 \\ H_1^2 p_1^2 + Kq^2 = R^2 \left[\frac{p_1^1}{p_2^1}\right] - H_2^2 p_2^2 - L^2 \\ \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ H_1^{N_T} p_1^{N_T} + Kq^{N_T} = R^{N_T} \left[\frac{p_1^{N_T-1}}{p_2^{N_T-1}}\right] + \\ \qquad\qquad - H_2^{N_T} p_2^{N_T} - L^{N_T} \end{array}\right].$$

Major component of the algorithm is estimation of Lagrange multipliers. There are numerous ways of estimating. In this case, the cutting plane method was used. We now discuss a strategy for solving the dual problem in which, at each iteration, a function that approximates the dual function is optimised.
Letting $z = \Theta(\boldsymbol{u})$, the inequality

$$z \leqslant f(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \tag{25}$$

must hold true for each $\boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R}^n$. Hence, the dual problem of maximising $\Theta(\boldsymbol{u})$ over $\boldsymbol{u} \geqslant \boldsymbol{0}$ is equivalent to the following problem:

$$\begin{gathered} \max z \\ \textit{subject to } z \leqslant f(\boldsymbol{x}, \boldsymbol{y}) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{x}, \boldsymbol{y}) \\ \boldsymbol{u} \geqslant \boldsymbol{0} \quad \textit{for} \quad \boldsymbol{x}, \boldsymbol{y} \in \boldsymbol{R}^n \end{gathered} \tag{26}$$

Note, that above problem is a linear in the variables z and $\boldsymbol{u}$.
Suppose, that we have the points $\boldsymbol{x}_1, \ldots, \boldsymbol{x}_{k-1}$, $\boldsymbol{y}_1, \ldots, \boldsymbol{y}_{k-1}$ in $\boldsymbol{R}^n$, and consider the following approximating problem:

$$\begin{gathered} \max z \\ \textit{subject to } z \leqslant f(\boldsymbol{x}_j, \boldsymbol{y}_j) + \boldsymbol{u}^T \boldsymbol{g}(\boldsymbol{x}_j, \boldsymbol{y}_j) \\ \textit{for } j = 1, \ldots, k-1 \\ \boldsymbol{u} \geqslant \boldsymbol{0} \end{gathered} \tag{27}$$

The above problem is a linear with a finite number of constraints and can be solved by simplex method.
Let $(z_k, \boldsymbol{u}_k)$ be an optimal solution to this approximating problem. If this solution satisfies Eq. (26), then it is an optimal solution to the Lagrangian dual problem. A cutting plane algorithm is used in the following way:

Initialisation step

Find a point $(\boldsymbol{x}_0, \boldsymbol{y}_0)$, $(\boldsymbol{x}_0, \boldsymbol{y}_0 \in \boldsymbol{R}^n)$ such that $\boldsymbol{g}(\boldsymbol{x}_0, \boldsymbol{y}_0) \leqslant \boldsymbol{0}$.
Let $k = 1$.

Main step

Solve the following problem (main problem):

$$\begin{aligned} &\max z \\ &\textit{subject to } z \leqslant f(x_j, y_j) + u^T g(x_j, y_j) \\ &\textit{for } j = 0, \ldots, k-1 \\ &u \geqslant 0 \end{aligned} \tag{28}$$

Let (z_k, u_k) be an optimal solution and solve the following subproblem:

$$\min f(x, y) + u^T g(x, y) \tag{29}$$

subject to:

$$x^{\min} \leqslant x \leqslant x^{\min}$$

$$\text{and } h(x, y) = 0$$

Let x_k be an optimal point, and let

$$\Theta(u_k) = f(x_k, y_k) + (u_k)^T g(x_k, y_k) \tag{30}$$

If $z_k = \Theta(u_k)$ then stop; u_k is an optimal dual solution. Otherwise, if $z_k > \Theta(u_k)$, then add the constraint $z \leqslant f(x, y) + u^T g(x_k, y_k)$ to the main problem, replace k by $k+1$ and repeat the main step.

5. Hierarchical algorithm

Alternatively, to solve above problem, hierarchical method has been used. An algorithm for the optimal control of a gas network with any configuration based upon hierarchical control and decomposition of the network has been developed.

Implicit in all of hierarchical system theory is the idea that it is generally easier to deal with several low order systems than with one system of high order. Since the gas transmission system has no apparent natural hierarchical structure, the decomposition should be carried out in whatever manner seems appropriate. One possibility is to employ spatial decomposition according to which, the network is divided into physically small subsystems. On this decomposition is imposed one constraint, namely each subsystem has to contain at least one operating compressor. A second possibility is decomposition in time. In this case a subsystem would be the complete system in one time step of optimisation. Time decomposition has several disadvantages:

(1) for most normal demand conditions on a gas network the control interval is subdivided into time steps which do not exceed 1 hour. For an overall time interval of 24 hours this could give at least 24 subsystems.

(2) the coordination will involve a large number of variables and hence much computation time.

(3) local optimisation of each subsystem is a difficult task.

For these reasons a spatial decomposition has been chosen to solve the given problem. After decomposition each subsystem is interconnected with some other subsystems.

Interconnection matrix gives a precise definition of the structure of an interconnected system and relate subsystem inputs to subsystem outputs, where it is explicity recognised that the inputs to a given subsystem are formed from the outputs of other subsystems.

The total (global) input and output vectors are taken composite vectors:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_1 \\ \alpha_2 \\ \vdots \\ \alpha_N \end{bmatrix} \qquad \boldsymbol{\beta} = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_N \end{bmatrix}$$

The interconnections are defined by the expressions:

$$\boldsymbol{\alpha}_i = \sum_{j=1}^{N} \boldsymbol{L}_{ij}\boldsymbol{\beta}_j = \boldsymbol{L}_i\boldsymbol{\beta}, \tag{31}$$

$$\boldsymbol{\alpha} = \boldsymbol{L}\boldsymbol{\beta}$$

where:

$\boldsymbol{\alpha}_i$ — input vector into i-th subsystem (nodal pressure at nodes incident to the other subsystem),

$\boldsymbol{\beta}_i$ — output vector from i-th subsystem (nodal pressures at nodes of i-th subsystem incident to the other subsystems),

N — number of subsystems,

$\boldsymbol{L}_{ij}, \boldsymbol{L}_i$ and $\boldsymbol{L}$ are Boolean matrices of interconnections.

To solve the given problem a spatial decomposition has been chosen. Discretization of the cost function yields:

$$I = \sum_{k=k_0}^{k_f-1} \Phi_1\left[\boldsymbol{x}(k), \boldsymbol{m}(k), k\right] \tag{32}$$

where:

$\boldsymbol{x}$ — state vector,

$\boldsymbol{m}$ — control vector,

k_f — final stage.

Constraints are taken by introducing Augmented Lagrangian function. Thus

$$I = \sum_{k=k_0}^{k_f-1} \Phi\left[\boldsymbol{x}(k), \boldsymbol{m}(k), \boldsymbol{x}_1(k), k\right], \tag{33}$$

where:

$$\Phi = \Phi_1 + \delta^T \cdot c(\boldsymbol{x}_1(k)) + \frac{\sigma}{2} \cdot c(\boldsymbol{x}_1(k))^T \cdot \boldsymbol{S}(k) \cdot c(\boldsymbol{x}_1(k))$$

$c(\boldsymbol{x}_1(k))$ — contains the constraints that are active at $\boldsymbol{x}_1$,
δ — Lagrange multiplier.

The Hamiltonian for the integrated system is:

$$H(k) = \Phi[\boldsymbol{x}(k), \boldsymbol{m}(k), \boldsymbol{x}_1(k), k] + \\ + \lambda^T(k+1) \cdot \boldsymbol{f}[\boldsymbol{x}(k), \boldsymbol{m}(k), \boldsymbol{x}_1(k), k]. \tag{34}$$

In terms of the subsystems, the Hamiltonian may be written as

$$H(k) = \sum_{i=1}^{N} \Bigg(\Phi_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \boldsymbol{\lambda}^T(k+1) \cdot \boldsymbol{f}_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \boldsymbol{\pi}_i^T(k) \cdot \Bigg(\sum_{j=1}^{N} \boldsymbol{L}_{ij} \cdot \beta_j(k) - \boldsymbol{\alpha}_i(k) \Bigg) \Bigg), \tag{35}$$

where: $\boldsymbol{\lambda}_i$, $\boldsymbol{\pi}_i$ — multipliers.
Because

$$\boldsymbol{\pi}_i^T(k) \cdot \sum_{j=1}^{N} \boldsymbol{L}_{ij} \cdot \boldsymbol{\beta}_j(k) = \sum_{i=1}^{N} \sum_{j=1}^{N} \boldsymbol{\pi}_i^T(k) \cdot \boldsymbol{L}_{ji} \cdot \boldsymbol{\beta}_j(k). \tag{36}$$

Eq. (35) can be written as follows:

$$H(k) = \sum_{i=1}^{N} (\Phi_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \boldsymbol{\lambda}^T(k+1) \cdot \boldsymbol{f}_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \boldsymbol{K}_i^T \cdot \boldsymbol{\beta}_j(k) - \boldsymbol{\pi}_i^T(k) \cdot \boldsymbol{\alpha}_i(k)), \tag{37}$$

where: $\boldsymbol{K}_i(t) = \sum_{j=1}^{N} \boldsymbol{L}_{ij} \cdot \boldsymbol{\pi}_j(k)$.
Thus for the i-th subsystem we have:

$$H(k) = \Phi_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \lambda^T(k+1) \cdot \boldsymbol{f}_i[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), \boldsymbol{x}_{1i}(k), k] + \\ + \boldsymbol{K}_i^T \cdot \boldsymbol{\beta}_j(k) - \boldsymbol{\pi}_i^T(k) \cdot \boldsymbol{\alpha}_i(k) \tag{38}$$

Then the optimisation problem for the i-th subsystem is the following: minimise wrt $\boldsymbol{m}_i$

$$I_i = \sum_{i=1}^{k_f - 1} (\Phi_i[\boldsymbol{x}_i(k) + \boldsymbol{K}_i^T(k) \cdot \boldsymbol{\beta}_j(k) - \boldsymbol{\pi}_i^T(k) \cdot \boldsymbol{\alpha}_i(k)]) \tag{39}$$

subject to constraints.

We set $\partial H(k)/\partial \boldsymbol{x}(k) = \boldsymbol{\lambda}(k)$ to obtain the adjoint equation

$$\boldsymbol{\lambda}_i(k) = \frac{\partial H_i(k)}{\partial \boldsymbol{x}_i(k)} = \frac{\partial \Phi_i(k)}{\partial \boldsymbol{x}_i(k)} + \left[\frac{\partial f_i^T}{\partial \boldsymbol{x}_i(k)}\right] \cdot \boldsymbol{\lambda}_i(k) + \left[\frac{\partial \beta_i^T(k)}{\partial \boldsymbol{x}_i(k)}\right] \cdot \boldsymbol{K}_i - \left[\frac{\partial \alpha_i^T(k)}{\partial \boldsymbol{x}_i(k)}\right] \cdot \boldsymbol{\pi}_i, \tag{40}$$

where the terminal condition the adjoint equation is:

$$\boldsymbol{\lambda}_i(k_f) = 0. \tag{41}$$

Local problems were solved using a gradient technique:
For given $\boldsymbol{m}_i(k)$, $k = k_0, ..., k_f$, $\boldsymbol{x}_i(k+1)$ is determined from:

$$\boldsymbol{x}_i(k+1) = \boldsymbol{f}_i\left[\boldsymbol{x}_i(k), \boldsymbol{m}_i(k), z_i(k), k\right] \tag{42}$$

$$\boldsymbol{x}_i(k_0) = \boldsymbol{x}_{i,0}.$$

This allows us to solve the adjoint equation, Eq. (40) backwards from stage k_f with the terminal condition of Eq. (41) to stage k_0.
Then we calculate

$$\frac{\partial H_i(k)}{\partial \boldsymbol{m}_i(k)} = \frac{\partial \Phi_i(k)}{\partial \boldsymbol{m}_i(k)} + \left[\frac{\partial f_i^T(k)}{\partial \boldsymbol{m}_i(k)}\right] \cdot \boldsymbol{\lambda}_i(k+1) \tag{43}$$

A search direction $\boldsymbol{s}$ is computed by a conjugate gradient technique. At first step

$$\boldsymbol{s}^0 = -\frac{\partial H_i(k_0)}{\partial \boldsymbol{m}_i(k_0)} \tag{44}$$

New value of control vector is computed according to the formula:

$$\boldsymbol{m}_i^{k+1} = \boldsymbol{m}_i^k + \tau \cdot \boldsymbol{s}_i^k. \tag{45}$$

For each stage k, we calculate euclidean norm:

$$E(k) = \sqrt{\sum_{i=1}^{s} \Delta m(i)^2}, \tag{46}$$

where:

i — is the number of compressors (compressor stations)

$$W(L) = \sum_{k=k_0}^{k_f} E(k), \tag{47}$$

where:

L — index number of cycle of optimisation.

If $W(L) \leqslant \varepsilon$ then stop.

Typical values used for ε are 0.025 MPa per node.

The goal coordination method was used to find the overall optimum.

To find the overall optimum the coordination variables (Π) for optimisation are introduced. The subproblems treat the coordination variables as known inputs which remain fixed until the coordinator supplies new values.

It can be seen that the main problem of coordinating subproblem solutions so that they solve the integrated problem is in the determination of Π^*.

If $\Pi \neq \Pi^*$, then the interconnection constraints are not satisfied, so that it is rational to examine the effect of Π on the interconnection error, which will be defined as:

$$\sum_{j=1}^{L} L_{ij}\beta_j(\Pi, k) - \alpha_i^*(\Pi, k) = e_i(\Pi, k) \tag{48}$$

For updating $\Pi(k)$ to minimise $e_i(\Pi, k)$ a gradient procedure to produce a new Π is applied.

6. Results

The algorithms described above have been implemented in double precision FORTRAN on Pentium II computer and some preliminary testing have been done. Variables and constraints are scaled to be of order unity. Two networks have been considered.

The first problem consists of a network of 23 nodes, 19 pipes, 3 compressor stations, 2 storage supply nodes and 1 source, was tested with 24 1-hour timesteps. The second network containing 64 nodes, 58 pipes, 4 compressor stations, 3 storage supply nodes, 1 regulator and 2 sources was tested with 24 1-hour timesteps.

To deal with hierarchical algorithm the networks were split into three and four systems respectively. The validity of the solutions was checked by comparing solutions with those from simulation programs run with appropriate control values. It was verified that the equations of gas flow were satisfied.

Second, the solutions obtained by varying objective function were checked for consistency and resonableness. Time of calculation (6 sec and 10 sec time per one hour of real time respectively for first and second network) and obrained results indicate that the application of the Lagrangian dual method could be very effective approach to the solution of the problem of optimal control of gas transportation systems.

Algorithm based upon decomposition — coordination technique takes about 3 times more computation time.

7. Conclusions

We have described two algorithms which are suitable for a general class of large non-linearly constrained optimisation problems. The results given for its application to the two gas network problems seem encouraging. Investigations have shown that developed algorithms work properly although algorithm based upon decomposition — coordination technique is much slower.

Careful analysis of the structure of the existing algorithms and amount of computation time needed, showed that certain modifications can be done to speed up calculations and to improve flexibility and accuracy of the algorithm.

It is very important to notice that the method based on decomposition — coordination is very suitable for parallel computation and hence implementation on parallel processes.

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