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# Optimal reorientation of spacecraft orbit

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The problem of optimal reorientation of the spacecraft orbit is considered. For solving the problem we used quaternion equations of motion written in rotating coordinate system. The use of quaternion variables makes this consideration more efficient. The problem of optimal control is solved on the basis of the maximum principle. An example of numerical solution of the problem is given.

**Key words:** spacecraft, optimal control, quaternion

## 1. Equations of motion

The motion of a spacecraft, which is considered as a material point $B$ of a variable mass, is studied in the coordinate system $OX_1X_2X_3(X)$ with an origin at the point of attraction $O$. The coordinate axes of this coordinate system are parallel to the axes of inertial frame of reference. Control $\boldsymbol{u}$ (vector of jet acceleration) is orthogonal to the orbit's plane. It is known that in this case form and dimensions of spacecraft orbit are fixed, and its orbit turns in the space like an unchangeable figure.

Let us consider the coordinate system $P\xi_1\xi_2\xi_3(\xi)$ with an origin at the orbit pericenter $P$. The axis $\xi_1$ of this coordinate system is directed along the radius vector $\boldsymbol{r}$ of a spacecraft, and the axis $\xi_3$ is aligned with the vector of spacecraft velocity moment $\boldsymbol{c} = \boldsymbol{r} \times d\boldsymbol{r}/dt = \boldsymbol{r} \times \boldsymbol{v}$. The angular position of the $\xi$ coordinate system is specified in the $X$ coordinate system by the normalized quaternion [3]

$$\boldsymbol{\Lambda} = \Lambda_0 + \Lambda_1 \boldsymbol{i}_1 + \Lambda_2 \boldsymbol{i}_2 + \Lambda_3 \boldsymbol{i}_3,$$

$$\|\boldsymbol{\Lambda}\|^2 = \Lambda_0^2 + \Lambda_1^2 + \Lambda_2^2 + \Lambda_3^2 = 1,$$

where $\boldsymbol{i}_1$, $\boldsymbol{i}_2$ and $\boldsymbol{i}_3$ are the unit vectors of a hypercomplex space (Hamilton imaginary units); $\Lambda_j$ $(j = \overline{0,3})$ are the components of the quaternion of orientation $\boldsymbol{\Lambda}$ (parameters of Rodrigue-Hamilton (Euler)), which are identical in the basis sets $X$ and $\xi$.

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Let us denote as $\Omega_k$, $c_k$ and $u_k$ $(k = 1, 2, 3)$ the projections of the vector of the absolute angular velocity $\boldsymbol{\Omega}$ of the $\xi$ coordinate system and of the vectors $\boldsymbol{c}$ and $\boldsymbol{u}$ onto the axes of the $\xi$ coordinate system. The following relations are valid for these quantities

$$\begin{aligned} &u_1 = u_2 = 0, \quad u_3 = u, \\ &c_1 = c_2 = 0, \quad c_3 = |\boldsymbol{c}| = c, \\ &\Omega_1 = u\frac{r}{c}\cos\varphi, \quad \Omega_2 = u\frac{r}{c}\sin\varphi, \quad \Omega_3 = 0, \end{aligned} \tag{1}$$

where $\varphi$ is the true anomaly (which characterizes the spacecraft position in its orbit) and $r = |\mathbf{r}|$.

Let us write equations of motion in the rotating coordinate system $\xi$ using the variables $r$, $c$, $\Lambda_j$, $j = \overline{0,3}$ [5, 6, 7, 8, 9, 10]

$$\begin{aligned} &2\frac{d\Lambda_0}{dt} = -\Omega_1\Lambda_1 - \Omega_2\Lambda_2, \quad 2\frac{d\Lambda_1}{dt} = \Omega_1\Lambda_0 - \Omega_2\Lambda_3, \\ &2\frac{d\Lambda_2}{dt} = \Omega_2\Lambda_0 + \Omega_1\Lambda_3, \qquad 2\frac{d\Lambda_3}{dt} = \Omega_2\Lambda_1 - \Omega_1\Lambda_2, \end{aligned} \tag{2}$$

$$r = \frac{p}{1 + e\cos\varphi}, \quad c = \text{const}, \quad \frac{d\varphi}{dt} = \frac{c}{r^2}, \tag{3}$$

where $p$ and $e$ are the orbit parameter and eccentricity.

It is notable that equations similar to (2) were used in [4, 11].

Subsystem (2) can be written in the quaternion form

$$2\frac{d\boldsymbol{\Lambda}}{dt} = \boldsymbol{\Lambda}\circ\boldsymbol{\Omega}_\xi,$$

$$\boldsymbol{\Omega}_\xi = \Omega_1\boldsymbol{i}_1 + \Omega_2\boldsymbol{i}_2 = u\frac{r}{c}\left(\cos\varphi\,\boldsymbol{i}_1 + \sin\varphi\,\boldsymbol{i}_2\right),$$

where $\boldsymbol{\Omega}$ is the vector of absolute angular velocity of the coordinate system $\xi$.

Here and below, the symbol "$\circ$" denotes quaternion multiplication, the quaternion $\boldsymbol{\Omega}_\xi$ is the mapping of the vector $\boldsymbol{\Omega}$ onto the basis set $\xi$.

The components $\Lambda_j$ of the quaternion $\boldsymbol{\Lambda}$ can be expressed through angular elements of an orbit (characterizing the orientation of the spacecraft instantaneous orbit in space) and the true anomaly. Let us denote the longitude of the ascending node as $\Omega_u$, the orbit inclination as $I$ and the pericenter angular distance as $\omega_\pi$.

Then we have

$$\begin{aligned} &\Lambda_0 = \cos\frac{I}{2}\cos\frac{\Omega_u + \omega_\pi}{2}, \quad \Lambda_1 = \sin\frac{I}{2}\cos\frac{\Omega_u - \omega_\pi}{2}, \\ &\Lambda_2 = \sin\frac{I}{2}\sin\frac{\Omega_u - \omega_\pi}{2}, \quad \Lambda_3 = \cos\frac{I}{2}\sin\frac{\Omega_u + \omega_\pi}{2}. \end{aligned} \tag{4}$$

Thus, the quaternion of orientation $\mathbf{\Lambda}$ of the coordinate system $\xi$ characterizes the orientation of the spacecraft instantaneous orbit.

Let us write (4) in the quaternion form

$$\mathbf{\Lambda} = \left(\cos\frac{\Omega_u}{2} + \boldsymbol{i}_3\sin\frac{\Omega_u}{2}\right)\circ\left(\cos\frac{I}{2} + \boldsymbol{i}_1\sin\frac{I}{2}\right)\circ\left(\cos\frac{\omega_\pi}{2} + \boldsymbol{i}_3\sin\frac{\omega_\pi}{2}\right).$$

For the sake of comparison we write below the equations in angular osculating elements [1, 12], which are usually employed in astrodynamics instead of (2)

$$\frac{d\Omega_u}{dt} = u\frac{r}{c}\sin\Sigma\csc I, \quad \frac{dI}{dt} = u\frac{r}{c}\cos\Sigma,$$
$$\frac{d\omega_\pi}{dt} = -u\frac{r}{c}\sin\Sigma\cot I,$$

where $\Sigma = \omega_\pi + \varphi$ (latitude argument).

## 2. Statement of problem

It is required to determine the bounded (in magnitude) control $\boldsymbol{u}$

$$-u_{max} \leqslant u \leqslant u_{max} < \infty, \quad u = \pm|\boldsymbol{u}|, \tag{5}$$

which transfers the spacecraft whose motion is described by equations (2), (3) from specified initial state

$$t = t_0 = 0, \quad \varphi(0) = \varphi_0, \quad \mathbf{\Lambda}(0) = \mathbf{\Lambda}^0 \tag{6}$$

into the final state

$$t = t^* = ?, \quad \varphi(t^*) = \varphi^*, \quad \mathbf{\Lambda}(t^*) = \pm\mathbf{\Lambda}^* \tag{7}$$

and minimizing the functional

$$J = \int_0^{t^*} \left(\alpha_1 + \alpha_2 u^2\right) dt, \quad \alpha_1, \alpha_2 = \text{const} \geqslant 0. \tag{8}$$

Here $\mathbf{\Lambda}^* = \text{const}$ is the quaternion of the orientation of the desired spacecraft orbit whose components $\Lambda_j^*$ can be expressed through the constant angular elements $\Omega_u^*$, $I^*$ and $\omega_\pi^*$ of this orbit using the relations similar to (4). The values of $c$, $p$, $e$, $\mathbf{\Lambda}^0$, $\mathbf{\Lambda}^*$ and $\varphi_0$ are assumed to be specified.

Functional (8) characterizes the energy consumption for a spacecraft transfer from the initial to final state and the time required for this transfer. At $\alpha_1 = 1$ and $\alpha_2 = 0$ functional $J = t^*$ and in this case the posed problem is a fast response problem.

The final time moment $t^*$ is not fixed and should be determined as a result of solving the problem, therefore, the problem under consideration is a problem with a movable right boundary.

Four components of the quaternion $\mathbf{\Lambda}$ satisfy condition $\Lambda_0^2+\Lambda_1^2+\Lambda_2^2+\Lambda_3^2=1$ so we should change condition (7) onto

$$\operatorname{vect}\left[\widetilde{\mathbf{\Lambda}}(t^*)\circ\mathbf{\Lambda}^*\right]=0, \tag{9}$$

where $\widetilde{\mathbf{\Lambda}}(t^*)$ is a conjugate of $\mathbf{\Lambda}(t^*)$. Vector condition (7) is equivalent to four scalar conditions, and vector condition (9) is equivalent to three scalar conditions. Condition (9) is more efficient for numerical solution of the posed problem.

Note that similar problems were considered earlier by S.A Ishkov and V.A Romanenko [13]; O.M. Kamel, B.E. Mabsout and A.S. Soliman [14, 15]; A. Miele and T. Wang [16]; S.Yu. Ryzhov and I.S. Grigoriev [17]. Unfortunately, most authors were deal with equations in angular elements (or Cartesian coordinates). Also they were often studied only transfers between coplanar or closed to each other orbits.

## 3. Necessary conditions of optimality

The posed problem will be solved based on the Pontryagin's maximum principle. We introduce an additional variable $g$, which satisfies the differential equation $dg/dt=\alpha_1+\alpha_2u^2$ with the boundary condition $g(0)=0$. We also introduce the quaternion conjugate variable $\mathbf{M}$ corresponding to the quaternion phase variable $\mathbf{\Lambda}$ and scalar conjugate variables $\chi$ and $\psi_0$ corresponding to the scalar phase variables $\varphi$ and $g$. Let us compose the Hamilton-Pontryagin function

$$\begin{aligned} H=\psi_0\sigma+\chi\frac{c}{r^2}+\frac{1}{2}[\mathrm{M}_0\left(-\Omega_1\Lambda_1-\Omega_2\Lambda_2\right)+\mathrm{M}_1\left(\Omega_1\Lambda_0-\Omega_2\Lambda_3\right)+\\ +\mathrm{M}_2\left(\Omega_2\Lambda_0+\Omega_1\Lambda_3\right)+\mathrm{M}_3\left(\Omega_2\Lambda_1-\Omega_1\Lambda_2\right)]=\\ =\psi_0\sigma+\chi\frac{c}{r^2}+u\frac{r}{2c}\left(\mathrm{N}_1\cos\varphi+\mathrm{N}_2\sin\varphi\right), \end{aligned} \tag{10}$$

where

$$\begin{aligned} \mathrm{N}_1&=-\mathrm{M}_0\Lambda_1+\mathrm{M}_1\Lambda_0+\mathrm{M}_2\Lambda_3-\mathrm{M}_3\Lambda_2,\\ \mathrm{N}_2&=-\mathrm{M}_0\Lambda_2-\mathrm{M}_1\Lambda_3+\mathrm{M}_2\Lambda_0+\mathrm{M}_3\Lambda_1 \end{aligned} \tag{11}$$

are the components of the quaternion $\mathbf{N}=\widetilde{\mathbf{\Lambda}}\circ\mathbf{M}$; $\mathrm{M}_j$, $j=\overline{0,3}$ are the components of the quaternion $\mathbf{M}$; $\sigma=\alpha_1+\alpha_2u^2$.

The system of equations for conjugate variables has the form

$$
\begin{aligned}
2\frac{d\mathrm{M}_0}{dt} &= -\Omega_1\mathrm{M}_1 - \Omega_2\mathrm{M}_2, \quad & 2\frac{d\mathrm{M}_1}{dt} &= \Omega_1\mathrm{M}_0 - \Omega_2\mathrm{M}_3,\\
2\frac{d\mathrm{M}_2}{dt} &= \Omega_2\mathrm{M}_0 + \Omega_1\mathrm{M}_3, \quad & 2\frac{d\mathrm{M}_3}{dt} &= \Omega_2\mathrm{M}_1 - \Omega_1\mathrm{M}_2,
\end{aligned}
\tag{12}
$$

$$
\frac{d\chi}{dt} = 2\frac{\chi}{r}\frac{dr}{dt} + u\frac{r}{2c}\left(\mathrm{N}_1\sin\varphi - \mathrm{N}_2\cos\varphi\right) - u\frac{r^2}{2c^2}\frac{dr}{dt}\left(\mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi\right). \tag{13}
$$

$$
\frac{d\psi_0}{dt} = 0,
$$

where $\mathrm{N}_1$ and $\mathrm{N}_2$ are determined by relations (11), and $\Omega_1$ and $\Omega_2$ by relations (1).

In accordance with the maximum principle $\psi_0(t^*) \leqslant 0$, therefore by virtue of the last equation of this system and homogeneity of equations, one can choose any $\psi_0(t) \equiv \mathrm{const} < 0$, redetermining the other variables appropriately. In what follows, the multiplier $\psi_0$ in expression (10) for the $H$ function is assumed to be equal to $-1$.

Note that subsystem (12) can be written in the quaternion form

$$
2\frac{d\mathbf{M}}{dt} = \mathbf{M}\circ\boldsymbol{\Omega}_\xi,
$$

$$
\boldsymbol{\Omega}_\xi = \Omega_1\boldsymbol{i}_1 + \Omega_2\boldsymbol{i}_2 = u\frac{r}{c}\left(\cos\varphi\,\boldsymbol{i}_1 + \sin\varphi\,\boldsymbol{i}_2\right),
$$

The optimal control $u^{opt}$ is found from the condition of a maximum in variable $u$ of the $H$ function, determined by relations (10), (11), with allowance made for constraint (5).

1. In the case $\sigma = 1$ (fast response problem)

$$
u^{opt} = u_{max}\,\mathrm{sign}\,k; \quad k = \mathrm{N}_1\cos\varphi + \mathrm{N}_2\sin\varphi. \tag{14}
$$

2. In the case $\sigma = \alpha_1 + \alpha_2 u^2$

$$
u^{opt} = \begin{cases} \dfrac{1}{4\alpha_2}\dfrac{r}{c}k, & \text{for } \dfrac{1}{4\alpha_2}\dfrac{r}{c}|k| \leqslant u_{max},\\ u_{max}\,\mathrm{sign}\,k, & \text{for } \dfrac{1}{4\alpha_2}\dfrac{r}{c}|k| > u_{max}. \end{cases} \tag{15}
$$

Here and below, by the optimal control is meant the control, satisfying the necessary conditions of optimality (Pontryagin's maximum principle). The optimal trajectory is the trajectory corresponding to this control.

Thus the posed problem is reduced to integration ten differential equations (2), (3), (12), (13), (14) (or (15)). When the system of equations is integrated, ten arbitrary constants will appear, the variable $t^*$ being eleventh unknown. For determining the constants we have eleven conditions: eight boundary conditions (6), (9), the relations

$$
t = t^*, \quad \Lambda_0^*\mathrm{M}_0 + \Lambda_1^*\mathrm{M}_1 + \Lambda_2^*\mathrm{M}_2 + \Lambda_3^*\mathrm{M}_3 = 0, \quad \chi = 0,
$$

following from the conditions of transversality, and the equality

$$H^{opt}\big|_{t^*} = H^{opt}\left(\ \mathbf{\Lambda}, \quad \mathbf{M}, \quad \chi, \quad u^{opt}\ \right)\Big|_{t^*} = 0,$$

which takes place for the optimal control $u^{opt}$ and the optimal spacecraft trajectory.

## 4. An example of numerical solution of problem

Figures 1–3 present the results of numerical solution of the boundary value problem of optimization described in Section 2. An algorithm for solving the problem numerically is realized with two methods to solve the boundary value problem: the Newton method and the method of gradient descent. For integration of phase and conjugate equations was used Runge-Kutta method.

For numerical solution of the problem, the equations and relations given in Section 2 were written in dimensionless form (components $\Lambda_j$, $\mathrm{M}_j$, $j = \overline{0,3}$ are dimensionless). The dimensionless variables $r^b$, $t^b$ and control $u^b$ are connected with dimension variables and control by the relations

$$r = Rr^b, \quad t = Tt^b, \quad u = u_{max}u^b,$$

where $R$ is the characteristic distance (the value close to the semi-major axis of the spacecraft orbit was taken as such distance); $V$ and $T$ are the characteristic velocity and time, respectively; they were defined by the relations

$$V = (fM/R)^{1/2}, \quad T = R/V,$$

where $f$ is the gravitational constant, $M$ is the mass of attracting body.

Note that when making conversion to dimensionless variables in the equations for phase and conjugate variables there appears a characteristic dimensionless parameter $N^b = u_{max}R^3/c^2$ and on the figures all variables are dimensionless (index “b” is omitted).

The quantities characterizing the forms and dimensions of spacecraft orbits, initial and final orientations of spacecraft orbit are equal to ($a_{or}$ is the semi-major axis of an orbit; $\Omega_u^0 = \Omega_u(0)$, $I^0 = I(0)$, $\omega_\pi^0 = \omega_\pi(0)$; $\Omega_u^* = \Omega_u(t^*)$, $I^* = I(t^*)$, $\omega_\pi^* = \omega_\pi(t^*)$ ):

$$a_{or} = 25500000\,\text{m.}, \quad u_{max} = 0.101907\,\text{m/s}^2, \quad N^b = 0.35;$$

initial spacecraft position ($\varphi_0 = 3.940323$ rad.)

$$\Omega_u^0 = 40.00^\circ, \quad I^0 = -70.57^\circ, \quad \omega_\pi^0 = 84.98^\circ;$$

$$\Lambda_0^0 = 0.679417, \quad \Lambda_1^0 = 0.245862, \quad \Lambda_2^0 = -0.539909, \quad \Lambda_3^0 = -0.353860;$$

final spacecraft position

variant 1 (small difference between initial and final spacecraft orbits):

$$\Omega_u^* = 36.70^\circ, \quad I^* = -69.06^\circ, \quad \omega_\pi^* = 86.57^\circ;$$

$$\Lambda_0^* = 0.678275, \quad \Lambda_1^* = -0.268667, \quad \Lambda_2^* = -0.577802, \quad \Lambda_3^* = -0.366116;$$

variant 2 (big difference between initial and final spacecraft orbits):

$$\Omega_u^* = 72.00^\circ, \quad I^* = 47.00^\circ, \quad \omega_\pi^* = 45.02^\circ;$$

$$\Lambda_0^* = -0.440542, \quad \Lambda_1^* = -0.522476, \quad \Lambda_2^* = -0.125336, \quad \Lambda_3^* = -0.719189.$$

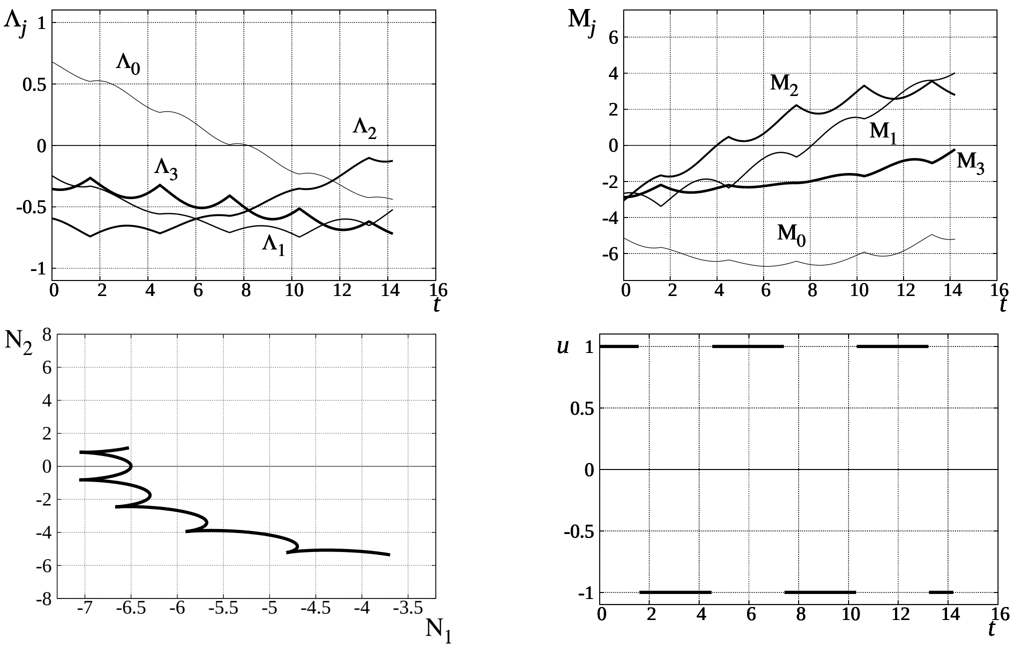


Figure 1. Circular orbit, variant 2, fast response problem.

Scaling factors are equal to:

$$R = 26000000 \text{ m.}, \quad V = 2751.405874\,\text{m/s}, \quad T = 9449.714506 \text{ s.}$$

Note that the problem of optimal control was solved for spacecraft whose initial and final coordinates and velocity projections were taken from [2].

In the case of fast response problem we have relay optimal control, so it is very difficult to solve that problem. In this case the time of flight of a controlled spacecraft

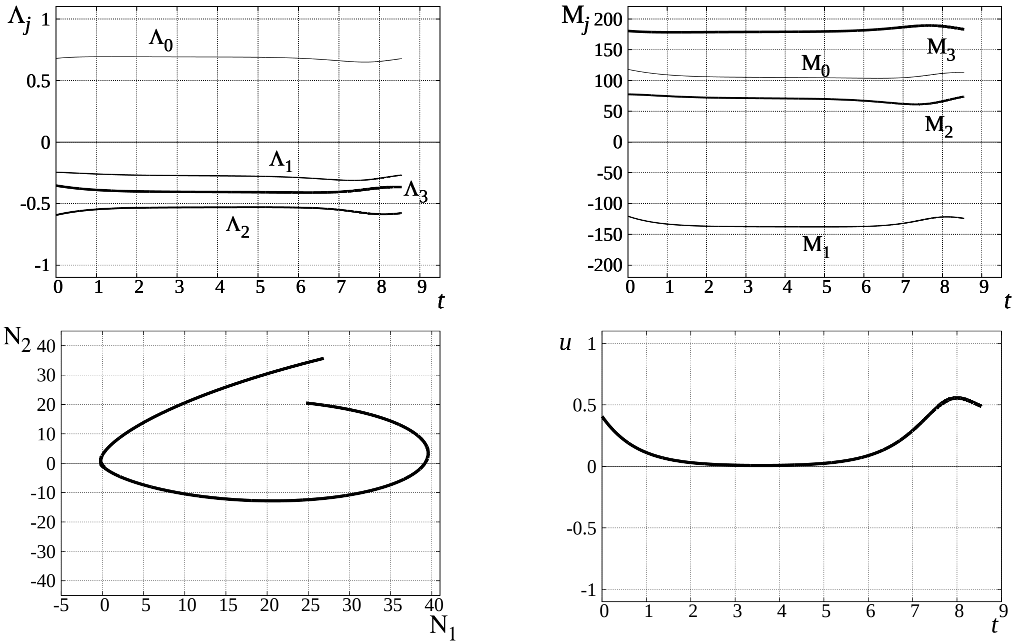


Figure 2. Elliptical orbit $(e = 0.5)$, variant 1, $\int_0^{t^*} (1 + 4.2u^2)\, dt \to \min$.

is 134276.134060 s. (37.298926 h.). Optimal control changes its sign five times during the flight.

The time of flight of a controlled spacecraft in the case when functional $J = = \int_0^{t^*} (1 + 4.2u^2)$ is minimized is equal to 80826.252534 s. (22.451737 h.) for variant 1 and 88049.132367 s. (24.458092 h.) for variant 2. In this case $J$ is equal to 10.817037 and 42.600058 respectively.

Note that when optimal control changes its sign dependence $N_2(N_1)$ changes sharply on the phase portraits.

We also note that durations of second – fifth boost phases in the case of fast response problem are almost equal.

We have found some features and patterns of optimal trajectories and control. When orbit's eccentricity increases from 0 to 0.5 the number of boost phases, their durations and the time of flight decrease. There are no boost phases where control reaches its maximum (in absolute value) in the case of minimizing functional $J$ when orbit's eccentricity is between 0.5 and 0.7.

In the case of little difference between initial and final orbits the time of flight and ranges of variation of the phase and conjugate variables are smaller than in the case of big difference between orbits.

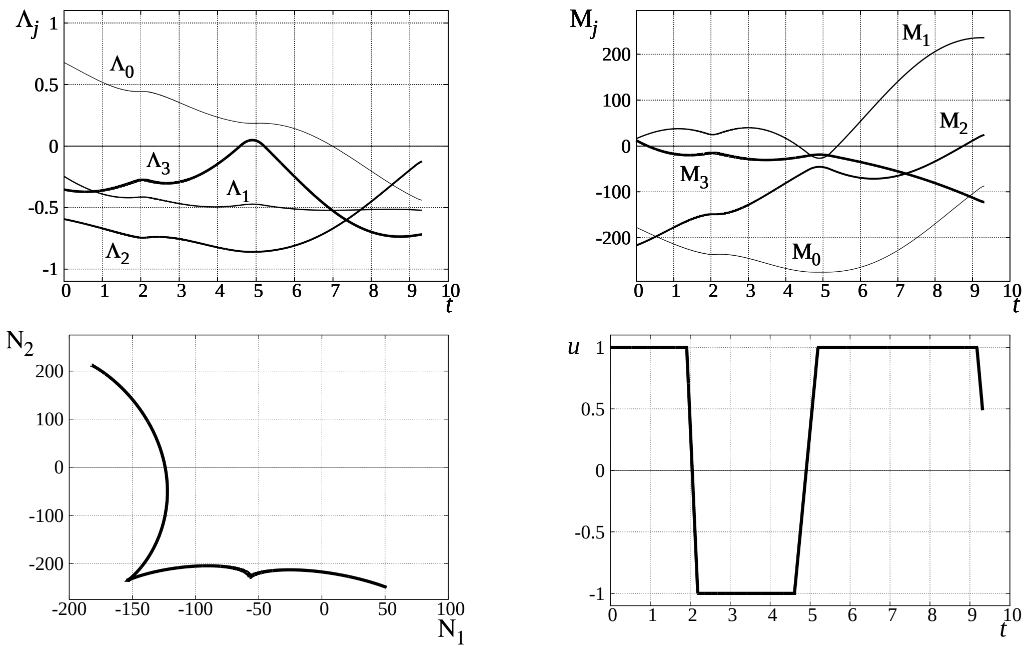


Figure 3. Elliptical orbit ($e = 0.5$), variant 2, $\int_0^{t^*} (1 + 4.2u^2)\, dt \to \min$.

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