10.2478/acsc-2013-0019

Archives of Control Sciences
Volume 23(LIX), 2013
No. 3, *pages 311–331*

Design and experimentation of a self-tuning PID control applied to the 3DOF helicopter

AHSENE BOUBAKIR, SALIM LABIOD, FARES BOUDJEMA and FRANCK PLESTAN

The paper presents design and experimental validation of a stable self-tuning PID controller for three degrees of freedom (3-DOF) helicopter. At first, it is proposed a self-tuned proportional-integral-derivative (PID) controller for a class of uncertain second order multi-input multi-output nonlinear dynamic systems to which the 3-DOF helicopter dynamic model belongs. Within this scheme, the PID controller is employed to approximate unknown ideal controller that can achieve control objectives. PID controller gains are the adjustable parameters and they are updated online with a stable adaptation mechanism designed to minimize the error between the unknown ideal controller and the used by PID controller. The stability analysis of the closed-loop system is performed using Lyapunov approach. It is proven that all signals in the closed-loop system are uniformly ultimately bounded. The proposed approach can be regarded as a simple and effective model-free control since the mathematical model of the system is assumed unknown. Experimental results are presented to verify the effectiveness of the proposed controller.

Key words: 3-DOF helicopter, PID control, adaptive control, model-free control, MIMO nonlinear systems

1. Introduction

Control design for helicopter systems has been a topic of active research in recent years due to their important potential applications. The 3-DOF helicopter prototype is often the system used in helicopter research and education for the design and implementation of control concepts. In our study we consider the 3-DOF helicopter laboratory produced by Quanser consulting Inc. [1]. Because this helicopter has nonlinear and unstable dynamics as well as significant cross-coupling between its control channels, the control of this multi-input multi-output (MIMO) system is a challenging task. Many researchers have investigated the control of 3-DOF helicopters. In [2], Liu *et al.* proposed an optimal tracking control strategy based on fuzzy logic and LQR. In [3], Hao

---

A. Boubakir and S. Labiod are with Faculty of Science and Technology, University of Jijel, BP. 98, Ouled Aissa, 18000, Jijel, Algeria, E-mails: ah_boubakir@yahoo.fr, labiod_salim@yahoo.fr. F. Boudjema is with Department of Automatic Control, National Polytechnic School, Avenue Pasteur, Hassen Badi, BP 182, El Harrach, Algiers, Algeria. E-mail: fboudjema@yahoo.fr.F. Plestan is with IRCCyN, UMR CNRS 6597, Ecole Centrale de Nantes, Nantes, France. E-mail: Franck.Plestan@irccyn.ec-nantes.fr

Received 7.11.2012.

10.2478/acsc-2013-0019

et al. suggested robust LQR attitude control method consisting of three parts: a nominal feed-forward controller, a nominal LQR state feedback controller and a robust compensator. Kiefer *et al.* [4] proposed a control scheme, to ensure the trajectory tracking of a 3-DOF helicopter under input and state constraints. It consists of an inversion-based feed-forward controller for trajectory tracking and a feedback controller for the trajectory error dynamics. In [5], Kutay *et al.* introduced an adaptive output feedback control method based on model inversion with feedback linearization and linearly parameterized neural networks to cancel modeling error. Other control approaches can be found in the literature such as fuzzy logic control [6], robust control [7], predictive control [8], H_∞ control [9], neural networks control [10], and adaptive control [11].

The most classical in automatic control field is the PID control algorithm. Since 1940, emerge of process control, PID controllers are used in most of the feedback loops of process industries despite continual advances in control theory. These controllers are preferred because of their versatility, simple structure, high reliability and easy implementation on the analog or digital platforms. Nowadays, about 90% of industrial objects are controlled by PID controllers [12]. The key idea of designing the PID controller is the choice of three parameters, i.e. proportional gain K_p, integral gain K_I, and derivative gain K_d. To yield satisfactory control results, the values of K_p, K_I and K_d must be tuned. Several approaches have been reported in literature for tuning the parameters of PID controllers [13-15]. Ziegler-Nichols and Cohen-Coon are the most commonly used conventional methods for tuning PID controllers. By reason of the progress in the industrial applications, there are many processes with time-variant or nonlinear characteristics and, hence, the PID controller tuned with conventional tuning methods becomes inefficient for these systems. In order to solve this problem, the adaptive PID controller design has received wide attention. The common design idea of adaptive PID controller is to adjust PID parameters according to varying system states to obtain better control effects.

The PID control of a 3-DOF helicopter has been studied in some papers [16-19]. In [16], Andrievsky *et al.* proposed an adaptive PID control law to ensure the pitch angle control. In [17], Fradkov *et al.* presented a PID control law for the 3-DOF helicopter using state estimation. In [18], Rios *et al.* developed a PID controller with sliding-mode observer used to compensate and identify the disturbance. Rios *et al.* proposed also in [19] a control structure based on PID controller combined with quasi-continuous controller. In the aforementioned papers, the PID controller is not used alone but combined with another control technique. Moreover, the gains of the used PID controller are constants which can be considered as a limitation of these approaches when it comes to strict requirement of tracking error and disturbance rejection.

In this paper, we develop a stable self-tuning PID controller for a 3-DOF helicopter system. Firstly, we introduce the proposed stable self-tuning PID control scheme for a class of uncertain MIMO second order nonlinear systems to which the 3-DOF helicopter dynamic model belongs. The basic idea of this control scheme is to use PID controllers to approximate unknown ideal controllers that can achieve control objectives. For that, the adaptive laws of the gains K_p, K_I and K_d are designed, based on the gradient descent method, to directly minimizing the error between the unknown ideal controllers and the

10.2478/acsc-2013-0019

used PID controllers. The overall closed-loop system stability is studied by using a Lyapunov approach. The proposed self-tuning PID controller guarantees the boundedness of all variables in the closed-loop system and the convergence of the output tracking error to a small neighborhood of the origin. Since the knowledge of the model is not required in this approach, the proposed self-tuning PID controller can be considered as a simple and effective model-free controller. Finally, we examined experimentally the effectiveness and feasibility of the proposed self-tuning PID controller applied to the 3-DOF helicopter.

The paper is organized as follows. Section 2 presents the description of the 3-DOF helicopter system. The proposed self-tuning PID controller scheme is developed in section 3 with its adaptive law and the stability analysis of the overall system. In section 4, the proposed control scheme is used to control in real time the 3-DOF helicopter. Section 5 concludes this article.

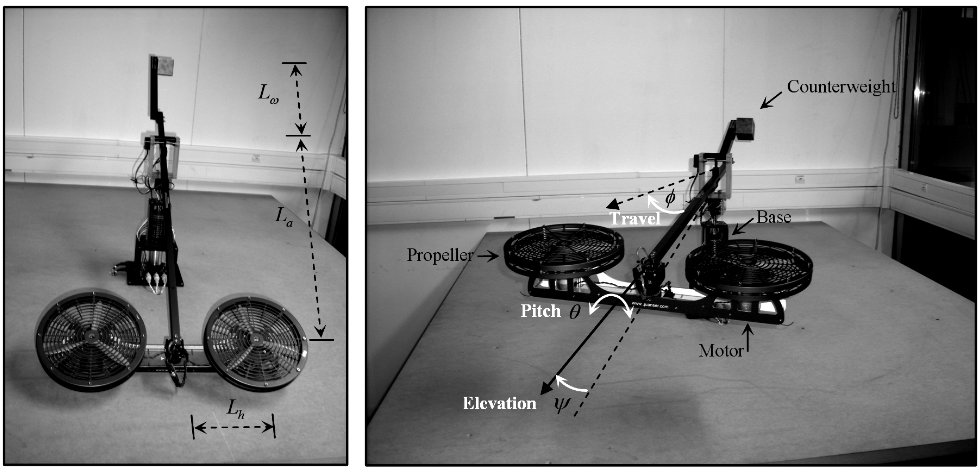

Figure 1. Quanser helicopter with three degrees of freedom.

2. 3-DOF helicopter description

The 3-DOF helicopter setup used in our work is manufactured by Quanser Consulting Inc. [1]. It is a platform technology for researching helicopter flight control system (Fig. 1). This setup is an excellent test-bed for advanced control methods and it consists of a base on which a long arm is mounted. The arm carries the helicopter body composed of two propellers on one end and a counterweight on the other end. Two DC motors are mounted below the propellers to create the forces which drive propellers. The motors axes are parallel and their thrust is vertical to the propellers. We have three degrees of freedom (DOF): elevation (ψ), pitch (θ) and travel (ϕ). To measure these angles, three encoders are installed on elevation axis, pitch axis and travel axis. The movement range

10.2478/acsc-2013-0019

of the elevation ψ and pitch θ angles is limited between around -1 and 1 [rad] due to the hardware restriction.

In the literature, the dynamic modeling of the 3-DOF helicopter was studied in several papers [6,9,10,20]. The equations of motion about axes ψ, θ and θ are given by:

$$\begin{aligned} J_\psi\ddot{\psi} &= -M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega + K_f(V_f+V_b)\cos(\theta)L_a - f_\psi(\psi) \\ J_\theta\ddot{\theta} &= K_f(V_f-V_b)L_h - f_\theta\left(\dot{\theta}\right) \\ J_\phi\ddot{\phi} &= K_f(V_f+V_b)\sin(\theta)L_a - f_\phi\left(\dot{\phi}\right) \end{aligned} \tag{1}$$

where J_ψ, J_θ, J_ϕ denote the moments of inertia, M_h – total mass of the helicopter, M_ω – the mass of the counterweight, L_a – the helicopter distance to pivot, L_ω – the counterweight distances to pivot, L_h – the motor distance to pitch, g – the gravity constant, K_f – the motor volt-to-thrust relationship constant, V_f and V_b the voltages applied to the front and back motors respectively, $f_\psi(\dot{\psi})$, $f_\theta(\dot{\theta})$ and $f_\phi(\dot{\phi})$ – the friction terms. Table 1 provides physical parameters of the helicopter model, taken from the Quanser 3-DOF Helicopter prototype installed in *IRCCyN* laboratory.

Table 1. 3-DOF helicopter parameter values

Parameter	Value	Units
V_f and V_b	[-24, +24]	[V]
K_f	0.1188	[N/V]
g	9.81	[ms^2]
M_h	1.426	[kg]
M_ω	1.87	[kg]
L_a	0.66	[m]
L_ω	0.47	[m]
L_h	0.178	[m]
J_ψ	1.0348	[kgm^2]
J_θ	0.0451	[kgm^2]
J_ϕ	1.0348	[kgm^2]

By setting $u_1 = (V_f + V_b)$ and $u_2 = (V_f - V_b)$, system (1) takes the form

$$\begin{aligned} \ddot{\psi} &= \frac{1}{J_\psi}\left(-M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega - f_\psi(\dot{\psi})\right) + \frac{1}{J_\psi}(K_f L_a)\cos(\theta)u_1 \\ \ddot{\theta} &= \frac{1}{J_\theta}\left(-f_\theta(\dot{\theta})\right) + \frac{1}{J_\theta}(K_f L_h)u_2 \\ \ddot{\phi} &= \frac{1}{J_\phi}\left(-f_\phi(\dot{\phi})\right) + \frac{1}{J_\phi}(K_f L_a)\sin(\theta)u_1. \end{aligned} \tag{2}$$

10.2478/acsc-2013-0019

In this study, our objective is to ensure the convergence of the elevation and travel angles (ψ, ϕ) to the desired trajectories (ψ_d, ϕ_d) while keeping the stability of the pitch angle θ. In the next section, we will develop a stable self-tuning PID control scheme for a class of second order MIMO nonlinear systems.

3. Proposed adaptive PID controller

3.1. Problem formulation

Consider the second order MIMO nonlinear dynamic systems Σ composed of q subsystems Σ_i, $i = 1, 2, \ldots, q$ represented in the following normal form

$$\Sigma_i \begin{cases} \dot{x}_{i1} = x_{i2} \\ \dot{x}_{i2} = f_i(\mathbf{x}) + g_i(\mathbf{x})\, u_i \\ y_i = x_{i1} \end{cases} \tag{3}$$

where $\mathbf{x} = [x_{11}, x_{12}, x_{21}, x_{22}, \ldots, x_{q1}, x_{q2}]^{\mathrm{T}} \in \Re^n$ with $n = 2q$, is the overall state vector which is assumed available for measurement, $\mathbf{u} = [u_1, \ldots, u_q]^{\mathrm{T}} \in \Re^q$ is the control input vector, $\mathbf{y} = [y_1, \ldots, y_q]^{\mathrm{T}} \in \Re^q$ is the output vector, $f_i(\mathbf{x})$ and $g_i(\mathbf{x})$, $i = 1, 2, \ldots, q$ are smooth unknown nonlinear functions.

Since the proposed self-tuning PID controller is a model-free control scheme, we will develop our controller directly for the MIMO nonlinear systems class given by (3).

Let us denote:

$$\mathbf{y}^{(2)} = \begin{bmatrix} y_1^{(2)} \\ \vdots \\ y_q^{(2)} \end{bmatrix}, \quad \mathbf{f}(\mathbf{x}) = \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_q(\mathbf{x}) \end{bmatrix}, \quad \text{and} \quad \mathbf{G}(\mathbf{x}) = \begin{bmatrix} g_1(\mathbf{x}) & 0 & \ldots & 0 \\ 0 & g_2(\mathbf{x}) & 0 & \vdots \\ \vdots & 0 & \ddots & 0 \\ 0 & \ldots & 0 & g_q(\mathbf{x}) \end{bmatrix}.$$

The dynamic system (3) can be rewritten in the following compact form

$$y_i^{(2)} = f_i(\mathbf{x}) + g_i(\mathbf{x})\, u_i. \tag{4}$$

Then, system Σ can be written as

$$\mathbf{y}^{(2)} = \mathbf{f}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\, \mathbf{u}. \tag{5}$$

In this section, our goal is to design a control law $u(t)$ that ensures the boundedness of all variables in the closed-loop system and guarantees output tracking of a specified desired trajectory $y_d(t) = \left[y_{d1}(t), \ldots, y_{dq}(t)\right]^{\mathrm{T}}$.

Throughout this paper we make the following assumptions.

10.2478/acsc-2013-0019

Assumption 1 The matrix $\mathbf{G}(\mathbf{x})$ is positive definite and bounded as $0 < \bar{g}_0\mathbf{I_q} < \mathbf{G}(\mathbf{x}) < \bar{g}_1\mathbf{I_q}$, where $\mathbf{I_q}$ is the $q \times q$ identity matrix, $\bar{g}_0$ and $\bar{g}_1$ are some positive constants.

Assumption 2 The desired trajectory $y_{di}(t)$, $i = 1,\ldots,q$, and its time derivatives $y_{di}^{(j)}(t)$, $j = 1,2$, are smooth and bounded.

Remark 1 The Assumption 1 is a sufficient condition ensuring that the matrix $\mathbf{G}(\mathbf{x})$ is always regular and, therefore, system (3) is feedback linearizable by a static state feedback. Note that the result of this paper can be easily adapted to the case of systems with $-\bar{g}_1\mathbf{I_q} < \mathbf{G}(\mathbf{x}) < -\bar{g}_0\mathbf{I_q} < 0$.

Let us define the tracking errors as

$$\begin{aligned} e_1(t) &= y_{d1}(t) - y_1(t), \\ &\vdots \\ e_q(t) &= y_{dq}(t) - y_q(t) \end{aligned} \tag{6}$$

and the filtered tracking errors as

$$\begin{aligned} s_1(t) &= \left(\tfrac{d}{dt} + \lambda_1\right) e_1(t), \quad \lambda_1 > 0, \\ &\vdots \\ s_q(t) &= \left(\tfrac{d}{dt} + \lambda_q\right) e_q(t), \quad \lambda_q > 0. \end{aligned} \tag{7}$$

From (7), $s_i(t) = 0$ represents a linear differential equation whose solution implies that the tracking error $e_i(t)$ converges to zero with a time constant $1/\lambda_i$. In addition, the derivative $\dot{e}_i(t)$ of the tracking $e_i(t)$ also converges to zero [21]. Thus, the control objective becomes the design of a controller to keep $s_i(t)$ at zero, $i = 1,\ldots,q$, therefore, the original stabilizing problem of the vector $[e_i(t), \dot{e}_i(t)]^{\mathrm{T}}$, $i = 1,\ldots,q$, is reduced to that of keeping the scalar $s_i(t)$ at zero. Moreover, bounds on $s_i(t)$ can be directly translated into bounds on the tracking error. Specifically, if we have $|s_i(t)| \leqslant \Phi_i$ where Φ_i is a positive constant, we can conclude that [21]: $\left|e_i^{(j)}(t)\right| \leqslant 2^j\lambda_i^{j-1}\Phi_i$, $j = 0,1$, $i = 1,\ldots,q$. These bounds can be reduced by increasing the design parameters λ_i.

The time derivatives of the filtered errors (7) can be rewritten as

$$\begin{aligned} \dot{s}_1 &= \Lambda_1 - f_1(\mathbf{x}) - g_1(\mathbf{x})\, u_1, \\ &\vdots \\ \dot{s}_q &= \Lambda_q - f_q(\mathbf{x}) - g_q(\mathbf{x})\, u_q \end{aligned} \tag{8}$$

10.2478/acsc-2013-0019

where $\Lambda_1, \ldots, \Lambda_q$, are given as follows

$$\begin{aligned} \Lambda_1 &= y_{d1}^{(2)} + \lambda_1 \dot{e}_1, \\ &\vdots \\ \Lambda_q &= y_{dq}^{(2)} + \lambda_q \dot{e}_q. \end{aligned} \tag{9}$$

Denote

$$\mathbf{s} = [s_1 \ldots s_q]^{\mathrm{T}}, \quad \mathbf{\Lambda} = [\Lambda_1 \ldots \Lambda_q]^{\mathrm{T}}.$$

Then, (8) can be written in the compact form

$$\dot{\mathbf{s}} = \mathbf{\Lambda} - \mathbf{f}(\mathbf{x}) - \mathbf{G}(\mathbf{x})\mathbf{u}. \tag{10}$$

If the nonlinear functions $\mathbf{f}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are known, to achieve the control goal, one can use the following nonlinear control law

$$\mathbf{u}^* = \mathbf{G}^{-1}(\mathbf{x}) \left(-\mathbf{f}(\mathbf{x}) + \mathbf{\Lambda} + \mathbf{K}\mathbf{s} + \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right)\right) \tag{11}$$

where, $\mathbf{K} = diag[k_1, \ldots, k_q]$, $\mathbf{K}_0 = diag[k_{01}, \ldots, k_{0q}]$, with $k_i > 0$ and $k_{0i} > 0$, for $i = 1, \ldots, q$, ε_0 is a small positive constant, and $\tanh(\cdot)$ is the hyperbolic tangent function defined for the vector $\mathbf{s} = [s_1, \ldots, s_q]^{\mathrm{T}}$ as

$$\tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) = \left[\tanh\left(\frac{s_1}{\varepsilon_0}\right), \ldots, \tanh\left(\frac{s_q}{\varepsilon_0}\right)\right]^{\mathrm{T}}. \tag{12}$$

Effectively, when we select the control input as $\mathbf{u} = \mathbf{u}^*$, equation (10) simplifies to

$$\dot{\mathbf{s}} = -\mathbf{K}\mathbf{s} - \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) \tag{13}$$

or, equivalently

$$\dot{s}_i = -k_i s_i - k_{0i} \tanh\left(\frac{s_i}{\varepsilon_0}\right), \quad i = 1, \ldots, q. \tag{14}$$

From which one can conclude that $s_i(t) \to 0$ as $t \to \infty$ and, therefore, $e_i(t)$ and $\dot{e}_i(t)$ converge to zero [21].

According to the above analysis, the ideal control law (11) is easily obtained if the nonlinear functions $\mathbf{f}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are known. However, in this paper, these nonlinear functions are considered unknown, so the ideal controller (11) cannot be implemented. In this work, we propose to design an adaptive PID control to approximate this unknown ideal controller.

10.2478/acsc-2013-0019

3.2. Adaptive PID Control design

In the previous subsection we have established that there exists an ideal control law $\mathbf{u}^*$ given by (11) that can achieve control objectives. However, this ideal controller cannot be used since it depends on unknown functions. In this subsection, to overcome this problem, we propose to use adaptive PID control for approximating this ideal controller. The error between the adaptive PID controller and the ideal controller will be used to update the free parameters of the PID controller.

3.2.1. Control law

To develop the control law, we assume that each component of the ideal input control vector $\mathbf{u}^* = \left[u_1^*, \ldots, u_q^*\right]^{\mathrm{T}}$ can be approximated by a PID controller u_{pid_i}, $i = 1, \ldots, q$, whose general form is given as follows:

$$u_{pid_i} = K_{p_i} e_i(t) + K_{I_i} \int_0^t e_i(\tau)\, d\tau + K_{d_i} \frac{de_i(t)}{dt} \tag{15}$$

where K_{p_i} is the proportional gain, K_{I_i} is the integral gain, and K_{d_i} is the derivative gain. For convenience, let $\Theta_i = [K_{p_i}, K_{I_i}, K_{d_i}]^{\mathrm{T}}$ and $\Pi_i(e_i) = \left[e_i(t), \int_0^t e_i(\tau)\, d\tau, \frac{de_i(t)}{dt}\right]^{\mathrm{T}}$. Hence, we can rewrite (15) as

$$u_{pid_i}(e_i, \Theta) = \Pi_i^{\mathrm{T}}(e_i)\, \Theta_i. \tag{16}$$

Moreover, we assume that there exists an optimal bounded time varying parameter vector Θ_i^* with a bounded time derivative such that the ideal control u_i^* fulfills

$$u_i^* = \Pi_i^{\mathrm{T}}(e_i)\, \Theta_i^* + \varepsilon_i(\mathbf{x}) \tag{17}$$

where $\varepsilon_i(\mathbf{x})$ is the approximation error, Θ_i^* is an unknown ideal parameter vector which minimizes the function $|\varepsilon_i(\mathbf{x})|$.

Let us denote

$$\varepsilon(\mathbf{x}) = [\varepsilon_1(\mathbf{x}), \ldots, \varepsilon_q(\mathbf{x})]\,\mathrm{T}, \;\; \Theta^* = \left[\Theta_1^{*T}, \ldots, \Theta_q^{*\mathrm{T}}\right]^{\mathrm{T}} \;\text{ and }\; \Pi(e) = \operatorname{diag}[\Pi_1(e_1), \ldots, \Pi_q(e_q)]$$

therefore, we can write

$$\mathbf{u}^* = \Pi^{\mathrm{T}}(e)\, \Theta^* + \varepsilon(\mathbf{x}). \tag{18}$$

Before proceeding we need to introduce an assumption about the approximation error $\varepsilon(\mathbf{x})$.

Assumption 3 The approximation error $\varepsilon(\mathbf{x})$ in (18) is bounded as

$$\varepsilon^{\mathrm{T}}(\mathbf{x})\, \mathbf{G}(\mathbf{x})\, \varepsilon(\mathbf{x}) \leqslant \bar{\varepsilon}_0 \mathbf{s}^{\mathrm{T}} \mathbf{G}(\mathbf{x})\, \mathbf{s} + \bar{\varepsilon}_1 \tag{19}$$

where $\bar{\varepsilon}_0$ and $\bar{\varepsilon}_1$ are two positive constants.

10.2478/acsc-2013-0019

Since the ideal parameter vector Θ^* is unknown, so it should be estimated by a suitable adaptation law. Let Θ be an estimate of the ideal vector Θ^* and define the control law as the adaptive PID approximation of the ideal controller (18), i.e., the control law for system (3) is chosen as

$$\mathbf{u} = u_{pid} = \Pi^T(e)\,\Theta. \tag{20}$$

After the specification of the controller structure, the next step should be the design of an adaptive law for the free parameters Θ such that the control law $\mathbf{u}$ approximates, as best as possible, the ideal controller $\mathbf{u}^*$. To this end, a gradient descend adaptation algorithm will be developed in the next subsection.

3.2.2. Adaptation law for PID control

Our goal in this subsection is to design an adaptive law for the parameter estimates Θ such that the PID controller (20) approximates the unknown ideal controller (18), i.e., the adaptive law should be designed to make the error between $\mathbf{u}^*$ and $\mathbf{u}$ as small as possible. Furthermore, the adaptive law should guarantee the boundedness of the parameters estimates. To this end, let us define the error between u^* and u as

$$\mathbf{e_u} = \mathbf{u}^* - \mathbf{u}. \tag{21}$$

The error e_u represents the actual deviation between the unknown function u^* and the control input u_{pid}. Using (18) and (20), (21) becomes

$$\mathbf{e_u} = \mathbf{u}^* - \Pi^{\mathrm{T}}(e)\,\Theta = \Pi^{\mathrm{T}}(e)\,\widetilde{\Theta} + \varepsilon_i(\mathbf{x}) \tag{22}$$

where $\widetilde{\Theta} = \Theta^* - \Theta$ is the parameter estimation error vector. Adding and subtracting $\mathbf{G}(\mathbf{x})\,\mathbf{u}^*$ to the right-hand side of (10), we obtain the error equation governing the closed-loop system

$$\dot{\mathbf{s}} = \mathbf{\Lambda} - \mathbf{f}(\mathbf{x}) - \mathbf{G}(\mathbf{x})\,\mathbf{u} + \mathbf{G}(\mathbf{x})\,\mathbf{u}^* - \mathbf{G}(\mathbf{x})\,\mathbf{u}^*. \tag{23}$$

With (11) and (22), (23) becomes

$$\dot{\mathbf{s}} = -\mathbf{K}\mathbf{s} - \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) + \mathbf{G}(\mathbf{x})\,\mathbf{e_u}. \tag{24}$$

Now, consider a quadratic cost function that measures the discrepancy between the ideal controller $\mathbf{u}^*$ and the actual PID controller u_{pid}, defined as

$$J(\Theta) = \frac{1}{2}\mathbf{e_u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\,\mathbf{e_u} = \frac{1}{2}\left(\mathbf{u}^* - \Pi^{\mathrm{T}}(e)\,\Theta\right)^{\mathrm{T}}\mathbf{G}(\mathbf{x})\left(\mathbf{u}^* - \Pi^{\mathrm{T}}(e)\,\Theta\right). \tag{25}$$

The gradient descent method is used here to minimize the cost function (25). Hence, by applying the gradient descent method [21][22], we obtain as an adaptive law for the parameters Θ, the following first order differential equation

$$\dot{\Theta} = -\eta\nabla_\theta J(\Theta) \tag{26}$$

10.2478/acsc-2013-0019

where η is a positive constant parameter. From (25), the gradient of $J(\Theta)$ with respect to Θ is

$$\frac{\partial J(\Theta)}{\partial\Theta} = -\Pi(e)\,\mathbf{G}(\mathbf{x})\,\mathbf{e_u}. \tag{27}$$

Therefore, the gradient descent algorithm becomes

$$\dot{\Theta} = \eta\Pi(e)\,\mathbf{G}(\mathbf{x})\,\mathbf{e_u}. \tag{28}$$

We recall here that the ideal controller $\mathbf{u}^*$ is unknown, so the error signal $\mathbf{e_u}$ defined in (21) is not available. Equation (24) will be used to overcome this difficulty. Indeed, from (24), we see that even if the error vector $\mathbf{e_u}$ is not available, the vector $\mathbf{G}(\mathbf{x})\,\mathbf{e_u}$ is available, and it is given by

$$\mathbf{G}(\mathbf{x})\,\mathbf{e_u} = \dot{\mathbf{s}} + \mathbf{K}\mathbf{s} + \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right). \tag{29}$$

Therefore, (28) becomes

$$\dot{\Theta} = \eta\Pi(e)\left\{\dot{\mathbf{s}} + \mathbf{K}\mathbf{s} + \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right)\right\}. \tag{30}$$

As shown in [23], the adaptive law (30) cannot guarantee the boundedness of the parameters $\widetilde{\Theta}$ in the presence of approximation errors that are unavoidable in such adaptive schemes. So, to improve the robustness of the adaptive law (30) in the presence of approximation errors, we modify it by introducing a σ-modification term as follows [23]

$$\dot{\Theta} = \eta\Pi(e)\left\{\dot{\mathbf{s}} + \mathbf{K}\mathbf{s} + \mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right)\right\} - \eta\sigma\Theta \tag{31}$$

where σ is a small positive constant. We notice that the adaptive law is modified so that the time derivative of the Lyapunov function used to analyze this adaptive law becomes negative in the space of the parameter estimates when these parameters exceed certain bound [23].

3.2.3. Stability of the closed-loop system

In order to analyze the tracking error convergence and the stability of the closed-loop system, let us consider the following Lyapunov-like function

$$V = \frac{1}{2}\mathbf{s}^{\mathrm{T}}\mathbf{s} + \frac{1}{2\eta}\widetilde{\Theta}^{\mathrm{T}}\widetilde{\Theta}. \tag{32}$$

The time derivative of (32) can be given as

$$\dot{V} = \mathbf{s}^{\mathrm{T}}\dot{\mathbf{s}} - \frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta} + \frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta}^*. \tag{33}$$

10.2478/acsc-2013-0019

Using (24), (29) and (31), (33) becomes

$$\dot{V}=\mathbf{s}^{\mathrm{T}}\left(-\mathbf{Ks}-\mathbf{K}_0\tanh\left(\frac{s}{\varepsilon_0}\right)+\mathbf{G}(\mathbf{x})\mathbf{e_u}\right)-\widetilde{\Theta}^{\mathrm{T}}\left(\Pi(e)\mathbf{G}(\mathbf{x})\mathbf{e_u}-\sigma\Theta\right)+\frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta}^{*}. \tag{34}$$

With (22), we can write

$$\begin{aligned}\dot{V} &= -\mathbf{s}^{\mathrm{T}}\mathbf{Ks}-\mathbf{s}^{\mathrm{T}}\mathbf{K}_0\tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right)+\mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}\\ &- \left(\mathbf{e}_\mathbf{u}^{\mathrm{T}}-\varepsilon^{\mathrm{T}}(\mathbf{x})\right)\mathbf{G}(\mathbf{x})\mathbf{e_u}+\sigma\widetilde{\Theta}^{\mathrm{T}}\Theta+\frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta}^{*}\end{aligned} \tag{35}$$

or,

$$\begin{aligned}\dot{V} &= -\mathbf{s}^{\mathrm{T}}\mathbf{Ks}-\mathbf{s}^{\mathrm{T}}\mathbf{K}_0\tanh\left(\frac{s}{\varepsilon_0}\right)+\mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}-\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}+\varepsilon^{\mathrm{T}}(\mathbf{x})\mathbf{G}(\mathbf{x})\mathbf{e_u}\\ &+ \sigma\widetilde{\Theta}^{\mathrm{T}}\Theta+\frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta}^{*}.\end{aligned} \tag{36}$$

Using the inequalities

$$\sigma\widetilde{\Theta}^{\mathrm{T}}\Theta=-\frac{\sigma}{2}\left\|\widetilde{\Theta}\right\|^2-\frac{\sigma}{2}\|\Theta\|^2+\frac{\sigma}{2}\left\|\widetilde{\Theta}+\Theta\right\|^2\leqslant-\frac{\sigma}{2}\left\|\widetilde{\Theta}\right\|^2+\frac{\sigma}{2}\|\Theta^{*}\|^2 \tag{37}$$

$$\begin{aligned}\varepsilon^{\mathrm{T}}(\mathbf{x})\mathbf{G}(\mathbf{x})\mathbf{e_u} &= -\left(\frac{1}{2}\mathbf{e_u}-\varepsilon(\mathbf{x})\right)^{\mathrm{T}}\mathbf{G}(\mathbf{x})\left(\frac{1}{2}\mathbf{e_u}-\varepsilon(\mathbf{x})\right)+\frac{1}{4}\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}\\ &+ \varepsilon^{\mathrm{T}}(\mathbf{x})\mathbf{G}(\mathbf{x})\varepsilon(\mathbf{x})\leqslant\frac{1}{4}\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}+\varepsilon^{\mathrm{T}}(\mathbf{x})\mathbf{G}(\mathbf{x})\varepsilon(\mathbf{x})\end{aligned} \tag{38}$$

$$\frac{1}{\eta}\widetilde{\Theta}^{\mathrm{T}}\dot{\Theta}^{*}\leqslant\frac{\sigma}{4}\left\|\widetilde{\Theta}\right\|^2+\frac{1}{\sigma\eta^2}\left\|\dot{\Theta}^{*}\right\|^2 \tag{39}$$

$$\begin{aligned}\mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u} &= -\left(\frac{1}{2}\mathbf{e_u}-\mathbf{s}\right)^{\mathrm{T}}\mathbf{G}(\mathbf{x})\left(\frac{1}{2}\mathbf{e_u}-\mathbf{s}\right)+\frac{1}{4}\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}+\mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{s}\\ &\leqslant \frac{1}{4}\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}+\mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{s}\end{aligned} \tag{40}$$

equation (36) can be bounded as

$$\begin{aligned}\dot{V} &\leqslant -\frac{1}{2}\mathbf{e}_\mathbf{u}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e_u}-\mathbf{s}^{\mathrm{T}}\mathbf{K}_0\tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right)-\mathbf{s}^{\mathrm{T}}\left(\mathbf{K}-\mathbf{G}(\mathbf{x})\right)\mathbf{s}-\frac{\sigma}{4}\left\|\widetilde{\Theta}\right\|^2\\ &+ \frac{\sigma}{2}\|\Theta^{*}\|^2+\frac{1}{\sigma\eta^2}\left\|\dot{\Theta}^{*}\right\|^2+\varepsilon^{\mathrm{T}}(\mathbf{x})\mathbf{G}(\mathbf{x})\varepsilon(\mathbf{x}).\end{aligned} \tag{41}$$

10.2478/acsc-2013-0019

Using the inequality(19), we have

$$\begin{aligned} \dot{V} &\leqslant -\frac{1}{2}\mathbf{e}_{\mathbf{u}}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e}_{\mathbf{u}} - \mathbf{s}^{\mathrm{T}}\mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) - \mathbf{s}^{\mathrm{T}}(\mathbf{K} - \mathbf{G}(\mathbf{x}))\mathbf{s} - \frac{\sigma}{4}\left\|\widetilde{\Theta}\right\|^2 \\ &+ \frac{\sigma}{2}\|\Theta^*\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\Theta}^*\right\|^2 + \varepsilon_0 \mathbf{s}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{s} + \bar{\varepsilon}_1 \end{aligned} \tag{42}$$

or,

$$\begin{aligned} \dot{V} &\leqslant -\frac{1}{2}\mathbf{e}_{\mathbf{u}}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e}_{\mathbf{u}} - \mathbf{s}^{\mathrm{T}}\mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) - \mathbf{s}^{\mathrm{T}}(\mathbf{K} - (1+\bar{\varepsilon}_0)\mathbf{G}(\mathbf{x}))\mathbf{s} - \frac{\sigma}{4}\left\|\widetilde{\Theta}\right\|^2 \\ &+ \frac{\sigma}{2}\|\Theta^*\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\Theta}^*\right\|^2 + \bar{\varepsilon}_1. \end{aligned} \tag{43}$$

Since the parameters Θ^* and its time derivative $\dot{\Theta}^*$, the functions $\varepsilon(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are assumed bounded in this paper, so we can define a positive constant bound Q as

$$Q = \sup_t \left(\frac{\sigma}{2}\|\Theta^*(t)\|^2 + \frac{1}{\sigma\eta^2}\left\|\dot{\Theta}^*(t)\right\|^2 + \bar{\varepsilon}_1\right). \tag{44}$$

Then, (43) can be simplified to

$$\dot{V} \leqslant -\frac{1}{2}\mathbf{e}_{\mathbf{u}}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e}_{\mathbf{u}} - \mathbf{s}^{\mathrm{T}}\mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) - \mathbf{s}^{\mathrm{T}}(\mathbf{K} - (1+\bar{\varepsilon}_0)\mathbf{G}(\mathbf{x}))\mathbf{s} - \frac{\sigma}{4}\left\|\widetilde{\Theta}\right\|^2 + Q. \tag{45}$$

Assuming that the design parameter k_i is chosen such that $k_i > (1+\bar{\varepsilon}_0)\bar{g}_1$, $i = 1,\ldots,q$, and $\gamma = \min\left(2\min_{1\leqslant i\leqslant p}(k_i - (1+\bar{\varepsilon}_0)\bar{g}_1), 0.5\sigma\eta\right)$, the inequality (45) can be written as follows

$$\dot{V} \leqslant -\frac{1}{2}\mathbf{e}_{\mathbf{u}}^{\mathrm{T}}\mathbf{G}(\mathbf{x})\mathbf{e}_{\mathbf{u}} - \mathbf{s}^{\mathrm{T}}\mathbf{K}_0 \tanh\left(\frac{\mathbf{s}}{\varepsilon_0}\right) - \frac{\gamma}{2}\mathbf{s}^{\mathrm{T}}\mathbf{s} - \frac{\gamma}{2\eta}\left\|\widetilde{\Theta}\right\|^2 + Q \tag{46}$$

or,

$$\dot{V} \leqslant -\gamma V + QAC. \tag{47}$$

Now we can prove the following theorem that shows the boundedness of all variables in the closed-loop system.

Theorem 1 *Consider the system* (3). *Suppose that Assumptions 1 and 2 are satisfied and the design parameter k_i is chosen such that $k_i > (1+\bar{\varepsilon}_0)\bar{g}_1$, $i = 1,\ldots,q$. Then the control law defined by* (20) *with the adaptation law given by* (31) *guarantees that the closed-loop system is UUB stable and the output tracking error converges to a small neighborhood of the origin.*

Proof From (47), we can have

$$V(t) \leqslant V(0)e^{-\gamma t} + \frac{Q}{\gamma}. \tag{48}$$

10.2478/acsc-2013-0019

Then from (47), it can be shown that for $V \geqslant Q/\gamma$ we have $\dot{V} \leqslant 0$. According to a standard Lyapunov theorem, the signals $s(t)$, $\widetilde{\Theta}(t)$ and $u(t)$ in the closed-loop system are bounded. Moreover, from (32) and (48) we can write $\|\mathbf{s}(t)\| \leqslant \sqrt{\|\mathbf{s}(0)\|^2 + \frac{1}{\eta}\left\|\widetilde{\Theta}(0)\right\|^2 e^{-0.5\gamma t}} + \sqrt{\frac{2Q}{\gamma}}$, and in order to achieve the tracking error convergence to a small neighborhood around zero, the parameters k_i, σ and η should be chosen appropriately. Then, it is possible to make $\sqrt{\frac{2Q}{\gamma}}$ as small as desired. Denote $\Phi = \sqrt{\frac{2Q}{\gamma}}$, it is easy to see that $\|s(t)\| \leqslant \Phi$ as $t \to \infty$. This implies that the tracking errors converge asymptotically to residual sets as: $\left|e_i^{(j)}(t)\right| \leqslant 2^j \lambda_i^{j-1}\Phi, j = 0,1, i = 1,\ldots,q$. This completes the proof. □

Remark 2 It is worth to point out that in the PID controller (20) there is no robustifying control term. In this paper, the term $\mathbf{K}_0 \tanh(\mathbf{s}/\varepsilon_0)$ in the parameter adaptive law plays, in some way, the role of a robustifying control term. Therefore, the robustness of the controller can be improved by selecting large positive values for the design parameter $\mathbf{K}_0$.

Remark 3 Because the aim of the σ-modification adaptive law (31) is to avoid parameter drift, it does not need to be active when the estimated parameters are within some acceptable bound. Therefore, a more reasonable modification would be to select σ as [23]: $\sigma = 0$, if $\|\Theta\| \leqslant M_\Theta$, $\sigma = \sigma_0$, otherwise; where M_Θ and σ_0 are design positive constants, and $M_\Theta \geqslant \sup_{t\geqslant 0}(\|\Theta^*(t)\|)$.

Remark 4 It is worth noticing that because of the integral structure of the adaptive law (31), this parameter updating law is implementable despite the presence of the time derivative $\dot{\mathbf{s}}(t)$. To show that, rewrite first the adaptation law (31) as

$$\dot{\Theta}_i = \eta\Pi_i(e_i)\left\{\dot{s}_i + k_i s_i + k_{0i}\tanh(s_i/\varepsilon_0)\right\} - \eta\sigma\Theta_i\ , i = 1,\ldots,q. \tag{49}$$

From (7) , the time derivative $\dot{s}_i(t)$ can be written as

$$\dot{s}_i = e_i^{(2)} + \lambda_i \dot{e}_i. \tag{50}$$

Then, (49) can be expressed as

$$\dot{\Theta}_i = \eta\Pi_i(e_i)e_i^{(2)} + \varphi_i \tag{51}$$

where $\varphi_i = \eta\Pi_i(e_i)\left\{\lambda_i\dot{e}_i + k_i s_i + k_{0i}\tanh(s_i/\varepsilon_0)\right\} - \eta\sigma\Theta_i$. From (51), one can obtain $\Theta_i(t)$ as

$$\Theta_i(t) = \Theta_i(0) + \int_0^t \varphi_i\, d\tau + \eta\int_0^t \left(\Pi_i(e_i)e_i^{(2)}\right) d\tau. \tag{52}$$

10.2478/acsc-2013-0019

The first term $\int_0^t \varphi_i d\tau$ is easily computable since φ_i depends only upon measurable signals. However, the second term $\eta \int_0^t \left(\Pi_i(e_i)\, e_i^{(2)}\right) d\tau$ raises the question of the availability of the signal $e_i^{(2)}(t)$. Since $\Pi_i(e_i) = [e_i(t), z_i(t), \dot{e}_i(t)]^{\mathrm{T}}$ with $z_i(t) = \int_0^t e_i(\tau)\, d\tau$, using integration by parts, the three entries of the vector $\int_0^t \left(\Pi_i(e_i)\, e_i^{(2)}\right) d\tau$ can be computed without using $e_i^{(2)}(t)$. The first entry $\int_0^t \left(e_i e_i^{(2)}\right) d\tau$ is computed as

$$\int_0^t \left(e_i e_i^{(2)}\right) d\tau = e_i \dot{e}_i \big|_0^t - \int_0^t (\dot{e}_i)^2 d\tau. \tag{53}$$

The second entry $\int_0^t \left(z_i e_i^{(2)}\right) d\tau$ is given by

$$\int_0^t \left(z_i e_i^{(2)}\right) d\tau = z_i \dot{e}_i \big|_0^t - \int_0^t (e_i \dot{e}_i)\, d\tau. \tag{54}$$

Finally, the third entry $\int_0^t \left(\dot{e}_i e_i^{(2)}\right) d\tau$ is obtained as

$$\int_0^t \left(\dot{e}_i e_i^{(2)}\right) d\tau = \frac{1}{2} \int_0^t \left(\frac{d\dot{e}_i^2}{dt}\right) d\tau = \frac{1}{2}\dot{e}_i^2(t) - \frac{1}{2}\dot{e}_i^2(0). \tag{55}$$

Consequently, the parameters of the PID controllers can be computed without the need of using $e_i^{(2)}(t)$.

4. Self-tuning PID control for the 3-DOF helicopter

The stable self-tuning PID controller developed in the previous section will be used to control the 3-DOF helicopter. Our objective is to ensure the convergence of the elevation and travel angles (ψ, ϕ) to the desired trajectories (ψ_d, ϕ_d). Since the control of the travel rotation require the pitch rotation control, another controller is used to ensure also the convergence of the angle (θ) to the desired angle (θ_d). In each case, a self-tuning PID controller will be used.

From (2), it can be seen that the travel rotation (ϕ) depends on the control input u_1. Indeed, u_1 is the designed total input vector oriented to obtain the desired elevation and

10.2478/acsc-2013-0019

travel angles. Let us define $u_3 = u_1 \sin(\theta)$ as the component of the vector u_1 responsible for the travel rotation. Then, the desired pitch angle θ_d ensuring the suitable travel rotation can be computed by

$$\theta_d = \arcsin\left(\frac{u_3}{u_1 + \Delta\varepsilon}\right) \tag{56}$$

where $\Delta\varepsilon$ is a small positive constant. The dynamic model (2) can be rewritten as follows

$$\begin{aligned}
\ddot{\psi} &= \frac{1}{J_\psi}\left(-M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega - f_\psi(\dot{\psi})\right) + \frac{1}{J_\psi}\left(K_f\cos(\theta)L_a\right)u_1 \\
\ddot{\theta} &= \frac{1}{J_\theta}\left(-f_\theta(\dot{\theta})\right) + \frac{1}{J_\theta}\left(K_f L_h\right)u_2 \\
\ddot{\phi} &= \frac{1}{J_\phi}\left(-f_\phi(\dot{\phi})\right) + \frac{1}{J_\phi}\left(K_f L_a\right)u_3
\end{aligned} \tag{57}$$

In order to simplify the application of the self-tuning PID controller developed in the previous section to the 3-DOF helicopter system, let us define $\mathbf{y} = [\psi, \theta, \phi]$ as the output vector, $\mathbf{u} = [u_1, u_2, u_3]^T$ as the vector of the control inputs and the state space vector by $\mathbf{x} = \left[\psi, \dot{\psi}, \theta, \dot{\theta}, \phi, \dot{\phi}\right]^T$, $\mathbf{f}(\mathbf{x}) = [f_1(\mathbf{x}), f_2(\mathbf{x}), f_3(\mathbf{x})]^T$ and $\mathbf{G}(\mathbf{x}) = diag[g_1(\mathbf{x}), g_2(\mathbf{x}), g_3(\mathbf{x})]^T$ where the elements $f_i(\mathbf{x})$ and $g_i(\mathbf{x})$, $i = 1:3$, are given as the following:

$$\begin{aligned}
f_1(\mathbf{x}) &= \left(1/J_\psi\right)\left(-M_h g\cos(\psi)L_a + M_\omega g\cos(\psi)L_\omega - f_\psi(\dot{\psi})\right), \\
f_2(\mathbf{x}) &= \left(1/J_\theta\right)\left(-f_\theta(\dot{\theta})\right), \\
f_3(\mathbf{x}) &= \left(1/J_\phi\right)\left(-f_\phi(\dot{\phi})\right), \\
g_1(x) &= \left(1/J_\psi\right)\left(K_f\cos(\theta)L_a\right), \\
g_2(x) &= \left(1/J_\theta\right)\left(K_f L_h\right), \\
g_3(x) &= \left(1/J_\phi\right)\left(K_f L_a\right),
\end{aligned}$$

Then, the 3-DOF helicopter system given by (2) can be expressed as

$$\ddot{\mathbf{y}} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} \tag{58}$$

which is in the general input-output form given by (5) with $q = 3$ in this case. Moreover, as long as $-1\,[\text{rad}] < \theta < +1\,[\text{rad}]$, we have $g_1(\mathbf{x}) > 0$ and consequently the matrix $\mathbf{G}(\mathbf{x})$ is positive definite.

The convergence of the angular positions (ψ, θ, ϕ) to the desired trajectory $(\psi_d, \theta_d, \phi_d)$ can be achieved with the control inputs (u_1, u_2, u_3). In this subsection, we describe the application of the control scheme presented in section 3 to control the elevation, the pitch and the travel angles. In the first step, let us define the tracking errors $(e_\psi, e_\theta, e_\phi)$ and the filtering errors $\left(s_\psi(t), s_\theta(t), s_\phi(t)\right)$ as follows

$$\begin{cases}
s_\psi(t) = \dot{e}_\psi(t) + \lambda_\psi e_\psi(t), & e_\psi = \psi - \psi_d \\
s_\theta(t) = \dot{e}_\theta(t) + \lambda_\theta e_\theta(t), & e_\theta = \theta - \theta_d \\
s_\phi(t) = \dot{e}_\phi(t) + \lambda_\phi e_\phi(t), & e_\phi = \phi - \phi_d.
\end{cases} \tag{59}$$

10.2478/acsc-2013-0019

The inputs (u_1, u_2, u_3) are chosen as the outputs of three PID controllers given by

$$\begin{cases} u_1 = K_{p_\psi} e_\psi(t) + K_{I_\psi} \int_0^t e_\psi(\tau)\, d\tau + K_{d_\psi} de_\psi(t)/dt \\ u_2 = K_{p_\theta} e_\theta(t) + K_{I_\theta} \int_0^t e_\theta(\tau)\, d\tau + K_{d_\theta} de_\theta(t)/dt \\ u_3 = K_{p_\phi} e_\phi(t) + K_{I_\phi} \int_0^t e_\phi(\tau)\, d\tau + K_{d_\phi} de_\phi(t)/dt \end{cases} \tag{60}$$

that are updated online with the following parameter adaptation laws

$$\begin{cases} \Theta_\psi = \left[K_{p_\psi}, K_{I_\psi}, K_{d_\psi}\right], & \dot{\Theta}_\psi = \eta \Pi(e_\psi) \left\{\dot{s}_\psi + k_\psi s_\psi + k_{0_\psi} \tanh\left(s_\psi/\varepsilon_0\right)\right\} - \eta \sigma \Theta_\psi \\ \Theta_\theta = \left[K_{p_\theta}, K_{I_\theta}, K_{d_\theta}\right], & \dot{\Theta}_\theta = \eta \Pi(e_\theta) \left\{\dot{s}_\theta + k_\theta s_\theta + k_{0_\theta} \tanh\left(s_\theta/\varepsilon_0\right)\right\} - \eta \sigma \Theta_\theta \\ \Theta_\phi = \left[K_{p_\phi}, K_{I_\phi}, K_{d_\phi}\right], & \dot{\Theta}_\phi = \eta \Pi(e_\phi) \left\{\dot{s}_\phi + k_\phi s_\phi + k_{0_\phi} \tanh\left(s_\phi/\varepsilon_0\right)\right\} - \eta \sigma \Theta_\phi. \end{cases} \tag{61}$$

As it has been demonstrated previously, the PID controllers given by (60) and updated with the adaptive law (61) guarantee the convergence of the angles (ψ, ϕ, θ) to the desired angles $(\psi_d, \phi_d, \theta_d)$. The control voltages V_f and V_b, applied to the front and back motors are computed from the command signals u_1 and u_2 as follows :

$$\begin{cases} V_f = 0.5(u_1 + u_2) \\ V_b = 0.5(u_1 - u_2). \end{cases} \tag{62}$$

4.1. Experiment results

In this section, we will verify experimentally the effectiveness of the proposed stable self-tuning PID controller applied to 3-DOF helicopter system. The control structure is illustrated in Fig. 2 and the parameters of the used helicopter are given in Table 1.

The control law was developed and implemented using Matlab Simulink and Real Time Workshop with a fixed step size of $\Delta t = 0.001$sec. The control objective consists of moving the helicopter from the initial position $(\psi = 0, \phi = 0)$ to the position $(\psi = 20^\circ, \phi = -20^\circ)$. In order to make the desired outputs as smooth curves, the reference trajectories chosen for $\psi_d(t)$, $\theta_d(t)$ and $\phi_d(t)$ are filtered respectively with a second order filter, a first order filter and a six order filter defined by the transfer functions $H_\psi = 1\big/(p+1)^2$, $H_\theta = 1/(p+1)$ and $H_\phi = 1/(p+1)^6$ where p is the *Laplace* variable. The controller parameters used in experiment study are: $\lambda_\psi = \lambda_\phi = 150$ and $\lambda_\theta = 100$, $k_\psi = k_\theta = 3$ and $k_\phi = 0.3$, $k_{0_\psi} = k_{0_\theta} = k_{0_\phi} = 0.1AC$, $\eta = 25$, $\eta = 25$, $\sigma = 0.001$, $\varepsilon_0 = 0.01$. The initial values of the parameter estimates are chosen as: $\Theta_\psi(0) = \Theta_\theta(0) = [0, 0, 0]^{\mathrm{T}}$ and $\Theta_\phi(0) = [0, 0, 50]^{\mathrm{T}}$.

The experiment results appear in Figs. 3-10. Fig. 3 illustrates the time evolution of the desired and actual elevation angle, Fig. 4 shows the desired and actual travel angle and Fig. 5 shows the desired and actual pitch angle. It can be seen from these figures that the actual trajectories $\{\psi(t), \theta(t), \phi(t)\}$ converge to the desired trajectories $\{\psi_d(t), \theta_d(t), \phi_d(t)\}$. The control voltages for front and back motors are shown in Figs. 6 and 7. Fig. 8 illustrates the time evolution of the gains K_{p_1}, K_{I_1} and K_{d_1} for the elevation

10.2478/acsc-2013-0019

angle controller, the gains K_{p_2}, K_{I_2} and K_{d_2} for the travel angle controller are given in Fig. 9 and the gains K_{p_3}, K_{I_3} and K_{d_3} for the pitch angle controller are given in Fig. 10. From the Figs. 8, 9 and 10, it can be seen that all the parameter estimates are bounded. These experimental results demonstrate the performances of the proposed self-tuning PID controller and its effectiveness for control tracking of helicopter systems.

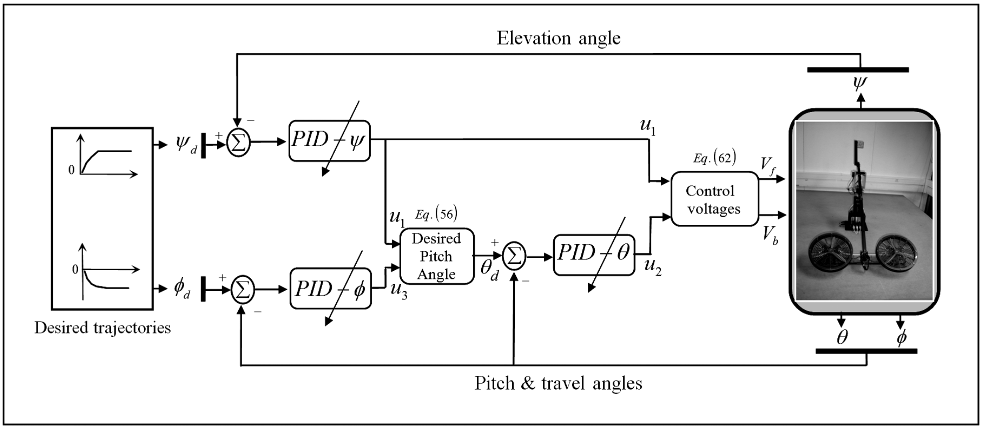

Figure 2. Synoptic scheme of the proposed controller.

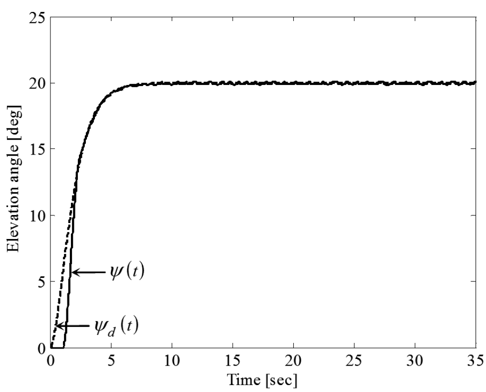

Figure 3. Trajectories $\psi(t)$-solid and $\psi_d(t)$-broken.

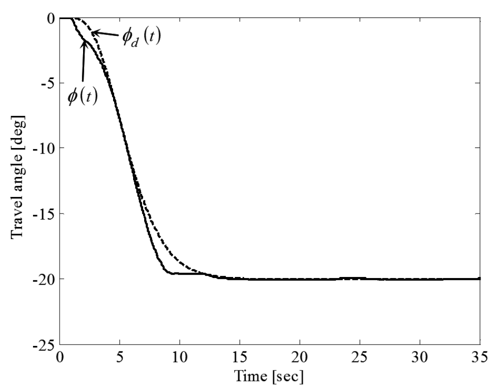

Figure 4. Trajectories $\phi(t)$-line and $\phi_d(t)$-broken.

10.2478/acsc-2013-0019

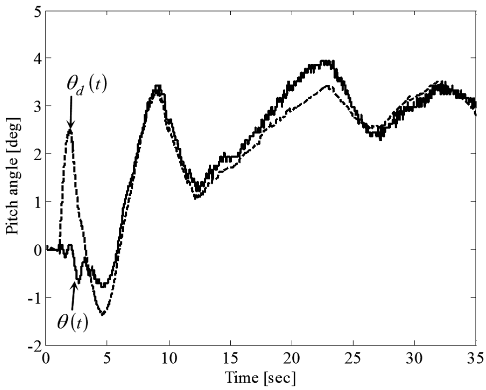

Figure 5. Trajectories $\theta(t)$-solid and $\theta_d(t)$-broken.

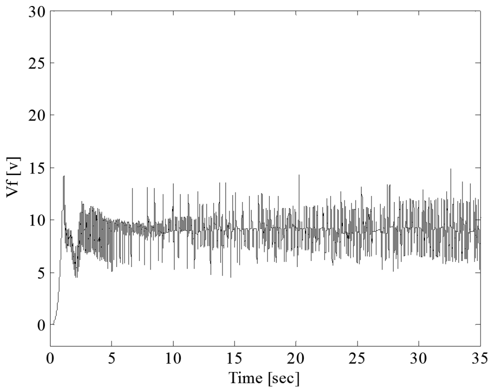

Figure 6. Control voltage V_f.

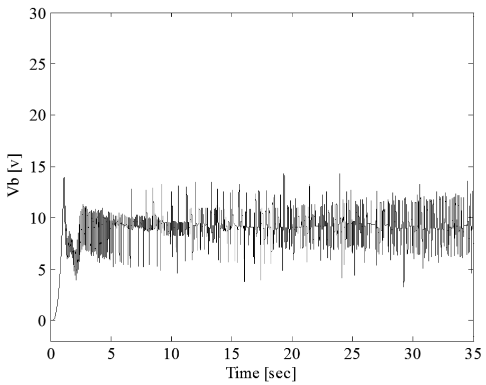

Figure 7. Control voltage V_b.

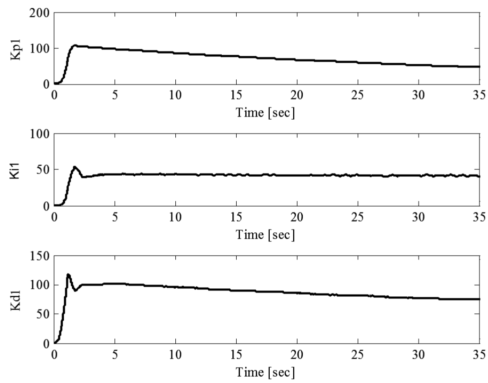

Figure 8. Evolution of the gains K_{p_1}, K_{I_1}, K_{d_1}.

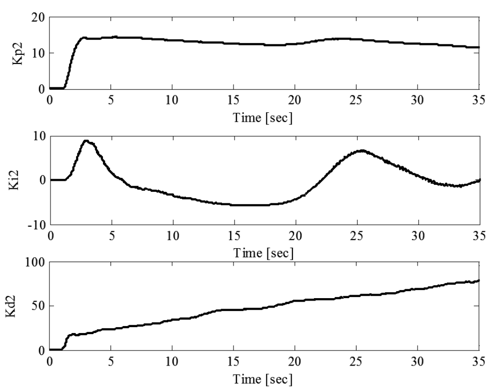

Figure 9. Evolution of the gains K_{p_2}, K_{I_2}, K_{d_2}.

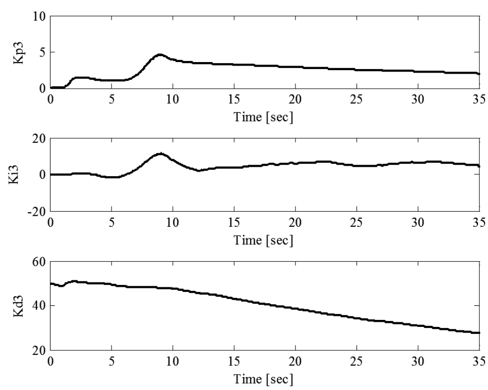

Figure 10. Evolution of the gains K_{p_3}, K_{I_3}, K_{d_3}.

10.2478/acsc-2013-0019

5. Conclusion

In this paper, a stable self-tuning PID controller was proposed and applied in real time for a 3-DOF helicopter which is a nonlinear system, coupled and under-actuated. Since the proposed self-tuning PID control scheme is a model-free control it has been developed for a second order MIMO nonlinear systems class in the general case and then applied to the 3-DOF helicopter system that belongs to this class. The scheme consists of an adaptive PID controller with its adaptive law. The PID algorithm is used to construct adaptively an unknown ideal controller and its adjustable parameters are updated, by using the gradient descent method, in order to minimize the error between the unknown controller and the used PID controller. The proposed control scheme does not require the knowledge of the mathematical model of the plant, guarantees the boundedness of all the signals in the closed-loop system, and ensures the convergence of the tracking errors to a neighborhood of the origin. The ability of the proposed controller has been experimentally examined and tested in the control of a helicopter system. Our objective is to drive the helicopter to a desired elevation and travel angles while keeping the stability of the pitch. Experiment results show the good performances of the proposed controller. Future works will focus to improve our approach by introducing a state observer to provide an estimate of the state vector.

References

[1] J. Apkarian: 3D Helicopter experiment manual. Canada: Quanser Consulting, 1998.

[2] Z. Liu, Z. Choukri El Haj and H. Shi: Control strategy design based on fuzzy logic and LQR for 3-DOF helicopter model. In: *Proc. Int. Conf. on Intelligent Control and Information Processing*, Dalian, China, (2010), 262-266.

[3] L. Hao, Y. Yao, L. Geng and Z. Yisheng: Robust LQR attitude control of 3DOF helicopter. In: *Proc. of the 29th Chinese Control Conference*, Beijing, China, (2010), 529-534.

[4] T. Kiefer, K. Graichen and A. Kugi: Trajectory tracking of a 3DOF laboratory helicopter under input and state constraints. *IEEE Trans. on Control Systems Technology*, **18**(4), (2010), 944-952.

[5] A.T. Kutay, A.J. Calise, M. Idan and N. Hovakimyan: Experimental results on adaptive output feedback control using a laboratory model helicopter. *IEEE Trans. on Control Systems Technology*, **13**(2), (2005), 196-202.

[6] F. Zhou, D. Li and P. Xia: Research of fuzzy control for elevation attitude of 3-DOF helicopter. In *Proc. 2009 Int. Conf. on Intelligent Human-Machine Systems and Cybernetics*, Hangzhou, Zhejiang, China, (2009), 367-370.

10.2478/acsc-2013-0019

[7] Y. Yu and Y. Zhong: Robust attitude control of a 3dof helicopter with multi-operation points. *J. of Systems Science and Complexity*, **22**(2), (2009), 207-219.

[8] J. Witt, S. Boonto and H. Werner: Approximate model predictive control of a 3-DOF helicopter. In *Proc. 46th IEEE Conf. on Decision and Control*, New Orleans, LA, USA, (2007), 4501-4506.

[9] W. Xiuyan, Z. Changli and L. Zongshuai: Robust H-infinity tracing control of 3-DOF helicopter model. In *Proc 2010 Int. Conf. on Measuring Technology and Mechatronics Automation*, Changsha City, China, (2010), 279-282.

[10] F.G. Marques de Carvalho and E.M. Hemerly: Adaptive elevation control of a three degrees-of-freedom model helicopter using neural networks by state and output feedback. *ABCM Symp. Series in Mechatronics*, **3** (2008), 106-113.

[11] M. Ishitobi, M. Nishi and K. Nakasaki: Nonlinear adaptive model following control for a 3-DOF tandem-rotor model helicopter. *Control Engineering Practice*, **18**(8), 2010, 936-943.

[12] H. Zhu, L. Li, Y. Zhao, Y. Guo and Y. Yang: CAS algorithm-based optimum design of PID controller in AVR System. *Chaos, Solitons and Fractals*, **42**(2), (2009), 792-800.

[13] V. Bobál: Technical note self-tuning Ziegler-Nichols PID controller. *Int. J. of Adaptive Control and Signal Processing*, **9**(2), (1995), 213-226.

[14] R. Dittmar, S. Gill, H. Singh and M. Darby: Robust optimization-based multi-loop PID controller tuning: A new tool and its industrial application. *Control Engineering Practice*, **20**(4), (2012), 355-370.

[15] S. Iplikci: A comparative study on a novel model-based PID tuning and control mechanism for nonlinear systems. *Int. J. of Robust and Nonlinear Control*, **20**(13), (2010), 1483-1501.

[16] B. Andrievsky, A. Fradkov and D. Peaucelle: Adaptive control experiments for LAAS "Helicopter" benchmark. In *Proc. Int. Conf. on Physics and Control*, Saint Petersburg, Russia, (2005), 760-766.

[17] A.L. Fradkov, B. Andrievsky and D. Peaucelle: Estimation and control under information constraints for LAAS helicopter benchmark. *IEEE Trans. on Control Systems Technology*, **18**(5), (2010), 1180-1187.

[18] H. Rios, A. Rosales, A. Ferreira and A. Davila: Robust regulation for a 3-DOF helicopter via sliding-modes control and observation techniques. In *2010 American Control Conference*, Baltimore, Maryland, USA, (2010), 4427-4432.

10.2478/acsc-2013-0019

[19] H. RIOS, A. ROSALES and A. DAVILA: Global non-homogeneous quasi-continuous controller for a 3-DOF Helicopter. In *2010 11th Int. Workshop on Variable Structure Systems*, Mexico City, Mexico, (2010), 475-480.

[20] J. SHAN, H.T. LIU and S. NOWOTNY: Synchronised trajectory-tracking control of multiple 3-DOF experimental helicopters. *IEE Proc. Control Theory Applications*, **152**(6), (2005), 683-692.

[21] J.E. SLOTINE and W. LI: Applied Nonlinear Control. Englewood Cliffs, NJ (USA), Prentice Hall, 1991.

[22] S. LABIOD and T.M. GUERRA: Direct adaptive fuzzy control for a class of MIMO nonlinear systems. *Int. J. of Systems Science*, **38**(8), (2007), 665-675.

[23] P.A. IOANNOU and J. SUN: Robust Adaptive Control. Englewood Cliffs, NJ (USA), Prentice Hall, 1996.