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Fusion Kalman filtration with *k*-step delay sharing pattern

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A fusion hierarchical state filtration with $k-$step delay sharing pattern for a multisensor system is considered. A global state estimate depends on local state estimates determined by local nodes using local information. Local available information consists of local measurements and $k-$step delay global information - global estimate sent from a central node. Local estimates are transmitted to the central node to be fused. The synthesis of local and global filters is presented. It is shown that a fusion filtration with $k-$step delay sharing pattern is equivalent to the optimal centralized classical Kalman filtration when local measurements are transmitted to the center node and used to determine a global state estimate. It is proved that the $k-$step delay sharing pattern can reduce covariances of local state errors.

Key words: fusion Kalman filtration, multisensor system, $k-$ step delay sharing pattern.

1. Introduction

It is well known that an optimal state estimate for a linear dynamic system can be determined by using a Kalman filter. Conventional Kalman filtration requires that all process measurements are sent to a central node which determines an estimate of the system state. The centralized architecture produces an optimal estimate in a minimum mean square error (MMSE) sense, but it may require high processing and communication loads or may imply low survivability.

A lot of real systems use a large number of sensors. These systems are known as multisensor systems. Practical applications of the multisensor systems find applications in many areas such as robotics, aerospace, image processing, military surveillance. The systems have an advantage over a systems with a single sensor e.g. improved reliability, robustness, extended coverage, improved resolution. In these systems a state filtration given the measurements is very important practical problem.

Theoretically, a classical Kalman filter may be carried out to determine a state estimate of the multisensor system. Because of some drawbacks of this approach [11] fusion algorithms and appropriate architectures are proposed.

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In many papers a centralized optimal state estimate is calculated from estimates determined by local nodes. The global estimate is equivalent to the optimal centralized one.

In [7] fusion algorithm without feedback (a fusion center does not broadcast information to local nodes) was presented. In [11] a fusion filtration algorithm with feedback (the fusion center transmits its latest global state estimate to the local nodes), suggested in [4], was analysed.

Fusion algorithms guaranteed only local optimality are presented in [1, 2, 3].

Different methodologies to obtain non-centralized state estimation algorithms and their implementations are discussed in [10].

A comprehensive review of the data state fusion state domain is given in [8].

In [5] a Kalman filter with one-step delay information structure, suggested in [4], was educed. In a hierarchical filtration local nodes compute state estimates basing on local current information and one step delayed global information from a fusion center. It was shown that Kalman filtration is optimal and is equivalent to the corresponding centralized one.

In this paper a hierarchical fusion system with $k-$ step delay sharing pattern is presented . These equations are educed by directly derivation of a Kalman filter. It is shown that for proposed architecture Kalman fusion is optimal and is equivalent to the corresponding centralized Kalman filtering formula. An advantage of this structure is analysed.

2. Preleminaries

Let us consider a multisensor system described by a state equation

$$x_{n+1} = A_n x_n + w_n \tag{1}$$

and measurement equations

$$y_n^j = C_n^j x_n + r_n^j, \quad j = 1, \ldots, M \tag{2}$$

where x_n, y_n^j are the state and the measurement from the jth sensor (jth local node), respectively; A_n, C_n^j are the system and observation models, w_n, r_n^j are the state and measurement noises, respectively.

It is assumed that $x_0 \sim N(\bar{x}_0, X_0), w_n \sim N(\bar{w}_n, W_n)$, $r_n^j \sim N(0, R_n^j)$ and $x_n \in R^k, w_n \in R^k, y_n^j \in R^{p^j}$, $r_n^j \in R^{p^j}$; $A_n \in R^{k\times k}$, $C_n^j \in R^{p^j\times k}$. Additionally, w_n, r_m^j are gaussian white noise processes independent of each other and of the gaussian initial state x_0.

Let us denote by $y_n = [y_n^{1T}, \ldots, y_n^{MT}]^T, C_n = [C_n^{1T}, \ldots, C_n^{MT}]^T, r_n = [r_n^{1T}, \ldots, r_n^{MT}]^T, R_n = E r_n r_n^T = \text{block diag}\{R_n^1, \ldots, R_n^M\}$.

The classical covariance Kalman filter $\hat{x}_{n+1|n+1} = E(x_{n+1}|y_0, y_1, \ldots, y_{n+1})$ is described by the equations [9]

$$\hat{x}_{n+1|n+1} = \hat{x}_{n+1|n} + K_{n+1}(y_{n+1} - C_{n+1}\hat{x}_{n+1|n}) \tag{3}$$

where

$$\hat{x}_{n+1|n} = E(x_{n+1}|y_0, y_1, \dots, y_n) = A_n \hat{x}_{n|n} + \bar{w}_n. \tag{4}$$

The matrix gain K_{n+1} is described as

$$K_{n+1} = P_{n+1|n} C_{n+1}^T (C_{n+1} P_{n+1|n} C_{n+1}^T + R_{n+1})^{-1} \tag{5}$$

where

$$P_{n+1|n} = A_n P_{n|n} A_n^T + W_n \tag{6}$$

and

$$P_{n|n} = (\mathbf{1} - K_n C_n) P_{n|n-1}. \tag{7}$$

An initial condition $\hat{x}_{0|0}$ results from the eqn. (3)

$$\hat{x}_{0|0} = \bar{x}_0 + K_0 (y_0 - C_0 \bar{x}_0). \tag{8}$$

The covariance matrix $P_{0|-1}$ can be determined as

$$P_{0|-1} = X_0. \tag{9}$$

Classical covariance filter presented above can be described in an information form [6] as

$$\begin{aligned}
\hat{x}_{n+1|n+1} \quad &= P_{n+1|n+1} \left[P_{n+1|n}^{-1} \hat{x}_{n+1|n} + C_{n+1}^T (R_{n+1})^{-1} y_{n+1} \right] = \\
&= P_{n+1|n+1} \left[P_{n+1|n}^{-1} \hat{x}_{n+1|n} + \sum_{j=1}^{M} C_{n+1}^{jT} (R_{n+1}^j)^{-1} y_{n+1}^j \right] \\
&\hat{x}_{n+1|n} = A_n \hat{x}_{n|n} + \bar{w}_n \\
&P_{n+1|n+1}^{-1} = P_{n+1|n}^{-1} + C_{n+1}^T (R_{n+1})^{-1} C_{n+1} = \\
&= P_{n+1|n}^{-1} + \sum_{j=1}^{M} C_{n+1}^{jT} (R_{n+1}^j)^{-1} C_{n+1}^j.
\end{aligned} \tag{10}$$

Information filter has some computational advantages in multisensor systems where the matrix $C_n^T (R_n)^{-1} C_n$ is usually of high dimension and nondiagonal.

The global estimate performed by the central node depends on information state vectors $C_{n+1}^{jT} (R_{n+1}^j)^{-1} y_{n+1}^j$ and information matrices $C_{n+1}^{jT} (R_{n+1}^j)^{-1} C_{n+1}^j, \quad j = 1, \dots, M$, that can be performed and transmitted by local nodes to the central node. It may increase processing speed.

Sometimes it is better to perform Kalman filtration by every local node upon its own available information and then transmit local state estimates to the fusion center, where a fusion is carried out.

3. Problem statement

Let us assume that local nodes perform local state estimates of the system (1) basing on assumed available local information.

It is known a solution in the case when local estimates $\hat{x}^j_{n|n}, \quad j = 1,\ldots,M$, are based on the local information $\vec{y}^j_n = \{y^j_0,\ldots,y^j_n\}$. It leads to distributed Kalman filter fusion without feedback [7].

In [5] the case when local estimates $\hat{x}^j_{n|n}, \quad j = 1,\ldots,M$, are based on the information $\vec{y}^j_n = \{y_0,\ldots,y_{n-1},y^j_n\}$ was considered. It leads to distributed Kalman filter fusion with one step delay feedback. Let us notice that the local node has global measurement information of the system with one step delay.

In the paper a synthesis of local filters with $k-$step delay feedback information is presented. In this case local estimates $\hat{x}^j_{n|n}, \quad j = 1,\ldots,M$, are based on the information $\vec{y}^j_n = \{y_0,\ldots,y_{n-k},y^j_{n-k+1},\ldots,y^j_n\}$.

The problem is to find

$$\hat{x}^j_{n|n} = E(x_n|\vec{y}^j_n). \tag{11}$$

Local estimates are sent to the central node to be fused and to obtain a global state estimate.

An advantage of the $k-$step feedback (in the sense of local filtering performance) will be discussed.

4. Kalman filtering with one step delay sharing pattern

Let us assume that the jth local estimate of the state x_{n+1} is based on the local information $\vec{y}^j_{n+1,1} = \{y_0,\ldots,y_n,y^j_{n+1}\} = \{\vec{y}_n,y^j_{n+1}\}$.

The local filtration problem for the jth mode is to find

$$\hat{x}^j_{n+1|n+1,1} = E(x_{n+1}|\vec{y}^j_{n+1,1}). \tag{12}$$

It was shown in [5] that

$$\hat{x}^j_{n+1|n+1,1} = A_n\hat{x}_{n|n} + \bar{w}_n + K^j_{n+1,1}\left[y^j_{n+1} - C^j_{n+1}(A_n\hat{x}_{n|n} + \bar{w}_n)\right] \tag{13}$$

where $\hat{x}_{n|n}$ is a global estimate sent by a fusion center to the jth local node with one step delay.

The matrix gain $K^j_{n+1,1}$ is described as

$$K^j_{n+1,1} = P_{n+1|n}C^{jT}_{n+1}(C^j_{n+1}P_{n+1|n}C^{jT}_{n+1} + R^j_{n+1})^{-1} \tag{14}$$

The covariance matrix $P_{n+1|n}$ is described by the eqn. (6).

Additionally it was shown that

$$C^{jT}_{n+1}(R^j_{n+1})^{-1}y^j_{n+1} = (P^j_{n+1|n+1,1})^{-1}\hat{x}^j_{n+1|n+1,1} - P^{-1}_{n+1|n}\hat{x}_{n+1|n} \tag{15}$$

and

$$C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1} = (P^j_{n+1|n+1,1})^{-1} - P^{-1}_{n+1|n} \tag{16}$$

where an inverse of a local covariance matrix $P^j_{n+1|n+1,1}$ defined as $P^j_{n+1n+1,1} = E(x_{n+1} - \hat{x}^j_{n+1|n+1,1})(x_{n+1} - \hat{x}^j_{n+1|n+1,1})^T$ has the form

$$(P^j_{n+1|n+1,1})^{-1} = P^{-1}_{n+1|n} + C^{jT}_{n+1}(R^j_{n+1})^{-1}C^j_{n+1}. \tag{17}$$

Using the eqn. (15) and (16) in the eqn. (10) gives

$$\begin{aligned}
\hat{x}_{n+1|n+1} &= P_{n+1|n+1}\left[\sum_{j=1}^{M}(P^j_{n+1|n+1,1})^{-1}\hat{x}^j_{n+1|n+1,1} - (M-1)P^{-1}_{n+1|n}\hat{x}_{n+1|n}\right] \\
\hat{x}_{n+1|n} &= A_n\hat{x}_{n|n} + \bar{w}_n \\
P^{-1}_{n+1|n+1} &= \sum_{j=1}^{M}(P^j_{n+1|n+1,1})^{-1} - (M-1)P^{-1}_{n+1|n}.
\end{aligned} \tag{18}$$

Equations (18) describe the fusion Kalman filter that generates optimal global state estimate according to (10). Local node needs its own local measurement and global information from the central node with one step delay to generate the local state estimate. Thus communication from central node to the local nodes is needed. That is why this algorithm is known as the fusion algorithm with one step delay feedback.

5. Local covariance Kalman filter with k-step delay sharing pattern

Let us assume that the jth local estimate of the state x_{n+1} is based on the local information $\vec{y}^j_{n+1,k} = \{\vec{y}_{n+1-k}, y^j_{n+2-k}, \ldots, y^j_n, y^j_{n+1}\}$, where $\vec{y}_{n+1-k} = \{y_0, \ldots, y_{n+1-k}\}$.

The local filtration problem for the jth mode is to find

$$\hat{x}^j_{n+1|n+1,k} = E(x_{n+1}|\vec{y}^j_{n+1,k}) = E(x_{n+1}|\vec{y}_{n+1-k}, y^j_{n+2-k}, \ldots, y^j_n, y^j_{n+1}). \tag{19}$$

For the system described by the eqn. (1) and (2) the random vector $[x^T_{n+1}, \vec{y}^T_{n+1-k}, y^{jT}_{n+2-k}, \ldots, y^{jT}_n, y^{jT}_{n+1}]^T$ is gaussian.

Thus the estimate $\hat{x}^j_{n+1|n+1,k}$ results from the relation [9]

$$\hat{x}^j_{n+1|n+1,k} = E(x_{n+1}|\vec{y}_{n+1-k}, y^j_{n+2-k}, \ldots, y^j_n) + E(x_{n+1}|\tilde{y}^j_{n+1|n,k-1}) - Ex_{n+1} \tag{20}$$

where

$$\widetilde{y}^{j}_{n+1|n,k-1} = y^{j}_{n+1} - E(y^{j}_{n+1}|\vec{y}_{n+1-k}, y^{j}_{n+2-k}, \ldots, y^{j}_{n}). \tag{21}$$

The random vector $[x^{T}_{n+1}, \widetilde{y}^{jT}_{n+1|n,k-1}]^{T}$ is gaussian, thus [9]

$$E(x_{n+1}|\widetilde{y}^{j}_{n+1|n,k-1}) =$$
$$= Ex_{n+1} + \overbrace{P_{x_{n+1}\widetilde{y}^{j}_{n+1|n,k-1}} P^{-1}_{\widetilde{y}^{j}_{n+1|n,k-1}\widetilde{y}^{j}_{n+1|n,k-1}}}^{K^{j}_{n+1,k}} (\widetilde{y}^{j}_{n+1|n,k-1} - E\widetilde{y}^{j}_{n+1|n,k-1}). \tag{22}$$

Inserting the eqn. (22) to the eqn. (20) yields

$$\hat{x}^{j}_{n+1|n+1,k} =$$
$$= E(x_{n+1}|\vec{y}_{n+1-k}, y^{j}_{n+2-k}, \ldots, y^{j}_{n}) + K^{j}_{n+1,k}(\widetilde{y}^{j}_{n+1|n,k-1} - E\widetilde{y}^{j}_{n+1|n,k-1}). \tag{23}$$

Let us notice that

$$E\widetilde{y}^{j}_{n+1|n,k-1} = E\left[y^{j}_{n+1} - E(y^{j}_{n+1}|\vec{y}_{n+1-k}, y^{j}_{n+2-k}, \ldots, y^{j}_{n})\right] = 0. \tag{24}$$

Thus the eqn. (23) can be written in the form

$$\hat{x}^{j}_{n+1|n+1,k} = \overbrace{E(x_{n+1}|\vec{y}_{n+1-k}, y^{j}_{n+2-k}, \ldots, y^{j}_{n})}^{\hat{x}^{j}_{n+1|n,k-1}} + K^{j}_{n+1,k}(y^{j}_{n+1} - C^{j}_{n+1}\hat{x}^{j}_{n+1|n,k-1}) \tag{25}$$

where

$$\hat{x}^{j}_{n+1|n,k-1} = E(x_{n+1}|\vec{y}_{n+1-k}, y^{j}_{n+2-k}, \ldots, y^{j}_{n}) = A_{n}\hat{x}^{j}_{n|n,k-1} + \bar{w}_{n}. \tag{26}$$

From the eqn. (21) and (2) we have

$$\widetilde{y}^{j}_{n+1|n,k-1} = y^{j}_{n+1} - C^{j}_{n+1}\hat{x}^{j}_{n+1|n,k-1} = C^{j}_{n+1}\overbrace{(x_{n+1} - \hat{x}^{j}_{n+1|n,k-1})}^{\widetilde{x}^{j}_{n+1|n,k-1}} + r^{j}_{n+1}. \tag{27}$$

Thus the matrix $P_{\widetilde{y}^{j}_{n+1|n,k-1}\widetilde{y}^{j}_{n+1|n,k-1}}$ in (22) can be derived as

$$P_{\widetilde{y}^{j}_{n+1|n,k-1}\widetilde{y}^{j}_{n+1|n,k-1}} =$$
$$= E(\widetilde{y}^{j}_{n+1|n,k-1}\widetilde{y}^{jT}_{n+1|n,k-1}) = C^{j}_{n+1}\overbrace{E\widetilde{x}^{j}_{n+1|n,k-1}\widetilde{x}^{jT}_{n+1|n,k-1}}^{P^{j}_{n+1|n,k-1}} C^{jT}_{n+1} + R^{j}_{n+1} \tag{28}$$

where

$$\widetilde{x}^j_{n+1|n,k-1} = x_{n+1} - \hat{x}^j_{n+1|n,k-1} = A_n \overbrace{(x_n - \hat{x}^j_{n|n,k-1})}^{\widetilde{x}^j_{n|n,k-1}} + w_n - \bar{w}_n. \tag{29}$$

The matrix $P_{x_{n+1}\widetilde{y}^j_{n+1|n,k-1}}$ in the eqn. (22) has the form

$$\begin{aligned} P_{x_{n+1}\widetilde{y}^j_{n+1|n,k-1}} &= E(x_{n+1} - \bar{x}_{n+1})\widetilde{y}^{jT}_{n+1|n,k-1} = \\ &= E\overbrace{(\widetilde{x}^j_{n+1|n,k-1} + \hat{x}^j_{n+1|n,k-1})}^{x_{n+1}}\widetilde{y}^{jT}_{n+1|n,k-1} = P^j_{n+1|n,k-1}C^{jT}_{n+1}. \end{aligned} \tag{30}$$

Thus the matrix gain $K^j_{n+1,k}$ defined in the eqn. (22) results from the eqn. (28) and (30) as

$$K^j_{n+1,k} = P^j_{n+1|n,k-1}C^{jT}_{n+1}(C^j_{n+1}P^j_{n+1|n,k-1}C^{jT}_{n+1} + R^j_{n+1})^{-1}. \tag{31}$$

The matrix $P^j_{n+1|n,k-1}$ defined in the eqn. (28) can be found as

$$P^j_{n+1|n,k-1} = A_n \overbrace{E\widetilde{x}^j_{n|n,k-1}\widetilde{x}^{jT}_{n|n,k-1}}^{P^j_{n|n,k-1}} A^T_n + W_n. \tag{32}$$

Using the eqn. (25) and (27) gives

$$\begin{aligned} &\widetilde{x}^j_{n|n,k-1} = x_n - \hat{x}^j_{n|n,k-1} = x_n - \hat{x}^j_{n|n-1,k-2} - K^j_{n,k-1}\widetilde{y}^j_{n|n-1,k-2} = \\ &= \widetilde{x}^j_{n|n-1,k-2} - K^j_{n,k-1}\widetilde{y}^j_{n|n-1,k-2} \end{aligned} \tag{33}$$

and

$$\begin{aligned} P^j_{n|n,k-1} &= E\widetilde{x}^j_{n|n-1,k-2}\widetilde{x}^{jT}_{n|n-1,k-2} - EK^j_{n,k-1}\widetilde{y}^j_{n|n-1,k-2}\widetilde{x}^{jT}_{n|n-1,k-2} = \\ &= (\mathbf{1} - K^j_{n,k-1}C^j_n)P^j_{n|n-1,k-2}. \end{aligned} \tag{34}$$

For any $n = 0, 1, .., k-1$, an available information for the jth node is defined as $\vec{y}^j_n = \{y^j_0, ..., y^j_n\}$. Thus the jth local filter may be determined from the eqn. (3)-(9) for the system described by the eqn. (1) and (2).

For any $n+1 \geqslant k$ the jth local state estimate $\hat{x}^j_{n+1|n+1,k}$ can be found by a recursive way starting with $\hat{x}^j_{n-k+2|n-k+2,1}$ (the estimate with one step delay feedback described in the section 4). Next we can determine the estimates $\hat{x}^j_{n-k+3|n-k+3,2}, ..., \hat{x}^j_{n|n,k-1}, \hat{x}^j_{n+1|n+1,k}$ from the eqn. (25)-(26) with (31), (32) and (34).

The local estimates $\hat{x}^j_{n+1|n+1,k}$, $j = 1, 2, ..., M$, should be sent to the central node to generate optimal global state estimate $\hat{x}_{n+1|n+1}$ according to the eqn. (10).

To do this an information form of the local covariance filter will be determined.

6. Information form of the local Kalman filter

Let us notice that the eqn. (25) can be written as

$$\hat{x}^j_{n+1|n+1,k} = (\mathbf{1} - K^j_{n+1,k}C^j_{n+1})\hat{x}^j_{n+1|n,k-1} + K^j_{n+1,k}y^j_{n+1}. \tag{35}$$

Let us transform $(\mathbf{1} - K^j_{n+1,k}C^j_{n+1})$ and $K^j_{n+1,k}$ to an appropriate form. We have

$$\mathbf{1} - K^j_{n+1,k}C^j_{n+1} = \overbrace{(\mathbf{1} - K^j_{n+1,k}C^j_{n+1})P^j_{n+1|n,k-1}}^{P^j_{n+1|n+1,k}(34)}(P^j_{n+1|n,k-1})^{-1}. \tag{36}$$

Denote by

$$O^j_{n+1,k} = C^j_{n+1}P^j_{n+1|n,k-1}C^{jT}_{n+1} + R^j_{n+1}. \tag{37}$$

Multiplying the both sides of the eqn. (31) by $O^j_{n+1,k}$ gives

$$K^j_{n+1,k}\overbrace{(C^j_{n+1}P^j_{n+1|n,k-1}C^{jT}_{n+1} + R^j_{n+1})}^{O^j_{n+1,k}(37)} = P^j_{n+1|n,k-1}C^{jT}_{n+1}(R^j_{n+1})^{-1}. \tag{38}$$

Thus

$$K^j_{n+1,k}R^j_{n+1} = (\mathbf{1} - K^j_{n+1,k}C^j_{n+1})P^j_{n+1|n,k-1}C^{jT}_{n+1} \tag{39}$$

and

$$K^j_{n+1,k} = P^j_{n+1|n+1,k}C^{jT}_{n+1}(R^j_{n+1})^{-1}. \tag{40}$$

Inserting the eqn. (36) and(40) to the eqn. (35) gives

$$\hat{x}^j_{n+1|n+1,k} = \\ = P^j_{n+1|n+1,k}(P^j_{n+1|n,k-1})^{-1}\hat{x}^j_{n+1|n,k-1} + P^j_{n+1|n+1,k}C^{jT}_{n+1}(R^j_{n+1})^{-1}y^j_{n+1}. \tag{41}$$

From the eqn. (41) we have

$$C^{jT}_{n+1}(R^j_{n+1})^{-1}y^j_{n+1} = (P^j_{n+1|n+1,k})^{-1}\hat{x}^j_{n+1|n+1,k} - (P^j_{n+1|n,k-1})^{-1}\hat{x}^j_{n+1|n,k-1}. \tag{42}$$

The relation (42) will be used in the eqn. (10) to determine the global state estimate $\hat{x}_{n+1|n+1}$.

Now we determine a recursive form of the covariance matrix $(P^j_{n+1|n+1,k})^{-1}$.

The covariance matrix $P^j_{n+1|n+1,k}$ results from the eqn. (34) and can be written by

$$P^j_{n+1|n+1,k} = P^j_{n+1|n,k-1} - K^j_{n+1,k} O^j_{n+1,k} \overbrace{(O^j_{n+1,k})^{-1} C^j_{n+1} P^j_{n+1|n,k-1}}^{K^{jT}_{n+1,k}(31)} = \\ = P^j_{n+1|n,k-1} - K^j_{n+1,k} O^j_{n+1,k} K^{jT}_{n+1,k}. \tag{43}$$

The eqn. (43) has the form

$$P^j_{n+1|n+1,k} = \\ P^j_{n+1|n,k-1} - K^j_{n+1,k} R^j_{n+1} K^{jT}_{n+1,k} - K^j_{n+1,k} C^j_{n+1} P^j_{n+1|n,k-1} C^{jT}_{n+1} K^{jT}_{n+1,k}. \tag{44}$$

But from the eqn. (36) we have

$$K^j_{n+1,k} C^j_{n+1} = \mathbf{1} - P^j_{n+1|n+1,k} (P^j_{n+1|n,k-1})^{-1}. \tag{45}$$

Inserting the eqn (45) to the eqn. (44) yields

$$P^j_{n+1|n+1,k} = \tag{46}$$
$$= P^j_{n+1|n+1,k} (P^j_{n+1|n,k-1})^{-1} P^j_{n+1|n+1,k} + P^j_{n+1|n+1,k} C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1} P^j_{n+1|n+1,k}.$$

Multiplying two-times the both sides of the eqn.(47) by $(P^j_{n+1|n+1,k})^{-1}$ gives

$$(P^j_{n+1|n+1,k})^{-1} = (P^j_{n+1|n,k-1})^{-1} + C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1}. \tag{47}$$

Thus

$$C^{jT}_{n+1} (R^j_{n+1})^{-1} C^j_{n+1} = (P^j_{n+1|n+1,k})^{-1} - (P^j_{n+1|n,k-1})^{-1}. \tag{48}$$

The relation (48) will be used in the eqn. (10) to determine the inverse of the covariance $P_{n+1|n+1}$.

7. Optimal global filtering with k-step delay sharing pattern

The jth local estimate can be determined from the eqn. (3)-(9), for $n = 0, 1, .., k-1$. Global state estimate results from the eqn. (10) and has the form

$$\hat{x}_{n+1|n+1} = \\ P_{n+1|n+1} \left\{ P^{-1}_{n+1|n} \hat{x}_{n+1|n} + \sum_{j=1}^{M} \left[(P^j_{n+1|n+1})^{-1} \hat{x}^j_{n+1|n+1} - (P^j_{n+1|n})^{-1} \hat{x}^j_{n+1|n} \right] \right\} \tag{49}$$

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{j=1}^{M} \left[(P^{j}_{n+1|n+1})^{-1} - (P^{j}_{n+1|n})^{-1} \right] \tag{50}$$

$$(P^{j}_{n+1|n+1})^{-1} = (P^{j}_{n+1|n})^{-1} + C^{jT}_{n+1}(R^{j}_{n+1})^{-1}C^{j}_{n+1} \tag{51}$$

$$P^{j}_{n+1|n} = A_n P^{j}_{n|n} A^{T}_{n} + W_n. \tag{52}$$

Using the eqn. (42) and (48) in the eqn. (10), for $n+1 \geqslant k$, gives

$$\hat{x}_{n+1|n+1} = \tag{53}$$
$$P_{n+1|n+1}\left\{ P^{-1}_{n+1|n}\hat{x}_{n+1|n} + \sum_{j=1}^{M}\left[(P^{j}_{n+1|n+1,k})^{-1}\hat{x}^{j}_{n+1|n+1,k} - (P^{j}_{n+1|n,k-1})^{-1}\hat{x}^{j}_{n+1|n,k-1} \right] \right\}$$

$$P^{-1}_{n+1|n+1} = P^{-1}_{n+1|n} + \sum_{j=1}^{M} \left[(P^{j}_{n+1|n+1,k})^{-1} - (P^{j}_{n+1|n,k-1})^{-1} \right]. \tag{54}$$

The local node needs its own local measurement and the global state estimate from the central node with $k-$step delay to generate the local state estimate. Thus a communication from the central node to the local nodes is need. That is why this algorithm may be named as the fusion algorithm with $k-$step delay feedback.

8. The quality of the k-step delay sharing pattern

Let us notice that the centralized Kalman filtering (10) and the fusion algorithm with k step delay sharing pattern are exactly equivalent. Thus the feedback does not improve the performance at the central node.

Does the $k-$step feedback reduce local state filtering error?

Let us compare local covariance matrices $P^{j}_{n+1|n+1,k}$ and $P^{j}_{n+1|n+1}$described by the eqn. (47) and (51), respectively.

It was shown [5], that in the one step delay information structure, local error state estimate may be reduced in the sense that

$$(P^{j}_{n+1|n+1,1}) \leqslant (P^{j}_{n+1|n+1}). \tag{55}$$

Thus for the $k-$step delay structure we have

$$P^{j}_{n-k+2|n-k+2,1} \leqslant P^{j}_{n-k+2|n-k+2} \tag{56}$$

and consequently

$$\begin{aligned}
&\overbrace{A_{n-k+2}P^j_{n-k+2|n-k+2,1}A^T_{n-k+2}+W_{n-k+2}}^{P^j_{n-k+3|n-k+2,1}(32)} \leqslant \overbrace{A_{n-k+2}P^j_{n-k+2|n-k+2}A^T_{n-k+2}+W_{n-k+2}}^{P^j_{n-k+3|n-k+2}(52)}\\
&(P^j_{n-k+3|n-k+2,1})^{-1} \geqslant (P^j_{n-k+3|n-k+2})^{-1}\\
&\overbrace{(P^j_{n-k+3|n-k+2,1})^{-1}+C^j_{n-k+3}(R^j_{n-k+3})^{-1}C^{jT}_{n-k+3}}^{(P^j_{n-k+3|n-k+3,2})^{-1}(47)} \geqslant\\
&\geqslant \overbrace{(P^j_{n-k+3|n-k+2})^{-1}+C^j_{n-k+3}(R^j_{n-k+3})^{-1}C^{jT}_{n-k+3}}^{(P^j_{n-k+3|n-k+3})^{-1}(51)}\\
&P^j_{n-k+3|n-k+3,2} \leqslant P^j_{n-k+3|n-k+3}
\end{aligned} \tag{57}$$

Working recursively we obtain

$$\begin{aligned}
&(P^j_{n|n,k-1}) \leqslant (P^j_{n|n})\\
&\overbrace{A_nP^j_{n|n,k-1}A^T_n+W_n}^{P^j_{n+1|n,k-1}(32)} \leqslant \overbrace{A_nP^j_{n|n}A^T_n+W_n}^{P^j_{n+1|n}(52)}\\
&(P^j_{n+1|n,k-1})^{-1} \geqslant (P^j_{n+1|n})^{-1}\\
&\overbrace{(P^j_{n+1|n,k-1})^{-1}+C^j_{n+1}(R^j_{n+1})^{-1}C^{jT}_{n+1}}^{(P^j_{n+1|n+1,k})^{-1}(47)} \geqslant \overbrace{(P^j_{n+1|n})^{-1}+C^j_{n+1}(R^j_{n+1})^{-1}C^{jT}_{n+1}}^{(P^j_{n+1|n+1})^{-1}(51)}\\
&P^j_{n+1|n+1,k} \leqslant P^j_{n+1,n+1}.
\end{aligned} \tag{58}$$

By the eqn. (58) it is seen that the $k-$step delay feedback information structure may reduce the local state filtering errors for $n+1 \geqslant k$.

9. Conclusion

A new hierarchical fusion filtration formula for a multisensor system with the $k-$step delay sharing pattern was presented. It was shown that the fusion algorithm with the $k-$ step delay information feedback is equivalent to the centralized Kalman filter, thus is optimal. Comparing with a structure without feedback this algorithm improves local performances in the sense that it reduces local state error covariances.

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