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Analysis and adaptive control of a novel 3-D conservative no-equilibrium chaotic system

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First, this paper announces a seven-term novel 3-D conservative chaotic system with four quadratic nonlinearities. The conservative chaotic systems are characterized by the important property that they are volume conserving. The phase portraits of the novel conservative chaotic system are displayed and the mathematical properties are discussed. An important property of the proposed novel chaotic system is that it has no equilibrium point. Hence, it displays hidden chaotic attractors. The Lyapunov exponents of the novel conservative chaotic system are obtained as $L_1 = 0.0395, L_2 = 0$ and $L_3 = -0.0395$. The Kaplan-Yorke dimension of the novel conservative chaotic system is $D_{KY} = 3$. Next, an adaptive controller is designed to globally stabilize the novel conservative chaotic system with unknown parameters. Moreover, an adaptive controller is also designed to achieve global chaos synchronization of the identical conservative chaotic systems with unknown parameters. MATLAB simulations have been depicted to illustrate the phase portraits of the novel conservative chaotic system and also the adaptive control results.

Key words: chaos, chaotic system, conservative chaotic system, adaptive control, synchronization.

1. Introduction

Chaos theory describes the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1].

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Lü-Chen system [7], Liu system [8], Cai system [9], T-

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system [10], etc. Many new chaotic systems have been also discovered like Li system [11], Sundarapandian systems [12, 13], Vaidyanathan systems [14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26], Pehlivan system [27], Jafari system [28], Sampath system [29], Pham systems [30, 31], etc.

Chaos theory has applications in several fields of science and engineering such as lasers [32], oscillators [33], chemical reactions [34, 35], biology [36], ecology [37], neural networks [38, 39], robotics [40, 41], fuzzy logic [42, 43], electrical circuits [44, 45, 46], cryptosystems [47, 48], memristors [49, 50, 51],etc.

In the chaos literature, there is an active interest in the discovery of conservative chaotic systems [52], which have the special property that the volume of the flow is conserved. If the sum of the Lyapunov exponents of a chaotic system is zero, then the system is conservative. Classical examples of conservative chaotic systems are Nosé-Hoover system [53], Hénon-Heiles system [54], etc. A recent example of a conservative chaotic system is Vaidyanathan-Pakiriswamy system [55].

In this paper, we announce a novel 3-D conservative chaotic system which does not possess any equilibrium point. Thus, the novel chaotic system belongs to the class of chaotic systems with hidden attractors [56]. Studying systems with hidden attractors is a new research direction because of their practical and theoretical importance [57, 58].

Next, this paper derives an adaptive control law that stabilizes the novel conservative chaotic system, when the system parameters are unknown. This paper also derives an adaptive control law that achieves global chaos synchronization of the identical novel conservative systems with unknown parameters.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the synchronization approaches, the master-slave or drive-response formalism is used. If a particular chaotic system is called the master or drive system and another chaotic system is called the slave or response system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In the chaos literature, an impressive variety of techniques have been proposed to solve the problem of chaos synchronization such as active control method [59, 60, 61], adaptive control method [62, 63, 64], backstepping control method [66, 67, 68], sliding mode control method [69, 70, 71], etc.

All the main adaptive results in this paper are proved using Lyapunov stability theory [72]. MATLAB simulations are depicted to illustrate the phase portraits of the novel conservative chaos system, adaptive stabilization and synchronization results for the novel 3-D conservative chaotic system.

2. A seven-term 3-D novel conservative chaotic system

In this section, we describe a seven-term novel conservative chaotic system with four quadratic nonlinearities, which is modeled by the 3-D dynamics

$$\begin{aligned} \dot{x}_1 &= ax_2 + x_1x_3 \\ \dot{x}_2 &= -bx_1 + x_2x_3 \\ \dot{x}_3 &= 1 - x_1^2 - x_2^2 \end{aligned} \tag{1}$$

where x_1, x_2, x_3 are the states and a, b are constant, positive, parameters of the system.

The system (1) exhibits a *conservative chaotic attractor* for the values

$$a = 0.05 \;\text{ and }\; b = 1. \tag{2}$$

For numerical simulations, we take the initial conditions of the state $x(t)$ as $x_1(0) = -1$, $x_2(0) = -1$ and $x_3(0) = 4$.

Fig. 1 shows the 3-D phase portrait of the conservative chaotic attractor of the system (1). Figs. 2–4 show the 2-D projection of the chaotic attractor of the system (1) on $(x_1, x_2), (x_2, x_3)$ and (x_1, x_3) planes, respectively.

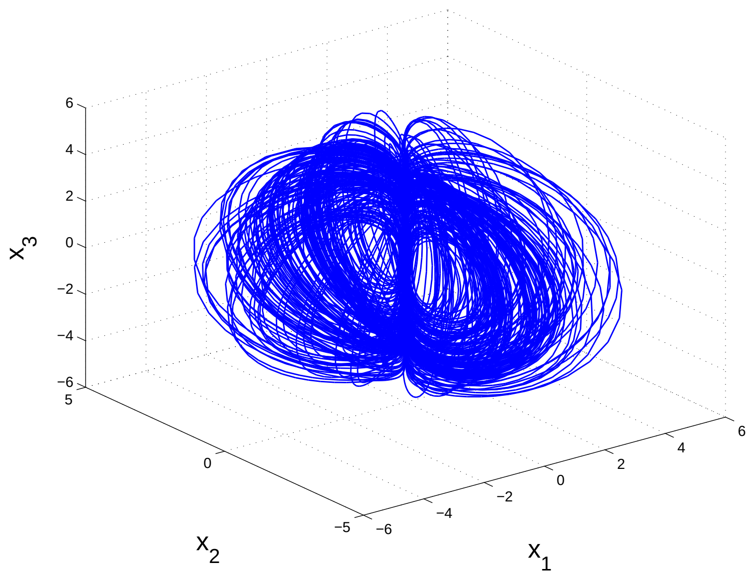

Figure 1: Phase portrait of the conservative chaotic System

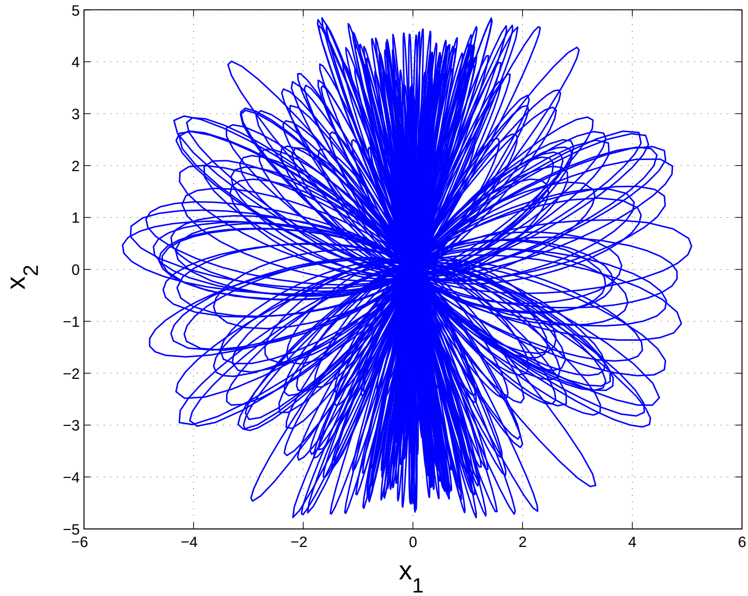

Figure 2: 2-D projection of the conservative chaotic system on the (x_1, x_2) plane

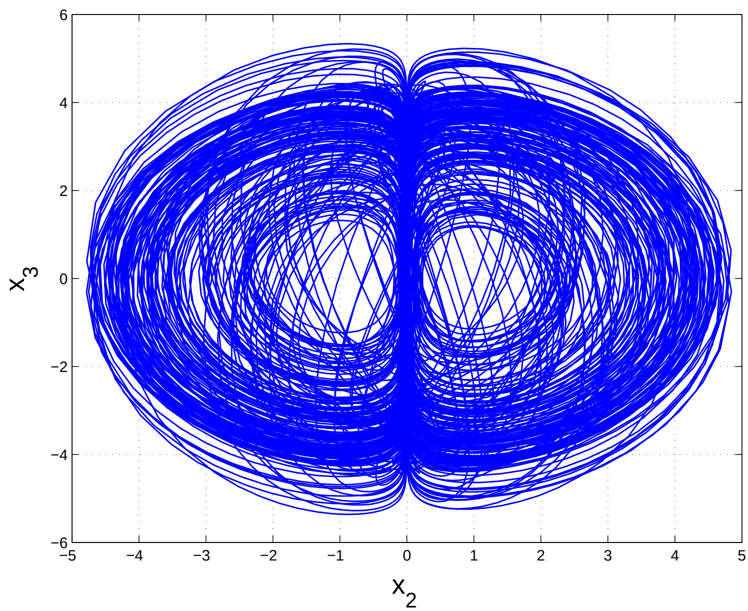

Figure 3: 2-D projection of the conservative chaotic system on the (x_2, x_3) plane

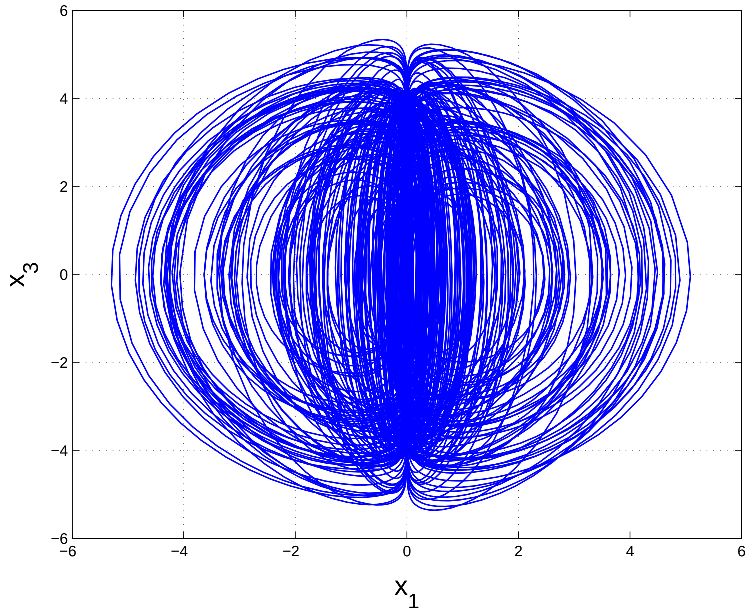

Figure 4: 2-D projection of the conservative chaotic system on the (x_1, x_3) plane

3. Analysis of the 3-D novel conservative chaotic system

3.1. Equilibrium points

The equilibrium points of the novel system (1) are obtained by solving the equations

$$ax_2 + x_1x_3 = 0 \tag{3a}$$
$$-bx_1 + x_2x_3 = 0 \tag{3b}$$
$$1 - x_1^2 - x_2^2 = 0. \tag{3c}$$

From (3a) and (3b), it follows that

$$x_1x_2x_3 = -ax_2^2 = bx_1^2$$

which gives

$$bx_1^2 + ax_2^2 = 0. \tag{4}$$

Since $a > 0$ and $b > 0$, the only solution of (4) is given by

$$x_1 = 0, \;\; x_2 = 0. \tag{5}$$

Since (3c) and (5) contradict each other, there is no equilibrium point to the novel system (1).

3.2. Rotation symmetry about the x_3-axis

We define a new set of coordinates as

$$\begin{aligned} z_1 &= -x_1 \\ z_2 &= -x_2 \\ z_3 &= x_3. \end{aligned} \tag{6}$$

We find that

$$\begin{aligned} \dot{z}_1 &= -ax_2 - x_1x_3 &= az_2 + z_1z_3 \\ \dot{z}_2 &= bx_1 - x_2x_3 &= -bz_1 + z_2z_3 \\ \dot{z}_3 &= 1 - x_1^2 - x_2^2 &= 1 - z_1^2 - z_2^2. \end{aligned} \tag{7}$$

This shows that the 3-D novel conservative chaotic system (1) is invariant under the change of coordinates

$$(x_1, x_2, x_3) \mapsto (-x_1, -x_2, x_3). \tag{8}$$

Since the transformation (8) persists for all values of the parameters, it follows that the 3-D novel conservative chaotic system (1) has rotation symmetry about the x_3-axis and that any non-trivial trajectory must have a twin trajectory.

3.3. Invariance

It is easy to see that the x_3-axis is invariant under the flow of the 3-D novel conservative chaotic system (1). The invariant motion along the x_3-axis is characterized by the scalar dynamics

$$\dot{x}_3 = 1 \tag{9}$$

which is unstable.

3.4. Lyapunov exponents and Kaplan-Yorke dimension

For the parameter values given in (2), the Lyapunov exponents of the novel chaotic system (1) are calculated as

$$L_1 = 0.0395, \quad L_2 = 0, \quad L_3 = -0.0395. \tag{10}$$

Clearly, the maximal Lyapunov exponent of the novel chaotic system (1) is given by $L_1 = 0.0395$, which is positive.

Since the sum of the Lyapunov exponents in (10) is zero, the novel chaotic system (1) is conservative.

The Kaplan-Yorke dimension of a chaotic system is defined as

$$D_{KY} = j + \sum_{i=1}^{j} \frac{L_i}{|L_{j+1}|}$$

where j is the maximum integer such that the sum of the j largest Lyapunov exponents is still non-negative. D_{KY} represents an upper bound for the information dimension of the system. It is easy to deduce that for the 3-D conservative chaotic system (1), the Kaplan-Yorke dimension is given by

$$D_{KY} = 3. \tag{11}$$

4. Adaptive control of the 3-D novel conservative chaotic system with unknown parameters

In this section, we use adaptive control design to derive an adaptive feedback control law for globally stabilizing the 3-D novel conservative chaotic system with unknown parameters.

Thus, we consider the 3-D novel conservative chaotic system given by

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & ax_2 + x_1 x_3 + u_1 \\ \dot{x}_2 & = & -bx_1 + x_2 x_3 + u_2 \\ \dot{x}_3 & = & 1 - x_1^2 - x_2^2 + u_3. \end{array} \right. \tag{12}$$

In (12), x_1, x_2, x_3 are the states and u_1, u_2, u_3 are adaptive controls to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for the unknown parameters a and b, respectively.

We consider the adaptive control law defined by

$$\left\{ \begin{array}{rcl} u_1 & = & -\hat{a}(t)x_2 - x_1 x_3 - k_1 x_1 \\ u_2 & = & \hat{b}x_1 - x_2 x_3 - k_2 x_2 \\ u_3 & = & -1 + x_1^2 + x_2^2 - k_3 x_3 \end{array} \right. \tag{13}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (13) into (12), we get the closed-loop plant dynamics as

$$\left\{ \begin{array}{rcl} \dot{x}_1 & = & [a - \hat{a}(t)]\, x_2 - k_1 x_1 \\ \dot{x}_2 & = & -\left[b - \hat{b}(t)\right] x_1 - k_2 x_2 \\ \dot{x}_3 & = & -k_3 x_3. \end{array} \right. \tag{14}$$

The parameter estimation errors are defined as

$$\left\{ \begin{array}{rcl} e_a(t) & = & a - \hat{a}(t) \\ e_b(t) & = & b - \hat{b}(t). \end{array} \right. \tag{15}$$

Using (15), we can simplify (14) as

$$\begin{cases} \dot{x}_1 &= e_a x_2 - k_1 x_1 \\ \dot{x}_2 &= -e_b x_1 - k_2 x_2 \\ \dot{x}_3 &= -k_3 x_3. \end{cases} \tag{16}$$

Differentiating (15) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) &= -\dot{\hat{a}}(t) \\ \dot{e}_b(t) &= -\dot{\hat{b}}(t). \end{cases} \tag{17}$$

We use adaptive control theory to find an update law for the parameter estimates. We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{x}, e_a, e_b) = \frac{1}{2}\left(x_1^2 + x_2^2 + x_3^2 + e_a^2 + e_b^2\right). \tag{18}$$

Clearly, V is a positive definite function on $\Re^5$.
Differentiating V along the trajectories of (16) and (17), we get

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[x_1 x_2 - \dot{\hat{a}}\right] + e_b\left[-x_1 x_2 - \dot{\hat{a}}\right]. \tag{19}$$

In view of (19), we take the parameter update law as follows:

$$\begin{aligned} \dot{\hat{a}} &= x_1 x_2 \\ \dot{\hat{b}} &= -x_1 x_2. \end{aligned} \tag{20}$$

Next, we state and prove the main result of this section.

Theorem 7 *The novel 3-D conservative chaotic system* (12) *with unknown system parameters is globally and exponentially stabilized for all initial conditions by the adaptive control law* (13) *and the parameter update law* (20)*, where* k_1, k_2, k_3 *are positive gain constants.*

Proof We prove this result using Lyapunov stability theory [72].

We consider the quadratic Lyapunov function defined by (18), which is a positive definite function on $\Re^5$.

By substituting the parameter update law (20) into (19), we obtain the time derivative of V as

$$\dot{V} = -k_1 x_1^2 - k_2 x_2^2 - k_3 x_3^2. \tag{21}$$

From (21), it is clear that $\dot{V}$ is a negative semi-definite function on $\Re^5$.

Thus, we can conclude that the state vector $\boldsymbol{x}(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\begin{bmatrix} x_1(t) & x_2(t) & x_3(t) & e_a(t) & e_b(t) \end{bmatrix}^T \in \mathcal{L}_\infty.$$

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (21) that

$$\dot{V} \leqslant -k\|\boldsymbol{x}(t)\|^2. \tag{22}$$

Thus, we have

$$k\|\boldsymbol{x}(t)\|^2 \leqslant -\dot{V}. \tag{23}$$

Integrating the inequality (23) from 0 to t, we get

$$k\int_0^t \|\boldsymbol{x}(\tau)\|^2 d\tau \leqslant V(0) - V(t). \tag{24}$$

From (24), it follows that $\boldsymbol{x} \in \mathcal{L}_2$.

Using (16), we can conclude that $\boldsymbol{x} \in \mathcal{L}_\infty$.

Using Barbalat's lemma [72], we conclude that $\boldsymbol{x}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $x(0) \in \Re^3$. This completes the proof. □

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (12) and (20), when the adaptive control law (13) is applied.

The parameter values of the novel conservative chaotic system are taken as in the chaotic case, *viz.* $a = 0.05$ and $b = 1$. We take the positive gain constants as $k_i = 5$ for $i = 1, 2, 3$.

Furthermore, as initial conditions of the novel conservative chaotic system (12), we take $x_1(0) = 7.2, x_2(0) = -5.3$ and $x_3(0) = 3.7$.

Also, as initial conditions of the parameter estimates $\hat{a}(t)$ and $\hat{b}(t)$, we take $\hat{a}(0) = 5.6$ and $\hat{b}(0) = 4.8$.

In Fig. 5, the exponential convergence of the controlled states of the 3-D conservative chaotic system (12) is depicted.

5. Adaptive synchronization of the 3-D novel conservative chaotic systems with unknown parameters

In this section, we use adaptive control method to derive an adaptive feedback control law for globally synchronizing identical 3-D novel conservative chaotic systems with unknown parameters.

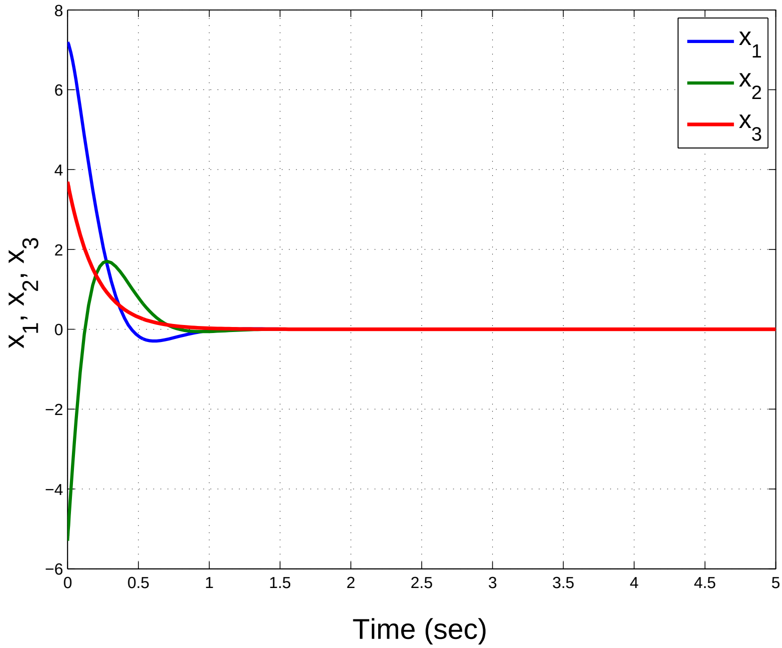

Figure 5: Time-history of the controlled states $x_1(t), x_2(t), x_3(t)$

As the master system, we consider the 3-D novel conservative chaotic system given by

$$\begin{cases} \dot{x}_1 &= ax_2 + x_1x_3 \\ \dot{x}_2 &= -bx_1 + x_2x_3 \\ \dot{x}_3 &= 1 - x_1^2 - x_2^2. \end{cases} \tag{25}$$

In (25), x_1, x_2, x_3 are the states and a, b are unknown system parameters.

As the slave system, we consider the 3-D novel conservative chaotic system given by

$$\begin{cases} \dot{y}_1 &= ay_2 + y_1y_3 + u_1 \\ \dot{y}_2 &= -by_1 + y_2y_3 + u_2 \\ \dot{y}_3 &= 1 - y_1^2 - y_2^2 + u_3. \end{cases} \tag{26}$$

In (26), y_1, y_2, y_3 are the states and u_1, u_2, u_3 are the adaptive controls to be determined using estimates $\hat{a}(t)$ and $\hat{b}(t)$ for the unknown parameters a and b, respectively.

The synchronization error between the novel 3-D conservative chaotic systems (25) and (26) is defined by

$$\begin{cases} e_1 &= y_1 - x_1 \\ e_2 &= y_2 - x_2 \\ e_3 &= y_3 - x_3. \end{cases} \tag{27}$$

Then the synchronization error dynamics is obtained as

$$\begin{cases} \dot{e}_1 &= ae_2 + y_1y_3 - x_1x_3 + u_1 \\ \dot{e}_2 &= -be_1 + y_2y_3 - x_2x_3 + u_2 \\ \dot{e}_3 &= -y_1^2 + x_1^2 - y_2^2 + x_2^2 + u_3. \end{cases} \tag{28}$$

We consider the adaptive feedback control law

$$\begin{cases} u_1 &= -\hat{a}(t)e_2 - y_1y_3 + x_1x_3 - k_1e_1 \\ u_2 &= \hat{b}(t)e_1 - y_2y_3 + x_2x_3 - k_2e_2 \\ u_3 &= y_1^2 - x_1^2 + y_2^2 - x_2^2 - k_3e_3 \end{cases} \tag{29}$$

where k_1, k_2, k_3 are positive gain constants.

Substituting (29) into (28), we get the closed-loop error dynamics as

$$\begin{cases} \dot{e}_1 &= [a - \hat{a}(t)]\, e_2 - k_1e_1 \\ \dot{e}_2 &= -\left[b - \hat{b}(t)\right] e_1 - k_2e_2 \\ \dot{e}_3 &= -k_3e_3. \end{cases} \tag{30}$$

The parameter estimation errors are defined as

$$\begin{cases} e_a(t) &= a - \hat{a}(t) \\ e_b(t) &= b - \hat{b}(t). \end{cases} \tag{31}$$

In view of (31), we can simplify the plant dynamics (30) as

$$\begin{cases} \dot{e}_1 &= e_ae_2 - k_1e_1 \\ \dot{e}_2 &= -e_be_1 - k_2e_2 \\ \dot{e}_3 &= -k_3e_3. \end{cases} \tag{32}$$

Differentiating (31) with respect to t, we obtain

$$\begin{cases} \dot{e}_a(t) &= -\dot{\hat{a}}(t) \\ \dot{e}_b(t) &= -\dot{\hat{b}}(t). \end{cases} \tag{33}$$

We use adaptive control theory to find an update law for the parameter estimates. We consider the quadratic candidate Lyapunov function defined by

$$V(\boldsymbol{e}, e_a, e_b) = \frac{1}{2}\left(e_1^2 + e_2^2 + e_3^2 + e_a^2 + e_b^2\right). \tag{34}$$

Differentiating V along the trajectories of (32) and (33), we obtain

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2 + e_a\left[e_1 e_2 - \dot{\hat{a}}\right] + e_b\left[-e_1 e_2 - \dot{\hat{b}}\right]. \tag{35}$$

In view of (35), we take the parameter update law as

$$\begin{cases} \dot{\hat{a}}(t) &= e_1 e_2 \\ \dot{\hat{b}}(t) &= -e_1 e_2. \end{cases} \tag{36}$$

Next, we state and prove the main result of this section.

Theorem 8 *The novel 3-D conservative chaotic systems* (25) *and* (26) *with unknown system parameters are globally and exponentially synchronized for all initial conditions by the adaptive control law* (29) *and the parameter update law* (36)*, where* k_1, k_2, k_3 *are positive gain constants.*

Proof We prove this result by applying Lyapunov stability theory [72].

We consider the quadratic Lyapunov function defined by (34), which is clearly a positive definite function on $\mathfrak{R}^5$.

By substituting the parameter update law (36) into (35), we obtain the time-derivative of V as

$$\dot{V} = -k_1 e_1^2 - k_2 e_2^2 - k_3 e_3^2. \tag{37}$$

From (37), it is clear that $\dot{V}$ is a negative semi-definite function on $\mathfrak{R}^5$.

Thus, we can conclude that the state vector $\boldsymbol{e}(t)$ and the parameter estimation error are globally bounded, *i.e.*

$$\begin{bmatrix} e_1(t) & e_2(t) & e_3(t) & e_a(t) & e_b(t) \end{bmatrix}^T \in \mathcal{L}_\infty.$$

We define $k = \min\{k_1, k_2, k_3\}$.

Then it follows from (37) that

$$\dot{V} \leqslant -k\|\boldsymbol{e}(t)\|^2. \tag{38}$$

Thus, we have

$$k\|\boldsymbol{e}(t)\|^2 \leqslant -\dot{V}. \tag{39}$$

Integrating the inequality (39) from 0 to t, we get

$$k\int_0^t \|\boldsymbol{e}(\tau)\|^2 \, d\tau \leqslant V(0) - V(t). \tag{40}$$

From (40), it follows that $\boldsymbol{e} \in \mathcal{L}_2$.

Using (32), we can conclude that $\dot{\boldsymbol{e}} \in \mathcal{L}_\infty$.

Using Barbalat's lemma [72], we conclude that $\boldsymbol{e}(t) \to 0$ exponentially as $t \to \infty$ for all initial conditions $\boldsymbol{e}(0) \in \Re^3$. This completes the proof. □

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the systems (25) and (26) and (36), when the adaptive control law (29) is applied.

The parameter values of the novel 3-D conservative chaotic systems are taken as in the chaotic case, *viz.* $a = 0.05$ and $b = 1$. We take the positive gain constants as $k_1 = 5, k_2 = 5$ and $k_3 = 5$.

Furthermore, as initial conditions of the master system (25), we take

$$x_1(0) = 5.7, \;\; x_2(0) = 3.9, \;\; x_3(0) = -7.4.$$

As initial conditions of the slave system (26), we take

$$y_1(0) = -4.2, \;\; y_2(0) = 8.5, \;\; y_3(0) = 6.4.$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 8.1, \;\; \hat{b}(0) = 3.4.$$

Figs. 6-8 describe the complete synchronization of the 3-D novel conservative chaotic systems (25) and (26), while Fig. 9 describes the time-history of the synchronization errors e_1, e_2, e_3.

6. Conclusions

In this research work, we detailed a seven-term novel 3-D conservative no-equilibrium chaotic system with four quadratic nonlinearities. In the chaos literature, there are very few conservative no-equilibrium chaotic systems. Thus, the proposed no-equilibrium conservative chaotic system is a valuable addition to the chaos literature. Next, we designed an adaptive controller to globally stabilize the novel conservative chaotic system with unknown parameters. We also designed an adaptive controller to achieve global chaos synchronization of the identical conservative chaotic systems with unknown parameters. MATLAB simulations were shown to illustrate all the main results derived in this research work.

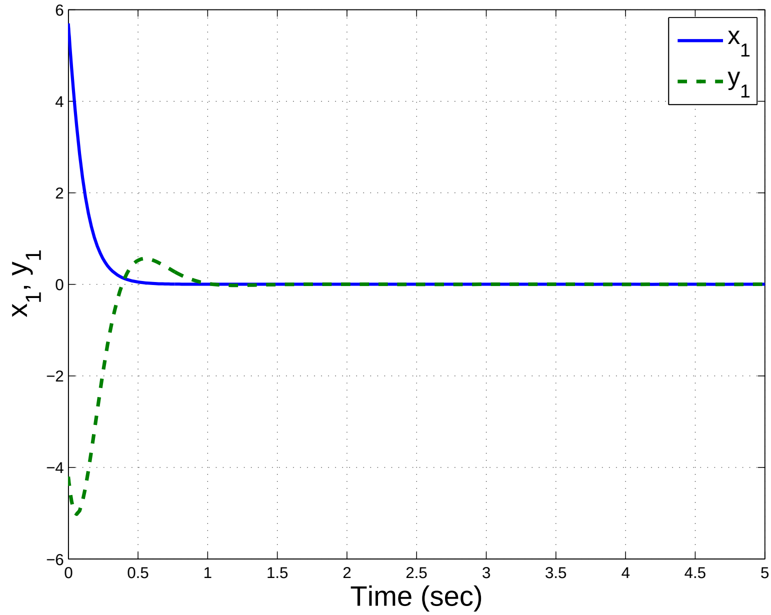

Figure 6: Complete synchronization of the states x_1 and y_1

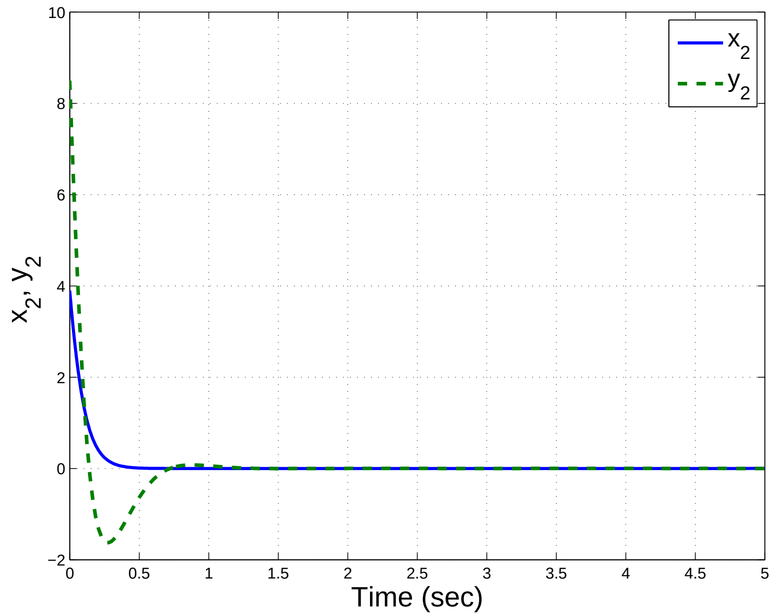

Figure 7: Complete synchronization of the states x_2 and y_2

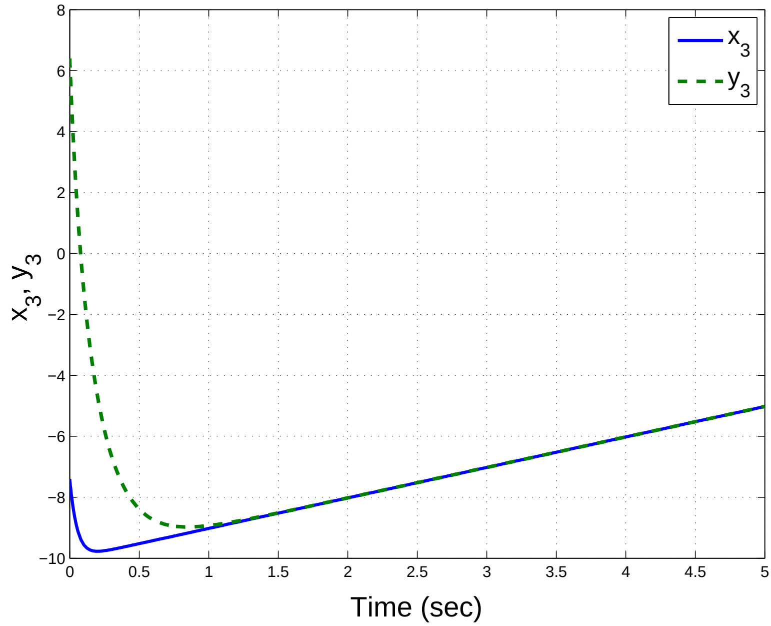

Figure 8: Complete synchronization of the states x_3 and y_3

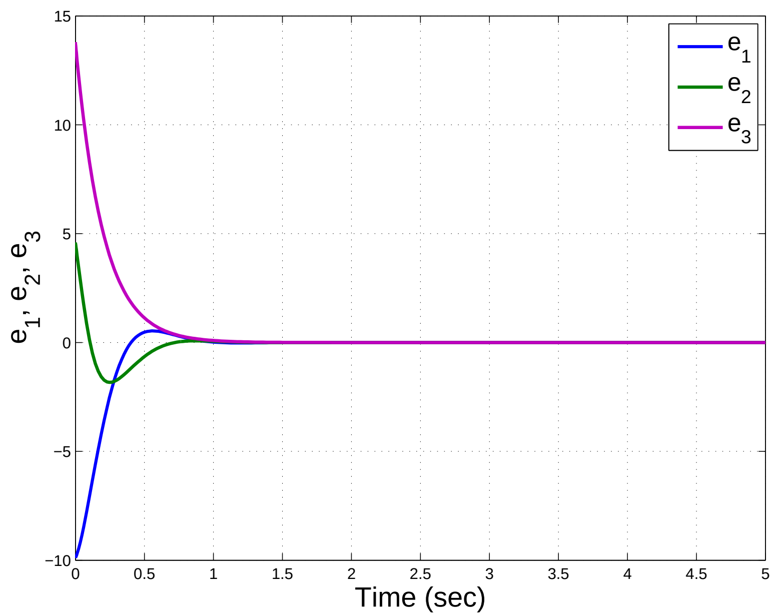

Figure 9: Time-history of the synchronization errors e_1, e_2, e_3

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