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CONTROL, INFORMATICS AND ROBOTICS

A Markov chain model to determine optimal maintenance policy for production process in automotive company

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Abstract. This paper considers an operating machine with deteriorating performance over time. Initially, functioning at 100% of its nominal capacity, the machine fails after a stochastic period, reducing its capacity to a proportion of the nominal level. In this degraded capacity state, three maintenance and repair policy options are available for evaluation. By modelling the system as a discrete-time Markov chain and analyzing the probability transition matrix between the system states, the costs associated with the loss of production, part replacement, and ongoing operation in each state can be quantified. The objective function representing the average cost per unit time of production is calculated to determine the optimal maintenance policy. Different policies are modelled by the Markov chain and the average cost of each policy is obtained. The results demonstrate the applicability of the proposed methodology to evaluating different policies.

Keywords: discrete-time Markov chain; maintenance policy; optimum policy; cost minimization.

1. INTRODUCTION

In general, maintenance and repair include performing planned and unplanned activities to maintain or return the system to an acceptable operational condition. The goal of an optimal maintenance and repair policy is to provide reliability and safe performance at the lowest cost [1]. In today's industries, given the automation of equipment and the fact that parts of machinery, including shafts, bearings, bushings, belts, etc., are subject to wear and tear, the failure of any of these components will stop the machine and production line. Therefore, to ensure the reliability of equipment and reduce downtime costs, the implementation of an optimal maintenance and repair policy is essential so that inadequate and incorrect maintenance and repair will be extremely costly, not only because of failure to meet equipment repair needs but also because of missed opportunities. Maintenance and repair processes are essential because they account for a massive portion of production costs, ranging from 15% to 60%, depending on the industry [2]. Due to the significant impact of random factors, such as sudden machinery failure, in production systems, determining the optimal maintenance and repair policies is critical. One of the critical goals of maintenance and repair policies is to minimize unplanned machine downtime and, as a result, bring it under control and increase machine productivity [3].

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2. LITERATURE REVIEW

In recent years, many studies have been conducted to optimize maintenance and repair systems. By reviewing articles published in recent years, we can conclude that various solution methods are used to solve optimization problems. The most important of these are as follows:

- Operational research models;
- Stochastic models;
- Markov models;
- Analytical models;
- Simulation models;
- Bayesian networks;
- Fuzzy models;
- Multiobjective models.

Qiu *et al.* [4] proposed a model for determining the optimal maintenance and repair policies for shipbuilding systems. Using mathematical modelling, this model considers potential orders, production fluctuations, and interdependency between machinery, which directly affect failure and, thus, maintenance and repair costs. The optimal maintenance and repair policy is obtained by minimizing the maintenance and repair costs. Fallahnezhad *et al.* [5] presented a statistical reliability model-based preventive maintenance method using Bayesian inference. In this study, the goal of Bayesian inference is to obtain the inspection point. By combining Bayesian inference and statistical-reliability model-based preventive maintenance methods, they sought to provide more accurate and practical preventive maintenance methods. Allal *et al.* [6] presented a simulation-optimization approach for optimizing wind turbine maintenance and repair planning to minimize costs and maximize equipment availability. The proposed model employs an ant colony algorithm to optimize the routing of maintenance and repair activities. Diallo *et al.* [7] proposed a model to determine the optimal maintenance and repair policies for multicomponent systems. A mathematical model with two objective functions was developed to optimize the tradeoff between the total maintenance cost and the system reliability based on the preferences of the decision-makers. Ghorbani *et al.* [8] proposed a model to determine the optimal maintenance and repair program for an operating multicomponent system with scheduled downtime and rest periods. The objectives of the proposed model are to minimize total cost and maximize equipment reliability by using a stochastic programming approach. Fallahnezhad *et al.* [9] presented a Markov model for a single two-state machine replacement problem, aiming to determine a threshold for optimal decision-making based on selecting whether to replace the machine, repair the machine, or continue production. Andersen *et al.* [10] presented an integrated model for the time- and condition-based maintenance of a multicomponent system to optimize the part replacement time. All system components exhibit ageing and deterioration properties and follow a multivariate gamma distribution. In this study, the CBM and TBM models are described as Markov decision processes, and dynamic programming is used to solve the final model and determine the optimal policy. Jin *et al.* [11] proposed a model for determining the optimal preventive maintenance period for a multistate deteriorating machine. Because the transition rate between machine states is unknown, reversible linear integral equations are used to calculate the transition matrix of the states. Finally, using the semi-Markov decision model and proposed algorithm, the optimal preventive maintenance period can be obtained. Tajiani *et al.* [12] developed an optimal maintenance policy simulation method for a single-component system by considering two types of failure: failure due to equipment wear and failure due to random events such as weather conditions and overload. Li *et al.* [13] presented an optimization model for scheduled and condition-based maintenance and repairs intending to minimize costs, and they designed an optimization algorithm based on Monte Carlo simulations to solve the model. Finally, an opportunistic maintenance strategy for CNC gear-grinding machines was developed. Ziolkowski *et al.* [14] proposed a mathematical model for the process of operating aviation fuel-supplying vehicles before flight. The phase space of the process was mapped by a seven-state directed graph of the operation process, and Markov chains and processes were used to calculate the technical readiness index. Oszczypala *et al.* [15] applied a stochastic method to a wide spectrum of technical objects. The three-state semi-Markov model was implemented for reliability analyses, and the Laplace transform was used to determine the reliability function, the failure probability density function, the failure intensity, and the expected time to failure. Knopik *et al.* [16] developed the semi-Markov model for age-replacements of technical objects. The model considered in this paper includes two types of repairs: perfect and minimal repairs. The asymptotic availability coefficient and profit per time unit are considered criteria for the quality of system operation. Jabash *et al.* [17] proposed a Markov fuzzy real-time demand-side manager to reduce the operating cost of the smart grid system and maintain a supply-demand balance in an uncertain environment.

By reviewing articles published in the field of maintenance and repair optimization, especially those that used Markov chain relationships for optimization, we found that the system state was considered continuous in all models, and continuous Markov process relationships were used to solve and optimize the problem. However, in this research, in addition to considering the erosive failure state, which is the absorbing state, the sudden failure state, which is also an absorbing state, is included for the machine as well. Since in reality, the number of states that can be moved for a machine is limited, unlike most research conducted in recent years that used continuous Markov model relations, in this article, discrete Markov model relations were used to determine the optimal policy. In contrast, in the real world, system states are discrete, and an absorbing state exists. Therefore, in this study, by considering a machine with five states, two of which are absorbing, and using discrete Markov chain relationships, the optimal maintenance and repair policy is determined such that the objective function, which is the average cost of the production process, is minimized. In addition, different repair and maintenance policies are modelled using an absorbing Markov chain, and the average cost of each policy is determined to allow us to compare the performance of different policies. In other words, by using the equations of the absorbing Markov chain and the transition probability matrix, the parameters of the objective function are determined. Then, the objective function of the problem, which is the minimization of the average cost of the production process, is calculated, and the optimal policy is selected. It should be noted that we have applied three contributions to develop the model. First, we modelled the operation process of the machine as an absorbing Markov chain not addressed before by this Markov modelling. Another contribution is to convert the absorbing Markov chain to an integrated Markov chain by an assumption of omitting the absorbing states to determine the limiting probabilities of each transient state. The third contribution is to develop the cost objective function that has two parts. The first part is the failure cost of the machine that should be obtained in time units. Thus, it is divided on time to failure that is obtained using equations of absorbing Markov chain and the second part is the operation cost of the machine that is obtained by multiplying the limiting probability of each state with the cost of each state.

When there are no absorbing states, the transition among all states is possible with a certain probability. Absorbing states should be considered in Markov models of maintenance problems because failure states are absorbing states, and the lifetime of the machine will end. The proposed method is based on the equations of the absorbing Markov chain and failure states are considered for the system in which if a machine enters one of these states, a new machine must be replaced. The condition of the machine is periodically and systematically reviewed and evaluated. These assessments are conducted using the Markov chain model, which analyzes transitions between different states of the machine (from reliable performance to failure). Additionally, since the machine performance directly affects the production, the amount of production decreases with its deterioration or ageing. Thus, the physical condition of the machine can also be interpreted through the number of produced goods.

The characteristic of memorylessness in state transitions significantly simplifies the objective function, and this assumption may ignore gradual changes in the system's state. However, many types of equipment used in industries, considering their mechanisms and components, do not experience gradual changes in state and transition from one state to another after a while. Due to difficulties in obtaining necessary data, expert estimates can be used as substitutes for real data. Also, these data can be estimated from historical data on machine failures and repair costs.

3. PROBLEM STATEMENT

The main purpose of this article is to optimize and determine the optimal maintenance and repair policy for industrial equipment, especially critical equipment in the steel industry. Some of the critical equipment of the steel industry, such as cooling towers, reformer tubes, clarifiers, etc. have erosive properties, and after starting up and with the failure of some of its minor parts, the performance and capacity of the equipment is reduced. With this low capacity, it is possible to continue the operation of the equipment. In this study, the Markov chain relationships were used to determine the optimal maintenance and repair policy for the equipment. To make the model more realistic, absorbing states were added to the model. The problem considered in this study involves an operating machine with deteriorating properties that can experience two types of failure: deterioration and sudden failure.

First, the machine is in a good state with 100% capacity; it deteriorates over time and enters a medium state, or the operating capacity of the machine can be reduced, which is called a bad state. In this case, it is possible to increase the capacity of the machine to a medium state by imperfect maintenance or to increase the capacity of the machine to the initial state (100% capacity), which is called a good state by perfect replacement.

The machine condition is periodically and systematically reviewed and evaluated.

The machine is in one of the following five states:

State 1: Operating in a good state (100% capacity).

State 2: Operating in a medium state.

State 3: Operating in a bad state.

State 4: Sudden failure.

State 5: Deteriorating failure and machine stoppage.

The machine state is obtained by inspection at the end of each stage.

Initially, the machine starts operating in State 1 (100% rated capacity or good state. With specific probabilities, the machine enters State 3 or 4 or remains within its current state. If the machine enters State 3, the following three policies are possible:

Policy 1: The production process is continued in a bad state.

Policy 2: When imperfect replacement is applied, the machine enters State 2 or the medium state.

Policy 3: Apply perfect replacement and the machine enters State 1 or a good state.

The main objective of this study is to determine and select the optimal policy when the machine enters State 3 such that the average cost of the production process is minimized.

If Policy 1 is selected and a decision is made to continue machine operation with specific probabilities, the machine can enter State 4 or 5 or remain in its current state. In this case, if the machine has not entered one of the failure states (sudden or deteriorating), it continues operating in a bad state.

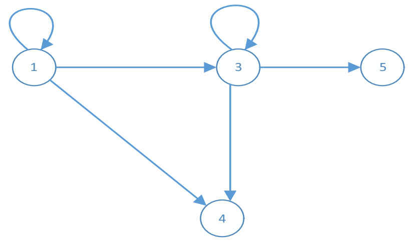

Fig. 1. State diagram of Policy 1

When the machine enters State 4 or State 5, there is no possibility of returning to other states. These are the absorbing states of the system in which the current machine is replaced with a new one.

If Policy 2 is selected and the decision for imperfect replacement is made, then with the incurred cost of machine downtime and imperfect maintenance, the machine enters State 2 and starts operating in good state. Subsequently, with specific probabilities, the machine enters State 3 or 4 or remains in its current state.

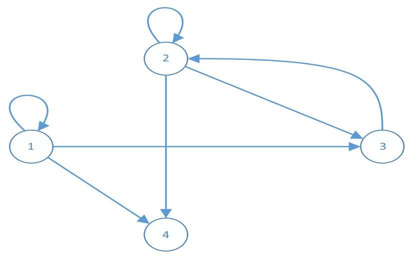

Fig. 2. State diagram of Policy 2

If Policy 3 is selected and a perfect replacement decision is made, the machine enters State 1 and starts operating at 100% capacity considering the replacement cost.

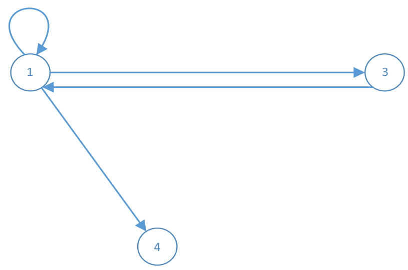

Fig. 3. State diagram of Policy 3

By considering the system state transition probability matrix, machine downtime, lost production cost, and maintenance cost, we can determine the objective function of this study, namely, to minimize the average cost of the production process. The average production and maintenance costs of each policy are determined, and the policy with the minimum average cost is selected.

4. PROBLEM PARAMETERS

The notations used in this paper are as follows:

- P_{ij} – Transition probability of the system from state i to state j
- C_i – Expected operating cost in state i
- F(C) – The objective function that is the average cost of the production process
- T – Time to failure vector
- π_i – Limiting probability of state i
- F_{ij} – Probability of absorption from transient state i to absorbing state j
- P_i – Transition probability matrix of system states at policy i
- C_{ir} – cost of imperfect replacement
- h_{ir} – Downtime due to imperfect replacement
- C_{pr} – cost of perfect replacement
- h_{pr} – Downtime due to perfect replacement
- C_p – the cost per hour of lost production
- Q – Transition probability matrix among the transient states
- R – Elements related to the rows of transient states and columns of absorbing states
- S – Probability of transition between the transient states of the system if the absorbing states are removed
- M – the number of transient states of the system
- N – the fundamental matrix for P, which denotes the expected number of states before being absorbed

5. PROBLEM FORMULATION

If Policy 1 is selected, matrix P, Q, N, T, R, F, S and π are expressed as follows:

$$P = \begin{bmatrix} P_{11} & P_{13} & P_{14} & 0 \\ 0 & P_{33} & P_{34} & P_{35} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \tag{1}$$

$$Q = \begin{bmatrix} P_{11} & P_{13} \\ 0 & P_{33} \end{bmatrix}, \tag{2}$$

$$N = [I - Q]^{-1}, \tag{3}$$

$$T = N * 1, \tag{4}$$

$$R = \begin{bmatrix} P_{14} & 0 \\ P_{34} & P_{35} \end{bmatrix}, \tag{5}$$

$$F = N * R, \tag{6}$$

$$S = \begin{bmatrix} P_{11} + P_{14} & P_{13} \\ P_{34} + P_{35} & P_{33} \end{bmatrix}. \tag{7}$$

When the system enters one of the failure states, then the perfect maintenance is implemented on the machine, and it returns to the new machine (State 1) thus transition probability to State 1 is $P_{11} + P_{14}$.

Limiting probability of each state is obtained using equilibrium equations as follows:

$$\pi * S = \pi, \tag{8}$$

$$\sum_{i=1}^{M} \pi_i = 1. \tag{9}$$

In this case, the objective function of the problem is expressed as follows:

$$F(C) = \frac{\text{Total failure cost}}{\text{Time to failure}} + \text{expected operation cost}$$
$$= \frac{C_4F_4 + C_5F_5}{T_1} + C_1\pi_1 + C_3\pi_3. \tag{10}$$

In the above equation, T_1 is the time of failure, F_4 is the probability of absorption to absorbing State 4, F_5 is the probability of absorption to absorbing State 5, π_1 is limiting the probability of State 1, and π_3 is the limiting probability of State 3.

If Policy 2 is selected, then three transient states and one absorbing state are obtained, thus matrix P is determined as follows:

$$P = \begin{bmatrix} P_{11} & 0 & P_{13} & P_{14} \\ 0 & P_{22} & P_{23} & P_{24} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}. \tag{11}$$

Other matrices are obtained as follows:

$$Q = \begin{bmatrix} P_{11} & 0 & P_{13} \\ 0 & P_{22} & P_{23} \\ 0 & 1 & 0 \end{bmatrix}, \tag{12}$$

$$N = [I - Q]^{-1}, \tag{13}$$

$$T = N * 1, \tag{14}$$

$$R = \begin{bmatrix} P_{14} \\ P_{24} \\ 0 \end{bmatrix}, \tag{15}$$

$$F = N * R, \tag{16}$$

$$S = \begin{bmatrix} P_{11} + P_{14} & 0 & P_{13} \\ P_{24} & P_{22} & P_{23} \\ 0 & 1 & 0 \end{bmatrix} \tag{17}$$

In this case, the objective function of the problem is expressed as follows:

$$F(C) = \frac{(h_{ir} \times C_p) + C_{ir} + C_4F_4}{T_1} + C_1\pi_1 + C_2\pi_2 + C_3\pi_3 . \quad (18)$$

In the above equation, T_1 is the time of failure, F_4 is the probability of absorption to absorbing State 4, π_1 is the limiting probability of State 1, π_2 is the limiting probability of State 2, and π_3 is the limiting probability of State 3.

If Policy 3 is selected, then two transient states and one absorbing state are obtained thus matrix P is determined as follows:

$$P = \begin{bmatrix} P_{11} & P_{13} & P_{14} \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (19)$$

Other matrices are obtained as follows:

$$Q = \begin{bmatrix} P_{11} & P_{13} \\ 1 & 0 \end{bmatrix}, \quad (20)$$

$$R = \begin{bmatrix} P_{14} \\ 0 \end{bmatrix}, \quad (21)$$

$$S = \begin{bmatrix} P_{11} + P_{14} & P_{13} \\ 1 & 0 \end{bmatrix}. \quad (22)$$

In this case, the objective function of the problem is expressed as follows:

$$F(C) = \frac{(h_{pr} \times C_p) + C_{pr} + C_4F_4}{T_1} + C_1\pi_1 + C_3\pi_3 . \quad (23)$$

In the above equation, T_1 is the time of failure, F_4 is the probability of absorption to the absorbing State 4, π_1 is the limiting probability of State 1, and π_3 is the limiting probability of State 3.

6. CASE STUDY

In the steel industry, some vital equipment, such as cooling towers, reformer tubes, clarifiers, etc. have erosive properties, and at first, they start operating at 100% capacity. After starting up and with the failure of some of its minor parts, the performance and capacity of the equipment are reduced, but it does not lead to the total failure of the equipment, and with this low capacity, it is possible to continue the operation of the equipment. Therefore, using Markov chain relations, the optimal maintenance and repair policy for such equipment is determined. A cooling tower, which is one of the vital components of the steel industry, consists of two fans, after the failure of the first fan, the capacity and performance of the equipment are reduced to 50% of its nominal capacity. In this case, it is possible to increase the performance of the equipment to 80% by performing imperfect maintenance or to 100% of the nominal capacity by perfect replacement of the equipment. The objective function of the problem, which is the minimization of the average cost of the production process, is calculated for all types of existing policies, and the optimal policy is determined based on minimum cost.

It should be mentioned that considering 80% and 50% capacity is for some specific machines such as cooling towers, tube reformers, clarifiers, etc. which are used in the steel industry and have erosive properties and initially start operating at 100% capacity and with the failure of one of its minor parts, the performance and capacity of the equipment is reduced to 80%.

The parameters of this machine are expressed as follows:

$$C = \begin{bmatrix} C_1 = 1000, \quad C_2 = 2000, \quad C_3 = 2200, \\ C_4 = 3000b, \quad C_5 = 3500 \end{bmatrix},$$

$$C_p = 300,$$

$$P_1 = \begin{bmatrix} 0.8 & 0.1 & 0.1 & 0 \\ 0 & 0.7 & 0.1 & 0.2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

When the machine state transits down from the good state to the bad state, then by applying an imperfect maintenance action with the cost of $C_{ir} = 500$ and considering $h_{ir} = 10$ hours of production stoppage, the performance of the machine improves, and the machine enters the medium state; thus, the probability transition matrix of the system states is as follows:

$$P_2 = \begin{bmatrix} 0.8 & 0 & 0.1 & 0.1 \\ 0 & 0.8 & 0 & 0.1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

When the machine state transits from a good state to a bad state, then, by applying a perfect replacement and with the cost of $C_{pr} = 1000$ and considering $h_{pr} = 40$ hours of production stoppage, the performance of the machine improves, and the machine enters a good state; thus, the probability transition matrix of the system states is as follows:

$$P_3 = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

According to the above discussion, the machine began operating at State 1 (100 percent capacity). The average production process cost is calculated for each of the three policies. Then, the optimal policy is selected.

Table 1

Values of the objective functions for different policies $C_3 = 2200$

Objective function / Policy	F(C)
1	1775.2
2	2160.9
3	2562.5

According to the results, it is clear that by selecting Policy 1, the average production process cost is equal to 1775.2, which is lower than that of the other two policies; thus, the optimal maintenance and repair policy is to continue the machine operation at a bad state.

If the expected operating cost in State 3 increases to 5000, the cost functions are calculated as follows:

Table 2

Values of the objective functions for different policies $C_3 = 5000$

Policy \ Objective function	F(C)
1	2475.2
2	2440.9
3	2814.5

According to the results, it is clear that by selecting Policy 2, the average production process cost is 2440.9, which is lower than that of the other two policies; thus, the optimal maintenance and repair policy is imperfect maintenance.

If the expected operating cost in State 3 increases to 45 000, the cost functions are calculated as follows:

Table 3

Values of the objective functions for different policies $C_3 = 45\,000$

Policy \ Objective function	F(C)
1	12475.2
2	6440.9
3	6414.5

According to the results, it is clear that by selecting Policy 3, the average production process cost is 6414.5, which is less than that of the other two policies; thus, the optimal maintenance and repair policy is a perfect replacement.

At the end of this section, a sensitivity analysis and determination of the optimal policy based on different values of cost parameter c3 is conducted. The average cost plot for different policies is denoted in Fig. 4. In this figure, the blue line represents the average cost of continuing the production process, orange represents the average cost of applying an imperfect maintenance policy, and gray represents the average cost of applying a perfect replacement policy. According to the figure, when C_3 is lower than 4.768, it is better to continue the production process, when C3 is between 4.768 and 42.400 it is better to apply an imperfect maintenance action on the production process, and when C_3 is more than 42.400, it is better to apply a perfect replacement action on the production process.

According to the numerical analysis, it is concluded that every maintenance and repair policy can be modelled using an absorbing Markov chain. The transition probabilities can be calculated using expert opinions or past information. Using the calculated probabilities, the cost of each policy can be determined to select the optimal decision.

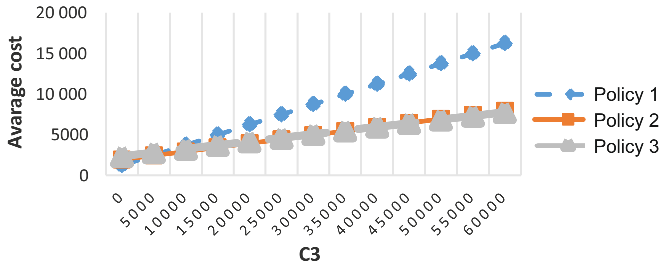

Fig. 4. Average cost of policies with different values of cost parameter C_3

7. CONCLUSIONS

Since the components of machinery are worn out and the failure of any of these parts stops the machine or the production line, and because maintenance and repair costs account for a significant portion of production costs, implementing an optimal maintenance and repair policy is essential for ensuring equipment reliability and reducing downtime costs. This study aimed to use discrete Markov chain relationships, a system-state transition probability matrix and all system costs to calculate the objective function, which minimizes the cost of average production processes and accordingly determines the optimal maintenance and repair policy. At the end of this article, a numerical example in which a machine starts operating at 100% of its rated capacity is given. After some time, and with a certain probability, it is positioned at its decreased capacity. Here, there are three maintenance and repair policies. In future research, assumptions such as considering several machines instead of a single machine and the effect of failure of a single machine on the performance of other machines can be added to the problem. The optimal maintenance and repair policy can then be calculated under different conditions. In addition to cost minimization, machine reliability maximization can be considered in the problem. The optimal maintenance and repair policy can then be determined for the multi-objective optimization problem.

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